

## ARTICLE TEMPLATE

# Why shouldn't you use numerical tests to diagnose the linear regression models?

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### ARTICLE HISTORY

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### ABSTRACT

Abstract to fill.

### KEYWORDS

visual inference; model diagnostics;

problem: residual plot diagnostics conventional test: too sensitive  
background:

1. residual plot for model diagnostics
  - a. residual is widely used
  - b. what are the types of residual plots
  - c. comparison
2. conventional test: F, BP
3. visual test: lineup, theory

desc of experiment: 1. simulation setup 2. experimental design 3. result

comparison of conventional tests: 1. power (visual test vs. conventional test) (visual test most different one (everything test, any departure)) 2. investigate the difference (gap), give examples 3. conventional is too sensitive 4. make conventional less sensitive (vary alpha)

conclusion: 1. too sensitive, visual test is needed/preferable 2. visual test is infeasible in large scale (expensive) 3. future work (role of computer vision)

## 1. Introduction

Regression diagnostics conventionally involve evaluating the fitness of the proposed model, detecting the presence of influential observations and outliers, checking the validity of model assumptions and many more. Common diagnostic techniques including summary statistics, hypothesis testing, and data plots are essential tools for a

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systematic and detailed examination of the regression model (Mansfield and Conerly 1987).

### 1.1. *Diagnostic plots*

Regression analysis is a field of study with at least a hundred years of history. Many of those regression diagnostic methods and procedures are mature and well-established in books first published in the twentieth century, such as Draper and Smith (2014), Montgomery, Peck, and Vining (2012), Belsley, Kuh, and Welsch (1980), Cook and Weisberg (1999) and Cook and Weisberg (1982). Regardless of the level of difficulty of the book, one will find the importance and usefulness of diagnostic plots being emphasized again and again. Checking diagnostic plots is also the recommended starting point for validating model assumptions like normality, homoscedasticity and linearity (Anscombe and Tukey 1963).

Graphical summaries in which residuals are plotted against fitted values or other functions of the predictor variables that are approximately orthogonal to residuals are referred to as standard residual plots. They are commonly used to identify patterns which are indicative of nonconstant error variance or non-linearity (Cook and Weisberg 1982). Raw residuals and studentized residuals are the two most frequently used residuals in standard residual plots. The debate on which type of residuals should be used always present. While raw residuals are the most common output of computer regression software package, by applying a scaling factor, the ability of revealing non-constant error variance in standard residual plots will often be enhanced by studentized residuals in small sample size (Gunst and Mason 2018).

As a two-dimensional representation of a model in a  $p$ -dimensional space, standard residual plots project data points onto the variable of the horizontal axis, which is a vector in  $p$ -dimensional space. Observations with the same projection will be treated as equivalent as they have the same position of the abscissa. Therefore, standard residual plots are often useful in revealing model inadequacies in the direction of the variable of the horizontal axis, but could be inadequate for detecting patterns in other directions, especially in those perpendicular to the variable of the horizontal axis. Hence, in practice, multiple standard residual plots with different horizontal axes will be examined.

Overlapping data points is a general issue in scatter plots not limited to standard residual plots, which often makes plots difficult to interpret because visual patterns are concealed. Thus, for relatively large sample size, Cleveland and Kleiner (1975) suggests the use of robust moving statistics as reference lines to give aids to eye in seeing patterns, which nowadays, are usually replaced with a spline or local polynomial regression line.

Other types of data plots that are often used in regression diagnostics include partial residual plots and probability plots. Partial residual plots are useful supplements to standard residual plots as they provide additional information on the extent of the non-linearity. Probability plots can be used to compare the sampling distribution of the residuals to the normal distribution for assessing the normality assumptions.

### 1.2. *Hypothesis testing*

In addition to diagnostic plots, researcher may also perform formal tests for detecting model defects. Depends on the alternative, variety of tests can be applied. For example,

for testing heteroskedasticity, one may use the White test (White 1980) or the Breusch-Pagan test (Breusch and Pagan 1979). And for testing non-linearity, there are RESET test (Ramsey 1969) and F-test.

As discussed in Cook and Weisberg (1982), most residual based tests for a particular type of departures from model assumptions are sensitive to other types of departures. Especially, outliers will often incorrectly trigger the rejection of null hypothesis despite the residuals are well-behaved (Cook and Weisberg 1999). This can be largely avoided in diagnostic plots as experienced analysts can evaluate the acceptability of assumptions flexibly, even in the presence of outliers. Furthermore, Montgomery, Peck, and Vining (2012) stated that based on their experience, statistical tests are not widely used in regression diagnostics. Most importantly, the same or even larger amount of information can be provided by diagnostic plots than the corresponding tests in most empirical studies. But still, the effectiveness of statistical tests shall not be disrespected. Statistical tests have chance to provide analysts with unique information. There are also situations where no suitable diagnostic plots can be found for a particular violation of the assumptions, or excessive diagnostic plots need to be checked. One will have no choice but to rely on statistical tests if there is any. A good regression diagnostic practice should be a combination of both methods.

### 1.3. *Visual inference*

Diagnostic plots are

However, unlike confirmatory data analysis built upon rigorous statistical procedures, e.g., hypothesis testing, visual diagnostics relies on graphical perception - human's ability to interpret and decode the information embedded in the graph (?), which is to some extent subjective. Further, visual discovery suffers from its unsecured and unconfirmed nature where the degree of the presence of the visual features typically can not be measured quantitatively and objectively, which may lead to over or under-interpretations of the data. One such example is finding an over-interpretation of the separation between gene groups in a two-dimensional projection from a linear discriminant analysis when in fact there are no differences in the expression levels between the gene groups and separation is not an uncommon occurrence (?).

Recently, a new branch of statistical inference

Visual inference was first introduced in a 1999 Joint Statistical Meetings (JSM) talk with the title "Inference for Data Visualization" by ? as an idea to address the issue of valid inference for visual discoveries of data plots (?). Later, in the Bayesian context, data plots was systematically considered as model diagnostics by taking advantage of the data simulated from the assumed statistical models (??).

## References

- Anscombe, F. J., and John W. Tukey. 1963. "The Examination and Analysis of Residuals." *Technometrics* 5 (2): 141–160. Accessed 2022-04-24. <http://www.tandfonline.com/doi/abs/10.1080/00401706.1963.10490071>.
- Belsley, David A, Edwin Kuh, and Roy E Welsch. 1980. *Regression diagnostics: Identifying influential data and sources of collinearity*. John Wiley & Sons.
- Breusch, T. S., and A. R. Pagan. 1979. "A Simple Test for Heteroscedasticity and Random Coefficient Variation." *Econometrica* 47 (5): 1287–1294. Publisher: [Wiley, Econometric Society], Accessed 2022-04-26. <http://www.jstor.org/stable/1911963>.

- Cleveland, William S, and Beat Kleiner. 1975. "A graphical technique for enhancing scatter-plots with moving statistics." *Technometrics* 17 (4): 447–454.
- Cook, R Dennis, and Sanford Weisberg. 1982. *Residuals and influence in regression*. New York: Chapman and Hall.
- Cook, R Dennis, and Sanford Weisberg. 1999. *Applied regression including computing and graphics*. John Wiley & Sons.
- Draper, Norman R, and Harry Smith. 2014. *Applied regression analysis*. 3rd ed. John Wiley & Sons.
- Gunst, Richard F, and Robert L Mason. 2018. *Regression analysis and its application: a data-oriented approach*. CRC Press.
- Mansfield, Edward R, and Michael D Conerly. 1987. "Diagnostic value of residual and partial residual plots." *The American Statistician* 41 (2): 107–116.
- Montgomery, Douglas C, Elizabeth A Peck, and G Geoffrey Vining. 2012. *Introduction to linear regression analysis*. 5th ed. John Wiley & Sons.
- Ramsey, J. B. 1969. "Tests for Specification Errors in Classical Linear Least-Squares Regression Analysis." *Journal of the Royal Statistical Society. Series B (Methodological)* 31 (2): 350–371. Publisher: [Royal Statistical Society, Wiley], Accessed 2022-04-26. <http://www.jstor.org/stable/2984219>.
- White, Halbert. 1980. "A Heteroskedasticity-Consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity." *Econometrica* 48 (4): 817–838. Publisher: [Wiley, Econometric Society], Accessed 2022-04-26. <http://www.jstor.org/stable/1912934>.