APPENDIX

A. Nomenclature

TABLE A1
DESCRIPTION OF THE ABBREVIATIONS IN THIS PAPER

Symbol	Meaning
IES	integrated energy system
IES-SR	security region of integrated energy system
ECSR	energy circuit security region
EPS-ECSR	ECSR of electric power system
NGS-ECSR	ECSR of natural gas system
IES-ECSR	ECSR of integrated energy system

TABLE A2
DESCRIPTION OF THE MATHEMATICAL SYMBOLS IN THIS PAPER

	Symbol	Meaning
Symbols of EPS-ECSR	$\boldsymbol{W}$	Operating point
	$\boldsymbol{\Omega}_{\text{EPS-ECSR}}$	ECSR of EPS
	$S_{\text{l,e}i}$	power of power load node i
	$S_{\text{l,c}m}$	power required to drive compressor m
	$\boldsymbol{I}_{\text{n}}$	vector of node current
	$\boldsymbol{I}_{\text{n}}^{\max}$, $\boldsymbol{I}_{\text{n}}^{\min}$	vectors of the upper and lower limits of node current
	$\boldsymbol{I}_{\text{b}}$	vector of branch current
	$\boldsymbol{I}_{\text{b}}^{\max}$	vector of the current-carrying capacity
	$\boldsymbol{U}_{\text{n}}$	vector of node voltage
	$\boldsymbol{U}_{\text{n}}^{\max}$, $\boldsymbol{U}_{\text{n}}^{\min}$	vectors of the upper and lower limits of node voltage
	$\boldsymbol{Y}_{\text{e}}$	node admittance matrix of EPS
	$\boldsymbol{K}_{\text{e}}$	transformer ratio matrix
Symbols of NGS-ECSR	$\boldsymbol{\Omega}_{\text{NGS-ECSR}}$	ECSR of NGS
	$G_{\text{l,g}j}$	flow rate of natural gas load node j
	$G_{\text{l,GT}n}$	natural gas input by the gas generator n
	$\boldsymbol{G}_{\text{n}}$	vectors of node flow of natural gas
	$\boldsymbol{G}_{\text{n}}^{\max}$, $\boldsymbol{G}_{\text{n}}^{\min}$	vector of the upper and lower limits of the node gas flow
	$\boldsymbol{p}_{\text{n}}$	vector of node pressure of natural gas
	$\boldsymbol{p}_{\text{n}}^{\max}$, $\boldsymbol{p}_{\text{n}}^{\min}$	vector of the upper and lower limits of node pressure
	$\boldsymbol{G}_{\text{b}}^{\max}$	vector of pipe capacity
	$\boldsymbol{K}_{\text{g}}$	matrix of compressor pressure ratio
	$\boldsymbol{Y}_{\text{g}}$	generalized node admittance matrix of NGS
	$\boldsymbol{A}_{\text{g, L}}^{-1}$	Left inverse matrix of $\boldsymbol{A}_{\text{g}}$
Symbols of IES-ECSR	$\boldsymbol{\Omega}_{\text{IES-ECSR}}$	ECSR of IES
	$\boldsymbol{P}_{\text{n}}$	vector of node pressure of natural gas
	$\boldsymbol{P}_{\text{n}}^{\max}$, $\boldsymbol{P}_{\text{n}}^{\min}$	vector of the upper and lower limits of node pressure
	$\boldsymbol{F}_{\text{n}}$	vector of energy flow
	$\boldsymbol{F}_{\text{b}}^{\max}$	vector of the upper limits of node flow
	$\boldsymbol{Y}_{\text{IES}}$	generalized node admittance matrix of IES
	ψ	efficiency of compressor
	K_{g}	pressure ratio of compressor
	a_{GT}, b_{GT}, c_{GT}, d_{GT}, e_{GT}	heat consumption coefficients of gas generator
	P_{GT}	active power output by the gas generator
	$P_{\text{GT}}^{\min}$	lower limit of P_{GT}
Symbols of solution and observation for ECSR	$\boldsymbol{Y}_{\text{g,pp}}$, $\boldsymbol{Y}_{\text{g,pg}}$, $\boldsymbol{Y}_{\text{g,gp}}$, $\boldsymbol{Y}_{\text{g,gg}}$	block the rearranged $\boldsymbol{Y}_{\text{g}}$
	$\boldsymbol{p}_{\text{n,p}}$	node pressure vector of the constant pressure node
	$\boldsymbol{G}_{\text{n,p}}$	injection flow vector of the constant pressure node
	$\boldsymbol{p}_{\text{n,g}}$	node pressure vector of the constant injection flow node
	$\boldsymbol{G}_{\text{n,g}}$	injection flow vector of the constant injection flow node
	$D^{\max}$	Distance error threshold for hyperplane fitting
	$\boldsymbol{H}^{0}$	initial hyperplane
	$\boldsymbol{H}_{j}^{k}$	The hyperplane j during the k-th segmented fitting
	$D_{\boldsymbol{W}_{\text{b}}\to\boldsymbol{H}_{j}^{k}}$	distance from $\boldsymbol{W}_{\text{b}}$ to $\boldsymbol{H}_{j}^{k}$
	$D_{\boldsymbol{W}_{\text{b}}\to\boldsymbol{H}^{k}}$	The minimum value of $D_{\boldsymbol{W}_{\text{b}}\to\boldsymbol{H}_{j}^{k}}$ during the k-th segment fitting
	$D_{\boldsymbol{W}_{\text{b}}\to\boldsymbol{H}^{k}}^{\max}$	The maximum value of a corresponding to all boundary points
	D_{i}	The distance from the origin to the corresponding hyperplane Hi at the safety boundary

B. ECSR modeling results of the cases

(1) EPS-ECSR

The EPS-ESSR of IEEE 14-node EPS is shown as equation (B1). In the equation, $\boldsymbol{Y}_{\text{e}}$ is the node admittance matrix, $\boldsymbol{U}_{\text{n}}$ and $\boldsymbol{I}_{\text{n}}$ are vectors composed of node voltage and current, $\boldsymbol{K}_{\text{e}}$ is the transformer ratio matrix.

$$
\begin{cases}
\boldsymbol{\Omega}_{\text{EPS-SR}}=\left\{\boldsymbol{W}_{\text{s}}=[S_{\text{l,e4}}\ S_{\text{l,e5}}\ S_{\text{l,e7}}\ S_{\text{l,e9}}\ S_{\text{l,e10}}\ S_{\text{l,e11}}\ S_{\text{l,e12}}\ S_{\text{l,e13}}\ S_{\text{l,e14}}]\,\middle|\, h(\boldsymbol{W}_{\text{s}})=0, g(\boldsymbol{W}_{\text{s}})\le 0\right\} \\
s.t.\begin{cases}
\boldsymbol{Y}_{\text{e}}\boldsymbol{U}_{\text{n}}=\boldsymbol{I}_{\text{n}} \\
[20\ 85\ 40\ 0\ 0\ 25\ 0\ 0\ 0\ 0\ 10]^{\text{T}}\le[S_{\text{l,e4}}\ S_{\text{l,e5}}\ S_{\text{l,e7}}\ S_{\text{l,e9}}\ S_{\text{l,e10}}\ S_{\text{l,e11}}\ S_{\text{l,e12}}\ S_{\text{l,e13}}\ S_{\text{l,e14}}]^{\text{T}} \\
\le[30\ 100\ 55\ 10\ 15\ 40\ 15\ 5\ 10\ 15\ 20]^{\text{T}} \\
[1.06\ 1.045\ 1.01\ 0.969\ 0.97\ 1.07\ 1.012\ 1.09\ 1.006\ 1.001\ 1.007\ 1.005\ 1\ 0.986]^{\text{T}}\le\boldsymbol{U}_{\text{n}} \\
\le[1.06\ 1.045\ 1.01\ 1.069\ 1.07\ 1.07\ 1.112\ 1.09\ 1.106\ 1.101\ 1.107\ 1.105\ 1\ 1.086]^{\text{T}} \\
[S_{\text{be1}}\ S_{\text{be2}}\ S_{\text{be3}}\ S_{\text{be4}}\ S_{\text{be5}}\ S_{\text{be6}}\ S_{\text{be7}}\ S_{\text{be8}}\ S_{\text{be9}}\ S_{\text{be10}}\ S_{\text{be11}}\ S_{\text{be12}}\ S_{\text{be13}}\ S_{\text{be14}}\ S_{\text{be15}}\ S_{\text{be16}}\ S_{\text{be17}}\ S_{\text{be18}}\ S_{\text{be19}}\ S_{\text{be20}}]^{\text{T}} \\
\le[200\ 100\ 100\ 75\ 50\ 50\ 75\ 50\ 25\ 75\ 25\ 25\ 25\ 25\ 50\ 10\ 25\ 10\ 5\ 10]^{\text{T}} \\
\boldsymbol{K}_{\text{e}}=\left[0.978\quad 0.969\quad 0.932\right]^{\text{T}}
\end{cases}
\end{cases}
\tag{B1}
$$

(2) NGS-ECSE

The NGS-ECSR of Belgian 9-node NGS is shown as equation (B2). In the equation, $\boldsymbol{p}_{\text{n}}$ and $\boldsymbol{G}_{\text{n}}$ are vectors composed of node gas pressure and gas flow, $\boldsymbol{Y}_{\text{g}}$ is the generalized node admittance matrix of the NGS [21,28]. When deriving $\boldsymbol{Y}_{\text{g}}$, the base value of natural gas flow is introduced, which bring the main error in the linearization process of ECSR modeling. The specific derivation and base value setting process of $\boldsymbol{Y}_{\text{g}}$ are detailed in [28].

$$\begin{cases}\boldsymbol{\Omega}_{\text{NGS-ECSR}}=\left\{\boldsymbol{W}_{\text{s}}=[G_{\text{l,e3}}\ G_{\text{l,e5}}\ G_{\text{l,e8}}\ G_{\text{l,e9}}]\middle| h(\boldsymbol{W}_{\text{s}})=0,\ g(\boldsymbol{W}_{\text{s}})\le 0\right\}\\ s.t.\begin{cases}\boldsymbol{Y}_{\text{g}}\boldsymbol{p}_{\text{n}}=\boldsymbol{G}_{\text{n}}\\ [69\ 126\ 0\ 0]^{\text{T}}\le[G_{\text{l,e3}}\ G_{\text{l,e5}}\ G_{\text{l,e8}}\ G_{\text{l,e9}}]^{\text{T}}\le[129\ 191\ 30\ 60]^{\text{T}}\\ [5.5\ 0\ 3\ 0\ 0\ 0\ 0\ 0\ 2.5]^{\text{T}}\le\boldsymbol{p}_{\text{n}}\le[6.5\ 6.5\ 6.5\ 6.5\ 6.5\ 6.5\ 6.3\ 6.5\ 6.5]^{\text{T}}\\ [G_{\text{bg1}}\ G_{\text{bg2}}\ G_{\text{bg3}}\ G_{\text{bg4}}\ G_{\text{bg5}}\ G_{\text{bg6}}\ G_{\text{bg7}}\ G_{\text{bg8}}\ G_{\text{bg9}}\ G_{\text{bg10}}\ G_{\text{bg11}}]^{\text{T}}\le[231.5\ 115.7\ 231.5\ 115.7\ 231.5\ 115.7\ 231.5\ 115.7\ 115.7\ 115.7\ 115.7]^{\text{T}}\\ [1\ 1\ 1]^{\text{T}}\le\boldsymbol{K}_{\text{g}}\le[1.2\ 1.2\ 1.2]^{\text{T}}\end{cases}\end{cases}\tag{B2}$$

(3) IES-ECSE

The NGS-ECSR of 23-node IES is shown as equation (B3). In the equation, $\boldsymbol{Y}_{\text{IES}}$ is the generalized node admittance matrix of IES; $\boldsymbol{P}_{\text{n}}$ and $\boldsymbol{F}_{\text{n}}$ are vectors composed of node pressure (voltage or gas pressure) and flow rate (current or gas flow) respectively; $\boldsymbol{S}_{\text{c}}$ is the power consumption of the compressor drive, p_0 and T_0 are the standard atmospheric pressure and temperature, Z and κ are the compression factor and adiabatic of natural gas, ψ and K_{g} are the efficiency and pressure ratio of compressor, V_{GH} is the total calorific value of natural gas.

$$\begin{cases}\boldsymbol{\Omega}_{\text{IES-ECSR}}=\left\{\boldsymbol{W}_{\text{s}}=[S_{\text{l,e4}}\ S_{\text{l,e5}}\ S_{\text{l,e7}}\ S_{\text{l,e9}}\ S_{\text{l,e10}}\ S_{\text{l,e11}}\ S_{\text{l,e12}}\ S_{\text{l,e13}}\ S_{\text{l,e14}}\ G_{\text{l,e3}}\ G_{\text{l,e5}}\ G_{\text{l,e8}}\ G_{\text{l,e9}}]\middle| h(\boldsymbol{W}_{\text{s}})=0,\ g(\boldsymbol{W}_{\text{s}})\le 0\right\}\\ s.t.(22-1)\begin{cases}\boldsymbol{Y}_{\text{IES}}\boldsymbol{P}_{\text{n}}=\boldsymbol{F}_{\text{n}}\\ [1.06\ \ 1.045\ \cdots\ 0.986\ \ 5.5\ \ 0\ \cdots\ 2.5]\le[U_1\ \ U_2\ \cdots\ U_{14}\ \ p_1\ \ p_2\ \cdots\ p_9]\le[1.06\ \ 1.045\ \cdots\ 1.086\ \ 6.5\ \ 6.5\ \cdots\ 6.5]\\ [20\ \ 85\ \cdots\ 10\ \ 69\ \ 126\ \ 0\ \ 0]\le[S_{\text{l,e4}}\ \ S_{\text{l,e5}}\ \cdots\ S_{\text{l,e14}}\ \ G_{\text{l,e3}}\ \ G_{\text{l,e5}}\ \ G_{\text{l,e8}}\ \ G_{\text{l,e9}}]\le[30\ \ 100\ \cdots\ 20\ \ 129\ \ 191\ \ 30\ \ 60]\\ [S_{\text{be1}}\ \ S_{\text{be2}}\ \cdots\ S_{\text{be20}}\ \ G_{\text{bg1}}\ \ G_{\text{bg2}}\ \cdots\ G_{\text{bg11}}]\le[200\ 100\ \cdots\ 10\ 231.5\ 115.7\ \cdots\ 115.7]\\ [0.978\ \cdots\ 0.932\ \ 1\ \cdots\ 1]\le\boldsymbol{K}_{\text{IES}}\le[0.978\ \cdots\ 0.932\ \ 1.2\ \cdots\ 1.2]\end{cases}\\ s.t.(22-2)\begin{cases}S_{\text{c}}=\dfrac{151.4653p_0ZTG_c\kappa}{\psi T_0(\kappa-1)}(K_{\text{g}}^{\frac{\kappa}{\kappa-1}}-1)\\ G_{\text{GT}}=\dfrac{1}{\text{V}_{\text{GH}}}(0.01P_{GT}^2+4P_{GT}+150+|15\sin(0.5(P_{\text{GT}}^{\min}-P_{GT}))|)\end{cases}\end{cases}\tag{B3}$$

C. Upper and lower boundary points in EPS-ECSR

TABLE C1
UPPER AND LOWER BOUNDARY POINTS IN EPS-ECSR OF IEEE 14-BUS SYSTEM

		($S_{\text{l,e2}}$, $S_{\text{l,e3}}$, $S_{\text{l,e4}}$, $S_{\text{l,e5}}$, $S_{\text{l,e6}}$, $S_{\text{l,e9}}$, $S_{\text{l,e10}}$, $S_{\text{l,e11}}$, $S_{\text{l,e12}}$, $S_{\text{l,e13}}$, $S_{\text{l,e14}}$)
Upper boundary point	$\boldsymbol{W}_{\text{b},1}$	(20, 85, 40, 0, 0, 25, 0, 5, 5, 0, 19.25)
	$\boldsymbol{W}_{\text{b},2}$	(20, 85, 40, 0, 0, 25, 0, 5, 5, 5, 17.18)
	$\boldsymbol{W}_{\text{b},3}$	(20, 85, 40, 0, 0, 25, 0, 5, 5, 10, 14.06)
	…	…
	$\boldsymbol{W}_{\text{b},19715}$	(30, 100, 45, 10, 15, 25, 0, 5, 0, 0, 13.74)
	$\boldsymbol{W}_{\text{b},19716}$	(30, 100, 45, 10, 15, 25, 0, 5, 0, 5, 11.88)
Lower boundary point	$\boldsymbol{W}_{\text{b},1}$	(30, 100, 45, 10, 15, 25, 0, 5, 0, 5, 10.00)
	$\boldsymbol{W}_{\text{b},2}$	(30, 100, 45, 10, 15, 25, 0, 5, 0, 0, 11.24)
	$\boldsymbol{W}_{\text{b},3}$	(30, 100, 45, 10, 15, 25, 0, 0, 5, 5, 10.87)
	…	…
	$\boldsymbol{W}_{\text{b},4254}$	(20, 100, 45, 0, 0, 25, 0, 0, 5, 15, 11.89)
	$\boldsymbol{W}_{\text{b},4255}$	(20, 100, 45, 0, 0, 25, 0, 0, 5, 10, 13.06)

TABLE C2
UPPER BOUNDARY POINTS OF THE SOUTHEASTERN BELGIUM NATURAL GAS SYSTEM

Upper boundary point	($G_{\text{l,g3}}$, $G_{\text{l,g5}}$, $G_{\text{l,g8}}$, $G_{\text{l,g9}}$)
$\boldsymbol{W}_{\text{b},1}$	(129, 126, 0, 4.87)
$\boldsymbol{W}_{\text{b},2}$	(124, 131, 0, 5.21)
$\boldsymbol{W}_{\text{b},3}$	(124, 126, 5, 5.33)
…	…
$\boldsymbol{W}_{\text{b},338}$	(104, 126, 20, 6.77)

TABLE VIII
UPPER AND LOWER BOUNDARY POINTS IN IES-ECSR OF 23-NODE IES

		($S_{\text{l,e2}}$, $S_{\text{l,e3}}$, $S_{\text{l,e4}}$, $S_{\text{l,e5}}$, $S_{\text{l,e6}}$, $S_{\text{l,e9}}$, $S_{\text{l,e10}}$, $S_{\text{l,e11}}$, $S_{\text{l,e12}}$, $S_{\text{l,e13}}$, $S_{\text{l,e14}}$, $S_{\text{l,c1}}$, $S_{\text{l,c2}}$, $S_{\text{l,c3}}$, $G_{\text{l,g3}}$, $G_{\text{l,g5}}$, $G_{\text{l,GT1}}$, $G_{\text{l,g8}}$, $G_{\text{l,GT2}}$, $G_{\text{l,g9}}$)
Upper boundary point	$\boldsymbol{W}_{\text{b},1}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 5, 15.7422)
	$\boldsymbol{W}_{\text{b},2}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 10, 10.7419)
	$\boldsymbol{W}_{\text{b},3}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 15, 5.7408)
	$\boldsymbol{W}_{\text{b},4}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 20, 0.7382)
	…	…
	$\boldsymbol{W}_{\text{b},10513152}$	(30, 100, 55, 10, 15, 40, 10, 5, 10, 5, 10, 4, 4, 3, 129, 126, 40, 0, 10, 4.4512)
Lower boundary point	$\boldsymbol{W}_{\text{b},1}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 5, 1.1266)
	$\boldsymbol{W}_{\text{b},2}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 10, 1.1269)
	$\boldsymbol{W}_{\text{b},3}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 15, 1.1281)
	$\boldsymbol{W}_{\text{b},4}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 20, 1.1304)
	…	…
	$\boldsymbol{W}_{\text{b},4725166}$	(30, 85, 50, 10, 15, 20, 10, 0, 10, 0, 10, 2, 2, 3, 94, 126, 15, 10, 10, 2.0852)

D. Comparison results between the proposed method and existing method

1) Solution results of boundary points of the IES region

TABLE D1

COMPARISON OF THE SOLUTION RESULTS OF BOUNDARY POINTS OF THE IES SECURITY REGION

		Operating points ($S_{l,e2}$,$S_{l,e3}$,$S_{l,e4}$,$S_{l,e5}$,$S_{l,e6}$,$S_{l,e9}$,$S_{l,e10}$,$S_{l,e11}$,$S_{l,e12}$,$S_{l,e13}$,$S_{l,e14}$,$S_{l,c1}$,$S_{l,c2}$,$S_{l,c3}$, $G_{l,g3}$,$G_{l,g5}$,$G_{l,GT1}$,$G_{l,g8}$,$G_{l,GT2}$,$G_{l,g9}$)		
		Traditional method [R2]	Proposed method	Relative deviation
Upper boundary point	$\boldsymbol{W}_{b,1}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 5, 15.7422)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 5, 15.7421)	0.001%
	$\boldsymbol{W}_{b,2}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 10, 10.7419)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 10, 10.7418)	0.001%
	$\boldsymbol{W}_{b,3}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 15, 5.7408)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 15, 5.7408)	-
	$\boldsymbol{W}_{b,4}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 20, 0.7382)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 5, 69, 126, 10, 0, 20, 0.7381)	0.014%
	…	…	…	
	$\boldsymbol{W}_{b,10513152}$	(30, 100, 55, 10, 15, 40, 10, 5, 10, 5, 10, 4, 4, 3, 129, 126, 40, 0, 10, 4.4512)	(30, 100, 55, 10, 15, 40, 10, 5, 10, 5, 10, 4, 4, 3, 129, 126, 40, 0, 10, 4.4511)	0.022%
Lower boundary point	$\boldsymbol{W}_{b,1}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 5, 1.1266)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 5, 1.1265)	0.009%
	$\boldsymbol{W}_{b,2}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 10, 1.1269)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 10, 1.1268)	0.009%
	$\boldsymbol{W}_{b,3}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 15, 1.1281)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 15, 1.1280)	0.009%
	$\boldsymbol{W}_{b,4}$	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 20, 1.1304)	(20, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 3, 3, 1, 69, 126, 10, 0, 20, 1.1303)	0.009%
	…	…	…	…
	$\boldsymbol{W}_{b,2951428}$	(25, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 1, 94, 131, 15, 0, 5, 0.3825)	(25, 85, 40, 0, 0, 25, 0, 0, 0, 0, 10, 2, 2, 1, 94, 131, 15, 0, 5, 0.3824)	0.026%
	…	…	…	
	$\boldsymbol{W}_{b,4725166}$	(30, 85, 50, 10, 15, 20, 10, 0, 10, 0, 10, 2, 2, 3, 94, 126, 15, 10, 10, 2.0852)	(30, 85, 50, 10, 15, 20, 10, 0, 10, 0, 10, 2, 2, 3, 94, 126, 15, 10, 10, 2.0851)	0.005%

Note: (1) Blue marks the deviation and maximum deviation generated when solving boundary points.

(2) To observe the relative deviation between the results of this paper and those of the traditional method, the accuracy of boundary points is improved to 10^{-4} in the table

2) online security analysis for operating points

First, randomly generate operating points $\boldsymbol{W}_1$~$\boldsymbol{W}_{10}$. Second, the full dimensional security boundary is calculated and the reduced dimensional sections of the existing method are taken respectively to analyze the security of $\boldsymbol{W}_1$~$\boldsymbol{W}_{10}$. Due to these points not being on the reduced dimensional section, the traditional method cannot analyze them, which means additional calculations are required. The Comparison results are shown in Table D1. It can be seen although the same analysis results are obtained, the calculation time of the proposed method is reduced by 10^6 times.

TABLE D1

COMPARISON OF ONLINE SECURITY ANALYSIS FOR RANDOM OPERATING POINTS OF 23-NODE IES

Operating point	($S_{l,e2}$,$S_{l,e3}$,$S_{l,e4}$,$S_{l,e5}$,$S_{l,e6}$,$S_{l,e9}$,$S_{l,e10}$,$S_{l,e11}$,$S_{l,e12}$,$S_{l,e13}$,$S_{l,e14}$,$S_{l,c1}$,$S_{l,c2}$,$S_{l,c3}$, $G_{l,g3}$,$G_{l,g5}$,$G_{l,GT1}$,$G_{l,g8}$,$G_{l,GT2}$,$G_{l,g9}$)	Proposed method		Traditional method	
		analysis result	time/s	analysis result	time/s
$\boldsymbol{W}_1$	(29.45, 96.58, 54.64, 5.67, 7.07, 28.81, 12.85, 0.66, 5.27, 7.19, 18.28, 134.0, 2.3, 28.05)	secure	8.54×10^{-3}	secure	1.87×10^{3}
$\boldsymbol{W}_2$	(27.14, 92.13, 48.33, 9.69, 8.14, 26.99, 13.5, 4.35, 8.94, 13.48, 15.07, 154.8, 22.22, 8.48)	insecure	8.11×10^{-3}	insecure	1.68×10^{3}
$\boldsymbol{W}_3$	(26.79, 95.21, 52.69, 8.25, 0.9, 33.18, 3.27, 3.01, 7.78, 14.02, 13.66, 184.4, 13.7, 4.09)	insecure	7.62×10^{-3}	insecure	1.79×10^{3}
$\boldsymbol{W}_4$	(29.59, 91.25, 46.12, 9.6, 9.87, 37.42, 1.15, 1.33, 0.69, 12.27, 12.27, 149.0, 20.05, 42.85)	insecure	7.98×10^{-3}	insecure	1.64×10^{3}
$\boldsymbol{W}_5$	(27.75, 90.7, 46.93, 6.46, 13.34, 37.56, 7.11, 4.32, 2.79, 10.63, 15.35, 133.8, 20.98, 18.48)	secure	8.36×10^{-3}	secure	1.69×10^{3}
$\boldsymbol{W}_6$	(26.08, 88.2, 52.39, 3.8, 1.64, 37.5, 12.53, 0.29, 3.79, 11.15, 12.89, 163.0, 17.14, 40.27)	insecure	7.47×10^{-3}	insecure	1.99×10^{3}
$\boldsymbol{W}_7$	(29.48, 90.74, 54.87, 4.77, 6.57, 28.06, 7.04, 2.29, 8.65, 13.5, 10.68, 182.9, 18.86, 39.15)	secure	7.55×10^{-3}	secure	1.81×10^{3}
$\boldsymbol{W}_8$	(20.6, 85.45, 47.86, 9.12, 4.2, 33.17, 6.21, 3.61, 4.2, 0.98, 10.85, 148.7, 26.33, 31.86)	insecure	8.02×10^{-3}	insecure	1.82×10^{3}
$\boldsymbol{W}_9$	(22.69, 92.08, 53.88, 0.15, 14.78, 38.12, 7.54, 1.7, 2.4, 5.04, 10.68, 128.7, 19.87, 42.91)	insecure	7.96×10^{-3}	insecure	1.78×10^{3}
$\boldsymbol{W}_{10}$	(29.87, 90.0, 51.09, 1.57, 9.13, 26.82, 1.88, 2.01, 5.98, 0.07, 14.1, 135.2, 26.26, 30.29)	insecure	8.17×10^{-3}	insecure	1.80×10^{3}