Analyzing Arguments with Truth Tables

- Use truth tables to determine validity of arguments.
- Recognize valid and invalid argument forms.
- Be familiar with the arguments of Lewis Carroll.

Truth Tables

Before, we used Euler diagrams were used to test the validity of arguments with the words "all," "some," or "not".

If these words are not present, it may be better to use a truth table than an Euler diagram to test validity.

Testing the Validity of an Argument with a Truth Table

Step 1 Assign a letter to represent each component statement in the argument.

Step 2 Express each premise and the conclusion symbolically.

Continued on the next slide…

Testing the Validity of an Argument with a Truth Table

Step 3 Form the symbolic statement of the entire argument by writing the *conjunction* of *all* the premises as the antecedent of a conditional statement, and the conclusion of the argument as the consequent.

Step 4 Complete the truth table for the conditional statement formed in Step 3. If it is a tautology, then the argument is valid; otherwise, it is invalid.

Testing the Validity of an Argument with a Truth Table: the old method vs. the new one

This method is equivalent to the method that we have used before:

- complete truth table for every premise and conclusion
- if for every row of true premises we get true conclusion -> the argument is valid
- if for some rows of true premises we get false conclusion -> the argument is invalid

Testing the Validity of an Argument with a Truth Table: the old method vs. the new one

Why is it equivalent?

The conjunction of premises is true iff all the premises are true.

The conditional statement: conjunction of premises implies conclusion, is false (the new criteria for determining invalidity of an argument) only when all the premises are true, but the conclusion is false (the previous criteria for determining invalidity of an argument).

Example: Using a Truth Table to Determine Validity

Is the following argument valid or invalid?

If there is a problem, then I must fix it.

There is a problem.

I must fix it.

Example: Truth Tables (Two Premises)

If there is a problem, then I must fix it.

There is a problem.

I must fix it.

Solution

Let p represent "There is a problem" and q represent "I must fix it."

$$
\begin{array}{l}
p \to q \\
\underline{p \qquad\quad} \\
q
\end{array}
$$

Example: Truth Tables (Two Premises)

$$\left[(p \to q) \quad \wedge \quad p \quad\right] \quad \to \quad q$$

Premise and premise implies conclusion.

The truth table is on the next slide.

Example: Truth Tables (Two Premises)

The truth table below shows that the argument is valid.

p	q	$[(p \to q) \wedge p] \to q$
T	T	T
T	F	T
F	T	T
F	F	T

Example: Using a Truth Table to Determine Validity

Is the following argument valid or invalid?

If I can avoid sweets, I can avoid the dentist.

I can’t avoid the dentist.

I can’t avoid sweets.

Example: Truth Tables (Two Premises)

If I can avoid sweets, I can avoid the dentist.

I can’t avoid the dentist.

I can’t avoid sweets.

Solution

Let p represent “I can avoid sweets” and q represent “I can avoid the dentist.”

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array}$$

Example: Truth Tables (Two Premises)

$$\left[(p \rightarrow q) \wedge \sim q \right] \rightarrow \sim p$$

$\uparrow \qquad \uparrow \qquad \uparrow \qquad \uparrow \qquad \uparrow$

Premise and premise implies conclusion

The truth table is on the next slide.

Example: Truth Tables (Two Premises)

The truth table below shows that the argument is valid.

p	q	$\left[(p \to q) \wedge \neg q\right] \to \neg p$
T	T	T
T	F	T
F	T	T
F	F	T

Valid Argument Forms

Modus Ponens	Modus Tollens	Disjunctive Syllogism	Reasoning by Transitivity
$$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$$	$$\begin{array}{l} p \to q \\ \sim q \\ \hline \sim p \end{array}$$	$$\begin{array}{l} p \vee q \\ \sim p \\ \hline q \end{array}$$	$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

Note: Here $\sim$ means $\neg$ (not).

Invalid Argument Forms (Fallacies)

Fallacy of the Converse	Fallacy of the Inverse
$$\begin{array}{l} p \to q \\ q \\ \hline p \end{array}$$	$$\begin{array}{l} p \to q \\ \sim p \\ \hline \sim q \end{array}$$

Note 1 : Here $\sim$ means $\neg$ (not).

Note 2 : See Lecture 4a slide 10 on converse and inverse.

Example: Truth Tables (More Than Two Premises)

Determine whether the argument is *valid* or *invalid*.

If Eddie goes to town, then Mabel stays at home. If Mabel does not stay at home, then Rita will cook. Rita will not cook. Therefore, Eddie does not go to town.

Example: Truth Tables (More Than Two Premises)

Determine whether the argument is *valid* or *invalid*.

If Eddie goes to town, then Mabel stays at home. If Mabel does not stay at home, then Rita will cook. Rita will not cook. Therefore, Eddie does not go to town.

Clicker answer:

A: valid

B: invalid

Example: Truth Tables (More Than Two Premises)

So we have $p \to q$

$$\neg q \to r$$

$$\neg r$$

$$\neg p$$

This leads to the statement

$$\left[(p \to q) \wedge (\neg q \to r) \wedge \neg r\right] \to \neg p.$$

The truth table is on the next slide.

Example: Truth Tables (More Than Two Premises)

p	q	r	$[(p \to q) \wedge (\sim q \to r) \wedge \sim r] \to \sim p$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

Example: Truth Tables (More Than Two Premises)

Because the final column does not contain all Ts, the statement is not a tautology and the argument is invalid.

Arguments from Literature

Poem	**Logical Form**
For want of a nail, the shoe was lost.	$\neg N \rightarrow \neg S$
For want of a shoe, the horse was lost.	$\neg S \rightarrow \neg H$
For want of a horse, the rider was lost.	$\neg H \rightarrow \neg R$
For want of a rider, the battle was lost.	$\neg R \rightarrow \neg B$
For want of a battle, the war was lost.	$\underline{\neg B \rightarrow WL}$
Therefore, for want of a nail, the war was lost.	$WL \rightarrow \neg N$

CQ: Is this argument valid? A: Yes B: No. Why?

Arguments of Lewis Carroll

Charles Lutwidge Dodgson (1832-1898)

was a mathematician, logician,

a lecturer in mathematics, writer essays, political pamphlets and poetry, was an avid photographer. However, he is most famous for being the author of ?

Supply a conclusion that yields a valid argument for the following premises.

Babies are illogical.
Nobody is despised who can manage a crocodile.
Illogical persons are despised.

Arguments of Lewis Carroll

Babies are illogical.
Nobody is despised who can manage a crocodile.
Illogical persons are despised.

Let p be "you are a baby," let q be "you are logical," let r be "you can manage a crocodile," and let s be "you are despised."

Example: Arguments of Lewis Carroll

With these letters, the statements can be written symbolically as:

$$p \to \sim q$$

$$r \to \sim s$$

$$\sim q \to s.$$

Swap 2nd and 3rd premise; exchange the 3rd statement for contrapositive:

$$p \to \sim q$$

$$\sim q \to s$$

$$s \to \sim r.$$

Example: Arguments of Lewis Carroll

Repeated use of reasoning by transitivity gives the conclusion:

$$p \rightarrow \sim r,$$

leading to a valid argument.

In words, the conclusion is "If you are a baby, then you cannot manage a crocodile."