CS131 Homework #9 (12 pts)

1) (1 pt) At least how many students would have their last names start with the same letter in a class of 100 students?
   By generalized pigeonhole principle the number of students that have last names beginning with the same letter $\geq\lceil 100/26\rceil = 4$
2) (1 pt) What should be the minimum number of US students in a college to make sure that there are at least 100 students from the same US state?
   By generalized pigeonhole principle we should select the smallest N, such that $\lceil N/50\rceil \geq 100$.
   For N divisible by 50: $N/50 \geq 100$ iff $N \geq 5000$.
   For N not divisible by 50, $\lceil N/50\rceil \geq 100$ iff $N/50 > 99$ iff N>99·50 iff N≥99·50+1=**4,951.**
3) (1 pt) If the password locking TV consists of 4 digits, how many different passwords my son will have to try in the worst case to unlock the TV?
   10^4=10,000
4) (1 pt) There are 4 different candidates for a governor of a state. In how many different orders can the names of the candidates be printed on a ballot?
   4!=24
5) (1 pt) How many different signals, each consisting of 6 flags hung in a vertical line, can be formed from 4 identical red flags and 2 identical blue flags?
   6!/(4!2!)=15 signals
6) (1 pt) Find the number of ways that an organization consisting of 10 members can elect a president, a treasurer, and a secretary (one person may be elected just for one position).
   10!/(10-3)!=10*9*8=720.
7) (1 pt) Find the number of ways that an organization consisting of 10 members can elect 3 representatives to a senate.
   10!/((10-3)!3!)=120
8) (2 pts) How many terms does $(x+y)^9$ have? What is the coefficient of the term x^3y^6?
   10 terms. 9!/(3!6!)=9*8*7/(3*2)=3*4*7=84
9) (1 pt) Suppose that an ice-cream café has 10 different flavors of ice cream. In how many different ways one can choose 3 scoops of ice-cream, so that order of flavors does not matter? (There is an unlimited amount of each flavor of ice-cream.)
   C(10+3-1,3)=C(12,3)=12!/(3!9!)=12*11*10/(3*2)=2*11*10=220.
10) (1 pt) There are 2504 CS students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Python, and 345 have taken a course in C++. Further, 876 have taken a courses in both Java and Python, 231 have taken courses in both Python and C++, and 290 have taken courses in both Java and C++. If 189 of these students have taken courses in Python, Java, and C++, how many of these 2504 students have not taken a course in any of these three programming languages?
    By inclusion-exclusion principle: 2504-(1876+999+345-876-231-290+189)=492
11) (1 pt) How many elements are there in the union of four sets if each of the sets has 100 elements, each pair of the sets shares 50 elements, each three of the sets share 25 elements, and there are 5 elements in all four sets?

By inclusion-exclusion principle, number of elements in the union =
C(4,1)*100-C(4,2)50+C(4,3)25-C(4,4)5=
=4*100-6*50+4*25-1*5=
=195