Combinatorial Structures

Introductory Lecture

Basic Notions

- **Combinatorics**
  - is a branch of mathematics concerning the study of countable or finite discrete structures.
- **Discrete Structures**:
  - abstract mathematical structures that represent discrete objects and the relationships between them, e.g. logic statements, sets, permutations, trees, graphs.

What are discrete objects?

- Discrete objects do not vary smoothly, but have distinct, separated values.
- Discrete mathematics is the part of mathematics devoted to the study of discrete objects.
- Calculus deals with continuous objects and is not part of discrete mathematics.
- Which objects are discrete?
  a) integers
  b) real numbers
  c) the plane curves between A and B
  d) the paths in a networks (graph) between A and B

Kinds of Problems Solved in this Course

- How many ways can a password be chosen following specific rules?
- How many valid Internet addresses are there?
- What is the probability of winning a particular lottery?
- Is there a link between two computers in a network?
- What is the shortest path between two cities using a transportation system?
- How can it be proved that a sorting algorithm always correctly sorts a list?
- How can we prove that there are infinitely many prime numbers?
- How can I encrypt a message so that no unintended recipient can read it?

CS131 is a Gateway Course

- Topics in this course will be important in many courses that you will take in the future:
  - **Computer Science**: Data Structures, Algorithms, Programming Languages, Compilers, Computer Architecture, Cryptography, Databases, Artificial Intelligence, Machine Learning, Networking, Graphics, Game Design, Natural Language Programming, …
  - **Other Disciplines**: You may find concepts learned here useful in business, economics, probability and statistics, bioinformatics, data science, etc.

Main topics of the course

- Logic
- Proof Strategies
- Number Theory
- Trees
- Graphs
- Sums and Products
- Asymptotic Notation
- Recurrences
- Counting

About me

- Instructor: Olga Lepsky
- Got BSc, MS, and PhD in Mathematics
- Wrote big computer programs for PhD
- Worked as a CAD software developer
- Taught many Math and Computer Science courses in many colleges (and one school)
- Love learning/teaching about applications of mathematics to computer science and algorithms

Logic

Is the argument valid?

- I will go to work tomorrow or today. (premise)
- I am going to stay at home today. (premise)
- Therefore, I will go to work tomorrow. (conclusion)

Is the argument valid?

- Either the butler is guilty or the maid is guilty. (premise)
- Either the maid is guilty or the cook is guilty. (premise)
- Therefore, either the butler is guilty or the cook is guilty. (conclusion)

Which argument is called valid?

Translating English Sentences

- Steps to convert an English sentence to a statement in a logical form
  - Identify atomic statements (propositional variables): P: the butler is guilty, Q: the maid is guilty, R: the cook is guilty
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Symbol	Meaning	Term
V	or	disjunction
∧	and	conjunction
¬	not	negation

  - To analyze validity of an argument – construct truth tables for all the values of P, Q, R. **Check whether when all the premises are true conclusion is also true (valid argument).**

Truth tables

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
F	T	F
T	F	F
F	F	F

P	Q	P V Q
T	T	T
F	T	T
T	F	T
F	F	F

a) How would the truth table for P ∧ Q should be modified for P+Q (exclusive or)?
b) How to express P+Q via ∧, V, ¬ ?

Truth tables for the first argument

		premise	premise	conclusion
P	Q			

The argument is _____ (valid/invalid).

Truth tables for the second argument

			premise	premise	conclusion
P	Q	R			

The argument is _____ (valid/invalid).