Translating from English to Logic

Example 1: Translate the following sentence into formal logic:

"Every student in this class has taken a course in Java."

Solution:

First decide on the domain U of the variable x.

Solution 1: If U is all students in this class, define a propositional function J(x) denoting "x has taken a course in Java" and translate as:

$\forall x\, J(x)$.

Solution 2: But if U is all people, also define a propositional function S(x) denoting "x is a student in this class" and translate as:

$\forall x\, (S(x) \rightarrow J(x))$

$\forall x\, (S(x) \wedge J(x))$ is not correct. It means: every person is both a student and took Java.

Translating from English to Logic

Example 2: Translate the following sentence into formal logic:

"Some student in this class has taken a course in Java."

Solution 1: If U is all students in this class, translate as

$$\exists x\, J(x)$$

Solution 2: But if U is all people, then translate as

$$\exists x\, (S(x) \wedge J(x))$$

Some people are students who took a course in Java.

Translation from English to Logic

Examples:

1. "Some student in this class has visited Mexico."

   Solution: Let $M(x)$ denote "x has visited Mexico" and $S(x)$ denote "x is a student in this class," and U be all people.

   $$\exists x \ (S(x) \land M(x))$$

2. "Every student in this class has visited Canada or Mexico."

   Solution: Add $C(x)$ denoting "x has visited Canada."

   $$\forall x \ (S(x) \rightarrow (M(x) \lor C(x)))$$

System Specification Example

- Predicate logic is used for specifying properties that systems must satisfy.
- For example, translate into predicate logic:
  - "Every mail message larger than one megabyte will be compressed."
  - Let $L(m, y)$ be "Mail message m is larger than y megabytes."
  - Let $C(m)$ denote "Mail message m will be compressed."

$$\forall m(L(m,1) \rightarrow C(m))$$

  - "If a user is active, at least one network link will be available."
  - Let $A(u)$ represent "User u is active."
  - Let $S(n, x)$ represent "Network link n is state x.

$$\exists u\, A(u) \rightarrow \exists n\, S(n, available)$$

Translating Mathematical Statements into Predicate Logic

Example : Translate "The sum of two positive integers is always positive" into a logical expression.

Solution:

1. Rewrite the statement to make the implied quantifiers and domains explicit:

   "For every two integers, if these integers are both positive, then the sum of these integers is positive."

2. Introduce the variables *x* and *y*, and specify the domain, to obtain:

   "For all positive integers *x* and *y*, *x* + *y* is positive."

3. The result is:

$$\forall x \forall y ((x > 0) \land (y > 0) \rightarrow (x + y > 0))$$

where the domain of both variables consists of all integers

Calculus in Logic (*optional*)

Example: Use quantifiers to express the definition of the limit of *a* real-valued function *f(x)* of a real variable *x* at a point *a* in its domain.

Recall the definition of the statement

$$\lim_{x \to a} f(x) = L$$

is

"For every real number ε > 0, there exists a real number δ > 0 such that |f(x) – L| < ε whenever 0 < |x –a| < δ."

Using quantifiers:

$$\forall \epsilon \exists \delta \forall x (0 < | x - a | < \delta \rightarrow | f(x) - L | < \epsilon)$$

where the domain for the variables ε and δ consists of all positive real numbers and the domain for *x* consists of all real numbers.