Conditionals

A **conditional** statement is a compound statement that uses the connective *if…then*.

The conditional is written with an arrow, so "if p then q" is symbolized: $p \rightarrow q$

We read the above as "p implies q" or "if p then q." The statement p is the **antecedent**, while q is the **consequent**.

Truth Table for The Conditional:

If *p*, then *q*

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example: If I am elected president, US will get peace and prosperity

p: if I am elected president   *q: US will get peace and prosperity*

Special Characteristics of Conditional Statements

1. $p \to q$ is false only when the antecedent is *true* and the consequent is *false*.
2. If the antecedent is *false*, then $p \to q$ is automatically *true*.
3. If the consequent is *true*, then $p \to q$ is automatically *true*.

Example: Determining Whether Conditionals Are True or False

Decide whether each statement is True or False (T represents a true statement, F a false statement).

a) $T \rightarrow (4 < 2)$ b) $(8 = 1) \rightarrow F$

Solution

a) False

b) True

Tautology

A statement that is always true, no matter what the truth values of the components, is called a **tautology**. They may be checked by forming truth tables.

Writing a Conditional as a Disjunction: "Or" Statement

$p \rightarrow q$ is equivalent to $\neg\, p \vee q$.

Q: How to prove it?

Example: Determining Statements Equivalent to Conditionals

Write the conditional as an equivalent statement without using *if* . . . *then*.

If the Indians win the pennant, then Johnny will go to the World Series.

Solution

Let p represent “The Indians win the pennant” and q represent “Johnny will go to the World Series.

Restate: The Indians do not win the pennant or Johnny will go to the World Series.

Negation of a Conditional $p \to q$

The negation of $p \to q$ is $p \wedge \neg q$.

Q: How to prove it?

Example: Determining Negations

Determine the negation of each statement.

If I’m hungry, I will eat.

Solution

I’m hungry and I will not eat.

Converse, Inverse, and Contrapositive

Conditional Statement	$p \to q$	If *p*, then *q*
Converse	$q \to p$	If *q*, then *p*
Inverse	$\neg p \to \neg q$	If not *p*, then not *q*
Contrapositive	$\neg q \to \neg p$	If not *q*, then not *p*

Equivalences

- A conditional statement and its contrapositive are equivalent.
- Also, the converse and the inverse are equivalent.

Example: Determining Related Conditional Statements

Given the conditional statement:

"If I am running, then I am moving"

a) the equivalent (contrapositive)

b) the converse

Solution

a) If I am not moving, then I am not running.

b) If I am moving, then I am running.

Alternative Forms of "If *p*, then *q*"

The conditional $p \to q$ can be translated in any of the following ways.

If p, then q.	p is sufficient for q.
If p, q.	q is necessary for p.
p implies q.	All p are q.
p only if q.	q if p.