CS131 Homework #7 (20 pts)

1) (6 pts) Find solution of the recurrence relations together with the initial conditions. Show your work, circle or put in bold each intermediate and final answer.

a. *(2 pts)* $a_n = 2a_{n-1} + 2n^2$ for $n \geq 2$, $a_1 = 4$

- *homogeneous rec. relation:* $a_n = 2a_{n-1}$
- *characteristic equation: r=2*
- *the solution of homogeneous recurrence relation:* $a_n = \alpha 2^n$
- *particular solution of nonhomogeneous recurrence relation:* $(p_2n^2 + p_1n + p_0)$
- *find coefficients of particular solution (from recurrence relation):* $p_2 = -2, p_1 = -8, p_0 = -12$
- *general solution of nonhomogenous recurrence relation:* $-2n^2 - 8n - 12 + \alpha 2^n$
- *find coefficients of general solution from* $a_1 = 4$: $\alpha = 13$
- *general solution of rec. relation with init. conditions:* $-2n^2 - 8n - 12 + 13 \cdot 2^n$

Grading remarks: if the answer is incorrect – indicate which step has the error or missing. (Also, please

b. *(2 pts)* $a_n = -5a_{n-1} - 6a_{n-2} + 42 \cdot 4^n$ for $n \geq 3$, $a_1 = 56$, $a_2 = 278$.

- *homogeneous rec. relation:* $a_n = -5a_{n-1} - 6a_{n-2}$
- *characteristic equation:* $r^2 = -5r - 6$
- *the solution of homogeneous recurrence relation:* $\alpha_1(-2)^n + \alpha_2(-3)^n$
- *particular solution of nonhomogeneous recurrence relation:* $C \cdot 4^n$
- *find coefficients of particular solution: C=16*
- *general solution of nonhomogenous recurrence relation:* $\alpha_1(-2)^n + \alpha_2(-3)^n + 16 \cdot 4^n$
- *find coefficients of general solution from initial conditions:* $(-2)^n + 2(-3)^n + 16 \cdot 4^n$

Grading remarks: if the answer is incorrect – indicate which step has the error or missing.

c. *(1 pt) f(n)=2f(n/3)+4* for any $n=3^k$ (k-integer, k≥1); $f(1)=1$.

$f(n) = 2f(n/3) + 4 =$

$=2(2f(n/3)+4) + 4=$

$= 2^k f(n/3^k) + 4(1+2+2^2 ... +2^{k-1})=$

$= 2^k f(1) + 4(2^k-1)/(2-1)=$

$= 2^k + 4\,(2^k - 1) =$

$= 5 \cdot 2^k - 4 =$

$= 5 \cdot 2^{\log_3 n} - 4 =$

$= 5 \cdot 3^{\log_3 2 \cdot \log_3 n} - 4 =$

$= 5 \cdot n^{\log_3 2} - 4.$

d. (1 pt) Derive *O()* estimate of *f(n),* true for any *n>1,* where *f(1)*=1 and *f(n)* is increasing function of *n*, satisfying recurrence relation *1c* for any $n=3^k$ (k-integer, k≥1).

$n>1 \rightarrow \exists k$ (k-integer, $k \geq 1$, $3^k < n \leq 3^{k+1}$)

f(n) is increasing function of $n \rightarrow f(n) \leq f(3^{k+1})=$

$= 5 \cdot 2^{k+1} - 4 =$

$= 5 * 2 * 2^k - 4 \leq$

$\leq 5 * 2 * 2a^{\log_3 n} - 4$ (since $3^k < n$)

$= 5 * 2 * n^{\log_3 2} - 4$

(since $2^{\log_3 n} = (3^{\log_3 2})^{\log_3 n} = (3^{\log_3 n})^{\log_3 2} = n^{\log_3 2}$)

Thus, $\boldsymbol{f(n) \in O(n^{\log_3 2})}$

2) (9 pts; each item – 1pt)
Give big-O estimate for f(n) satisfying the following recursive relation:

a. *f(n)=f(n/b)+C, b is an integer >1   By Theorem 1: O(log n)*
b. *f(n)=f(n/b)+Cn, b is an integer >1 By Master Theorem with a=1, b>1, d=1, (a<b^d)→f(n) is O(n)*
c. *f(n)=2f(n/2)+C By Theorem 1: O(n)*
d. *f(n)=3f(n/3)+C By Theorem 1: O(n)*
e. *f(n)=4f(n/2)+C By Theorem 1: O(n^2)*
f. *f(n)=2f(n/2)+Cn By Master Theorem with a=2, b=2, d=1, (a=b^d)→f(n) is O($n \log n$)*
g. *f(n)=2f(n/2)+Cn^2 By Master Theorem with a=2, b=2, d=2, (a<b^d)→f(n) is O(n^2)*
h. *f(n)=7f(n/2)+Cn^2 By Master Theorem with a=7, b=2, d=2, (a>b^d)→f(n) is O($n^{\log_2 7}$)*
i. *f(n)=2f(n/2)+C$\sqrt{n}$ By Master Theorem with a=2, b=2, d=0.5, (a>b^d)→f(n) is O(n)*

3) (5 pts: each item -1pt) Which of the recurrence relations in 2) describe the number of operations performed by
   a. binary search - 2a
   b. merge sort – 2f
   c. recursive min-max algorithm – 2c
   d. fast matrix multiplication – 2h
   e. the closest pair problem – 2f