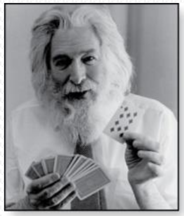
Propositions

- A *proposition* is a declarative sentence that is either true or false.
- Examples of propositions:
  a) The Moon is made of green cheese.
  b) Trenton is the capital of New Jersey.
  c) Toronto is the capital of Canada.
  d) $1 + 0 = 1$
  e) $0 + 0 = 2$
- Examples that are not propositions.
  a) Sit down!
  b) What time is it?
  c) $x + 1 = 2$
  d) $x + y = z$

Propositional Logic

- Constructing Propositions
  - Propositional Variables correspond to atomic propositions: P, Q, R, S, …
  - Compound Propositions; constructed from logical connectives and other propositions
    - Negation $\neg$
    - Conjunction $\wedge$
    - Disjunction $\vee$
    - exclusive or, nor, nand
    - Implication $\rightarrow$
    - Biconditional $\leftrightarrow$

Logic Puzzles



Raymond Smullyan (Born 1919)

- An island has two kinds of inhabitants, *knights*, who always tell the truth, and *knaves*, who always lie.
- You go to the island and meet A and B.
  - A says "B is a knight."
  - B says "The two of us are of opposite types."

Example: What are the types of A and B?

Logic Puzzles

- **Solution:** Let p and q be the statements that A is a knight and B is a knight, respectively. So, then $\neg p$ represents the proposition that A is a knave and $\neg q$ that B is a knave.
  - If A is a knight, then p is true. Since knights tell the truth, q must also be true. Then $(p \wedge \neg q) \vee (\neg p \wedge q)$ would have to be true, but it is not.
  - If A is a knave, then B must not be a knight since knaves always lie. So, then both $\neg p$ and $\neg q$ hold since both are knaves.

Equivalent propositions

Two compound propositions p and q are **equivalent** if and only if the columns in a truth table giving their truth values agree.

Key Logical Equivalences

- Identity Laws: $p \wedge T \equiv p$, $p \vee F \equiv p$
- Domination Laws: $p \vee T \equiv T$, $p \wedge F \equiv F$
- Idempotent laws: $p \vee p \equiv p$, $p \wedge p \equiv p$
- Double Negation Law: $\neg(\neg p) \equiv p$
- Negation Laws: $p \vee \neg p \equiv T$, $p \wedge \neg p \equiv F$

Key Logical Equivalences (*cont*)

- Commutative Laws: $p \lor q \equiv q \lor p$, $p \land q \equiv q \land p$
- Associative Laws: $(p \land q) \land r \equiv p \land (q \land r)$
  $(p \lor q) \lor r \equiv p \lor (q \lor r)$
- Distributive Laws: $(p \lor (q \land r)) \equiv (p \lor q) \land (p \lor r)$
  $(p \land (q \lor r)) \equiv (p \land q) \lor (p \land r)$
- Absorption Laws: $p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$

De Morgan's Laws

Augustus De Morgan

1806-1871

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Proof of the 2nd law.

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T