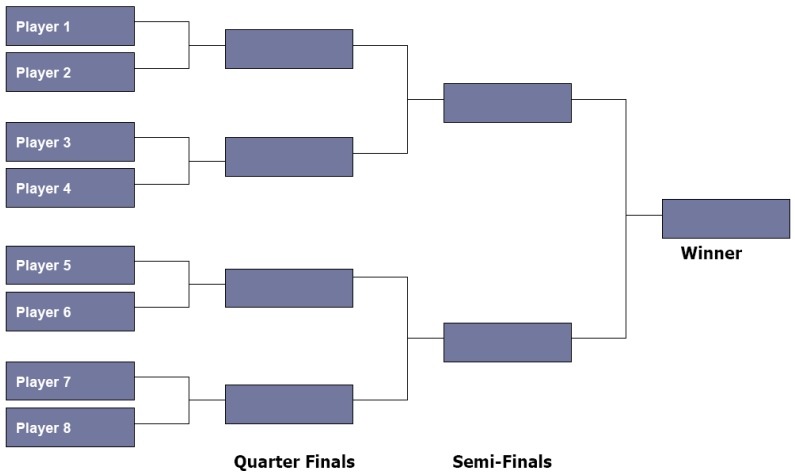
CS131 Homework #8 (17 pts)

1) (6 pts) Consider the three algorithms for computing a^n (n – natural number)
   Algorithm 1: iterative (a^0=1, if n>0 multiply a by itself n times)
   Algorithm 2: recursive, described at the slide 41 of Lecture 10
   Algorithm 3: recursive, divide-and-conquer:
   a^0=1
   $a^n = (a^{n/2}) \cdot (a^{n/2})$ if n is even;
   $a^n = a \cdot (a^{\lfloor n/2 \rfloor}) \cdot (a^{\lfloor n/2 \rfloor})$ if n is odd
   a. (2 pts) For the recursive algorithms write recurrence relations describing number of operations f(n) for natural n for algorithm 2 and even n for algorithm 3. (Hint: for algorithm 2 the recurrence relation is linear non-homogeneous of 1st degree: f(n)=…f(n-1)+…; for algorithm 3 the recurrence relation is of divide-and-conquer form: f(n)=…f(n/…)+…)
   b. (3 pts) Estimate O(g(n)) complexity of each of the three algorithms. (Hint: for algorithm 2 solve as nonhom. lin. rec. relation as in slides 34-37 of Lecture 11 or using backward substitution as in slide 10 of Lecture 11; for algorithm 3 use Master theorem)
   c. (1 pt) Which one is the fastest?

2) (5 pts) Suppose that there are $n = 2^k$ teams in an elimination tournament, where there are $n/2$ games in the first round, with $n/2$ winners playing in the second round, and so on.



   a. Develop a recurrence relation for $f(n)$ – the number of games in the tournament.
   b. Give a big-O estimate of $f(n)$, explaining your result.
   c. Solve the recurrence relation via the method of backward substitution, get the formula for $f(n)$ via n.
   d. Check whether your formula matches the result obtained in b. (If it does not – fix the errors.)

e. The tournament diagram for 8 players is shown above. Make tournament diagrams n=2, 4. Check whether your formula correctly counts the number of games when the number of teams n=2, 4, 8. (The number of games produced by the formula should be equal to the number of games shown on the tournament diagrams; if it doesn't – fix the errors.)

3) (4 pts) There are four possibilities for each base in DNA: A, C, G, and T. How many 5-element DNA sequences of bases
   a. end with A?
   b. start with T and end with G?
   c. contain only A and T?
   d. do not contain C?

4) (1 pt) A committee is formed consisting of one representative from each of the 50 states in the US, where a representative from a state is either the governor or one of the 2 senators from that state. How many ways are there to form this committee?

5) (1 pt) How many license plates can be made using either 3 uppercase English letters followed by 3 digits or 4 uppercase English letters followed by 2 digits?