CS1 Homework 1 (21 pts)

1. (9 pts) Write the statements in a logical form. Use negation rules, whenever possible.
   1. The fact A implies the fact B.

A → B

* 1. The conclusion C follows from the three premises: P, Q, and R.

P∧Q∧R → C

* 1. A is both sufficient and necessary for B.

A ↔ B

* 1. A is necessary for B, but not sufficient.

B → A ∧﹁(A→ B)

* 1. C is sufficient for B, but not necessary.

C → B∧﹁(B→ C)

* 1. It is sufficient to get good grades in order to get a good job.

Get good grades → Get a good job

* 1. It is **not** sufficient to get good grades in order to get a good job.

﹁(Get good grades → Get a good job) ≡ Get good grades ∧﹁Get a good job

* 1. It is necessary to get good grades in order to get a good job, but not sufficient.

(Good Job -> Good grades) ∧ ¬(Good Grades -> Good Job)

* 1. I get good grades if and only if I study hard.

Get good grades ↔ I study hard

1. (6 pts) Decide whether each statement is true or false. Explain your reasoning using the truth tables for conditional/biconditional statements.
   1. Pluto is called a planet if and only if many sky bodies orbiting the Sun located in Kuiper Belt are called planets.

P: Pluto is called a planet

S: many sky bodies orbiting the Sun located in Kuiper Belt are called planets

|  |  |  |
| --- | --- | --- |
| P | S | P↔S |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | T |

True

Because Pluto is not called a planet (P) is False and many sky bodies orbiting the Sun located in Kuiper Belt are called planets (S) is also False.

* 1. If Mars is a planet then Pluto is a planet.

M: Mars is a planet

P: Pluto is a planet

|  |  |  |
| --- | --- | --- |
| M | P | M→P |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

False

Because Pluto is a planet (P) is False and Mars is a planet (M) is True.

* 1. If it is raining or snowing, I am studying at Boston University.

R: it is raining

S: it is snowing

B: I am studying at Boston University

|  |  |  |  |
| --- | --- | --- | --- |
| R | S | B | (R∨S) → B |
| T | T | T | T |
| T | T | F | T |
| T | F | T | T |
| T | F | F | T |
| F | T | T | T |
| F | T | F | T |
| F | F | T | F |
| F | F | F | T |

False

**Counterexample**: There is a weekend when it is snowing and I am not studying at BU.

* 1. For every x: if x4>10,000, then x>10.

Q(X): x4>10,000

P(X): x>10

|  |  |  |
| --- | --- | --- |
| P(X) | Q(X) | Q(X) → P(X) |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

False

**Counterexample**: x = -100, (-100) 4>10,000, but -100 < 10.

* 1. x>10 implies x4>10,000.

P: x4>10,000

Q: x>10

|  |  |  |
| --- | --- | --- |
| P | Q | Q → P |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

True

Because x > 10, so that x\*x >10x, so that x4> 10xxx, … , because 10xxx > 10000, so x4>10000.

* 1. x>0: log x>0 iff x>1.

L: log x>0

O: x>1

Z: x>0

|  |  |  |  |
| --- | --- | --- | --- |
| L(x) | O(x) | Z(x) | ∀Z(x): L(x) ↔ O(x) |
| T | T | T | T |
| T | T | F | F |
| T | F | T | F |
| T | F | F | F |
| F | T | T | F |
| F | T | F | F |
| F | F | T | T |
| F | F | F | F |

True

log x>0 if x>1 is True.

x>1 if log x>0 is True.

1. (6 pts) Decide whether the following arguments are valid, using either Truth tables or known forms of valid/invalid arguments.
   1. If Pedro Martinez pitches, the Red Sox win.

The Red Sox do not win.

Pedro Martinez does not pitch.

P: Pedro Martinez pitches

R: the Red Sox win

Modus Tollens:

P→R

﹁R

＿＿

﹁P

Valid

* 1. If you got a good grade for the course, then you did well on the final exam.

You did well on the final exam.

You got a good grade for the course.

G: you got a good grade for the course

E: you did well on the final exam

G→E

E

＿＿

G

(Fallacy of the Converse)

|  |  |  |
| --- | --- | --- |
| G | E | ((G→E) ∧ E) → G |
| T | T | T |
| T | F | T |
| F | T | F |
| F | F | T |

Invalid

* 1. If there is a snow storm, there is no electricity in my house.

The snow is not falling.

My house has electricity.

S：there is a snow storm

E：no electricity in my house

S→E

﹁S

＿＿

E

|  |  |  |
| --- | --- | --- |
| S | E | ((S→E) ∧ ﹁S) → E |
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Invalid

Because there might be other reasons that cause no electricity in my house.

* 1. My friend wants to major in CS or biology or chemistry.

She decided that biology and chemistry are not for her.

My friend wants to major in CS.

S: major in CS

B: major in biology

C: major in chemistry

S∨B∨C ≡ S∨(B∨C)

﹁B∧﹁C ≡﹁(B∨C)

＿＿＿＿

S

(Disjunctive Syllogism)

Valid

* 1. If parents go to town, then nanny supervises kids.

If nanny does not supervise kids, then nanny will cook.

Nanny cooked today.

Therefore, parents did not go to town today.

P: parents go to town

S: nanny supervises kids

C: nanny will cook

P→S

﹁S→C

C

＿＿

﹁P

|  |  |  |  |
| --- | --- | --- | --- |
| P | S | C | (P→S)∧(﹁S→C)∧C → ﹁P |
| T | T | T | F |
| T | T | F | T |
| T | F | T | F |
| T | F | F | T |
| F | T | T | T |
| F | T | F | T |
| F | F | T | T |
| F | F | F | T |

False

* 1. There is an old anecdote:

A man rides in a train, and his fellow traveler asks him what time it is. The man responds, “I won’t tell you.” Then he explains:

“If I tell you the time, then we’ll start chatting.

If we start chatting, then I’ll have to invite you to my house.

If I invite you to my house, then you’ll meet my daughter and you both will fall in love.

If you both fall in love, you’ll marry my daughter.

Since I don’t want a son-in-law who can’t afford a watch, I won’t tell you the time.”

Is his argument valid?

T: I tell you the time

C: we’ll start chatting

I: I’ll have to invite you to my house

D: you’ll meet my daughter

F: you both fall in love

M: you’ll marry my daughter

T→C

C→I

I→D

D→F

F→M

＿＿

T→M

(Reasoning by Transitivity)

Valid

T→C∧ C→I ≡ T→I

T→I ∧ I→D ≡ T→D

T→D∧ D→F ≡ T→F

T→F ∧ F→M≡ T→M