CS131 Homework #4 (18 pts)

Prove the following statements. To prove them, start by writing givens and goals, modifying givens and goals, as discussed in the lectures, and then write the proof reasoning going from givens to goals.

1. Suppose a, b, c, and d are real numbers, 0 < a < b, and d > 0. Prove that if ac ≥ bd then c > d.
2. Suppose A ⊆ C, and B and C are disjoint. Prove that if x A then x B.
3. Suppose that y + x = 2y − x, and x and y are not both zero. Prove that y ≠ 0.
4. For all real numbers x and y there is a real number z such that x + z = y − z.
5. For all integers a and b there is an integer c such that a | c and b | c.

(x|y means x divides y)

1. If A is non-emptyset and A ⊆ B \ C then A and C are disjoint and B⊆ C.
2. For every integer x, the remainder when x4 is divided by 8 is either 0 or 1.
3. For every real number a>0, either a≤1 or f(x)=ax increases with x.

(Hint: function increases if f’(x)>0, (ax)’ = ax ln a )

1. For every real number x, if | x − 3 | > 3 then x2 > 6x.

(Hint: If x − 3 ≥ 0 then | x − 3 | = x − 3, and if x − 3 < 0 then | x − 3 | = 3 − x. Assume that x − 3 ≥ 0 in case 1, and x − 3 < 0 in case 2; break your proof into cases.)











