Programming

(a)

1. Optimization Problem

(1) From basic form, we have the Lagrange function $\mathcal{L}(w,b,\alpha) = \frac{1}{2}\|w\|^2 + \sum_{i=1}^{m} \alpha_i[1 - y_i(w^T x_i + b)]$,

(2) Stationarity: $\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = 0 \Rightarrow \mathbf{w} = \sum_{i}^{m} \alpha_i y_i \mathbf{x}_i$ and $\frac{\partial \mathcal{L}}{\partial b} = 0 \Rightarrow \sum_{i}^{m} \alpha_i y_i = 0$

(3) Feasibility: $\alpha_i \geq 0,\ 1 - y_i\left(\mathbf{w}^\top \mathbf{x}_i + b\right) \leq 0,\ \forall i$

(4) Complementary slackness: $\alpha_i\left[1 - y_i\left(\mathbf{w}^\top \mathbf{x}_i + b\right)\right] = 0,\ \forall i$

(5) we get the dual problem and replace all stationary conditions: $\max_{\alpha} \sum_{i=1}^{m} \alpha_i - \frac{1}{2}\sum_{i,j}^{m} \alpha_i \alpha_j y_i y_j x_i^T x_j \quad \text{s.t.} \quad \sum_{i=1}^{m} \alpha_i y_i = 0,\quad \alpha_i \geq 0,\quad \forall i$

(6) calculate w and b using the optimal solution $w = \sum_{i}^{m} \alpha_i y_i \mathbf{x}_i$, $b = \frac{1}{|\mathcal{S}|}\sum_{j \in \mathcal{S}}\left(y_j - \sum_{i}^{m} \alpha_i y_i \mathbf{x}_i^\top \mathbf{x}_j\right)$ where $\mathcal{S} = \{i \mid \alpha_i > 0\}$

2. result:

```
training_error: 0.0
testing_error: 0.027777777777779
w_of_wine_type_0: [ 0.63965167  0.67303317  2.0641877  -0.18931348  0.00445548 -0.33694301
  0.47339836  0.46928252 -0.20912794  0.25357763 -0.19575102  0.30897943
  0.00505666]
b_of_wine_type_0: -17.25747015585557
support_vector_indices_of_wine_type_0: [  4  36  99 124]
w_of_wine_type_1: [ 0.04419171  0.10853448  0.07231087 -0.07818079  0.01099053  0.29594697
  0.50169748  0.00176555  0.17125891 -0.12386278  0.04927243  0.38871159
  0.00172167]
b_of_wine_type_1: -4.16716409877467
support_vector_indices_of_wine_type_1: [ 26  34  35  39  71 125]
w_of_wine_type_2: [-2.29263446e-01 -3.43621886e-01 -9.27411858e-02  1.00041193e-01
 -4.92036571e-02  2.99912226e-01  1.33760006e+00  1.48576076e-01
  6.21064338e-01 -1.09194327e+00  1.85563437e-01  5.10299052e-01
 -3.36868279e-04]
b_of_wine_type_2: 7.980927081155945
support_vector_indices_of_wine_type_2: [ 12  21  27  58  74  93 102 112]
```

(b)

1. Optimization Problem

(1) $\min_{w,b,\xi} \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{m}\xi_i \quad \text{s.t.} \quad 1 - \xi_i - y_i(w^T x_i + b) \leq 0,\quad \forall i \quad -\xi_i \leq 0 \quad \forall i$

(2) Lagrange function: $\mathcal{L}(w,b,\alpha) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{m}\xi_i + \sum_{i=1}^{m}[\alpha_i(1 - \xi_i - y_i(w^T x_i + b)) + \mu_i(-\xi_i)]$ where $\alpha_i, \mu_i \geq 0,\quad \forall i$

(3) stationarity: $\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = 0 \Rightarrow \mathbf{w} = \sum_{i}^{m} \alpha_i y_i \mathbf{x}_i$, $\frac{\partial \mathcal{L}}{\partial b} = 0 \Rightarrow \sum_{i}^{m} \alpha_i y_i = 0$,

$\frac{\partial \mathcal{L}}{\partial \xi_i} = 0 \Rightarrow \alpha_i = C - \mu_i \quad \forall i$

(4) feasibility: $\alpha_i \geq 0,\ 1 - \xi_i - y_i\left(\mathbf{w}^\top \mathbf{x}_i + b\right) \leq 0,\ \xi_i \geq 0,\ \mu_i \geq 0,\ \forall i$

(5) complementary slackness: $\alpha_i \left(1 - \xi_i - y_i \left(\mathbf{w}^\top \mathbf{x}_i + b\right)\right) = 0, \ \mu_i \xi_i = 0, \ \forall i$

(6) by above equations, we get: $\max_{\alpha} \ \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i,j}^{m} \alpha_i \alpha_j y_i y_j x_i^T x_j \quad \text{s.t.} \quad \sum_{i=1}^{m} \alpha_i y_i = 0$

$0 \le \alpha_i \le C, \quad \forall i$

(7) w and b: $w = \sum_{i}^{m} \alpha_i y_i \mathbf{x}_i$, $b = \frac{1}{|\mathcal{M}|} \sum_{j \in \mathcal{M}} \left(y_j - \sum_{i}^{m} \alpha_i y_i \mathbf{x}_i^\top \mathbf{x}_j \right)$ where $\mathcal{M} = \{i \mid 0 < \alpha_i < C\}$

2. result

if C = 0.1

training_error: 0.024193548387096753

testing_error: 0.03703703703703709

w_of_wine_type_0: [0.31780258 0.46333307 0.19600666 −0.1289372 −0.0078446

4 0.09288631

  0.11118361 0.01299479 −0.08894368 0.36526926 −0.07445056 0.17801977

  0.00512369]

b_of_wine_type_0: −8.702745624377638

support_vector_indices_of_wine_type_0: [18   43   68 106 116 117]

number_of_slack_variable_of_wine_type_0: 39

w_of_wine_type_1: [−0.00708274 0.07949818 0.07843626 −0.04683579 0.0035180

2 0.20616001

  0.37579988 −0.01044131 0.12928718 −0.11749999 0.05679649 0.32644161

  0.00302703]

b_of_wine_type_1: −3.5746931208668187

support_vector_indices_of_wine_type_1: [8 11 13 16 17 21 47 53 72 104 107

 114]

number_of_slack_variable_of_wine_type_1: 77

w_of_wine_type_2: [−0.08487372 −0.29295446 −0.10102865 −0.03953776 −0.0025316

3 0.07631926

  0.45465227 −0.03370602 0.12052404 −0.60369919 0.0893062   0.22714005

 −0.00146847]

b_of_wine_type_2: 5.1633090824552506

support_vector_indices_of_wine_type_2: [3   9 50 56 75 84 94 112]

number_of_slack_variable_of_wine_type_2: 50

if C = 0.2

training_error: 0.016129032258064502

testing_error: 0.03703703703703709

w_of_wine_type_0: [0.3119302   0.49548314 0.23396967 −0.132782   −0.00749298

  0.12103827

  0.15741617 0.03194166 −0.10723996 0.35452434 −0.09896029 0.14769604

  0.00517536]

b_of_wine_type_0: −8.775883291890398

support_vector_indices_of_wine_type_0: [18 43 68 106 117]

number_of_slack_variable_of_wine_type_0: 35

w_of_wine_type_1: [1.68234971e-02   1.18946416e-01   7.91512469e-02 -7.37104833e-02

  1.24840480e-02   2.87091806e-01   4.90470774e-01   1.94710974e-04

  1.61058517e-01 -7.36290129e-02   5.47327016e-02   3.97290762e-01

  1.73546784e-03]

b_of_wine_type_1: -4.301830989772964

support_vector_indices_of_wine_type_1: [8   11   16   17   21   47   53   72 107 114]

number_of_slack_variable_of_wine_type_1: 79

w_of_wine_type_2: [-0.12301814 -0.23566756 -0.08240382 -0.04330842 -0.01206244   0.08105994

  0.5276736   0.00276846   0.16579102 -0.8305954   0.12611743   0.25133672

  -0.00108722]

b_of_wine_type_2: 6.880851721089387

support_vector_indices_of_wine_type_2: [2   3   9   40   56   75   94 112]

number_of_slack_variable_of_wine_type_2: 44

if C = 0.3

training_error: 0.016129032258064502

testing_error: 0.01851851851851849

w_of_wine_type_0: [0.30804032   0.57205641   0.30013497 −0.14638295 −0.0043861

7   0.18116906

  0.229077   0.04006673 −0.1794601   0.35053322 −0.11042658 0.16872574

  0.00494689]

b_of_wine_type_0: −9.128417661087653

support_vector_indices_of_wine_type_0: [18   68 106]

number_of_slack_variable_of_wine_type_0: 31

w_of_wine_type_1: [1.68391388e−02   1.19689548e−01   7.94370884e−02 −7.65770600

e−02

  1.23664508e−02   2.89055319e−01   4.94061488e−01   1.10894479e−04

  1.62174883e−01 −7.39410261e−02   5.49859618e−02   3.99967081e−01

  1.76252096e−03]

b_of_wine_type_1: −4.265220743562575

support_vector_indices_of_wine_type_1: [8   11   16   17   21   47   53 107 114]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [−1.53328135e−01 −3.22761792e−01 −9.85858832e−02   1.14183799

e−02

 −2.61691430e−02   1.66490897e−01   6.08977353e−01   2.34195390e−02

  2.55872518e−01 −9.88409084e−01   1.92685568e−01   3.18248826e−01

−5.95473860e−04]

b_of_wine_type_2: 7.691454337946711

support_vector_indices_of_wine_type_2: [2   3   9 40 56 75 94 112]

number_of_slack_variable_of_wine_type_2: 39

if C = 0.4

training_error: 0.016129032258064502

testing_error: 0.01851851851851849

w_of_wine_type_0: [3.57243162e−01   5.90207037e−01   3.78414022e−01 −1.47537202

e−01

   2.86183880e−04   1.61204548e−01   2.13493638e−01   7.03854009e−02

 −2.16449878e−01   4.27347775e−01 −1.27002291e−01   1.47309516e−01

   4.96123975e−03]

b_of_wine_type_0: −10.612331642429297

support_vector_indices_of_wine_type_0: [18   68 106]

number_of_slack_variable_of_wine_type_0: 35

w_of_wine_type_1: [1.68391388e−02   1.19689548e−01   7.94370884e−02 −7.65770600

e−02

   1.23664508e−02   2.89055319e−01   4.94061488e−01   1.10894479e−04

   1.62174883e−01 −7.39410261e−02   5.49859618e−02   3.99967081e−01

1.76252096e−03]

b_of_wine_type_1: −4.265220743562575

support_vector_indices_of_wine_type_1: [8 11 16 17 21 53 107 114]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [−1.83230978e−01 −3.07635191e−01 −9.10515665e−02 3.45624309e−02

 −3.02377121e−02 3.15785891e−01 6.89670275e−01 5.38043742e−02

  3.43727917e−01 −9.98964129e−01 2.15646182e−01 3.86882362e−01

 −4.75519122e−04]

b_of_wine_type_2: 7.360435194794495

support_vector_indices_of_wine_type_2: [2   3   9 40 56 75 94 112]

number_of_slack_variable_of_wine_type_2: 31

if C = 0.5

training_error: 0.008064516129032251

testing_error: 0.01851851851851849

w_of_wine_type_0: [0.38278408 0.62030645 0.46656202 −0.14868433 0.00225128 0.13063458

  0.19177645 0.09856444 −0.23396638 0.4574861 −0.14199567 0.13526616

  0.00520143]

b_of_wine_type_0: −11.542658809249613

support_vector_indices_of_wine_type_0: [18   41   68 106]

number_of_slack_variable_of_wine_type_0: 35

w_of_wine_type_1: [1.68391388e−02   1.19689548e−01   7.94370884e−02 −7.65770600

e−02

  1.23664508e−02   2.89055319e−01   4.94061488e−01   1.10894479e−04

  1.62174883e−01 −7.39410261e−02   5.49859618e−02   3.99967081e−01

  1.76252096e−03]

b_of_wine_type_1: −4.265220743562575

support_vector_indices_of_wine_type_1: [8   11   16   17   21   53 107 114]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [−1.89339427e−01 −2.96004132e−01 −4.50001086e−02   4.2706180

8e−02

 −3.26511816e−02   3.94737009e−01   7.30508774e−01   8.57030611e−02

  4.15286987e−01 −1.00662249e+00   2.38447362e−01   4.29168880e−01

 −3.98897992e−04]

b_of_wine_type_2: 7.007567364295701

support_vector_indices_of_wine_type_2: [2   3   9   40   56   75   84   94 112]

number_of_slack_variable_of_wine_type_2: 28

if C = 0.6

training_error: 0.008064516129032251

testing_error: 0.01851851851851849

w_of_wine_type_0: [0.39261044 0.61265853 0.54233229 −0.15029922 0.0027472
6   0.11261475

  0.20802282 0.1155824 −0.23793632 0.45174522 −0.14372429 0.14178792

  0.00509501]

b_of_wine_type_0: −11.751631013385984

support_vector_indices_of_wine_type_0: [18 41 68 81 106]

number_of_slack_variable_of_wine_type_0: 36

w_of_wine_type_1: [1.68391388e−02 1.19689548e−01 7.94370884e−02 −7.65770600
e−02

  1.23664508e−02 2.89055319e−01 4.94061488e−01 1.10894479e−04

  1.62174883e−01 −7.39410261e−02 5.49859618e−02 3.99967081e−01

  1.76252096e−03]

b_of_wine_type_1: −4.265220743562575

support_vector_indices_of_wine_type_1: [8 16 17 21 53 107]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [−1.90597104e−01 −3.09128723e−01 −3.38791777e−02 5.0233284

9e−02

 −3.72816006e−02 4.16802014e−01 7.57652325e−01 1.10192991e−01

  4.87648647e−01 −1.06020346e+00 2.14084397e−01 4.79767643e−01

 −4.30213548e−04]

b_of_wine_type_2: 7.3731075335810425

support_vector_indices_of_wine_type_2: [2   3   9   40   56   75   84   94 112]

number_of_slack_variable_of_wine_type_2: 28

if C = 0.7

training_error: 0.008064516129032251

testing_error: 0.01851851851851849

w_of_wine_type_0: [0.40203582   0.61221937   0.60831136 −0.1513033   0.00279686

  0.09204772

  0.23661761   0.1314835   −0.23779759   0.43946813 −0.14178896   0.13278732

  0.00506569]

b_of_wine_type_0: −11.93794155147734

support_vector_indices_of_wine_type_0: [18   68   81 106]

number_of_slack_variable_of_wine_type_0: 36

w_of_wine_type_1: [1.68391388e-02 1.19689548e-01 7.94370884e-02 -7.65770600e-02

  1.23664508e-02 2.89055319e-01 4.94061488e-01 1.10894479e-04

  1.62174883e-01 -7.39410261e-02 5.49859618e-02 3.99967081e-01

  1.76252096e-03]

b_of_wine_type_1: -4.265220743562575

support_vector_indices_of_wine_type_1: [8 16 17 21 53 107]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [-2.01333163e-01 -3.15028511e-01 -4.06264965e-02 5.90225555e-02

 -3.96948625e-02 3.87101929e-01 8.76827418e-01 1.19242537e-01

  5.09756241e-01 -1.06295386e+00 2.06314328e-01 4.98747351e-01

 -3.86878377e-04]

b_of_wine_type_2: 7.505439090492528

support_vector_indices_of_wine_type_2: [2   3   9 40 56 75 84 94 112]

number_of_slack_variable_of_wine_type_2: 23

if C = 0.8

training_error: 0.008064516129032251

testing_error: 0.01851851851851849

w_of_wine_type_0: [0.40621999 0.61428146 0.68260029 −0.15261375 0.00279309

  0.06902623

  0.25096099 0.14881178 −0.2374554   0.42790529 −0.14315415 0.13559532

  0.00507091]

b_of_wine_type_0: −12.090285059492206

support_vector_indices_of_wine_type_0: [18 68 81 106]

number_of_slack_variable_of_wine_type_0: 35

w_of_wine_type_1: [1.68391388e−02 1.19689548e−01 7.94370884e−02 −7.65770600

e−02

  1.23664508e−02 2.89055319e−01 4.94061488e−01 1.10894479e−04

  1.62174883e−01 −7.39410261e−02 5.49859618e−02 3.99967081e−01

  1.76252096e−03]

b_of_wine_type_1: −4.265220743562575

support_vector_indices_of_wine_type_1: [8 16 17 21 53 107]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [−2.12281776e−01 −3.20423098e−01 −4.71610010e−02 6.75105337

e−02

 −4.19412817e−02 3.56576414e−01 9.96353283e−01 1.28281404e−01

  5.29876082e−01 −1.06389942e+00 1.98191168e−01 5.16004035e−01

 −3.46170365e−04]

b_of_wine_type_2: 7.628838304595769

support_vector_indices_of_wine_type_2: [2 3 9 40 56 75 84 94 112]

number_of_slack_variable_of_wine_type_2: 21

if C = 0.9

training_error: 0.008064516129032251

testing_error: 0.01851851851851849

w_of_wine_type_0: [0.41034883 0.61641119 0.75687163 −0.15392003 0.0027856

 0.0460054

  0.26530976 0.16613221 −0.23713964 0.41620586 −0.14450432 0.13838364

  0.0050763]

b_of_wine_type_0: −12.241209595229506

support_vector_indices_of_wine_type_0: [18 68 81 106]

number_of_slack_variable_of_wine_type_0: 35

w_of_wine_type_1: [1.68391388e−02 1.19689548e−01 7.94370884e−02 −7.65770600

e−02

  1.23664508e−02 2.89055319e−01 4.94061488e−01 1.10894479e−04

  1.62174883e−01 −7.39410261e−02 5.49859618e−02 3.99967081e−01

  1.76252096e−03]

b_of_wine_type_1: −4.265220743562575

support_vector_indices_of_wine_type_1: [8 16 17 21 53 107]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [−2.23041106e−01 −3.25886636e−01 −5.38290106e−02 7.6047460

5e−02

 −4.42138178e−02 3.26061261e−01 1.11590454e+00 1.37289411e−01

  5.50180761e−01 −1.06509542e+00 1.90107340e−01 5.33021050e−01

 −3.05593092e−04]

b_of_wine_type_2: 7.7533370130956145

support_vector_indices_of_wine_type_2: [2   3   9 40 56 75 84 94 112]

number_of_slack_variable_of_wine_type_2: 18

if C = 1.0

training_error: 0.008064516129032251

testing_error: 0.01851851851851849

w_of_wine_type_0: [0.41447743 0.61854224 0.83114279 −0.15522428 0.00277819

 0.02298327

  0.2796578   0.18345294 −0.23682282 0.40450718 −0.1458546   0.14117062

  0.00508172]

b_of_wine_type_0: −12.392211420443639

support_vector_indices_of_wine_type_0: [18 68 81 106]

number_of_slack_variable_of_wine_type_0: 34

w_of_wine_type_1: [1.68391388e−02 1.19689548e−01 7.94370884e−02 −7.65770600e−02

  1.23664508e−02 2.89055319e−01 4.94061488e−01 1.10894479e−04

  1.62174883e−01 −7.39410261e−02 5.49859618e−02 3.99967081e−01

  1.76252096e−03]

b_of_wine_type_1: −4.265220743562575

support_vector_indices_of_wine_type_1: [8 16 17 21 53 107]

number_of_slack_variable_of_wine_type_1: 78

w_of_wine_type_2: [−2.27262027e−01 −3.35530999e−01 −7.45347814e−02 8.92078289e−02

 −4.69926089e−02 3.09840491e−01 1.24112524e+00 1.43924882e−01

  5.88702441e−01 −1.07904077e+00 1.87032479e−01 5.22874883e−01
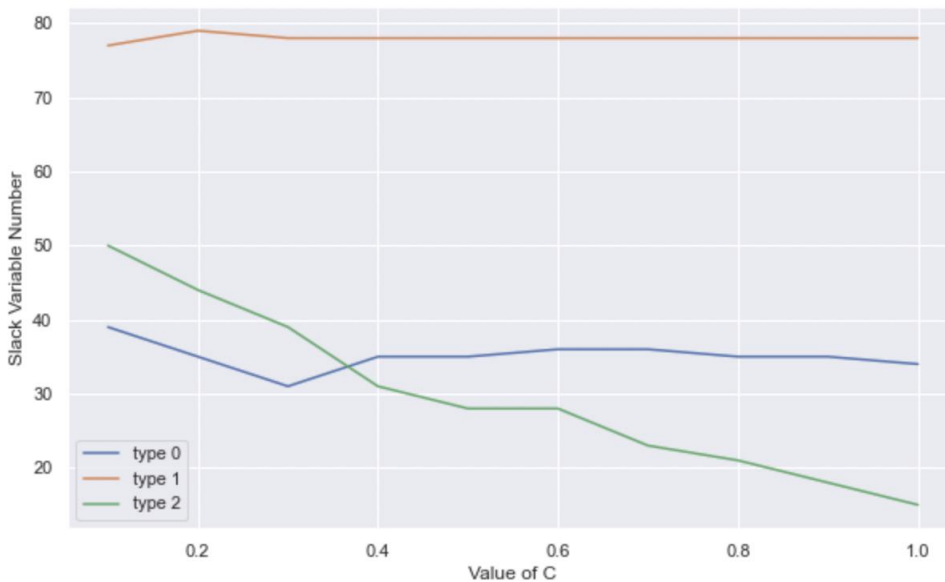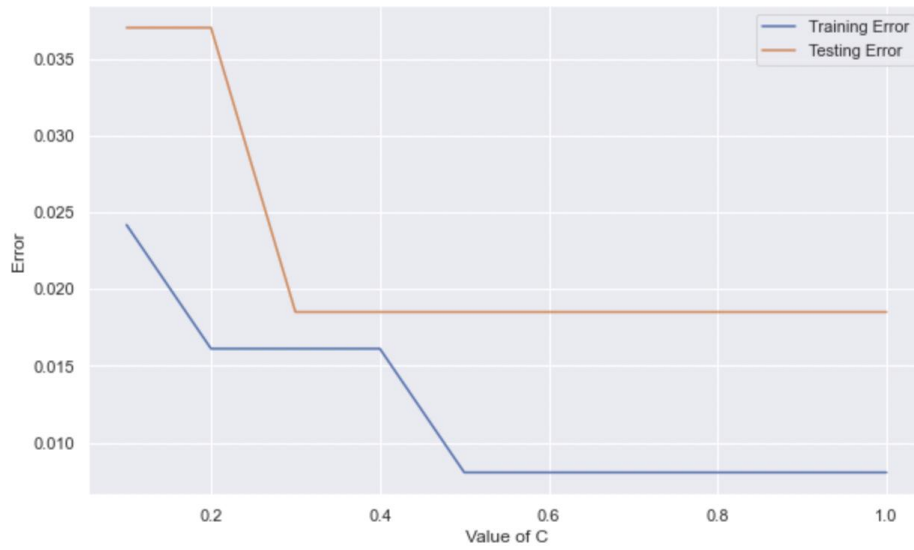
 −3.18311321e−04]

b_of_wine_type_2: 7.882122412125398

support_vector_indices_of_wine_type_2: [2 3 9 40 56 75 84 94]

number_of_slack_variable_of_wine_type_2: 15

3. impact of parameter settings

Hyperparameter C acts as the penalty intensity to the misclassified data. It can also be considered as the trade-off between minimizing training error and reducing the complexity of the decision surface. With larger C, the model becomes more complex and may overfit the training data, leading to lower train error but potentially higher test error. Conversely, if C is too small, the model risks underfitting the data and exhibiting high train and test errors.

The number of slack variables corresponds to the number of training examples that are either misclassified or within the margins. As the value of the hyperparameter C increases, so does the penalty imposed on non-zero slack variables. This can result in a decrease in the number of slack variables. However, this relationship is not that obvious in this illustration, which may result from the range of C is rather narrow.

(c)

1. Optimization Problem

$$\max_{\alpha} \quad \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i,j}^{m} \alpha_i \alpha_j y_i y_j k(x_i, x_j)$$

$$\text{s.t.} \quad \sum_{i=1}^{m} \alpha_i y_i = 0$$

$$0 \le \alpha_i \le C, \quad \forall i$$

The solution of b is

$$b = \frac{1}{|\mathcal{M}|} \sum_{j \in \mathcal{M}} \left(y_j - \sum_{i}^{m} \alpha_i y_i k(x_i, x_j) \right) \quad \text{where } \mathcal{M} = \{i \mid 0 < \alpha_i < C\}$$

$$k(x_i, x_j) = \phi(x_i)^{\top} \phi(x_j)$$

The prediction of new data x is

$$w^T x + b = \sum_{i}^{m} a_i y_i k(x_i, x) + b$$

(1.1)
$$k(x, x_i) = (1 + x^T x_i)^2$$

(1.2)result

```
training_error: 0.32258064516129037
testing_error: 0.2777777777777778
b_of_wine_type_0: -2.6161813228231923
support_vector_indices_of_wine_type_0: [ 18  41  43  68  87  93 106 110 116 117 122]
b_of_wine_type_1: -2.7733381875345637
support_vector_indices_of_wine_type_1: [  8  10  13  15  16  17  20  21  24  26  31  35  38
 45  47  51  53  54
  55  62  66  67  72  74  91  99 100 104 105 107 113 114 115 119 120]
b_of_wine_type_2: 1.4900266788899899
support_vector_indices_of_wine_type_2: [  2   3   5   6   9  12  22  23  27  28  30  40  42
 50  56  58  59  63
  71  75  78  79  82  84  90  92  94  95  96 102 103 108 109 112 123]
```

(2.1)

$$k(x, x_i) = (1 + x^T x_i)^3$$

(2.2)result

```
training_error: 0.33870967741935487
testing_error: 0.2592592592592593
b_of_wine_type_0: -1.976997063086833
support_vector_indices_of_wine_type_0: [ 18  43  68  87 106 110 116 117 122]
b_of_wine_type_1: -2.0295836011072295
support_vector_indices_of_wine_type_1: [  8  10  13  15  16  17  20  21  24  26  31  35  38
 45  47  51  53  54
  55  62  66  67  72  74  91  99 100 104 105 107 113 114 115 119 120]
b_of_wine_type_2: 1.2131788283586504
support_vector_indices_of_wine_type_2: [  2   3   5   6   9  12  22  23  27  28  30  40  42
 50  56  58  59  63
  71  75  78  79  82  84  90  92  94  95  96 102 103 108 109 112 123]
```

(3.1) $k\left(x, x_i\right) = e^{-\frac{1}{2}\|x - x_i\|^2}$

(3.2)result

```
training_error: 0.0
testing_error: 0.5925925925925926
b_of_wine_type_0: -0.18367346934656772
support_vector_indices_of_wine_type_0: [  0   7  14  18  19  29  36  37  39  41  43  44  46
 49  52  57  60  61
  64  65  68  69  70  73  76  80  81  83  86  87  89  93  97 106 110 111
 116 117 118 122]
b_of_wine_type_1: 0.1249927141085359
support_vector_indices_of_wine_type_1: [  1   4   8  10  11  13  15  16  17  20  21  24  25
 26  31  32  33  34
  35  38  45  47  48  51  53  54  55  62  66  67  72  74  77  85  88  91
  98  99 100 101 104 105 107 113 114 115 119 120 121]
b_of_wine_type_2: 0.28571428567827056
support_vector_indices_of_wine_type_2: [  2   3   5   6   9  12  22  23  27  28  30  40  42
 50  56  58  59  63
  71  75  78  79  82  84  90  92  94  95  96 102 103 108 109 112 123]
```

(4.1) $k\left(x, x_i\right) = \dfrac{1}{1 + e^{-x^T x_i - b}}$

(4.2)result

```
training_error: 0.6048387096774194
testing_error: 0.5925925925925926
b_of_wine_type_0: -1.0
support_vector_indices_of_wine_type_0: [  0   7  14  18  19  29  36  37  39  41  43  44  46
 49  52  57  60  61
  64  65  68  69  70  73  76  80  81  83  86  87  89  93  97 106 110 111
 116 117 118 122]
b_of_wine_type_1: 1.0
support_vector_indices_of_wine_type_1: [ 20  21  24  25  26  31  32  33  34  35  38  45  47
 48  51  53  54  55
  62  66  67  72  74  77  85  88  91  98  99 100 101 104 105 107 113 114
 115 119 120 121]
b_of_wine_type_2: 1.0
support_vector_indices_of_wine_type_2: [  2   3   5   6   9  12  22  23  27  28  30  40  42
 50  56  58  59  63
  71  75  78  79  82  84  90  92  94  95  96 102 103 108 109 112 123]
```