1、a) min $12y_1+8y_2+10y_3$

s.t. $2y_1+y_2 \geq 3$

$y_1+y_2-y_3 \geq 4$

$-y_1+y_2+2y_3 \geq 3$

$y_1+y_2+y_3 \geq 6$

$y_1 \leq 0$ $\quad y_2$ free $\quad y_3 \geq 0$

The optimal sol. of primal problem is $(4,0,0,4)$

$$\left.\begin{array}{l} 4(2y_1+y_2-3)=0 \Rightarrow 2y_1+y_2=3 \\ 0(y_1+y_2-y_3-4)=0 \\ 0(-y_1+y_2+2y_3-3)=0 \\ 4(y_1+y_2+y_3-6)=0 \Rightarrow y_1+y_2+y_3=6 \\ y_1(2\times4+0-0+4-12)=0 \\ y_2(4+0+0+4-8)=0 \\ y_3(-0+2\times0+4-10)=0 \Rightarrow y_3=0 \end{array}\right\} \Rightarrow \begin{cases} y_1=-3 \\ y_2=9 \\ y_3=0 \end{cases}$$

$\therefore y^* = (-3,9,0)^T$

Since the primal sol. is unique, the complementarity conditions make the dual problem have unique solution

b) ① change on C, check the reduced cost still be $\geq 0 \Rightarrow \tilde{c}_N^T - \tilde{c}^T A_B^{-1}A_N \geq 0$

$A_B^{-1}A_N = \begin{pmatrix} 0 & -2 & -1 \\ 1 & 3 & 1 \\ -2 & -1 & -1 \end{pmatrix}$ $\quad \tilde{c}^T = (-3,-4,-1,-6,0,0)$

Compute $\tilde{c}_N^T - \tilde{c}_B^T A_B A_N = (-4,-1,0)-(-3,-6,0)\begin{pmatrix} 0 & -2 & -1 \\ 1 & 3 & 1 \\ -2 & -1 & -1 \end{pmatrix} = (2,11,3) \geq 0$

The optimal solution and value will not change

② $\tilde{c}^T = (-3,-4,-12,-6,0,0)$

$\tilde{c}_N^T - \tilde{c}_B^T A_B^{-1} A_N = (-4,-12,0)-(-3,-6,0)\begin{pmatrix} 0 & -2 & -1 \\ 1 & 3 & 1 \\ -2 & -1 & -1 \end{pmatrix} = (2,0,3) \geq 0$

The optimal solution and value will not change

③ $\tilde{c}^T = (-1,-4,-3,-6,0,0)$

$\tilde{c}_N^T - \tilde{c}_B^T A_B^{-1} A_N = (-4,-3,0)-(-1,-6,0)\begin{pmatrix} 0 & -2 & -1 \\ 1 & 3 & 1 \\ -2 & -1 & -1 \end{pmatrix} = (2,13,5) \geq 0$

The optimal solution will not change, the optimal value will change to $4\times1+4\times6=28$

④ $\tilde{c}^T = (-7,-4,-3,-6,0,0)$

$\tilde{c}_N^T - \tilde{c}_B^T A_B^{-1} A_N = (-4,-3,0)-(-7,-6,0)\begin{pmatrix} 0 & -2 & -1 \\ 1 & 3 & 1 \\ -2 & -1 & -1 \end{pmatrix} = (2,1,-1)$ it doesn't subject to every element ≥ 0

The optimal sol. and value will change.

c) $x^* + \lambda A_B^{-1} e_3 \geq 0$

$\begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4-\lambda \\ 4+2\lambda \\ 6-2\lambda \end{pmatrix} \geq 0$ $\quad \therefore -2 \leq \lambda \leq 3$

The coefficient for the second constraint can be $[6,11]$

2. (a) $r_N^T = (1, \frac{7}{2}, \frac{1}{2})$ $\quad A_B^{-1}A_N = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -1 & -2 & -1 \\ 1 & -1 & -1 \end{pmatrix}$

$r_N^T + \lambda e_i = (1, \frac{7}{2}, \frac{1}{2}) + \lambda(1,0,0) \geq 0 \quad \Rightarrow \quad \lambda \geq -1$

The change $\Delta c_1 \geq -1$

c_1 from 1 to 100 $\quad \lambda = 99 > -1$

$$\tilde{c}_N^T - \tilde{c}_B^T A_B^{-1} A_N = (100, 2, 0) - (2,1,0)\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -1 & -2 & -1 \\ 1 & -1 & -1 \end{pmatrix} = (99, 1, 0) \geq 0$$

The optimal sol. and value doesn't change. $\begin{cases} \text{optimal sol: } (0, \frac{1}{2}, 0, 1, 3, 0) \\ \text{optimal value } 1.5 \end{cases}$

(b) let $r_N^T - \lambda e_j^T A_B^{-1} A_N \geq 0$

$$(1, \frac{7}{2}, \frac{1}{2}) - \lambda(1,0,0)\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ -1 & -2 & -1 \\ 1 & -1 & -1 \end{pmatrix} = (1-\lambda, \frac{7}{2} - 2\lambda, \frac{1}{2} - \frac{1}{2}\lambda) \geq 0$$

$\lambda \leq 1 \quad \Delta c_2 : (-\infty, 1] \qquad c_2$ range : $(-\infty, 2]$

(c) change on b_3.

$$\begin{pmatrix} \frac{1}{2} \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} - \frac{1}{2}\lambda \\ 1 + \lambda \\ 3 + \lambda \end{pmatrix} \geq 0 \qquad -1 \leq \lambda \leq 1 \qquad \Delta b_3 : [-1, 1]$$

b_3 range : $[0, 2]$

(d) b_3 from 1 to $\frac{3}{2}$, still in the range, since $b_3 \in [0, 2]$

The optimal value are the same $\Delta b = (0, 0, \frac{1}{2})$ $\Delta V = \Delta b^T y^*$

$$\tilde{x}_B = \frac{1}{2} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & 2 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 3/2 \end{pmatrix} = \begin{pmatrix} 0.25 \\ 1.5 \\ 3.5 \end{pmatrix}$$

new optimal sol. : $(0, 0.25, 0, 1.5)$

$$c_B^T A_B^{-1} = (1, 1, 0)\begin{pmatrix} 1 & 0 & -1 \\ 0 & 0 & 2 \\ 0 & -2 & -2 \end{pmatrix} \cdot \frac{1}{2}$$

$$= (0.5, 0, 0.5)$$

new dual sol $(0.5, 0, 0.5)^T$