1. reformulate this LP as standard form

$$\min \ -500x_1 - 250x_2 - 600x_3$$
$$\text{s.t.} \quad 2x_1 + x_2 + x_3 + s_1 = 240$$
$$3x_1 + x_2 + 2x_3 + s_2 = 150$$
$$x_1 + 2x_2 + 4x_3 + s_3 = 180$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

B	-500	-250	-600	0	0	0	0
4	2	1	1	1	0	0	240
5	$\boxed{3}$	1	2	0	1	0	150
6	1	2	4	0	0	1	180

B	0	-250/3	-800/3	0	500/3	0	25000
4	0	1/3	-1/3	1	-2/3	0	140
1	1	1/3	2/3	0	1/3	0	50
6	0	$\boxed{5/3}$	10/3	0	-1/3	1	130

B	0	0	-100	0	150	50	31500
4	0	0	-1	1	-3/5	-1/5	114
1	1	0	0	0	2/5	-1/5	24
2	0	1	$\boxed{2}$	0	-1/5	3/5	78

B	0	0	0	0	140	80	35400
4	0	1/2	0	1	-7/10	1/10	153
1	1	0	0	0	2/5	-1/5	24
3	0	1/2	1	0	-1/10	3/10	39

Step	Basic set	Basic Solution	objective value
1	{4, 5, 6}	{0, 0, 0, 240, 150, 180}	0
2	{4, 1, 6}	{50, 0, 0, 140, 0, 130}	-25000
3	{4, 1, 2}	{24, 78, 0, 114, 0, 0}	-31500
4	{4, 1, 3}	{24, 0, 39, 153, 0, 0}	-35400

The original LP: the optimal solution (24, 0, 39) optimal value 35400

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B	-2	-3	1	12	0	0	0
5	-2	-9	1	9	1	0	0
6	$\boxed{1/3}$	1	-1/3	-2	0	1	0

B	0	3	-1	0	0	6	0
5	0	-3	-1	-3	1	6	0
1	1	3	-1	-6	0	3	0

All the elements of this vector $\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$ are negative.

The problem is unbounded.

3、Find initial BFS:

$$\min \quad x_6+x_7+x_8$$

$$\text{s.t. } \begin{aligned} x_1+3x_2 \quad +4x_4+x_5+x_6 \quad &=2 \\ x_1+2x_2 \quad -3x_4+x_5 \quad +x_7 \quad &=2 \\ -x_1-4x_2+3x_3 \quad +x_8 &=1 \end{aligned}$$

B									
	-1	-1	-3	-1	-2	0	0	0	-5
6	$\boxed{1}$	3	0	4	1	1	0	0	2
7	1	2	0	-3	1	0	1	0	2
8	-1	-4	3	0	0	0	0	1	1

B									
	0	2	-3	3	-1	1	0	0	-3
1	1	3	0	4	1	1	0	0	2
7	0	-1	0	-7	0	-1	1	0	0
8	0	-1	$\boxed{3}$	4	1	1	0	1	3

B									
	0	1	0	7	0	2	0	1	0
1	1	3	0	4	1	1	0	0	2
7	0	-1	0	-7	0	-1	1	0	0
3	0	-1/3	1	4/3	1/3	1/3	0	1/3	1

The optimal solution is $(2,0,1,0,0,0,0,0)$

The initial BFS is $(2,0,1,0,0)$

We notice degeneracy happens, Let basis be $\{1,2,3\}$

B						
	0	0	0	3	-5	-7
1	1	0	0	-17	$\boxed{1}$	2
2	0	1	0	7	0	0
3	0	0	1	11/3	1/3	1

B						
	5	0	0	-82	0	3
5	1	0	0	-17	1	2
2	0	1	0	$\boxed{7}$	0	0
3	-1/3	0	1	28/3	0	1/3

B						
	5	82/7	0	0	0	3
5	1	17/7	0	0	1	2
4	0	1/7	0	1	0	0
3	-1/3	-4/3	1	0	0	1/3

optimal solution is $(0,0,\frac{1}{3},0,2)$

optimal value is -3

4. ① $\begin{cases} \delta < 0 \\ \alpha \le 0 \\ \gamma \le 0 \\ \beta \geqslant 0 \end{cases}$

② $\beta > 0$

The current solution is feasible $\Rightarrow \beta \geqslant 0$,
but not optimal $\Rightarrow$ the solution can move along $x_2 \Rightarrow \beta > 0$

③ $\begin{cases} \beta = 0 \\ \delta \geqslant -\frac{2}{3}\gamma \end{cases}$

B	δ	-2	0	0	0	-10
3	-1	η	1	0	0	4
4	α	-4	0	1	0	1
5	γ	3	0	0	1	0

$\Rightarrow$

B	$\delta+\frac{2\gamma}{3}$	0	0	0	$2/3$	-10
3	$-1-\frac{\eta\gamma}{3}$	0	1	0	$-\eta/3$	4
4	$\alpha+\frac{4\gamma}{3}$	0	0	1	$4/3$	1
5	$\gamma/3$	1	0	0	$1/3$	0

Let $\delta+\frac{2\gamma}{3} \geqslant 0$

5 ① $\beta \geqslant 0$

② $\begin{cases} \beta < 0 \\ \alpha \geqslant 0 \end{cases}$

if $x_i \geqslant 0$, $a_i \geqslant 0$ for $i=1,2,\cdots,7 \Rightarrow \sum_{i=1}^{7} a_i x_i \geqslant 0$
$x_2 + \alpha x_4 + x_5 + 3x_7 = \beta$, $\alpha \geqslant 0 \Rightarrow \beta \geqslant 0$, but $\beta < 0$, so it's infeasible.

③ $\begin{cases} \beta > 0 \\ \xi < 0 \end{cases}$ or $\begin{cases} \beta > 0 \\ \delta < 0 \end{cases}$ or $\begin{cases} \beta > 0 \\ \gamma < 0 \end{cases}$

④ $\begin{cases} \beta \geqslant 0 \\ \delta < 0 \\ \alpha \le 0 \end{cases}$

⑤ $\begin{cases} \beta \geqslant 0 \\ \eta > \frac{4}{3} \\ \gamma < 0 \end{cases}$

x_6 in, x_3 out $\Rightarrow \begin{cases} \gamma < 0 \\ \eta > 0 \\ \frac{2}{\eta} < \frac{3}{2} \end{cases} \Rightarrow \begin{cases} \gamma < 0 \\ \eta > \frac{4}{3} \end{cases}$

The basic solution is feasible $\Rightarrow \beta \geqslant 0$

$\Rightarrow \begin{cases} \beta \geqslant 0 \\ \eta > \frac{4}{3} \\ \gamma < 0 \end{cases}$