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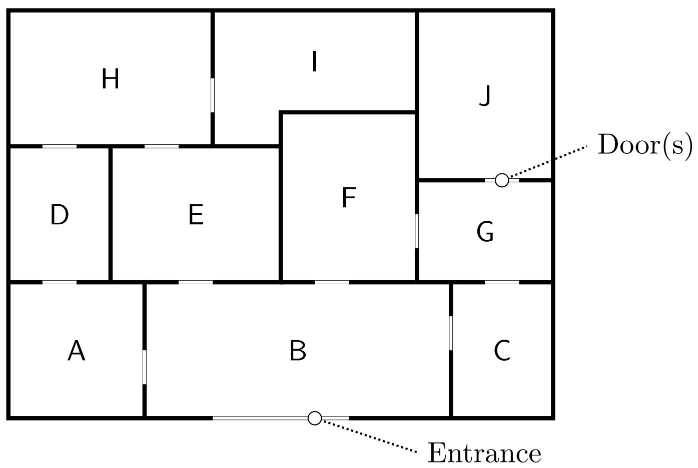
MAT 3007 — Optimization

Exercise Sheet 9

Problem 1 (Museum Guards): (approx. *50* pts)
A museum director must decide how many guards should be employed to control a newly opened wing of a museum. Budget cuts have forced the director to station guards at the doors between rooms — guarding two rooms at once. A map of the wing and different rooms (A—J) is shown below:



a) Formulate an integer optimization problem to minimize the total number of required guards (while each of the rooms is controlled by at least one guard).

   Describe and explain your optimization model briefly.

b) Implement and solve your model using `CVX`, `MATLAB`, or `Python`. How many museum guards are needed? What is the solution of the LP relaxation of your model?

Problem 2 (Branch-and-Bound Method): (approx. *50* pts)
Use the branch-and-bound method to solve the following integer program:

$$\begin{array}{lll} \text{maximize} & 17x + 12y & \\ \text{subject to} & 10x + 7y & \leq 40 \\ & x + y & \leq 5 \\ & x, y & \geq 0 \\ & x, y \in \mathbb{Z}. & \end{array}$$

Form the branch-and-bound tree and indicate the solution associated with each node (similar to the procedures introduced in the lecture). You can use an LP solver to solve the linear programming relaxation. Please include your calculations and/or code and the solution outputs in your answer.

Problem 3 (Multiple Knapsacks):
Suppose we have a set of n many items and a set of m different knapsacks. For each item i and knapsack j, the following information is given:

- The item i has value (preference) v_i.
- The weight of item i is a_i.
- The capacity of knapsack j is at most C_j.

a) Formulate an integer program to maximize the total value of items that can be packed in the different knapsack while adhering to the capacity constraint (i.e., the total weight of items in each bag j is not allowed to be larger than C_j).

Hint: You can introduce variables x_{ij} to denote whether item i is placed in knapsack j.

b) Consider the following list of items and bags:

Item	Laptop	T-Shirt	Swim. Trunks	Sunglasses	Apples	Opt. Book	Water
Value	2	1	3	2	1	4	2
Weight	2	0.5	0.5	0.1	0.5	1	1.5

Knapsack 1	Knapsack 2
$C_1 = 3$	$C_2 = 2$

Formulate the corresponding IP in that case. What are the optimal solutions to the IP and its LP relaxation (you can use `CVX`, `MATLAB` or `Python` to solve the problems)? Is there an integrality gap in this case?

Problem 4 (Manufacturing Company):
A manufacturing company plans to build new factories (variables x_1 and x_2) and warehouses (variables x_3 and x_4) in Shenzhen and/or Beijing. The company wants to solve the following binary integer program to determine the location and number of the potential factories and warehouses:

$$\begin{array}{lll} \text{maximize} & 9x_1 + 5x_2 + 6x_3 + 4x_4 & \\ \text{subject to} & 6x_1 + 3x_2 + 5x_3 + 2x_4 & = 10 \\ & x_3 + x_4 & \leq 1 \\ & x_3 - x_1 & \leq 0 \\ & x_4 - x_2 & \leq 0 \\ & x_1, x_2, x_3, x_4 & \in \{0, 1\}. \end{array}$$

a) Discuss and interpret the meaning of the constraints "$x_3 + x_4 \leq 1$", "$x_3 - x_1 \leq 0$", and "$x_4 - x_2 \leq 0$".

b) Use the branch-and-bound method to solve the integer problem. You are allowed to use an LP solver to solve each of the relaxed linear programs. Please specify the branch-and-bound tree and what you did at each node

Remark: Please note that only your solutions to problem 1 and 2 will be considered for grading. Problem 3 and 4 are listed in case you would like to have additional practice.

Sheet 9 is due on **May, 10th**. Submit your solutions before **May, 10th**, **12:00 pm (noon)**.