CS 515
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HW4:

Problem1:

Because the amount of max flow = capacity of minimum cut

For vertex a, b

a∈S ∪ S' ➔ a∈S or a∈ S'

b∈T ∩ T' ➔ b∈T or b∈T'

for a∈S, we have (a, b) ∈ (S, S')

for b∈S, we have (a, b) ∈ (T, T')

So, the edge is saturated in the maximum flow.

Depending on where a is, we have (b, a) ∈ (S, S') or (b, a) ∈ (T, T')

And it has no more flow because these cuts are minimal.

So, (S ∪ S', T ∩ T') is a minimum cut.

In same way, (S ∩ S', T ∪ T') also a minimum cut.

Problem2:

IDK

Problem3:

a.

Assume a directed graph $G = (V, E)$ with s, t, s is the source, t is the target and $s, t \in E$. For an edge in this graph, $(u, v) \in E$ has capacity $c(u, v)$, flow $f(u, v)$, and cost $k(u, v)$.

Because we need to use linear-programming formulation to find a feasible circulation whose total cost is as small as possible.

For the definition:

Minimize $\sum_{(u,v)\in E} k(u, v) * f(u, v)$.

Constraints:

$$f(u, v) \le c(u, v)$$
$$f(u, v) = -f(v, u)$$
$$\sum_{w\in V} f(u, w) = 0, \text{ for all } u \ne s, t$$
$$\sum_{w\in V} f(s, w) = \sum_{w\in V} f(w, t)$$
$$\sum_{i\in V} f(k, i) - \sum_{j\in V} f(j, k) = b_k, \ \forall k \in V$$
$$f(u, v) \ge 0, c(u, v) \ge 0$$

b.

Because of problem a, we can have this:

Minimize $\sum_{(u,v)\in E} k(u,v) * f(u,v)$ => $k^T * f$

Constraints:

$\sum_{i\in V} f(k,i) - \sum_{j\in V} f(j,k) = b_k, \ \forall k \in V$ $Af = b$

$\sum_{w\in V} f(u,w) = 0$, for all $u \neq s,t$

$\sum_{w\in V} f(s,w) = \sum_{w\in V} f(w,t)$ => $q \leq f \leq c$

$f(u,v) \leq c(u,v)$

$f(u,v) = -f(v,u)$

$f(u,v) \geq 0, c(u,v) \geq 0$ => $q = 0$

So, the dual of problem a is that:

Maximize $b^T * \mu + q^T * \beta$

Constraints:

$\mu\ free$

$\beta \leq 0$

$A^T \mu + \beta \leq k$

Because the capacity has no constraints on edges, and $q = 0$. So, we can get that:

Maximize $b^T * \mu$

s.t. $\mu\ free$

$A^T \mu \leq k$

Problem4:

IDK