1.

Proof:

If L is context-free language, then L^R is also a context-free language.

For context-free language L has grammar G as its grammar.

If G_1 = context free grammar, so G_1 creates L_1 (of language).

Form G_1 we can get a new grammar G' as follows:

For every production X $\rightarrow$ V of G_1, we add the production X $\rightarrow$ V^R in G'.

Where X is a variable and V is string of terminals and variables (non terminals)

Then

It is easy to see that a string W is generated by grammar G, if string W^R is generated by G'

Therefore, grammar G' generates language L^R, and the language L^R is context-free.

Therefore, context-free language is closed under reversal.

2.

Proof:

As L_1 and L_2 are context-free languages. The difference is $L_1 - L_2 = L_1 \cap \overline{L_2}$

Because L_1 is CFL, L2 is CFL, but L_2' need not be CFL because complementation is not closed.

Then intersection and complement are not closed for context-free language.

Therefore, context-free language is not closed under difference.

3.

Proof:

L1 = $\{a^n b^m,$: n = $2^m\}$.

If L1 is context-free language, then by pumping lemma,

For w ∈ L1, with |w| ≥ m can decomposed into w = uvxyz,

Such as |vy|≥1, |vxy|≤ m, and $w^i = uv^i xy^i z$ ∈ L1, for i=0,1,2,…

Pick $a^{2^m} b^m$, |w| ≥ m

case1:

v and y are all a's |vy|=K,

$w_0 = a^{2^m} b^m$ |w|≥m,

case 2:

v and y are all b's |vy|=K,

$w_0 = a^{2^m-k}b^m, \notin$L

case 3:

v and y are all contain a's, b's

w are out of order.

Case 4:

V is all a's, y is all b's, $|v|=k_1$, $|y|=k_2$, $2 \le k_1+k_2 \le m$,

W2 = $a^{2^m+k}b^{m+k}$,

Then, $2^m + 1 \le\ 2^m + k_1 \le\ 2^m + m,$

$$2^{m+k_2} = 2^m * 2^{k_2}, 1 \le k_1 \le \mathrm{m}, 1 \le k_2 \le \mathrm{m},$$

Therefore,

$$2^m + k_1 < 2^{m+k_2}$$

So, W2 = $a^{2^m+k}b^{m+k}\ \notin \mathrm{L}$.

All case lead to a contradiction, L is not a CFL.

4.

Proof:

L2= { $a^n b^n c^j$, : n ≤ j }.

If L2 is context-free language, then by pumping lemma,

For w ∈ L2, with |w| ≥ m can decomposed into w = uvxyz,

Like |vy|≥1, |vxy|≤ m, and $w^i = uv^ixy^iz$ ∈ L1, for i=0,1,2,…

Pick $a^m b^m c^{m+1}$, |w| ≥ m

Case 1:

v and y and all a's, |vy|=k.

$w_0 = a^{m-k}b^m c^{m+1} \notin \mathrm{L}$

Case 2:

v and y and all b's, |vy|=k.

$w_0 = a^m b^{m-k} c^{m+1} \notin \mathrm{L}$

Case 3:

V and y all is c's, |vy| = k,

$w_0 = a^m b^m c^{m+1-k} \notin \mathrm{L}$, since k ≥1

Case 4:

For either v or y both is a’s and b’s.

W are out of order.

Case 5:

For either v or y both contain b’s and c’s.

W are out of order.

Case 6:

V is all b’s, y is all b’s, |v|=k1, |y|= k2,

$w_2 = a^{m+k_1}b^{m+k_2}c^{m+1} \notin \mathrm{L}$, since k_1 ≥1, k_2 ≥1

Case 7:

V is b’s, y is c’s, |v|=k1, |y|= k2,

$w_2 = a^{m}b^{m+k_1}c^{m+1+k_2} \notin \mathrm{L}$, since k_1 ≥1, k_2 ≥1

All case lead to a contradiction, therefor L is not CFL.

5.

Proof:

L3 = { w: w ∈{a,b,c}* and na(w) < nb(w) < nc(w) }.

L ∩ $a^*b^*c^*$ = $\{a^n b^{2n} c^{3n}\}$

Then, wu just need prove L’ = $\{a^n b^{2n} c^{3n}\}$ is not a context-free language.

If $a^n b^{2n} c^{3n}$ is a CFL, L’ is an infinite language, then by the pumping lemma,

For w ∈ L’, with |w| ≥ m can decomposed into w = uvxyz,

Like |vy|≥1, |vxy|≤ m, and $w^i = uv^i xy^i z$ ∈ L1, for i=0,1,2,…

Pick $a^n b^{2n} c^{3n}$, |w|≥ m,

Case 1:

v and y and all a’s, |vy|=k.

$w_0 = a^{m-k}b^{2m}c^{3m} \quad \notin \mathrm{L}'$

Case 2:

v and y and all b’s, |vy|=k.

$w_0 = a^{m}b^{2m-k}c^{3m} \quad \notin \mathrm{L}'$

Case 3:

V and y all is c's, |vy| = k,

$w_0 = a^m b^{2m} c^{3m-k} \quad \notin \mathrm{L}'$

Case 4:

For either v or y both is a's and b's.

W are out of order.

Case 5:

For either v or y both contain b's and c's.

W are out of order.

Case 6:

V is all b's, y is all b's, |v|=k1, |y|= k2,

$w_2 = a^{m+k_1} b^{2m+k_2} c^{3m} \notin \mathrm{L}$, since 3($m + k_1$) >3m

Case 7:

V is b's, y is c's, |v|=k1, |y|= k2,

$w_2 = a^m b^{2m+k_1} c^{3m+k_2} \notin \mathrm{L}$, since 3m< $3m + k_2$

All cases lead to a contradiction, therefor L' is not a CFL.

Therefor,

Based on L ∩ $a^*b^*c^*$ = {$a^n b^{2n} c^{3n}$}

Because $a^*b^*c^*$ is a regular language, since L' = {$a^n b^{2n} c^{3n}$} is not CFL,

So L is not CFL.