CS527 Final ; Tenghuan Li ;

1. As the problem say, n is the length and $n=15$; r is the correcting up to two errors, $r=2$;

Assume $C(n,k)$ is a binary code of length n ;

So, as $|C| \le \dfrac{2^n}{\sum_{i=0}^{r} \binom{n}{i}}$

$n=15 \quad r=2$.

$$\Rightarrow |C| \le \frac{2^{15}}{\sum_{i=0}^{2} \binom{15}{i}}$$

$$\le \frac{2^{15}}{\binom{15}{0}+\binom{15}{1}+\binom{15}{2}}$$

$$\le \frac{32768}{1+15+105} = \frac{32768}{121}$$

$$\le 270$$

$|C| \le 270$ codewords.

2. Because the code digits are over $\mathbb{Z}_5$.

We need to design a code capable of correcting limited magnitude errors $l=1$ and $l=2$.

a. Because $k=10$, and digits over $\mathbb{Z}_5$.

$$(5-1)10+(5-1)r \le 5^r-1.$$

$$40+1+4r \le 5^r$$

$$41 \le 5^r-4r$$

$$r \text{ at least } = 3.$$

So. $H_{sys} = \left[\begin{array}{cccccccc|ccc} 0&0&0&0&1&1&1&1&1&0&0 \\ 1&1&1&1&0&0&0&0&0&1&0 \\ 1&2&3&4&1&2&3&4&0&0&1 \end{array}\right]$

$\qquad P_{3\times 8}. \qquad\qquad I_{3\times 3}$

b. When $l=1$.

$$r \geq \left\lceil \frac{10 \log 2}{\log \lceil \frac{5}{2} \rceil} \right\rceil = 7.56.$$

when $l=2$

$$r \geq \left\lceil \frac{k \log(\gamma+1)}{\log\lceil \frac{5}{3} \rceil} \right\rceil = \frac{10 \log 3}{\log\lceil \frac{5}{3}\rceil} = \lceil 8.4 \rceil.$$

$\therefore r \geq 9.$

C. Because 0000 0000 11.

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3 & 4 & 6 & 1 & 2 & 3 \end{bmatrix}$$

$$G = [I \quad (-P^T)] = \begin{bmatrix} & 0 & 0 & -1 \\ & 0 & -1 & 0 \\ & 0 & -1 & -1 \\ & 0 & -1 & -2 \\ I_{10\times 10} & 0 & -1 & -3 \\ & 0 & -1 & -4 \\ & -1 & 0 & 0 \\ & -1 & 0 & -1 \\ & -1 & 0 & -2 \\ & -1 & 0 & -3 \end{bmatrix} \bmod 5.$$

$$= \begin{bmatrix} & 0 & 0 & 4 \\ & 0 & 4 & 0 \\ & 0 & 4 & 4 \\ & 0 & 4 & 3 \\ I_{10\times 10} & 0 & 4 & 2 \\ & 0 & 4 & 1 \\ & 4 & 0 & 0 \\ & 4 & 0 & 4 \\ & 4 & 0 & 3 \\ & 4 & 0 & 2 \end{bmatrix}$$

So, $1\times1+1\times1+p_1=0$.

$1\times0+1\times0+p_2=0$

$1\times2+1\times3+p_3=0$.

So $p_1=3$ $p_2=0$ $p_3=0$.

So, $[0\,0\,0\,0 \quad 0\,0\,0\,0 \quad 1\,1\,\underline{3\,0\,0}]$

3. Because x^2+x+2

So we can get $\alpha^2+\alpha+2=0$.

So $\alpha^2=-\alpha-2=2\alpha+1$.

So $\alpha^2=2\alpha+1$

So,

α^0	0 1	1
α^1	1 0	α^1
α^2	2 1	$2\alpha+1$
α^3	2 2	$2\alpha^2+\alpha=5\alpha+2$
α^4	0 2	$5\alpha^2+2\alpha=12\alpha+5=2$
α^5	2 0	2α
α^6	1 2	$4\alpha+2$
α^7	1 1	$4\alpha^2+2\alpha=10\alpha+4=\alpha+1$.

a. single error correcty of length 8.

α and α^2 as the root.

So for α, α^3

$$(x-\alpha)(x-\alpha^3)=x^2-(\alpha+\alpha^3)x+\alpha^4$$

$$=x^2+x+2$$

α^2 $(x-\alpha^2)(x-\alpha^6)=x^2-(\alpha^2+\alpha^6)x+\alpha^8$

$$=x^2+1$$

$$\therefore g(x)=(x^2+x+2)(x^2+1)$$

$$=x^4+x^3+x+2$$

So the number of check digit $r=4$.
Then $n=8,\ r=4 \Rightarrow k=8-4=4$.

b. Becase least significant 22.
So $I(x) = (00\ 00\ 00\ 22)$

Becaye $g(x) = x^4 + x^3 + x^2 + x + 2$.
So. $x^4 I(x) = g(x) q(x) + R(x)$.
$\alpha^3 x^4 = g(x) q(x) + R(x)$

So. $x^4 + x^3 + x + 2 \overline{)\ \alpha^3 x^4}\ (\alpha^3$
$\alpha^3 x^4 + \alpha^3 x^3 + \alpha^3 x + 2\alpha^3$
$\Rightarrow \alpha^7 x^3 + \alpha^7 x + \alpha^3$

$\therefore R(x) = \alpha^7 x^3 + \alpha^7 x + \alpha^3$
$\therefore$ code word is $(00\ 00\ 00\ 22\ 11\ 00\ 11\ 22)$.

c. $(00\ 00\ E_1\ E_2\ 11\ 00\ 11\ 22)$
$A'(x) = E_1 x^5 + E_2 x^4 + \alpha^7 x^3 + \alpha^7 x + \alpha^3$
$A'(\alpha) = E_1 \alpha^5 + E_2 \alpha^4 + \alpha^2 + \alpha^8 + \alpha^3$
$A'(\alpha^2) = E_1 \alpha^{10} + E_2 \alpha^8 + \alpha^{13} + \alpha^9 + \alpha^3$

4. Because for this binary cyclic code has 1 and α as root of the generator poly. α is primitive root of $GF(2^4)$.

SO, from the question we can get

x^4+x+1.

$\Rightarrow \alpha^4+\alpha+1=0$

$\alpha^4=\alpha+1$.

So. the construct the table:

	α^3	α^2	α^1	α^0
0	0	0	0	0
α^0	0	0	0	1
α^1	0	0	1	0
α^2	0	1	0	0
α^3	1	0	0	0
α^4	0	0	1	1
α^5	0	1	1	0
α^6	1	1	0	0
α^7	1	0	1	1
α^8	0	1	0	1
α^9	1	0	1	0
α^{10}	0	1	1	1
α^{11}	1	1	1	0
α^{12}	1	1	1	1
α^{13}	1	1	0	1

$\alpha 14$	1	0	0	1
$\alpha 15$	0	0	0	1

as the table show. if two error in 34 colume

$(\alpha^2+\alpha^3, 0) \Rightarrow (\alpha^i, 0)$

if - - - in 4.4 colum,

$(\alpha^3+\alpha^4, 0) \Rightarrow (\alpha^j, 0)$

X^i and X^j are different, so we can correat any time betnen error.