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Assignment No, 1 :

Q_1: a) when $n=5$, $0 \to 00000$ so the wrong message occurs if # of errors ≥ 3.

$1 \to 11111$

so, $P_E = \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p) + \binom{5}{5} p^5 = 10(0.1)^3(0.9)^2 + 5(0.1)^4(0.9) + (0.1)^5$

$= 0.00856 \approx \boxed{0.856\%}$ ------ Q_1 the first answer.

b). when $n=7$,

$0 \to 0000000$ it same as $n=5$, the wrong message occurs if # of errors ≥ 4.

$1 \to 1111111$

$\therefore P_E = \binom{7}{4} p^4 (1-p)^3 + \binom{7}{5} p^5 (1-p)^2 + \binom{7}{6} p^6 (1-p) + \binom{7}{7} p^7$

$= 35 \cdot (0.1)^4 (0.9)^3 + 21 \cdot (0.1)^5 (0.9)^2 + 7(0.1)^6(0.9) + (0.1)^7$

$= 0.002726 \approx \boxed{0.273\%}$ ---- Q_1 the second answer.

Q_2: Because the number of selection is: $\{0, 1, 2, 3, 4, 5, 6, 7\}$.

X is denote the number of selection, if we pick 3 is stop.

So. $p(X=1) = \frac{1}{8}$ (first get 3)

$p(X=2) = \frac{7}{8} \times \frac{1}{7} = \frac{1}{8}$

$p(X=3) = \frac{7}{8} \times \frac{6}{7} \times \frac{1}{6} = \frac{1}{8}$

$\vdots$

$p(X=8) = \frac{7}{8} \times \frac{6}{7} \times \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{1} = \frac{1}{8}$

So $H(X) = -\sum_{i=1}^{8} P_i \cdot \log_2 P_i$ (when P_i all $= \frac{1}{8}$).

$= -\sum_{i=1}^{8} P \log_2 P = -\left[\frac{1}{8}\log_2\frac{1}{8} + \cdots + \frac{1}{8}\log_2\left(\frac{1}{8}\right)\right]$

$= 8 \cdot \left(\frac{1}{8}\log_2 8\right)$

$= \boxed{3 \text{ bits.}}$ ----- Q_2 answer

Q3: Because a box have 50 semicondutor chip, and it include 3 defective chips

$\therefore$ X represents number of defective chips: 0, 1, 2, 3.

$$\therefore P(X=0) = \frac{\binom{47}{5}\cdot\binom{3}{0}}{\binom{50}{5}} = 0.72397$$

$$P(X=1) = \frac{\binom{47}{4}\binom{3}{1}}{\binom{50}{5}} = 0.2525$$

$$P(X=2) = \frac{\binom{47}{2}\binom{3}{2}}{\binom{50}{5}} = 0.02296$$

$$P(X=3) = \frac{\binom{47}{2}\binom{3}{3}}{\binom{50}{5}} = 0.00051$$

$$H(X) = -\sum_{i=0}^{3} P(X)\log_2 P(X)$$

$$= -[0.72397\log_2 0.72397 + 0.2525\log_2 0.2525 + 0.02296\log_2 0.02296 + 0.00051\cdot\log_2 0.00051]$$

$$= \boxed{0.9692 \text{ bits}}$$ ---- Q3 answer.

Q4: Because the world series is 7 game-series and A, B team is independent, which team get 4 wins will win this game;

X represents outcomes; Y represents number of played games {4, 5, 6, 7}.

a, we stared H(Y).

$$P(Y=4) = A\text{ win or }B\text{ win} = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \left(\frac{1}{2}\right)^4 \times 2 = \frac{1}{8}.$$

when we played more than 4, A or B need win mean the last read must be A, B.

like ~~__~~ when Y=5, if A need win _ _ _ _ A. is $\binom{4}{3}$ for A win

B need win _ _ _ _ B. is $\binom{4}{3}$ for B win.

$$\therefore P(Y=5) = \binom{4}{3}\cdot\left(\frac{1}{2}\right)^5 + \binom{4}{3}\left(\frac{1}{2}\right)^5 = \frac{1}{4}.$$

$$P(Y=6) = \binom{5}{3}\cdot\left(\frac{1}{2}\right)^6 + \binom{5}{3}\cdot\left(\frac{1}{2}\right)^6 = \frac{5}{16} = 0.3125.$$

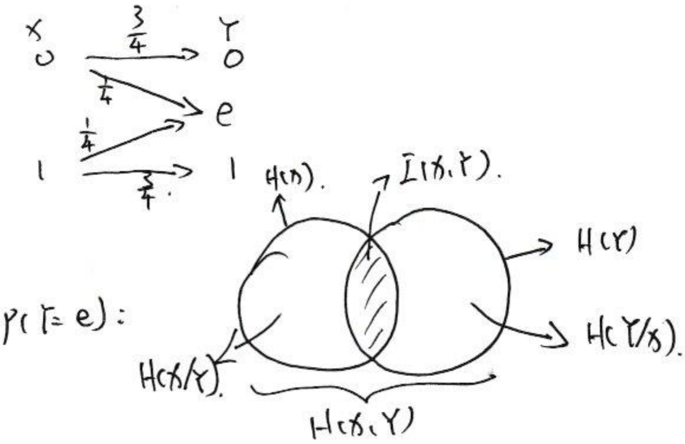
$$P(Y=7) = \binom{6}{3}\left(\frac{1}{2}\right)^7 + \binom{6}{3}\left(\frac{1}{2}\right)^7 = \frac{5}{16} = 0.3125$$

$$\therefore H(Y) = -\sum_{i=4}^{7} P(Y_i)\log_2 P(Y_i) = -\left[\frac{1}{8}\log_2\frac{1}{8} + \frac{1}{4}\log_2\frac{1}{4} + \frac{5}{16}\times 2\log_2\frac{5}{16}\right] = \boxed{1.9237 \text{ bits}}$$

$$H(X) = \sum_{x\in X} P(x)\log_2\frac{1}{P(x)} = -\left[2\cdot\left(\frac{1}{2}\right)^4\log_2\left(\frac{1}{2}\right)^4 + 2\binom{4}{3}\left(\frac{1}{2}\right)^5\log_2\left(\frac{1}{2}\right)^5 + 2\binom{5}{3}\left(\frac{1}{2}\right)^6\log_2\left(\frac{1}{2}\right)^6 + 2\binom{6}{3}\left(\frac{1}{2}\right)^7\log_2\left(\frac{1}{2}\right)^7\right]$$

$$= \boxed{\frac{93}{16}} = \boxed{5.8125 \text{ bits}}$$

Q5: $p(x=0)=\frac{2}{3}$, $p(x=1)=\frac{1}{3}$ $p=\frac{1}{4}$.



1. $H(X)=\sum_{x_i} P(x_i)\log_2\frac{1}{P(x_i)}=\frac{2}{3}\log_2\frac{3}{2}+\frac{1}{3}\log_2 3$

$\boxed{=0.9183 \text{ bit}}$

for H(Y), we need to know $P(Y=0)$, $P(Y=1)$, $P(Y=e)$:

so. $P(Y=0)=P(X=0)\cdot(1-P)=\frac{2}{3}\cdot\frac{3}{4}=\frac{1}{2}$

$P(Y=1)=P(X=1)\cdot(1-P)=\frac{1}{3}\cdot\frac{3}{4}=\frac{1}{4}$

$P(Y=e)=P(X=0)\cdot P+P(X=1)\cdot P=\frac{2}{3}\cdot\frac{1}{4}+\frac{1}{3}\cdot\frac{1}{4}=\cancel{\frac{3}{2}\cdot\frac{1}{4}}=\frac{1}{4}$

$\therefore H(Y)=\sum_{Y_i}P(Y_i)\log_2\frac{1}{P(Y_i)}=\frac{1}{2}\log_2 2+\frac{1}{4}\log_2 4+\frac{1}{4}\log_2 4=\boxed{\frac{3}{2}\text{ bit}}$

so $\boxed{H(X)=0.9183 \text{ bit}}$. $\boxed{H(Y)=\frac{3}{2}\text{ bit}}$. - - - - - - - - - - Answer 1.

2. $\because H(Y/X)=H(Y/X=0)\,P(X=0)+H(Y/X=1)\cdot P(X=1)$ - - - I.

① $\because H(Y/X=0)=P(Y=1/X=0)\log_2\frac{1}{P(Y=1/X=0)}+P(Y=0/X=0)\log_2\frac{1}{P(Y=0/X=0)}+P(Y=e/X=0)\log_2\frac{1}{P(Y=e/X=0)}$

$=0+\frac{3}{4}\log_2\frac{4}{3}+\frac{1}{4}\log_2 4=0.8112$ bits

② $H(Y/X=1)=P(Y=1/X=1)\log_2\frac{1}{P(Y=1/X=1)}+P(Y=0/X=1)\log_2\frac{1}{P(Y=0/X=1)}+P(Y=e/X=1)\log_2\frac{1}{P(Y=e/X=1)}$

$=\frac{3}{4}\log_2\frac{4}{3}+0+\frac{1}{4}\log_2 4=0.8112$ bits.

$\therefore$ ①② put into I. $\rightarrow$ $H(Y/X)=(0.8112)\cdot(\frac{2}{3})+(0.8112)\cdot(\frac{1}{3})=\boxed{0.8112 \text{ bits}}$ - - Answer 2.

$H(X/Y)=H(X)-H(Y)+H(Y/X)=0.9183-1.5+0.8112=\boxed{0.2295 \text{ bits}}$

3. $\because H(X,Y)=H(X)+H(Y/X)$ or $H(Y)+H(X/Y)$.

$\therefore H(X,Y)=H(X)+H(Y/X)=0.9183+0.8112=\boxed{1.729 \text{ bits}}$ - - - - Answer 3.

4. $I(X,Y)=H(X)-H(X/Y)$ or $H(Y)-H(Y/X)$

$=0.9183-0.23=\boxed{0.6883 \text{ bits}}$ - - - - - - Answer 4.

Q6. Because it is a coin, and until the first head occurs.

X denote the number of flips required.

so $P(H) = P(T) = \frac{1}{2}$.

$\therefore P(X=n) = (\frac{1}{2})^{n-1} \cdot (\frac{1}{2}) = (\frac{1}{2})^n$ ($(\frac{1}{2})^{n-1}$ mean the first $(n-1)$ is trails. The $(\frac{1}{2})$ means the n^{th} is Head).

$\therefore H(X) = \sum_{n=1}^{\infty} (\frac{1}{2})^n \log_2 \frac{1}{(\frac{1}{2})^n} = \sum_{n=1}^{\infty} (\frac{1}{2})^n \log_2 2^n = \sum_{n=1}^{\infty} (\frac{1}{2})^n \cdot n = \frac{\frac{1}{2}}{(1-\frac{1}{2})^2} = \boxed{2 \text{ bits}}$

($\because$ exploiting $\sum_{n=1}^{\infty} (k)^n \cdot n = \frac{k}{(1-k)^2}$).