CS527 ; HW #5.

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1.

a. Because we need to find the maximum number of non-systematic codewords.

$n=2$; $m=8$. ; $D=4$.

So, $|C| = \lceil \frac{m}{D} \rceil^n$

$= \lceil \frac{8}{4} \rceil^2 = \underline{4}$ $r=2$.

So. $C = \{(00), (04), (40), (44)\}$.

b. Because this question, we know that $k=4$; $l=3$; $m=8$ we want to find the number of check digits required to correct all limited magnitude.

so. $r \geq \frac{k \log D}{\log \lceil \frac{m}{D} \rceil}$ $k=4$; $l=3$; $m=8$ $D=4$

$\therefore \Rightarrow r \geq \frac{4 \log 4}{\log \frac{8}{4}} = 8. \Rightarrow \underline{r \geq 8}$

C. Because as a and b part, we get that

$k=4$, $m=8$, $l=3$, $D=4$, $r=8$.

For now, $X=(7246)$.

① $(7246) \bmod (l+1)$

$\Rightarrow (7246) \bmod 4 \Rightarrow (3202)$

② $\because (3202)$

$z = 3\cdot 4^3 + 2\cdot 4^2 + 0\cdot 4^1 + 2\cdot 4^0 = 226$.

③ $\because z = (z_{r-1}, z_{r-2} \dots z_0)$ by binary.

So. $z = (1,1,1,0,0,0,1,0)$

④ check symbol:

$(l+1)\cdot z$

$= 4\cdot(1,1,1,0,0,0,1,0)$.

$\therefore$ result it is that.

codeword $X = (7,2,4,6,4,4,4,0,0,0,4,0)$.

D. Because as c part we know that.

$(7,2,4,6,4,4,4,0,0,0,4,0)$

Then get the code (5134).

and check digits have -1 errors in all positions.

so,

$$(4\,4\,4\,0\,0\,0\,4\,0) \xrightarrow{\text{mode. }4} (1\,1\,1\,0\,0\,0\,1\,0).$$

Then $2 = 1\cdot 2^7 + 1\cdot 2^6 + 1\cdot 2^5 + 0\cdot 2^4 + 0\cdot 2^3 + 0\cdot 2^2 + 1\cdot 2^1 + 0\cdot 2^0$

$= 226.$

Then decimal to quaternary.

$226 \Rightarrow (3, 2, 0, 2) \Rightarrow \overline{b} = (2, 1, 1, 2)$

$\therefore$ we get is $(5, 1, 3, 4)$.

$X = (5\ 1\ 3\ 4) + (2\ 1\ 1\ 2)$

$= 7246.$

2. For the question, we can get

n = length of the code ; r = number of check bits.

a. $n = 16$. because. $n \le 2^r - 1$.

we can get $\underline{\underline{r = 5}}$.

so. $r = 5 \Rightarrow \binom{5}{3} + \binom{5}{5} + \binom{5}{1} = 16.$

$\therefore H = 10 + 1 + 5.$

(Table at next page)

$$
H = \begin{array}{cccccccccc|c|ccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16. \\
\end{array}
$$

$$
H = \left[\begin{array}{cccccccccc|c|ccccc}
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
$$

b. For the question, we need to show that this code is capable of correcting single errors and detecting double errors,

So, we need to find the min distance of code.

$d_{min} = 4 \Rightarrow t + d + 1 = 4$

Then we know that $t = 1$, $d = 2$.

① single error:

assume $A' = E + A$.

so $A'H^T = EH^T + AH^T$

$\because AH^T = 0 \Rightarrow A'H^T = EH^T$.

So we have one error, the matching result by EH^T will find the location of the single error:

②. double error:

Same as ① part, but in this proble, we will have two error. So in this if we add this two in H. will get a new colum that don't match H. So for the two error we just can detect. we can't correct.

c. $\underset{n-r\times n}{G} = [I \;\; p^T] = [I_{11\times11} \;\; P^T_{11\times5}]$

$$= \left[\begin{array}{c|ccccc} & 0 & 0 & 1 & 1 & 1 \\ & 1 & 1 & 1 & 0 & 0 \\ & 1 & 1 & 0 & 1 & 0 \\ & 1 & 1 & 0 & 0 & 1 \\ & 0 & 1 & 1 & 0 & 1 \\ I_{11\times11} & 0 & 1 & 1 & 1 & 0 \\ & 0 & 1 & 0 & 1 & 1 \\ & 1 & 0 & 0 & 1 & 1 \\ & 1 & 0 & 1 & 1 & 0 \\ & 1 & 0 & 1 & 0 & 1 \\ & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

3. as the question; $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$.

a. $G = [I \;\; p] = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$ <u>change 3&4</u>

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} r_1 \\ r_2 \\ r_3 \end{matrix} \xrightarrow{r_2 = r_2 + r_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$I_{3\times3}$ $\quad$ $P_{3\times3}$.

b. we need to find H

So. $H = [P^T \; I] = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

c.

① $\because A = (1\,1\,0)$

$C = A \cdot G = [1\,1\,0] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$= [1\,1\,0\,0\,1\,0]$

②. $C \cdot H^T = (1\,1\,0\,0\,1\,0) \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (0, 0, 0)$.

4.

a. as problem, we know 2s, $k = 8$.
and correct single errors.

as $\begin{cases} n \leq \dfrac{p^r - 1}{p - 1} \\ n = k + r \end{cases} \Rightarrow r = 3.$

So $H_{3\times 11} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 \end{bmatrix}$

$\Rightarrow H_{sys} = \left[\begin{array}{cccccccc|ccc} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 2 & 3 & 4 & 1 & 2 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$

$\qquad\qquad P_{3\times 8}. \qquad\qquad\qquad I_{3\times 3}$

b. $G_{8\times 11} = [\, I_{8\times 8} \quad -P^T_{8\times 3} \,]$

$= \left[\begin{array}{c|ccc} & 0 & 4 & 4 \\ & 0 & 4 & 3 \\ & 0 & 4 & 2 \\ I_{8\times 8}. & 0 & 4 & 1 \\ & 4 & 0 & 4 \\ & 4 & 0 & 3 \\ & 4 & 0 & 2 \\ & 4 & 0 & 1 \end{array}\right]$

c. $\because A = (12041123)$.

$C = A \cdot G$. (part b into this).

$= ((1204\ 1123] \cdot G_{8\times 11}$, mod 5)

$\Rightarrow [1204\ \ 1123\ \ 3\ 3\ 3]$

d. ∵ as C question. we get that

$c = [1\ 2\ 0\ 4\ \ 1\ 1\ 2\ 3\ 3\ 3\ 3\ 3]$

now the 4 to 2. we get new c'.

$c' = [1\ 2\ 0\ \underline{\underline{2}}\ \ 1\ 1\ 2\ 3\ 3\ 3\ 3\ 3]$

So. $S = c' \cdot H^T = [10\ \ 8\ \ 37]_{\bmod 5}$

$= [0\ \ 3\ 2]$

$= 3^{-1} \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} = 3h_4$.

So. $c = c' - E$

$= [1\ 2\ 0\ 2\ \ 1\ 1\ 2\ 3\ 3\ 3\ 3\ 3] - [0\ 0\ 0\ 3\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$

$= [1\ 2\ 0\ \underline{-1}\ \ 1\ 1\ 2\ 3\ 3\ 3\ 3\ 3]\ \underline{\bmod 5}$

$= [1\ 2\ 0\ \underline{4}\ \ 1\ 1\ 2\ 3\ 3\ 3\ 3\ 3]$

5. I don't know how to do.