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1. Because X be the random variable that represents the outcomes ; Y be the random variable that represents the number of sets played, which can be 2 or 3.

① the # of sets played = 2.

$\therefore P(Y) = 2 \times (\frac{1}{2} \cdot \frac{1}{2}) = \frac{1}{2}$

(because two players).

② the # of sets played = 3

$P(Y) = 2(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}) = \frac{1}{2}$.

$\therefore H(Y) = -\sum_{i=2}^{3} P(i) \log_2 P(i)$

$= -(\frac{1}{2} \cdot \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2})$

$= 1$ bits.

$H(X) = -\sum P(x) \log_2 P(x)$

$= -2((\frac{1}{2})^2 \log_2 (\frac{1}{2})^2 + \binom{2}{1} (\frac{1}{2})^3 \log_2 (\frac{1}{2})^3)$

$$= -2\left(\frac{1}{4}\cdot\log_2 4^{-1} + \frac{1}{4}\log_2 8^{-1}\right)$$

$$= 2\left(\frac{1}{4}\times 2 + \frac{1}{4}\times 3\right)$$

$$= \frac{5}{2} \text{ bits}.$$

So $\underline{H(Y) = 1 \text{ bits} \; ; \; H(X) = \frac{5}{2} \text{ bits}}$.

2. Because the fair coin is tossed,
So we need to know for top and bottom.

(1) for top. $P(Y=h) = P(Y=t) = \frac{1}{2}$.
(2) for Bottom $P(Y=h) = P(Y=t) = \frac{1}{2}$.

So. $I(X,Y) = H(Y) - H(Y/X)$.

$$H(Y) = \sum P(Y)\log\frac{1}{P(Y)}$$

$$= \frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2$$

$$= 1 \text{ bits}.$$

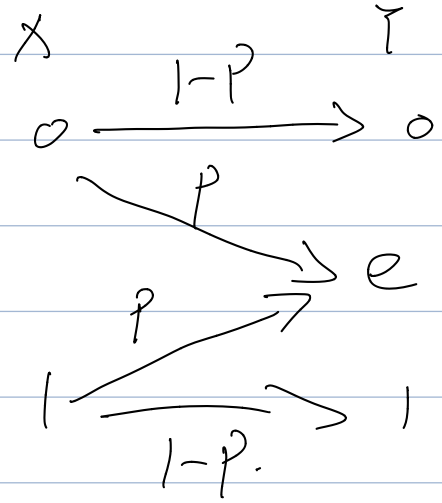
$$H(Y/X) = \sum P(x_i)\, H(Y/x_i).$$

But if X is out that occurs, the amount of uncertainty is 0,
So we can know that $H(Y/X) = 0$

$\therefore \underline{I(X,Y) = 1 - 0 = 1 \text{ bits}}$

3. Because that $C = \max_{P(x)} I(X;Y) = \max_{P(x)} [H(Y) - H(Y|X)]$.

we can know that:



Assume $P(x=0) = 1-q$

$P(x=1) = q$.

So $P(Y=0) = P(X=0) \cdot (1-P)$

$= (1-q)(1-P)$ --- ①

$P(Y=1) = P(X=1) \cdot (1-P)$

$= q \cdot (1-P)$. --- ②

$P(Y=e) = P(X=0) \cdot P + P(X=1) \cdot P$

$= (1-q) P + q \cdot P$

$= P$. --- ③.

$\because H(Y) = \sum P(y) \log_2 \frac{1}{P(y)}$ --- ④ ; ① ② ③ into ④.

$H(Y) = (1-q)(1-p) \log_2 \frac{1}{(1-q)(1-p)} + q(1-p) \log_2 \frac{1}{q(1-p)}$

$+ p \log_2 \frac{1}{p}$. --- (A).

$H(Y/X) = \sum_i P(x_i) \cdot H(Y/x_i)$

$= P(X=0) H(Y/x=0) + P(x=1) H(Y/x=1)$ --- (I)

$\because H(Y/x=0) = P(Y=0/x=0) \log_2 \frac{1}{P(Y=0/x=0)} + P(Y=e/x=0) \log_2 \frac{1}{P(Y=e/x=0)}$

$+ P(Y=1/x=0) \log_2 \frac{1}{P(Y=1/x=0)}$

$$= (1-p)\log_2\frac{1}{1-p} + p\log_2\frac{1}{p} + 0 \quad \cdots (\mathrm{II})$$

$$H(Y/X=1) = P(Y=1/X=1)\log_2\frac{1}{P(Y=1/X=1)} + P(Y=e/X=1)\log_2\frac{1}{P(Y=e/X=1)}$$

$$+ P(Y=0/X=1)\log_2\frac{1}{P(Y=0/X=1)}$$

$$= (1-p)\log_2\frac{1}{1-p} + p\log_2\frac{1}{p} + 0 \quad \cdots (\mathrm{III})$$

put II, III into I: we can get $H(Y/X)$

So, $$H(Y/X) = P(X=0)\left((1-p)\log_2\frac{1}{1-p} + p\log_2\frac{1}{p}\right)$$

$$+$$

$$P(X=1)\left((1-p)\log_2\frac{1}{1-p} + p\log_2\frac{1}{p}\right)$$

$$= \left((1-q) + q\right)\cdot\left((1-p)\log_2\frac{1}{1-p} + p\log_2\frac{1}{p}\right)$$

$$H(Y/X) = (1-p)\log_2\frac{1}{1-p} + p\log_2\frac{1}{p} \quad \cdots \text{B}$$

So make A and B together:

$$C = \max_{q(x)}\left[H(Y) - H(Y/X)\right]$$

$$= \max_{q(x)}\left[(1-q)(1-p)\log_2\frac{1}{(1-q)(1-p)} + q(1-p)\log_2\frac{1}{q(1-p)} + p\log_2\frac{1}{p} - (1-p)\log_2\frac{1}{1-p} - p\log_2\frac{1}{p}\right]$$

$$= \max_{q(x)} \left[(1-q)(1-p) \log_2 \frac{1}{(1-q)(1-p)} + q(1-p) \log_2 \frac{1}{q(1-p)} - (1-p) \log_2 \frac{1}{1-p} \right].$$

$$= \max_{q(x)} \left[(1-q)(1-p)\left(\log_2 \frac{1}{(1-q)} + \log_2 \frac{1}{(1-p)}\right) + q(1-p)\left(\log_2 \frac{1}{q} + \log_2 \frac{1}{1-p}\right) - (1-p)\log_2 \frac{1}{(1-p)} \right].$$

$$= \max_{q(x)} \left[(1-q)(1-p) \log_2 \frac{1}{1-q} + q(1-p) \log_2 \frac{1}{q} \right]$$

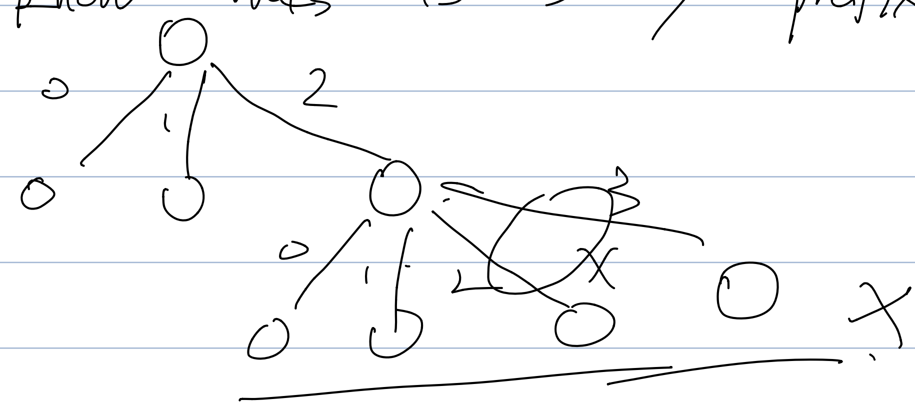
$$= \max_{q(x)} \left[(1-p)\left((1-q) \log_2 \frac{1}{1-q} + q \log_2 \frac{1}{q}\right). \right] \quad (*)$$

So, when $q(x) = \frac{1}{2}$ we can get the C_{max}.

$$\therefore \quad C = \max_{q(x)=\frac{1}{2}} (1-p) \quad \text{bits/transmission}.$$

4.

a). we know this is 3-ary prefix code.



if this code lengths is 1, 1, 2, 2, 2, 2

is not 3-ary prefix code.

so is not possible.

b). Because it is 4-ary Huffman code. D=4.

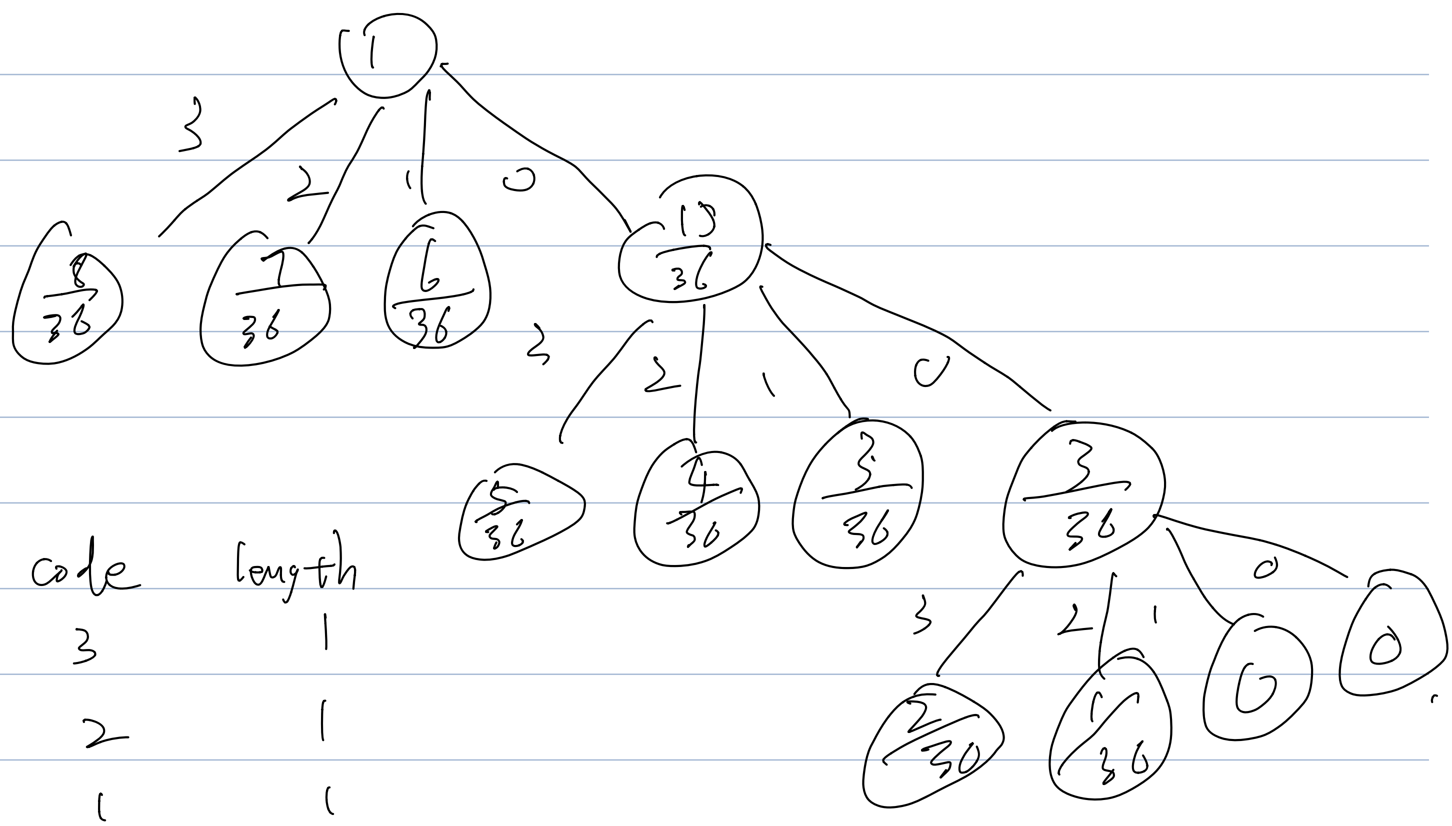
$$p = \left(\frac{8}{36}, \frac{7}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{2}{36}, \frac{1}{36}\right)$$

$$(N+j) = 1 \mod (D-1)$$

$$8+j = 1 \mod 3$$

so $j = 2$.

so $p = \left(\frac{8}{36}, \frac{7}{36}, \frac{6}{36}, \frac{5}{36}, \frac{4}{36}, \frac{3}{36}, \frac{2}{36}, \frac{1}{36}, 0, 0\right\}$



P(x)	code	length
8/36	3	1
7/36	2	1
6/36	1	1
5/36	0 3	2
4/36	0 2	2

3/36	01	2
2/36	003	3
1/36	002	3

So. we can know the code is

Code set = { 3 , 2 , 1 , 03 , 02 , 01 , 003 , 002 }