

1.

a. $L1 = \{ w \in \{0, 1\}^* \mid w \text{ contains at least three 1s} \}$

$S \rightarrow A1A1A1$

$A \rightarrow 0A \mid 1A \mid \lambda$

b. $L2 = \{ w \in \{0, 1\}^* \mid w = w^R \text{ and } |w| \text{ is even} \}$

$S \rightarrow 0S0 \mid 1S1 \mid \lambda$

c. $L3 = \{ a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } i = k \}$

$S \rightarrow AB \mid D$

$A \rightarrow aAb \mid \lambda$

$B \rightarrow cB \mid \lambda$

$D \rightarrow aDc \mid \lambda$

$E \rightarrow bE \mid \lambda$

2.

By the rules we can know:

$S \rightarrow \emptyset \mid a \mid b, S \rightarrow SS, S \rightarrow S+S, S \rightarrow (S), S \rightarrow S^*$

Then $S \rightarrow a \mid b \mid (S) \mid S+S \mid SS \mid S^*$

Therefore, CFG, $G=(V,T,S,P)$ $T=\{a,b,(,),+,*,\emptyset\}$ $V=\{S\}$

Where

S = starting variable

P = production set

Derivation of $(a + b)^*$:

$S \rightarrow S^* \Rightarrow (S)^* \Rightarrow (S + S)^* \Rightarrow (a + S)^* \Rightarrow (a + b)^*$

3.

a.

because $S \rightarrow SS \mid AAA \mid \lambda$ and $A \rightarrow aA \mid Aa \mid b$

so $(a^*ba^*ba^*ba^*)^*$

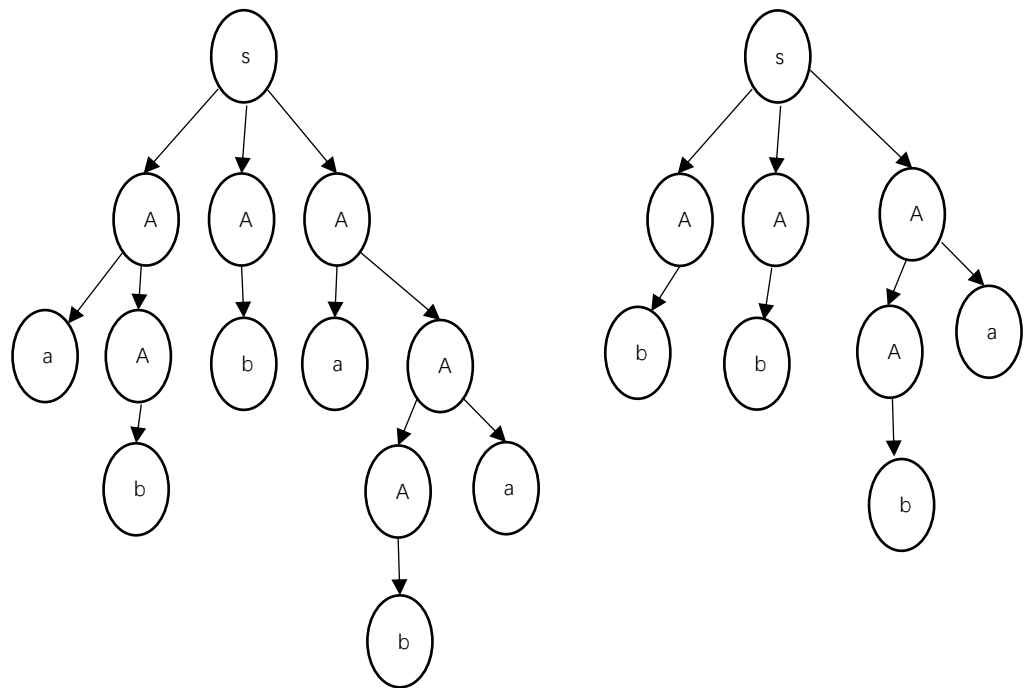
b.

because require left-most derivation strings is abbaba.

$S \rightarrow abbaba$, left most derivation:

$S \rightarrow AAA \Rightarrow aAAA \Rightarrow abAAA \Rightarrow abbA \Rightarrow abbaAa \Rightarrow abbaba$

c.



- d. $S \rightarrow A | \lambda$
 $A \rightarrow aA | bB$
 $B \rightarrow aB | bC$
 $C \rightarrow aC | bD$
 $D \rightarrow aD | A | \lambda$

4.
 Because $n \geq 1$
 $S \rightarrow aAb$
 $A \rightarrow aAb$
 $A \rightarrow b$

5.
 a.

$$L = \{a^n b^n : n \geq 0\}$$

$$\text{Then } L^2 = \{a^n b^n a^n b^n : n \geq 0\}$$

$G = (V, T, S, P)$ with set of variables

$V = \{S, A\}$, where S is the start variable; set terminals $T = \{a, b\}$

$S \rightarrow SS$

$S \rightarrow aSb$

$S \rightarrow \lambda$

Therefore, L^2 is a context-free language.

b.

$$L = \{a^n b^n : n \geq 0\}$$

$$\text{Then } L^* = \{(a^n b^n)^* : n \geq 0\}$$

$G = (V, T, S, P)$ with set of variables

$V = \{S, A\}$, where S is the start variable; set terminals $T = \{a, b\}$

$$S \rightarrow S^*$$

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Therefore, L^* is a context-free language