1.

a.

if L is regular language, so $L = \{0^n : n = 2^k \ \ for\ some\ k > 1\}$

m be the constant in the pumping lemma, so $\omega = \ 0^{2^m} \epsilon\ L\ ,\ |\omega| = 2^m \geq m,$

so all possible x,y,z $\omega = xyz, |xy| \leq m, |y| \geq 1.$

Case:

$x = 0^r\ ,\ y = 0^s\ ,\ z = 0^{2^m - r - s}\ , r + s \leq m\ , s \geq 1$

Let i =2, so $\omega = \ xy^2z$=$0^{2^m + s}$

Because $2^m < 2^m + s \leq 2^m + m < 2^m + 2^m = 2^{m+1}$

$2^m + s \ \neq 2^K$, so i=2, $\omega = 0^{2^m + s} \notin L$, so L is not regular language.

b.

if L is regular language , so L = {abb, aab, aaab,……}

$because\ n(a) \neq n(b)$, so m be the constant in the pumping lemma,

$\omega = \ a^{\mathrm{m}} b^{\mathrm{m-s}}$,

Case when $\omega =$ aaabbbb, x=aa,y=a,z=bbbb,

i=2, $\omega = \ xy^2z = (aa)(a)^2(bbbb) = aaaabbbb,$

Then n(a)=n(b)=4;

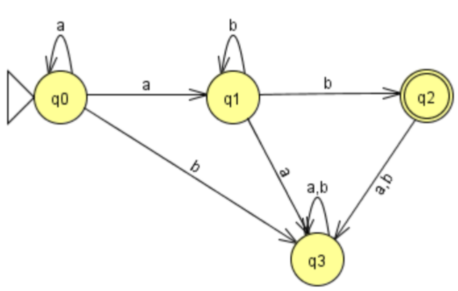
So L is not regular language.

2.

a.

let L1 = $\{a^{\mathrm{n}} b^{\mathrm{n}} : n > 0\ \}$, L2 = $\{a^{\mathrm{n}} b^{\mathrm{m}} : n > 0, m > 0\ \}$;

then because L2 is regular language



L2 is regular by the L2 = aa^*bb^*;

So L = L1∪L2 = $\{a^{\mathrm{n}}b^{\mathrm{n}}: n > 0\}$∪$\{a^{\mathrm{n}}b^{\mathrm{m}}: n > 0, m > 0\}$ is regular language;

b.

because L = { $a^n b^m$: n ≤ m ≤ 2n}, if L is regular language

let m be as in pumping length

w = $a^m b^m$,then let w = xyz be in pumping lemma,

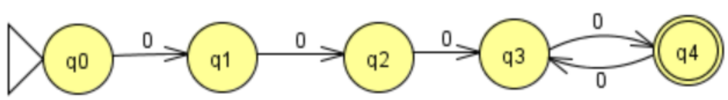
When i=2, y=a^{k},w = xy^2z = $a^{\mathrm{m+k}}a^{\mathrm{m}}$

Because m+k > m ,so L is not regular language.

c.

L is regular language, so $L = \{0^n: n = 2k \ \ for\ some\ k > 1\}$

The regular expression is L = 000$(00)^*0$;

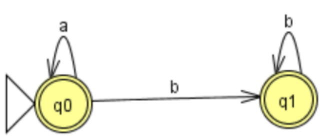


3.

As when L1 = $\{a^{\mathrm{n}}b^{\mathrm{m}}: n = m\}$,L1 is not a regular language

L2 = $\{a^{\mathrm{n}}b^{\mathrm{m}}: n \neq m\}$, L2 is not a regular language

But when L1∪ L2, the expression is L = a^*b^*;



This is regular language;

4.

The definition of symmetric difference of two sets that:
$S_1 \ominus S_2$= $(S1 \cap \overline{S2}) \cup (S2 \cap \overline{S1})$.
Because of since regular sets are closed under union, intersection, and complement.