CS 527
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HW 4

Problem 1. a. ∵ the data bits length 8, we need to design single error correcting - double error detecting code.

so $k, r, n \rightarrow$ (k: data bits ; r: check bits ; n: code length).

$\Rightarrow k+r \le n-1$

$k+r \le 2^r - 1$

$r = 4$. Then, 4 check-bits are required in this case.

b. ∵ $k+r = 8+4 = 12$ bits.

so.

p_1	p_2	m_3	p_4	m_5	m_6	m_7	p_8	m_9	m_{10}	m_{11}	m_{12}
0	1	1	0	0	1	1	1	1	1	1	0.

∴ $p_1 \oplus m_3 \oplus m_5 \oplus m_7 \oplus m_9 \oplus m_{11} \Rightarrow p_1 = 0$.

$p_2 \oplus m_3 \oplus m_6 \oplus m_7 \oplus m_{10} \oplus m_{11} \Rightarrow p_2 = 1$

$p_4 \oplus m_5 \oplus m_6 \oplus m_7 \oplus m_{12} \Rightarrow p_4 = 0$

$p_8 \oplus m_9 \oplus m_{10} \oplus m_{11} \oplus m_{12} \Rightarrow p_8 = 1$.

c. Because as the question say, if the first information bit is in error ~~explain~~. so it mean $\Rightarrow$ 0011 1110.

So, $\Rightarrow$

p_1	p_2	$\boxed{m_3}$	p_4	m_5	m_6	m_7	p_8	m_9	m_{10}	m_{11}	m_{12}
0	1	$\boxed{0}$	0	0	1	1	1	1	1	1	0

so. $e_0 = p_1 \oplus m_3 \oplus m_5 \oplus m_7 \oplus m_9 \oplus m_{11} = 1$

$e_1 = p_2 \oplus m_3 \oplus m_6 \oplus m_7 \oplus m_{10} \oplus m_{11} = 1$

$e_2 = p_4 \oplus m_5 \oplus m_6 \oplus m_7 \oplus m_{12} = 0$

$e_3 = p_8 \oplus m_9 \oplus m_{10} \oplus m_{11} \oplus m_{12} = 0$.

∵ the $S = (e_3 \underline{e_2 e_1 e_0})$ $e_2 e_1 e_0$ is the error location.

So $S = (0\ \underline{0\ 1\ 1})$

D. As the question, we can know the 1st and 5st are error.

p_1	p_2	$\boxed{m_3}$	p_4	m_5	m_6	m_7	p_8	$\boxed{m_9}$	m_{10}	m_{11}	m_{12}
0	1	$\boxed{0}$	0	0	1	1	1	$\boxed{0}$	1	1	0

$e_0 = p_1 \oplus m_3 \oplus m_5 \oplus m_7 \oplus m_9 \oplus m_{11} = 0$

$e_1 = p_2 \oplus m_3 \oplus m_6 \oplus m_7 \oplus m_{10} \oplus m_{11} = 1$

$e_2 = p_4 \oplus m_5 \oplus m_6 \oplus m_7 \oplus m_{12} = 0.$

$e_3 = p_8 \oplus m_9 \oplus m_{10} \oplus m_{11} \oplus m_{12} = 1.$

So, $e = p_1 \oplus p_2 \oplus m_3 \oplus p_4 \oplus m_5 \oplus m_6 \oplus m_7 \oplus p_8 \oplus m_9 \oplus m_{10} \oplus m_{11} \oplus m_{12} = 0.$

so, $s = (0, \underline{1010})$.

so, we can know the $1010 \neq 0$, and all $e=0$. so it will deleces two errors. but we can't correct them.

problem 2. As the problem, we need to proof $d_{min}(X, Y) \geq e+1$ in two-way.

① First of all, $d_{min}(X, Y) \geq e+1$.

Assume for code C the codeword is X and Y.

we have e erasure errors ; X' is X the easures.

so. $d(X, X') \leq e$. if X is error will be X'. but if X want to get the Y error, we don't know which one is error in $\{X', Y\}$

so. only when $\underline{d_{min}(X, Y) \geq e+1}$, we can correct the code. with e erasure errors.

②. Then, we need to prove correcting e errors $\Rightarrow d_{min}(X, Y) \geq e+1$.

Assume for code C the codeword is X and Y. $D(X, Y) \leq e$.

So we have. $S_X = \{X' \mid D(X, X') \leq e\}$

$S_Y = \{Y' \mid D(Y, Y') \leq e\}$.

$\forall X, Y \in C$, and $S_X \cap S_Y = \emptyset$.

if $D(X, X') \leq e$ and $D(Y, Y') \leq e$. if we delete erasure errors of X' and Y' call it $\bar{X}$ and $\bar{Y}$.

since $D(X, Y) \leq e$ we can get that $\bar{X} = \bar{Y}$, which is contradiction with assume. so we can prove that $\underline{d_{min}(X, Y) \geq e+1}$

Problem 3. As the question say, we can know that code C can correcting t errors and e erasures when $d_{min}(X,Y) \geq 2t+e+1$.

Assume $d_{min}(X,Y) \geq 2t+e+1$, we can correct t errors and e erasure errors.

I. Fist, if we don't have e erasure errors, For code C. the $d_{min}(X;Y) = d_m(X,Y) - e$.

$\therefore$ we need to prove: $d_{min}(X,Y) \geq 2t+1$

Let C be code with $2t+1$. $S_X = \{X' \mid D(X,X') \leq t\}$

$S_Y = \{Y' \mid D(Y,Y') \leq t\}$.

$\forall X,Y \in C, \quad S_X \cap S_Y = \emptyset$. ~~so $D(X',Y') \geq 1$~~.

$\because D(X,X') + D(X',Y) \geq D(X,Y)$.

$\therefore D(X',Y) \geq D(X,Y) - D(X,X')$

$\geq (2t+1) - t \geq t+1$ ①.

$D(X',Y') + D(Y',Y) \geq D(X',Y)$

$\therefore D(X',Y') \geq D(X',Y) - D(Y',Y)$

$\geq (t+1) - t \geq 1$.

then, we can prove that t errors can be correcd. which that deleted e components erased can be recovered.

II. when $d_{min}(X,Y) \geq e+1$. in the problem we prove that is correct. And. only one in the code. with erased. can be recovered.

So. when $d_{min}(X,Y) \geq (2t+1)+(e+1)$ together, 1 is share

So $d_{min}(X,Y) \geq 2t+e+1$. is can correcting t errors and. e erasures.

problem 4. As the problem say, $N(X,Y)$ is the number of $1 \to 0$ crossovers from X to Y, $D_A = \max\{N(X,Y), N(Y,X)\}$.

Assume that $D_A = \max\{N(X,Y), N(Y,X)\} \geq t+1$ can correctiy the t asymmetric errors. let call $D_A(X,Y) = \max\{N(X,Y), N(Y,X)\}$.

so we need to prove $D_A(X,Y) \geq t+1$.

$S_X = \{X' \mid N(X,X') \leq t\}$

$S_Y = \{Y' \mid N(Y,Y') \leq t\}$.

so, if $N(X,Y) > N(Y,X)$ that mean the X sub have $(t+1)$ bits.

if $N(Y,X) > N(X,Y)$ is same meaning the y sub have $(t+1)$ bits

if Y sucb. is $(t+1)$ bits, then $Y_i = 1$, $X_i = 0$. the only asymmetric. errors. we call the error is K, that $X_i = 0 \Rightarrow K_i = 0$. in Y sub.

So, when $\forall$ $\bar{K}$ and $\bar{Y}$ we can get the $K_i = 0$, $Y_i = 1$. ($X_i = 0 \Rightarrow K_i = 0$).

so ~~$d_A(X,Y)$~~ $d_A(Y,X) \geq t+1$. ~~$\bar{K} \notin \bar{Y}$~~. $\bar{K} \notin S_Y$

so $S_X \cap S_Y = \emptyset$.

when $d_A = \max\{N(X,Y), N(Y,X)\} \geq t+1$, we can correct t asymmetric errors.

Problem 5. As the problem, we can get $D_{max}(X,Y) = \max\{|X_i - Y_i|\}$ for all i.

we need to prove the minimum of $D_{max}(X,Y) \geq l+1$ can correctiy all symmetric errors of limited magnitude l.

~~So, $S_X = \{X' \mid D(X,X') \leq l\}$ $S_Y = \{Y' \mid D(Y,Y') \leq l\}$~~.

Because X, and Y are vector, so assume $\vec{X'} \in (\vec{X}, l)$ is the error. vecty, $\vec{Y'} \in (\vec{Y}, l)$ is same as $\vec{X'}$

Assume $X_i > Y_i$, so the $D_{max} = X_i - Y_i \Rightarrow \vec{X'} \in (\vec{X}, l)$ $\vec{X'} = (X_i - l, \cdots)$

$\therefore$ $D_{max}(X,Y) \geq l+1$.

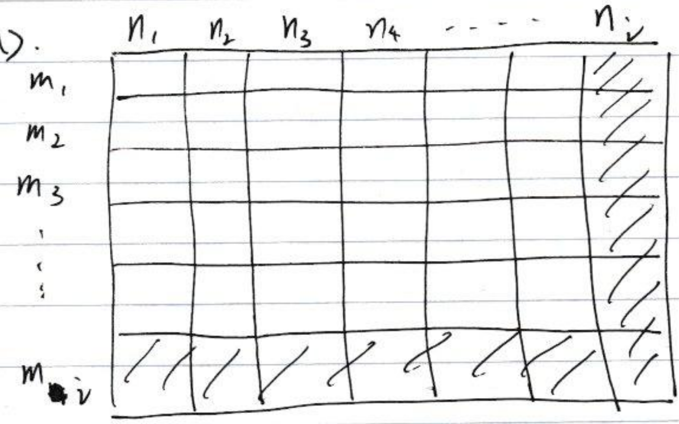
$X_i - Y_i \geq l+1$.

$X_i' - Y_i \geq l+1-l$

$X_i' - Y_i \geq 1$

So, it is meaning ~~when~~ the error vector $\vec{x'}$ didn't come Y, by the l. it come from the X. so $S_x \cap S_y = \emptyset$.
we can prove that. $D_{max}(x, Y) = \max\{|x_i - Y_i|\} \geq l+1$ can correctly all symmetric errors of limited magnitude l.

problem 6. a).



for the question, we can know that the last row and colum are the even parity

For single error correction:

m \ n	1	2	3	4	
1	0	0	0	1	1
2	1	$\boxed{0}$	0	1	0
3	1	1	0	1	1
4	ϕ	0	0	0	1
	1	1	0	1	

if the single error at $m_2 n_2$

m \ n		↓			
	0	0	0	1	1
	1	1	0	1	0
	1	1	0	1	1
	1	0	0	0	1
	1	1	0	1	

As the example shows, the $m_2 n_2$ will. at n_2 and m_2 ~~take the~~ check the even parity get the result are affected. then, the m_2 and n_2 intersection will find the single error location, then we can check it.

if the error location is $m_i n_j$, the last row and colum can find the error and check it to corrected it.

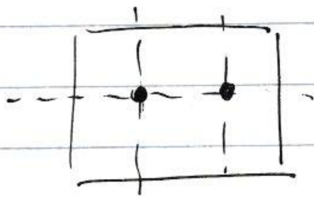
For , double error correction :

we have three of the following ;

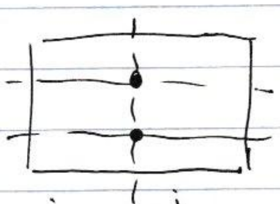
1 two errors are in same row different colum

2 two errors are in same colum different row

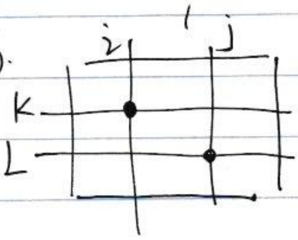
3 two errors are in different row and different colum.

1. 

also the row parity is not affected, but the colum get the affected, so we can find the double error detected.

2 

the same colum diff row is same as 1. the colum parity didn't affected but row can find the double errors.

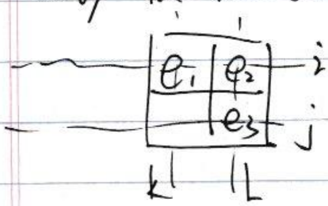
3. 

the different row and colum, we can get 4 point the {Ki, Kj, Li, Lj} the two error in this sub, so it be detected.

b). Because we need to find misdetected of 3 errors are miscorrected. as a part, we can know, the 3 error on one row or one colum can be find ;

when 3error on different row and difference colum can be detected.

only in this case : ~~two of 3error on one row~~



two error share a row and two share a colum. Then we can't find them.

in this case. i the parity no affected. only find j.

k L no affected only find K.

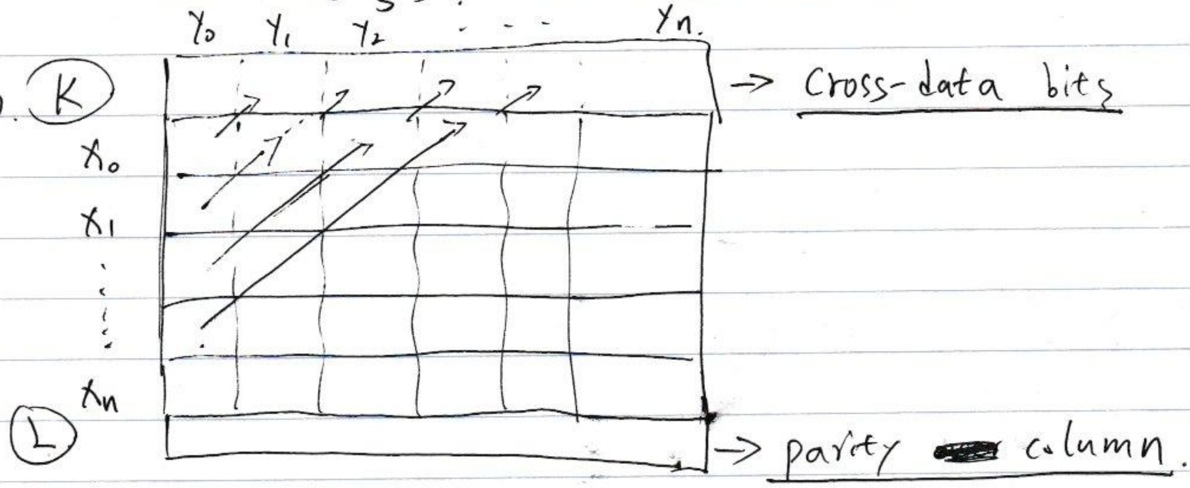
so in this case we find a light code be error both

In this case, we can't find the ~~mis~~location, that is 3error miscorrected and/or misdetected.

The A = mis correction B = triple error. for this case.

$$P(A|B) = \frac{m \cdot n \cdot (m-1) \cdot (n-1)}{C^{mn}_{3}}.$$

Problem 7. a)



As the question, we can know that One of the parity rows is the even parity over the column information bits. The other pairty row and the parity column are the parities over the cross-data bits.

For the any errors in a row ☆
we can use K (cross-data bits parity) and L (colum parity) to find the affected location.

b). if in a ~~line~~ row have the error.
we can use L (parity of column) find the location of column,
Then; we use the K (cross-data) parity can find the location of the row. Then we can correct the error code in a row.