CS 321 HW 4 – 30 points

1. (9 pts) Give context-free grammars that generate the following languages.
   a. $L_1 = \{ w \in \{0, 1\}^* \mid w \text{ contains at least three 1s} \}$
   b. $L_2 = \{ w \in \{0, 1\}^* \mid w = w^R \text{ and } |w| \text{ is even} \}$
   c. $L_3 = \{ a^i b^j c^k \mid i, j, k \geq 0, \text{ and } i = j \text{ or } i = k \}$

2. (5 pts) Consider the set of terminals $T = \{a, b, (,), +, *, \varnothing\}$. Construct a context-free grammar $G=(V,T,S,P)$ that generates all strings in T^* that are regular expressions over $\Sigma = \{a, b\}$. Use the grammar to derive the regular expression $(a+b)^*$.

   Recall the rules of regular expressions.

   1. $\varnothing$ and each member of Σ is a regular expression.
   2. If r_1 and r_2 are regular expressions, then so is $r_1 r_2$
   3. If r_1 and r_2 are regular expressions, then so is $r_1 + r_2$.
   4. If r_1 is a regular expression, then so is r_1^*.
   5. If r_1 is a regular expression, then so is (r_1).
   6. Nothing else is a regular expression.

3. (9 pts) Consider the following grammar $G = (\{S, A\}, \{a, b\}, S, P\}$ where P is defined below

   $S \rightarrow SS \mid AAA \mid \lambda$

   $A \rightarrow aA \mid Aa \mid b$

   a. Describe the language generated by this grammar.
   b. Give a left-most derivation for the terminal string abbaba.
   c. Show that the grammar is ambiguous by exhibiting two distinct derivation trees for some terminal string.

   d. If this language is regular, give a regular grammar generating it. If the language is not regular, prove that it is not.

4. (3 pts) Find an s-grammar for $L = \{a^n b^{n+1} : n \geq 1 \}$.

5. (4 pts) Let $L = \{ a^n b^n : n \geq 0 \}$

   a. Show that L^2 is a context-free language.

   b. Show that L^* is a context-free language.