

1.

Because :

$S \rightarrow A \mid ABD \mid 0BB$   
 $A \rightarrow 0 \mid BAA$   
 $B \rightarrow BB \mid 1 \mid 2 \mid \lambda$   
 $C \rightarrow CD \mid 0$   
 $D \rightarrow D1 \mid DD$

First, because production C can never be reached, so remove C.

$S \rightarrow A \mid ABD \mid 0BB$   
 $A \rightarrow 0 \mid BAA$   
 $B \rightarrow BB \mid 1 \mid 2 \mid \lambda$   
 $D \rightarrow D1 \mid DD$

Second, remove  $\lambda$

$S \rightarrow A \mid ABD \mid 0BB \mid AD \mid 0B \mid 0$   
 $A \rightarrow 0 \mid BAA \mid AA$   
 $B \rightarrow BB \mid 1 \mid 2 \mid B$   
 $D \rightarrow D1 \mid DD$

Then, Remove unit production

$S \rightarrow BAA \mid AA \mid ABD \mid 0BB \mid AD \mid 0B \mid 0$   
 $A \rightarrow 0 \mid BAA \mid AA$   
 $B \rightarrow BB \mid 1 \mid 2$   
 $D \rightarrow D1 \mid DD$

Next, remove 0,1

$S \rightarrow BAA \mid AA \mid ABD \mid X_1BB \mid AD \mid X_1B \mid 0$   
 $A \rightarrow 0 \mid BAA \mid AA$   
 $B \rightarrow BB \mid 1 \mid 2$   
 $D \rightarrow DX_2 \mid DD$   
 $X_1 \rightarrow 0$   
 $X_2 \rightarrow 1$

Final, remove the production which have more than one non-terminals in them.

$S \rightarrow X_3A \mid AA \mid X_4D \mid X_1X_5 \mid AD \mid X_1B \mid 0$   
 $A \rightarrow 0 \mid X_3A \mid AA$   
 $B \rightarrow BB \mid 1 \mid 2$

$D \rightarrow DX_2 \mid DD$

$X_1 \rightarrow 0$

$X_2 \rightarrow 1$

$X_3 \rightarrow BA$

$X_4 \rightarrow AB$

$X_5 \rightarrow BB$

2.

Check the  $w_1 = babbc$  :

$S \rightarrow AB \mid AD \mid AC$

$A \rightarrow AA \mid a$

$B \rightarrow BB \mid AB \mid b$

$C \rightarrow AC \mid DC \mid c$

$D \rightarrow DD \mid b \mid c$

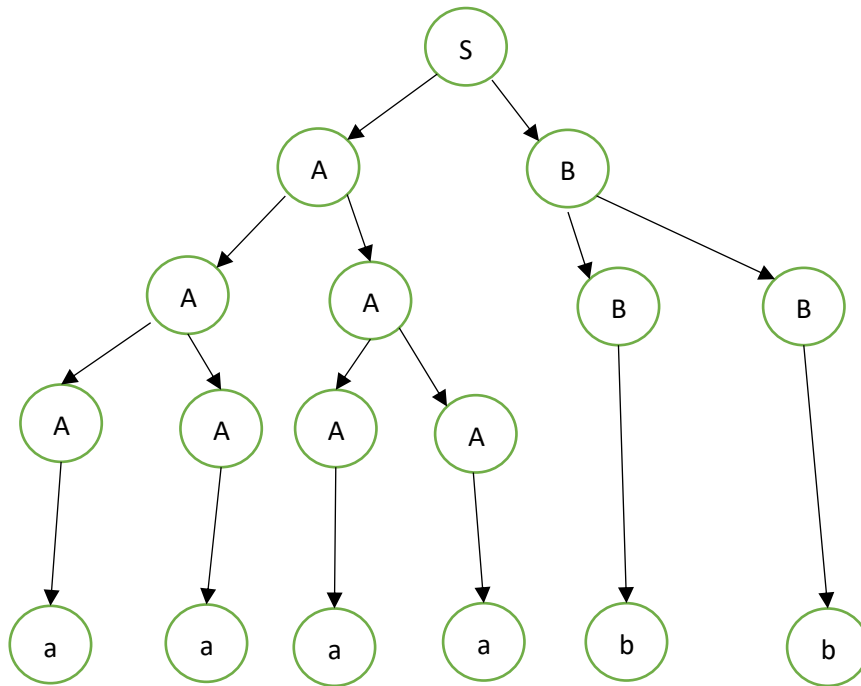
i/j	1	2	3	4	5
1	B,D	$\emptyset$	B	B	C
2		A	S,B	S,B	S,C
3			B,D	B,D	C,D
4				B,D	C,D
5					C,D

So,  $w_1 = babbc$ ,  $w_1$  is not in this language

Check the ,  $w_2 = aaaabb$ :

i/j	1	2	3	4	5	6
1	A	A	A	A	S,B	S,B
2		A	A	A	S,B	S,B
3			A	A	S,B	S,B
4				A	S,B	S,B
5					B,D	B,D
6						B,D

Therefore,  $w_2 = \text{aaaabb}$  is in this language,



3.

a.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{a, bb, \$\}$$

$$\delta =$$

$$(q_0, \lambda, \lambda) = \{q_1, \$\}$$

$$(q_1, a, \lambda) = \{q_1, a\}$$

$$(q_1, \lambda, \lambda) = \{q_2, \lambda\}$$

$$(q_2, bb, a) = \{q_2, \lambda\}$$

$$(q_2, \lambda, \$) = \{q_3, \$\}$$

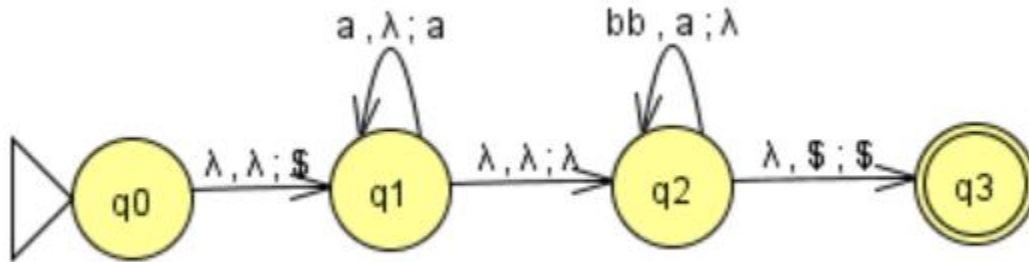
$$q_0 = \{q_0\}$$

$$Z = \text{empty}$$

$$F = \{q_3\}$$

In this question,

$L = \{ a^n b^{2n} : n \geq 0 \}$ , the b number is double a numbers, therefore, when we push a then we pop a, we need push double number b to pop a. When this is no more a in this stack then will end. Then the number of b is double number of a.



b.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b, c\}$$

$$\Gamma = \{a, b, c, \% \}$$

$$\delta =$$

$$(q_0, \lambda, \lambda) = \{q_1, \% \}$$

$$(q_1, a, \lambda) = \{q_1, 0\}$$

$$(q_1, a, 1) = \{q_1, \lambda\}$$

$$(q_1, b, 00) = \{q_1, \lambda\}$$

$$(q_1, b, \lambda) = \{q_1, 11\}$$

$$(q_1, c, \lambda) = \{q_1, \lambda\}$$

$$(q_1, b, 0\%) = \{q_1, 1\%\}$$

$$(q_1, \lambda, \% ) = \{q_2, \% \}$$

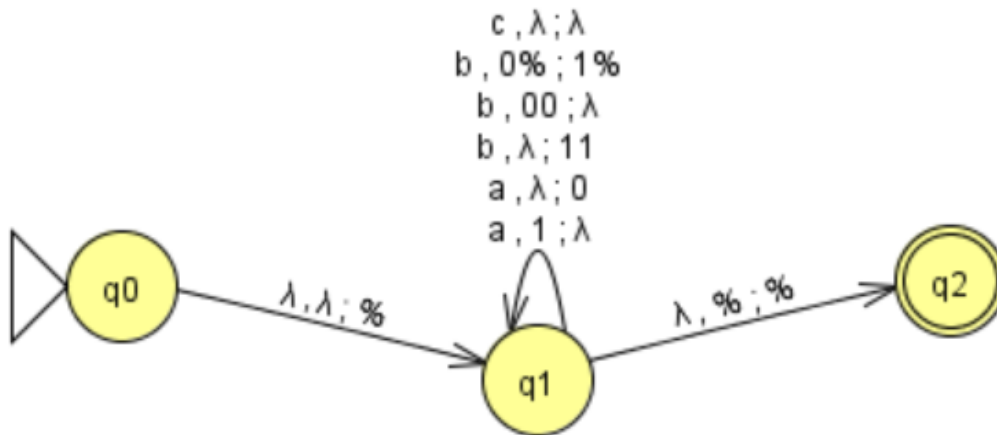
$$q_0 = \{q_0\}$$

$$Z = \text{empty}$$

$F = \{q_2\}$

In this language,

$L = \{ w : n_a(w) = 2n_b(w) \}$ , step1: when we read a, we push 0 if nothing in the stack. when we read a, we pop 1 if b in the stack. Step2: when we read b, we push 11 if nothing in the stack, when we read b, we pop 00 if stack have 00. Step3, we push 1% if we read 0% in the stack. When we read C, we do nothing, because doesn't in this language. When this is no more a in this stack then will end. Then the number of a is double number of b.



c.

$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{a, b, \$\}$

$\delta =$

$(q_0, \lambda, \lambda) = \{q_1, \$\}$

$(q_1, a, \lambda) = \{q_1, a\}$

$(q_1, b, \lambda) = \{q_1, b\}$

$(q_1, c, \lambda) = \{q_2, \lambda\}$

$(q_2, b, b) = \{q_2, \lambda\}$

$(q_2, a, a) = \{q_2, \lambda\}$

$(q_2, \lambda, \$) = \{q_3, \$\}$

$q_0 = \{q_0\}$

$Z = \text{empty}$

$F = \{q_3\}$

$L = \{ w c w_R : w \in \{a,b\}^* \}$ , step1: when we read a ,we push a if noting in the stack. when we read b ,we push b. Step2: when we read c, we go to next state, when we read a, we pop a or we read b pop b. When this is no more a in this stack then will end.

