XXZ自旋模型

模型的哈密顿量为：

$$H = -J\sum_i \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z\right)$$

利用自旋的升降算符可以将x和y方向的自旋化简如下：

$$\begin{cases} S^+ = S^x + iS^y \\ S^- = S^x - iS^y \end{cases} \Rightarrow \begin{cases} S^x = \frac{1}{2}(S^+ + S^-) \\ S^y = -\frac{i}{2}(S^+ - S^-) \end{cases}$$

$$S_i^x S_j^x = \frac{1}{4}(S_i^+ + S_i^-)(S_j^+ + S_j^-)$$

$$= \frac{1}{4}(S_i^+ S_j^+ + S_i^+ S_j^- + S_i^- S_j^+ + S_i^- S_j^-)$$

$$S_i^y S_j^y = \frac{-1}{4}(S_i^+ - S_i^-)(S_j^+ - S_j^-)$$

$$= -\frac{1}{4}(S_i^+ S_j^+ - S_i^+ S_j^- - S_i^- S_j^+ + S_i^- S_j^-)$$

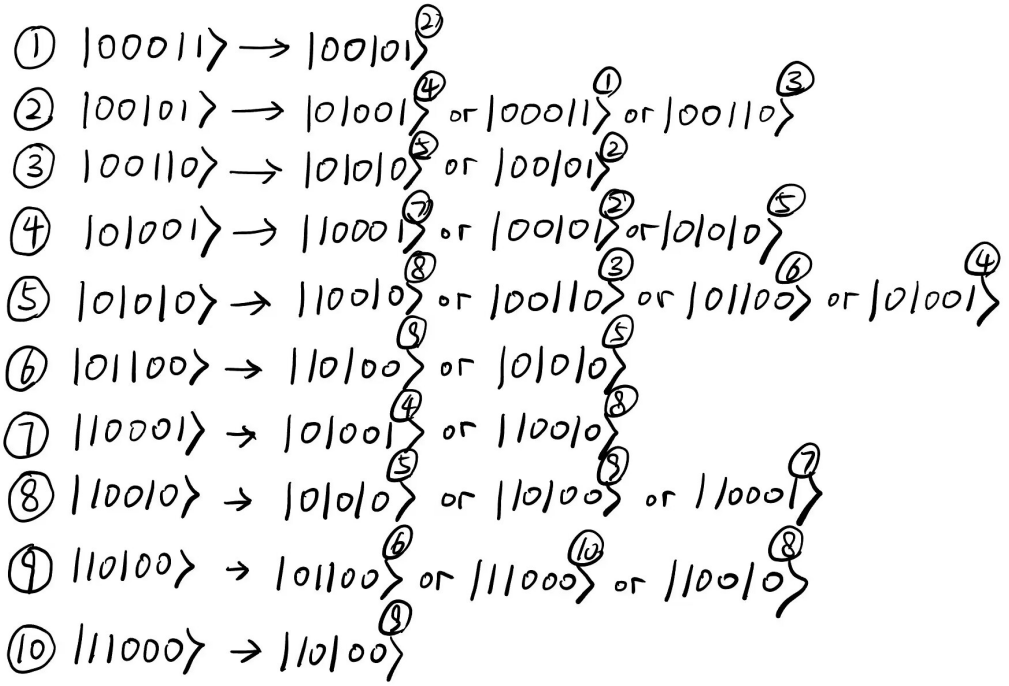
$$\Rightarrow S_i^x S_j^x + S_i^y S_j^y = \frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+)$$

如果我们将自旋向上定义为1态，向下定义为0态，那么上式会贡献如下的变换：

$$|01\rangle \rightarrow |10\rangle$$

$$|10\rangle \rightarrow |01\rangle$$

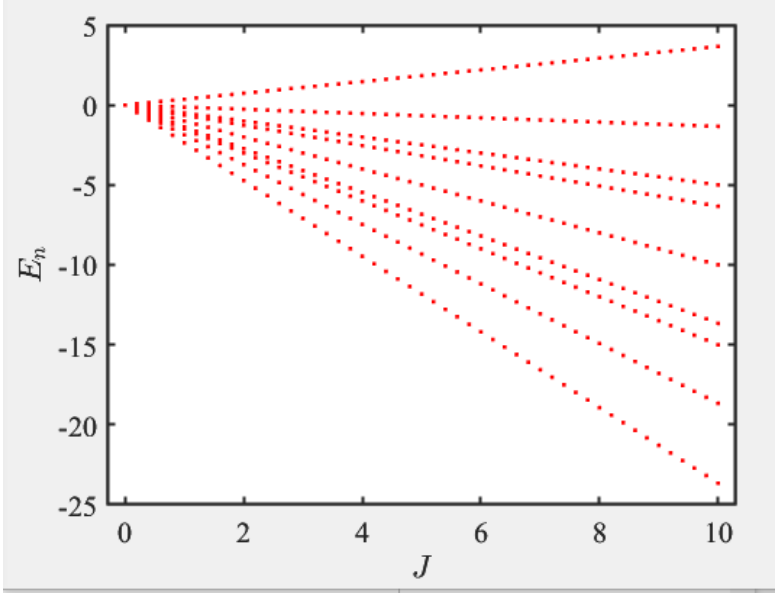
因而据此，我们列出L=5，N=2的OBC下的所以可能情况：

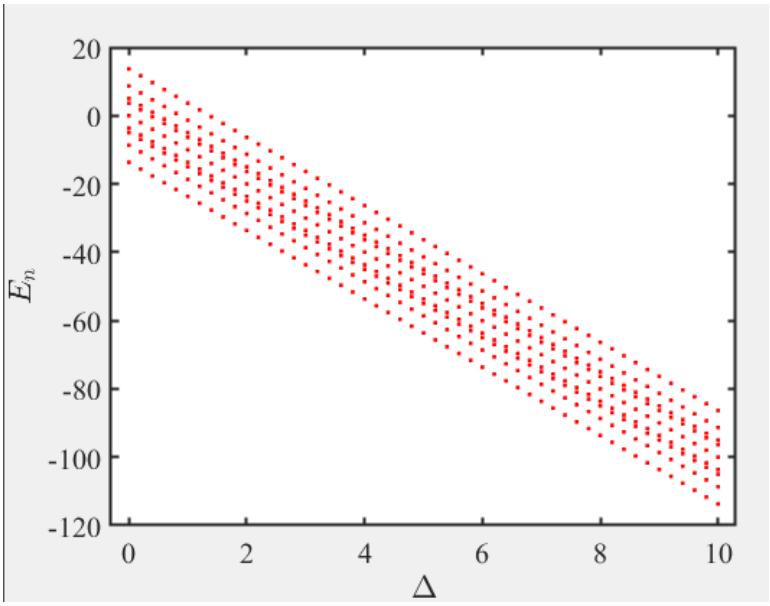


从而，我们可以写出如下的matlab程序进行对角化，求出能谱：

```
clear
dim = 10;
delta = 1;

for J = 0:0.2:10

Hz = diag(ones(1,dim)*(-J)*delta,0);

Hxy = zeros(dim,dim);
Hxy(1,2) = -0.5*J;
Hxy(2,1) = -0.5*J; Hxy(2,3) = -0.5*J; Hxy(2,4) = -0.5*J;
Hxy(3,2) = -0.5*J; Hxy(3,5) = -0.5*J;
Hxy(4,2) = -0.5*J; Hxy(4,5) = -0.5*J; Hxy(4,7) = -0.5*J;
Hxy(5,3) = -0.5*J; Hxy(5,4) = -0.5*J; Hxy(5,6) = -0.5*J; Hxy(5,8) = -0.5*J;
Hxy(6,5) = -0.5*J; Hxy(6,9) = -0.5*J;
Hxy(7,4) = -0.5*J; Hxy(7,8) = -0.5*J;
Hxy(8,5) = -0.5*J; Hxy(8,7) = -0.5*J; Hxy(8,9) = -0.5*J;
Hxy(9,6) = -0.5*J; Hxy(9,8) = -0.5*J; Hxy(9,10) = -0.5*J;
Hxy(10,9) = -0.5*J;

H = Hz + Hxy;

E = eig(H);

plot(J*ones(1,dim),E,'r.');
xlabel(['$J$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
```

```
hold on;

end


for delta = 0:0.2:10

Hz = diag(ones(1,dim)*(-J)*delta,0);

Hxy = zeros(dim,dim);
Hxy(1,2) = -0.5*J;
Hxy(2,1) = -0.5*J; Hxy(2,3) = -0.5*J; Hxy(2,4) = -0.5*J;
Hxy(3,2) = -0.5*J; Hxy(3,5) = -0.5*J;
Hxy(4,2) = -0.5*J; Hxy(4,5) = -0.5*J; Hxy(4,7) = -0.5*J;
Hxy(5,3) = -0.5*J; Hxy(5,4) = -0.5*J; Hxy(5,6) = -0.5*J; Hxy(5,8) = -0.5*J;
Hxy(6,5) = -0.5*J; Hxy(6,9) = -0.5*J;
Hxy(7,4) = -0.5*J; Hxy(7,8) = -0.5*J;
Hxy(8,5) = -0.5*J; Hxy(8,7) = -0.5*J; Hxy(8,9) = -0.5*J;
Hxy(9,6) = -0.5*J; Hxy(9,8) = -0.5*J; Hxy(9,10) = -0.5*J;
Hxy(10,9) = -0.5*J;

H = Hz + Hxy;

E = eig(H);

figure(2)
plot(delta*ones(1,dim),E,'r.');
xlabel(['$\Delta$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
hold on;

end
```





可以看到，随着J的增大，能级的劈裂越来越大；而z方向的delta仅仅类似于化学势一样，改变绝对能量大小，却不改变相对能差。

Spinless Fermi Hubbard

体系哈密顿量为

$$H = -J\sum_i \left(c_i^\dagger c_{i+1} + \text{h. c.}\right) + U n_i n_{i+1} - \mu n_i$$

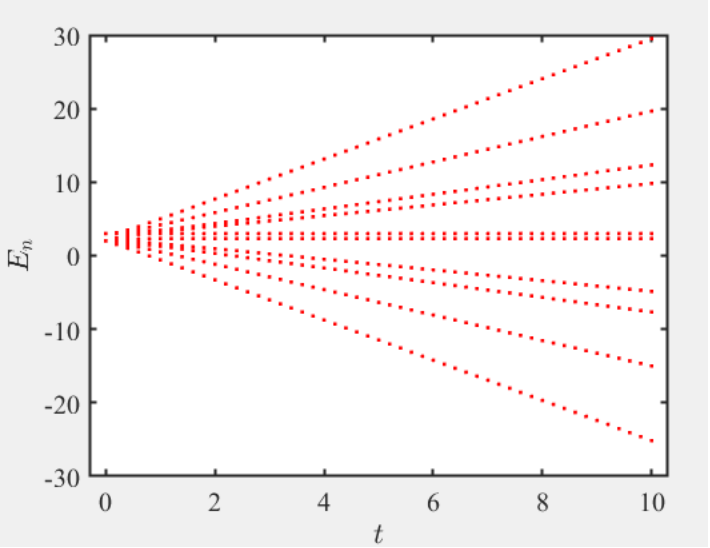
利用泡利不相容原理，可以列出L=5，N=2的所有态，并考虑OBC的边条,与之前类似，可以写出如下的matlab代码：

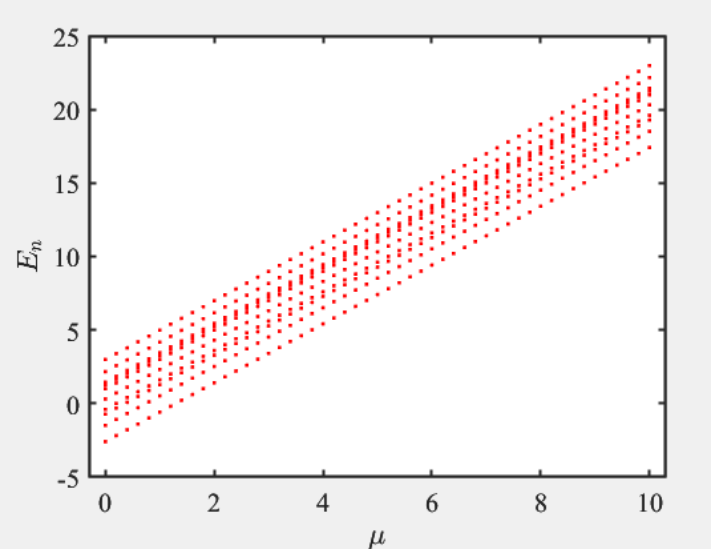
```
clear;
N = 2;
L = 5;
```

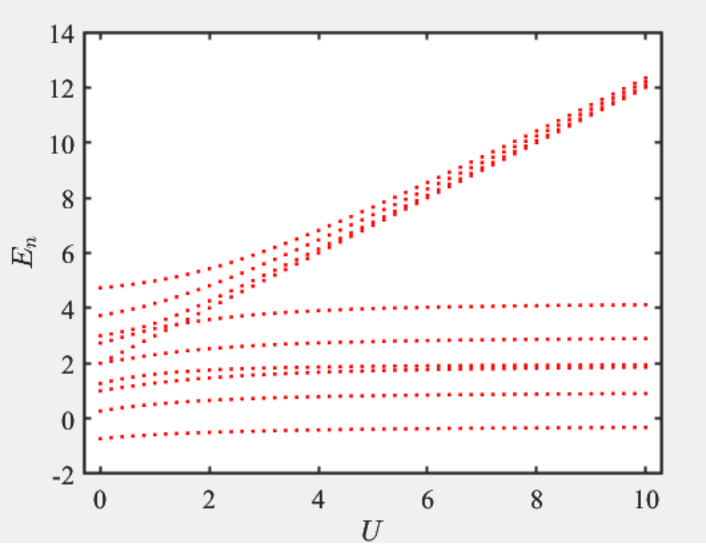
```
t = 1;
mu = 1;

ii = 1;
for n = 0:2^L-1
base = dec2bin(n,L);
if sum(base-'0') == N
basis(ii,:) = base;
basiss(ii) = n;
ii = ii+1;
end
end
dim = length(basis);

Hmu = N*diag(ones(1,dim),0);

Hu = zeros(dim,dim);
for di =1:dim
Hu(di,di) = sum(diff(find(basis(di,:)-'0'==1))==1);
end

Ht = zeros(dim,dim);
for di = 1:dim
for dj = di:dim
for li = 1:L-1
bij = abs(basiss(dj) - basiss(di));
if abs(bij - 2^(li-1)) < 0.00001
Ht(di,dj) = Ht(di,dj) + 1;
end
end
Ht(dj,di) = Ht(di,dj);
end
end

for U = 0:0.2:10
H = mu*Hmu+(-t)*Ht+U*Hu;
E = eig(H);
plot(U*ones(1,length(E)),E,'r.');
xlabel(['$U$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
hold on

end

U = 1;

for mu = 0:0.2:10
H = mu*Hmu+(-t)*Ht+U*Hu;
E = eig(H);
```

```
figure(2);
plot(mu*ones(1,length(E)),E,'r.');
xlabel(['$\mu$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
hold on

end

mu = 1;

for t = 0:0.2:10
H = mu*Hmu+(-t)*Ht+U*Hu;
E = eig(H);
figure(3);
plot(t*ones(1,length(E)),E,'r.');
xlabel(['$t$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
hold on

end


tlist = 0:0.1:2*pi;
for ti = 1:length(tlist)
theta = tlist(ti);
t = sin(theta);
U = cos(theta);
H = mu*Hmu+(-t)*Ht+U*Hu;
E(:,ti) = eig(H);
end

figure(4)
plot(tlist,E,'r.');
xlabel(['$\theta$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0,2*pi]);
```

分别变化t, μ, U，并保持其他参数为1，可以绘制如下能谱：

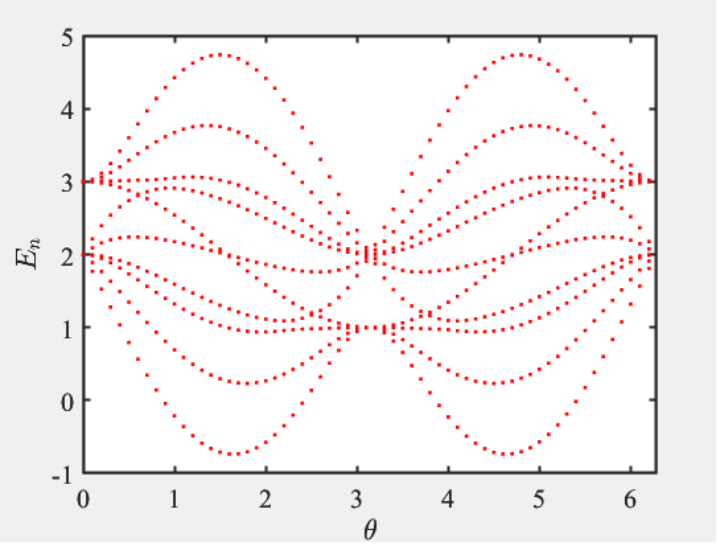






可以看到，随着t的增大，能级的劈裂越来越大；而化学势改变绝对能量大小，却不改变相对能差。对于相互作用能U，其能谱难以简单概况，有一部分能级相互靠近且不断升高，而剩下

的几乎保持不变，这也许就是Hubbard model的困难和微妙的体现。如果我们设t=sinθ而U=cosθ，可以绘制如下的能谱图：



Spinless Bose Hubbard

体系的哈密顿量为

$$H = -J\sum_i \left(c_i^\dagger c_{i+1} + \text{h. c.}\right) + \frac{U}{2} n_i(n_i - 1) \; - \mu n_i$$

由于boson不受泡利不相容原理的限制，因此同一个态上允许存在多个粒子，从而matlab程序改写如下：

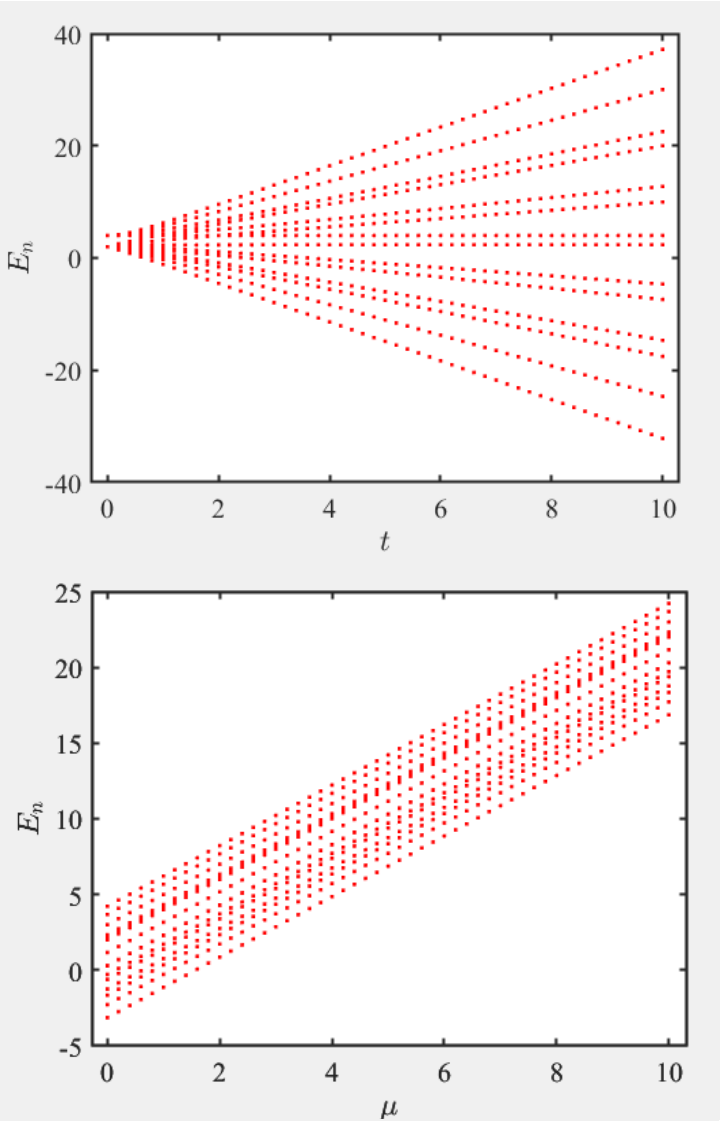
```
clear;
N = 2;
L = 5;
t = 1;
mu = 1;
Q = 3;

ii = 1;
for n = 0:Q^L-1
base = dec3bin(n,L);
if sum(base-'0') == N
basis(ii,:) = base;
basiss(ii) = n;
ii = ii+1;
end
end
dim = length(basis);

Hmu = N*diag(ones(1,dim),0);

Hu = zeros(dim,dim);
for di =1:dim
base = [];
for li =1:L
base = [base str2num(basis(di,li))];
```

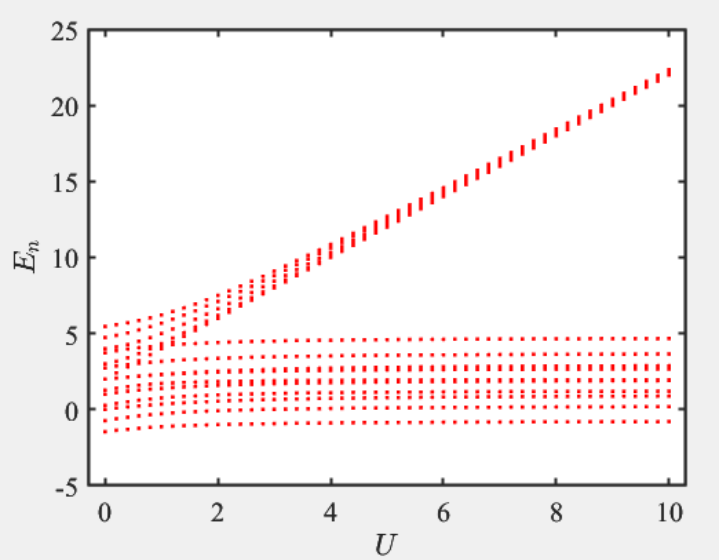
```
end

Hu(di,di) = sum(base.*(base-1));
end

Ht = zeros(dim,dim);
for di = 1:dim
for dj = di:dim
for li = 1:L-1
bij = basiss(dj) - basiss(di);
if abs(bij - 2*3^(li-1)) < 0.00001
state = basis(dj,:);
aa = sqrt(str2num(state(L-li+1))+1)*...
sqrt(str2num(state(L-li)));
Ht(di,dj) = Ht(di,dj) + aa;
end
end
Ht(dj,di) = Ht(di,dj);
end
end

for U = 0:0.2:10
H = mu*Hmu+(-t)*Ht+U*Hu;
E = eig(H);
plot(U*ones(1,length(E)),E,'r.');
xlabel(['$U$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
hold on

end

U = 1;

for mu = 0:0.2:10
H = mu*Hmu+(-t)*Ht+U*Hu;
E = eig(H);
figure(2);
plot(mu*ones(1,length(E)),E,'r.');
xlabel(['$\mu$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
hold on

end
```

```
mu = 1;

for t = 0:0.2:10
H = mu*Hmu+(-t)*Ht+U*Hu;
E = eig(H);
figure(3);
plot(t*ones(1,length(E)),E,'r.');
xlabel(['$t$'],'Interpreter','latex');
ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0-0.3,10+0.3]);
hold on

end

tlist = 0:0.1:2*pi;
 for ti = 1:length(tlist)
 theta = tlist(ti);
 t = sin(theta);
 U = cos(theta);
 H = mu*Hmu+(-t)*Ht+U*Hu;
 E(:,ti) = eig(H);
 end
 figure(4)
 plot(tlist,E,'r.');
 xlabel(['$\theta$'],'Interpreter','latex');
 ylabel(['$E_n$'],'Interpreter','latex');
fonts=15;
set(gca,'FontSize',fonts);
set(gca,'FontName','Times');
set(gca,'LineWidth',1.5)
xlim([0,2*pi]);



function out = bin3dec( a )

out = 0;
for ii = 1:length(a)
out = out+ str2num(a(ii))*3^(length(a)-ii);
end

end


function out = dec3bin(b,L)
out=[];
while (b>0)
c=mod(b,3);
out=[num2str(c) out];
```

```
b=(b-c)/3;
end

for ii = 1:L-length(out)
out = ['0' out];
end
end
```

分别变化t, μ, U，并保持其他参数为1，可以绘制如下能谱：





可以看到，随着t的增大，能级的劈裂越来越大；而化学势改变绝对能量大小，却不改变相对能差。对于相互作用能U，其能谱难以简单概况，有一部分能级相互靠近且不断升高，而剩下的几乎保持不变，这也许就是Hubbard model的困难和微妙的体现。如果我们设t=sinθ而U=cosθ，可以绘制如下的能谱图：