Bogoliubov变换

In[12]:= $$H=\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+\frac{1}{2}k(x_1^2+x_2^2)+\lambda x_1x_2$$

Out[12]= $$H=\frac{p_1^2}{2m_1}+\frac{p_2^2}{2m_2}+\frac{1}{2}k(x_1^2+x_2^2)+\lambda x_1x_2$$

1.运动方程法

利用海森堡方程，并设约化普朗克常数为1，求得哈密顿量矩阵为：

In[13]:=
```
H = I{{0,0,1/m1,0},{0,0,0,1/m2},{-k,-λ,0,0},{-λ,-k,0,0}};
H//TraditionalForm
```

Out[14]//TraditionalForm=

$$\begin{pmatrix} 0 & 0 & \frac{i}{\mathrm{m1}} & 0 \\ 0 & 0 & 0 & \frac{i}{\mathrm{m2}} \\ -i\,k & -i\,\lambda & 0 & 0 \\ -i\,\lambda & -i\,k & 0 & 0 \end{pmatrix}$$

Out[6]//TraditionalForm=

$$\begin{pmatrix} 0 & 0 & \frac{i}{\mathrm{m1}} & 0 \\ 0 & 0 & 0 & \frac{i}{\mathrm{m2}} \\ -i\,k & -i\,\lambda & 0 & 0 \\ -i\,\lambda & -i\,k & 0 & 0 \end{pmatrix}$$

求得其本征值为：

In[15]:=
```
H//Eigenvalues//Simplify
```

Out[15]= $$\left\{-\frac{i\sqrt{-\mathrm{m1\,m2}\left(k\,(\mathrm{m1}+\mathrm{m2})+\sqrt{k^2\,(\mathrm{m1}-\mathrm{m2})^2+4\,\mathrm{m1\,m2}\,\lambda^2}\right)}}{\sqrt{2}\,\mathrm{m1\,m2}},\right.$$
$$\frac{i\sqrt{-\mathrm{m1\,m2}\left(k\,(\mathrm{m1}+\mathrm{m2})+\sqrt{k^2\,(\mathrm{m1}-\mathrm{m2})^2+4\,\mathrm{m1\,m2}\,\lambda^2}\right)}}{\sqrt{2}\,\mathrm{m1\,m2}},$$
$$-\frac{i\sqrt{\mathrm{m1\,m2}\left(-k\,(\mathrm{m1}+\mathrm{m2})+\sqrt{k^2\,(\mathrm{m1}-\mathrm{m2})^2+4\,\mathrm{m1\,m2}\,\lambda^2}\right)}}{\sqrt{2}\,\mathrm{m1\,m2}},$$
$$\left.\frac{i\sqrt{\mathrm{m1\,m2}\left(-k\,(\mathrm{m1}+\mathrm{m2})+\sqrt{k^2\,(\mathrm{m1}-\mathrm{m2})^2+4\,\mathrm{m1\,m2}\,\lambda^2}\right)}}{\sqrt{2}\,\mathrm{m1\,m2}}\right\}$$

2.Bogoliubov变换

首先将哈密顿量用产生湮灭算符代替：

$$H=\sqrt{\frac{k}{m_1}}(a^\dagger a)+\sqrt{\frac{k}{m_2}}(b^\dagger b)+\frac{\lambda}{2\sqrt{k}\sqrt{\sqrt{m_1 m_2}}}(a^\dagger b^\dagger+ab+ab^\dagger+ba^\dagger)+Const.$$

由此矩阵可以写为：

In[16]:=

$$\text{B=}\frac{1}{2}\left\{\left\{\sqrt{\frac{\text{k}}{\text{m1}}},\frac{\lambda}{2\sqrt{\text{k}}\sqrt{\sqrt{\text{m1*m2}}}},0,\frac{\lambda}{2\sqrt{\text{k}}\sqrt{\sqrt{\text{m1*m2}}}}\right\},\left\{\frac{\lambda}{2\sqrt{\text{k}}\sqrt{\sqrt{\text{m1*m2}}}},\sqrt{\frac{\text{k}}{\text{m2}}},\frac{\lambda}{2\sqrt{\text{k}}\sqrt{\sqrt{\text{m1*m2}}}},0\right\},\left\{0,\frac{}{2\sqrt{\text{k}}\,\cdots}\right.\right.$$

```
B//TraditionalForm
```

Out[17]//TraditionalForm=

$$\begin{pmatrix} \frac{\sqrt{\frac{k}{\text{m1}}}}{2} & \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} & 0 & \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} \\ \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} & \frac{\sqrt{\frac{k}{\text{m2}}}}{2} & \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} & 0 \\ 0 & \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} & \frac{\sqrt{\frac{k}{\text{m1}}}}{2} & \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} \\ \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} & 0 & \frac{\lambda}{4\sqrt{k}\sqrt[4]{\text{m1 m2}}} & \frac{\sqrt{\frac{k}{\text{m2}}}}{2} \end{pmatrix}$$

由于谐振子为可以看作是玻色子，因此需要乘以 Σ 矩阵再进行本征值的求解：

In[18]:=

```
Simplify[{{1,0,0,0},{0,1,0,0},{0,0,-1,0},{0,0,0,-1}}.B//Eigenvalues,Assumptions→{m1>0,m2>0,k
```

Out[18]=

$$\left\{-\frac{\sqrt{\text{k}\sqrt{\text{m1 m2}}\,(\text{m1}+\text{m2})-\sqrt{\text{m1 m2}\left(\text{k}^2(\text{m1}-\text{m2})^2+4\,\text{m1 m2}\,\lambda^2\right)}}}{2\sqrt{2}\,(\text{m1 m2})^{3/4}},\right.$$
$$\frac{\sqrt{\text{k}\sqrt{\text{m1 m2}}\,(\text{m1}+\text{m2})-\sqrt{\text{m1 m2}\left(\text{k}^2(\text{m1}-\text{m2})^2+4\,\text{m1 m2}\,\lambda^2\right)}}}{2\sqrt{2}\,(\text{m1 m2})^{3/4}},$$
$$-\frac{\sqrt{\text{k}\sqrt{\text{m1 m2}}\,(\text{m1}+\text{m2})+\sqrt{\text{m1 m2}\left(\text{k}^2(\text{m1}-\text{m2})^2+4\,\text{m1 m2}\,\lambda^2\right)}}}{2\sqrt{2}\,(\text{m1 m2})^{3/4}},$$
$$\left.\frac{\sqrt{\text{k}\sqrt{\text{m1 m2}}\,(\text{m1}+\text{m2})+\sqrt{\text{m1 m2}\left(\text{k}^2(\text{m1}-\text{m2})^2+4\,\text{m1 m2}\,\lambda^2\right)}}}{2\sqrt{2}\,(\text{m1 m2})^{3/4}}\right\}$$

再经过手动化简，不难发现，两种方法求出的结果是一致的！可见，Bogoliubov变换在保证bose子对易关系的情况下，用产生湮灭算符将哈密顿量做了重新的表达，而背后的物理是不会变的，因此两种方法的本征值是一致的。

In[19]:=

In[20]:=