OpenFOAM

Catalyst

Project Work

Minimal Surfaces

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TensorFields

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LaTeX template from [1]

1 Project Work

Assume a wire, shaped into a closed frame, e.g. a circle. If you put it into dilute liquid soap and then take it out, a soap film will form which tries to *minimize its area* due to surface tension, Figure 1.1. Our goal is to find that final surface shape, but for a fluctuating circular wire rather than a static one.

Figure 1.1: Thin film [2]

1.1 Problem Statement

Assume a circular wire frame in the $x - y$ plane whose boundary has a sinusoidal, time-dependent motion in the z direction, described by Eq.(1.2). The minimal surface, described by $z = f(x, y, t)$ and formed by a soap film is the solution of the PDE (1.1),

$$-\nabla.(\frac{1}{\sqrt{1 + |\nabla z|^2}}\nabla z) = 0 \quad \text{[3]} \qquad \text{in the domain} \qquad (1.1)$$

$$z = \sin(2\pi(x + y + t)) \quad \text{[3]} \qquad \text{on the boundary, i.e., on } x^2 + y^2 = R^2 \qquad (1.2)$$

where x, y and z are conventional coordinates, t is time, and R is the radius of the boundary. We assumed that adopting the new shape by the film is immediate (time-independent).

1.2 Expected solution

You are expected to

- Implement a solver that solves 1.1 constrained by 1.2; it should be compilable on a typical linux machine by running `wmake` without needing users to arrange for any extra setup;
- Prepare a test case along with a script that runs the solver;
- Create a short video showing the time-varying solution, or an image showing a snapshot of the solution.

A typical solution for $t = 0$ looks like Figure 1.2.

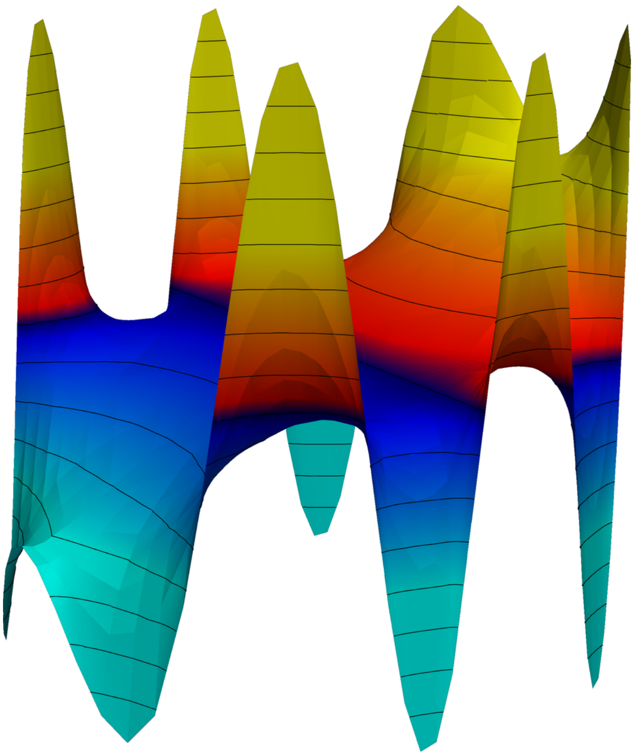

Figure 1.2: A typical solution of 1.1 and 1.2, [3]

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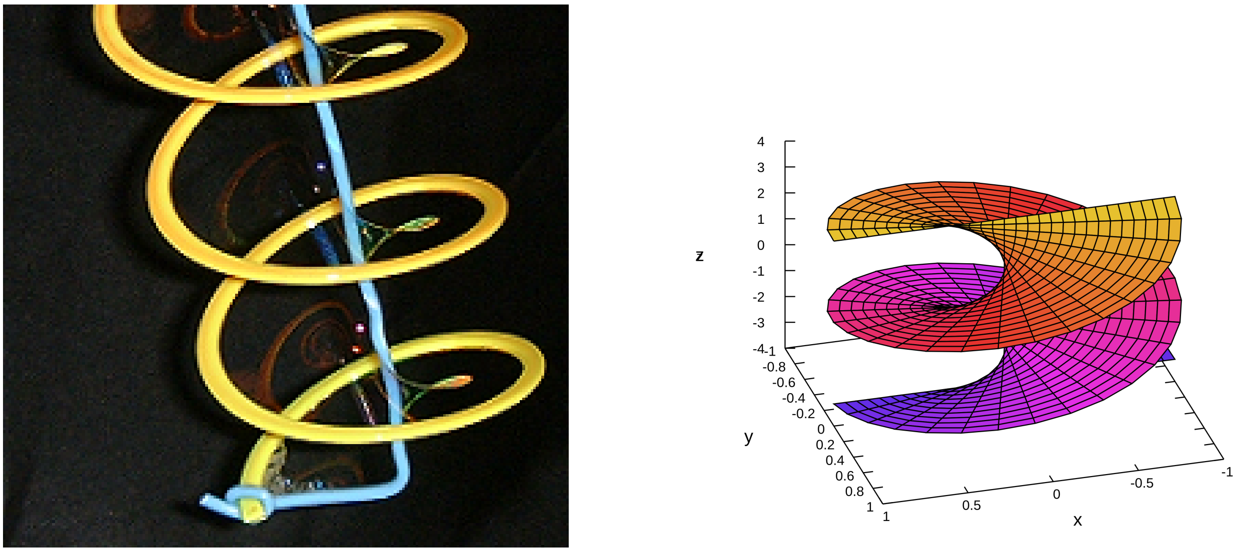
1.3 Possible Extensions

You can try other boundary conditions, e.g., helicoid, shown in Figure 1.3, and described by

$$
\begin{aligned}
&z = 0 \quad \text{at} \quad x = y = 0, \\
&z = \alpha \arctan(\frac{y}{x}) \quad \text{at} \quad x^2 + y^2 = R
\end{aligned}
\tag{1.3}
$$

where α and R are constant, and R is the outer radius of the helicoid.



(a) **(b)**

Figure 1.3: A soap film helicoid (a) [4], and an analytical one (b) [5].

Bibliography

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[2] UNSW School of Physics, "Soap Bubbles and Their Colors." https://www.animations.physics.unsw.edu.au/jw/light/soap-bubbles.htm, 2024. Accessed: 2024-06-07.

[3] S. Wetterauer, "deal.II, Step-15: Parallel Computation of Eigenvalues and Eigenvectors." https://dealii.org/current/doxygen/deal.II/step_15.html, 2012. Accessed: 2024-12-07.

[4] Wikipedia, "Minimal Surface." https://en.wikipedia.org/wiki/Minimal_surface, 2024. Accessed: 2024-12-07.

[5] Wikipedia, "Helicoid." https://en.wikipedia.org/wiki/Helicoid, 2024. Accessed: 2024-12-07.