

• ATNN / Lab 1 / Multiclass Logistic Regression

$$x = [1, 3, 0]$$

$$W = \begin{bmatrix} 0.3 & 0.1 & -2 \\ -0.6 & -0.5 & 2 \\ -1 & -0.5 & 0.1 \end{bmatrix}$$

$$b = [0.1, 0.1, 0.1]$$

$$y = [0, 1, 0]$$

$$\rightarrow z = W^T \cdot x + b \Rightarrow \begin{aligned} z_1 &= (0.3 \cdot 1 + (-0.6) \cdot 3 + (-1) \cdot 0) + 0.1 = \\ &= -1.5 + 0.1 = -1.4 \end{aligned}$$

$$\begin{aligned} z_2 &= (0.1 \cdot 1 + (-0.5) \cdot 3 + (-0.5) \cdot 0) + 0.1 = \\ &= -1.4 + 0.1 = -1.3 \end{aligned}$$

$$\begin{aligned} z_3 &= ((-2) \cdot 1 + 2 \cdot 3 + 0.1 \cdot 0) + 0.1 = \\ &= 4 + 0.1 = 4.1 \end{aligned}$$

$$z = [-1.4, -1.3, 4.1]$$

$$\hat{y} = \text{softmax}(z) = \frac{e^{z_i}}{\sum_{j=1}^3 e^{z_j}} \approx [0.004, 0.0045, 0.9915]$$

$$\begin{aligned} \nabla_z L &= \hat{y} - y = [0.004, 0.0045, 0.9915] - [0, 1, 0] = \\ &= [0.004, -0.9955, 0.9915] = \nabla_b L \end{aligned}$$

$$\begin{aligned} \nabla_W L &= \nabla_z L \cdot x^T = 0.004 \cdot 1 + (-0.9955) \cdot 3 + 0.9915 \cdot 0 = \\ &= 0.004 - 2.9865 = -2.9825 \end{aligned}$$

$$\begin{aligned} \text{let } \eta &= 0.2 \Rightarrow W = W - \eta \cdot \nabla_W L & \Leftrightarrow & W = W + 0.5965 = \begin{bmatrix} 0.8965 & 0.6965 & -1.4035 \\ -0.0035 & 0.0965 & 2.5965 \\ -0.4035 & 0.0965 & 0.6965 \end{bmatrix} \\ b &= b - \eta \cdot \nabla_b L & & b = b - [0.0008, 0.1991, 0.1983] \\ & & & = [0.0992, 0.2991, -0.0983] \end{aligned}$$