Theory of Evolutionary Algorithms

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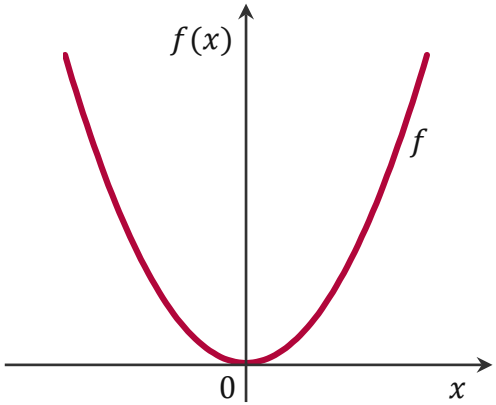
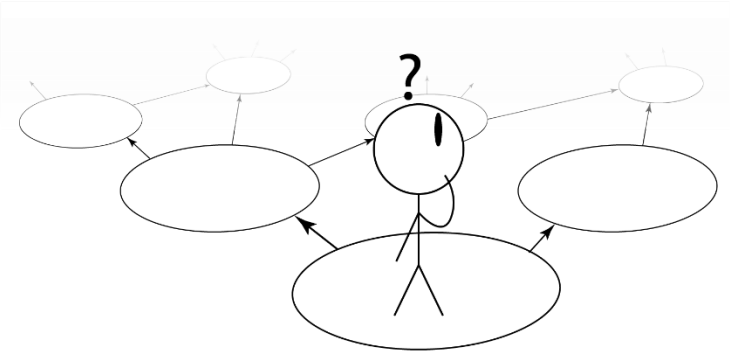
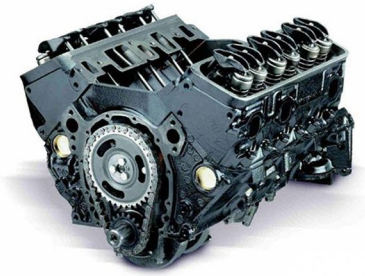
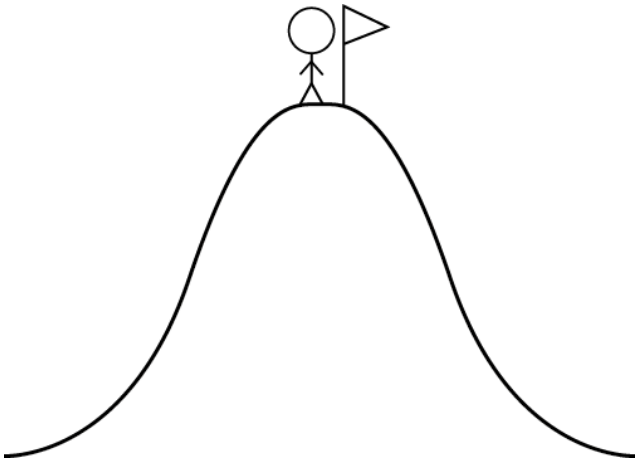
Mathematical Optimization

$$g(x,y,z) = (x,y,z)\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

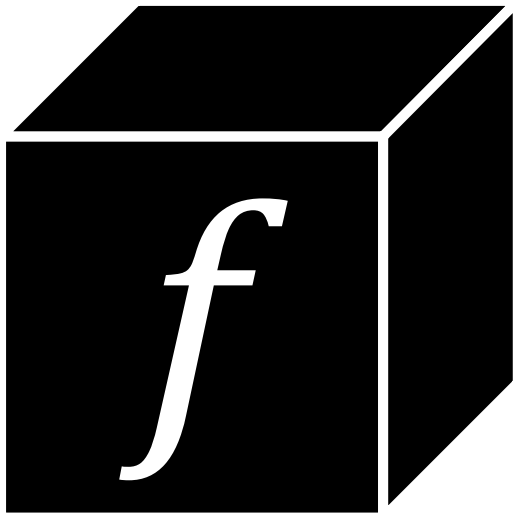






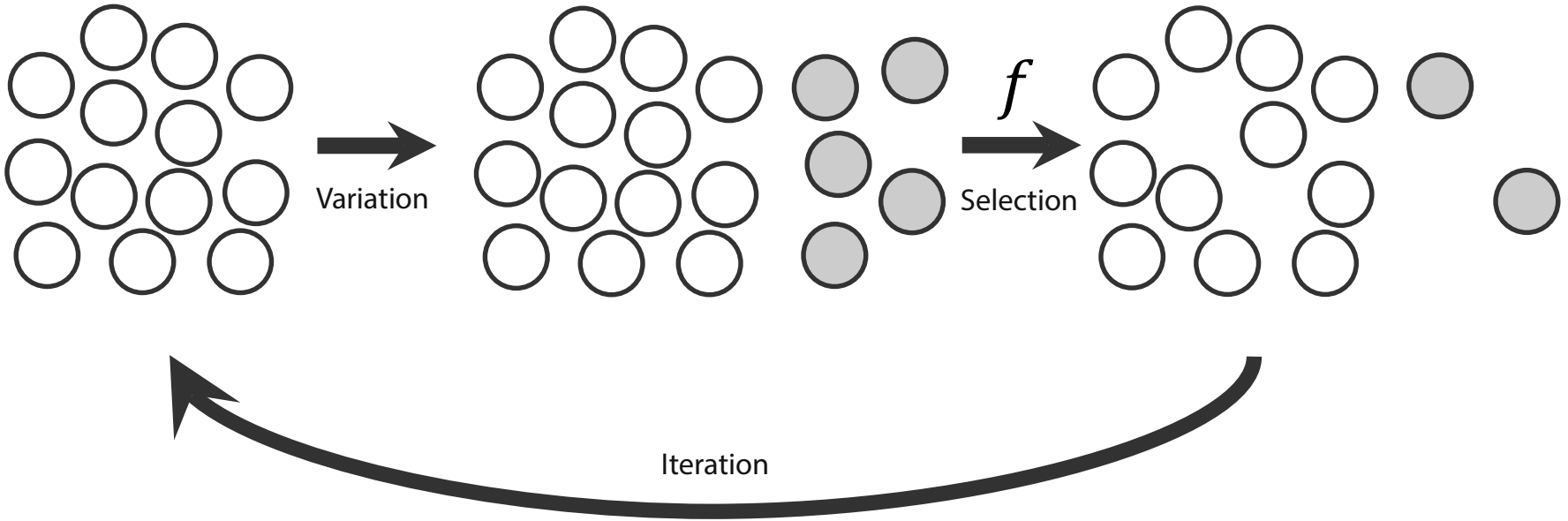


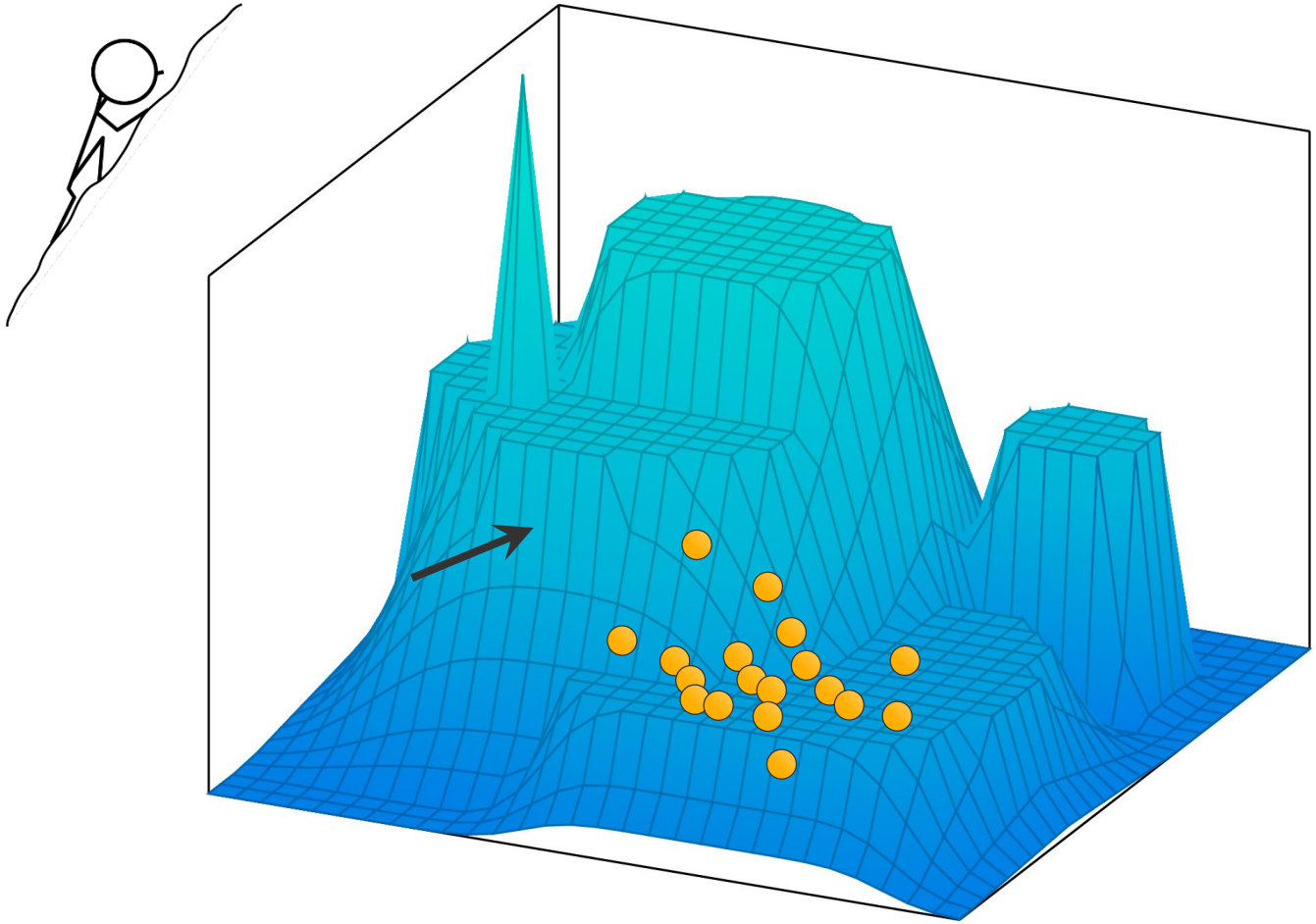
Optimization

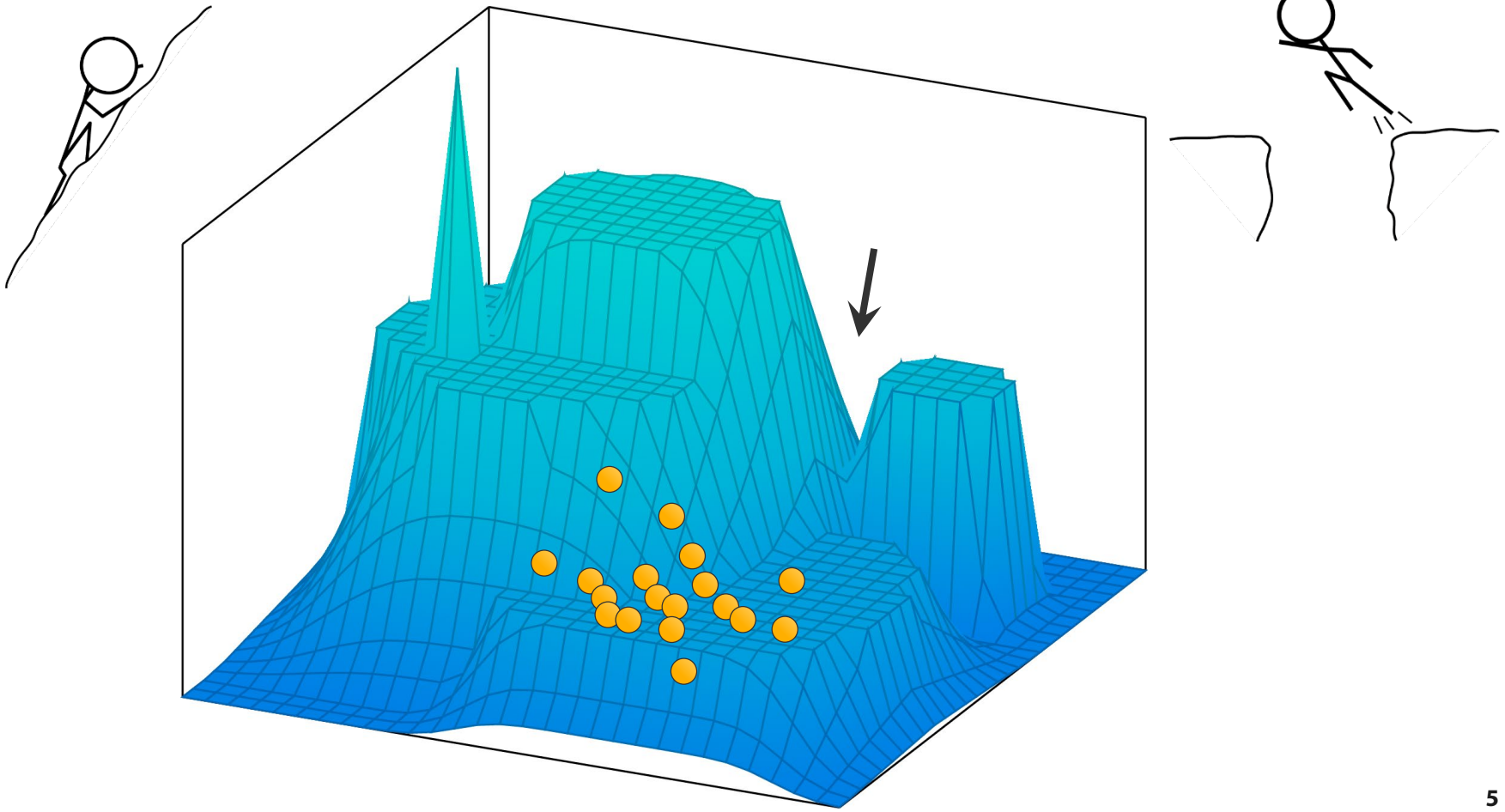


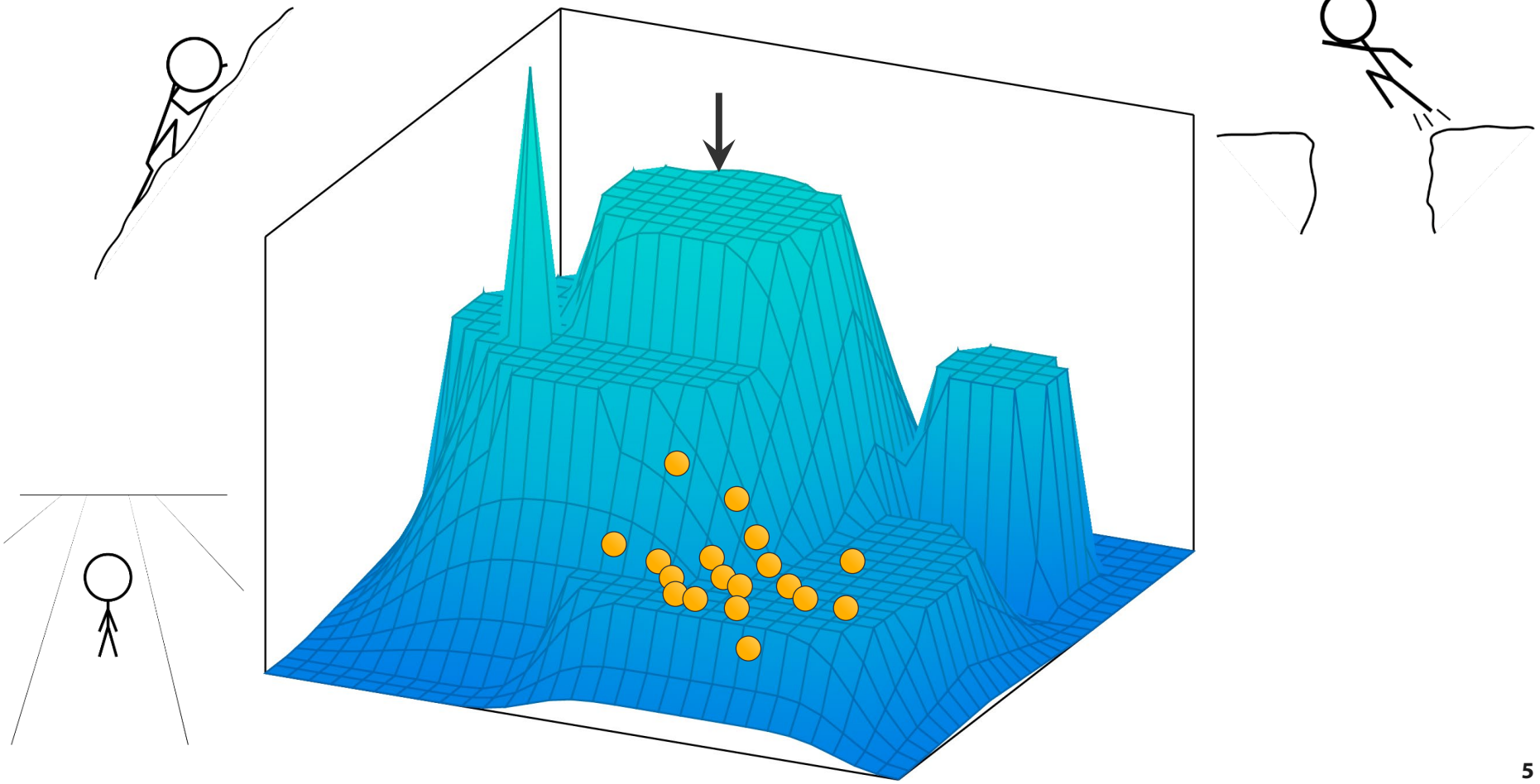
Black-box Optimization

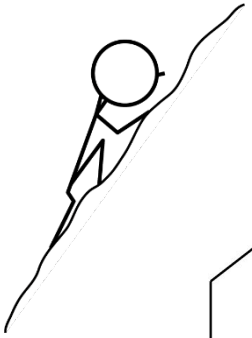
Evolutionary Algorithms

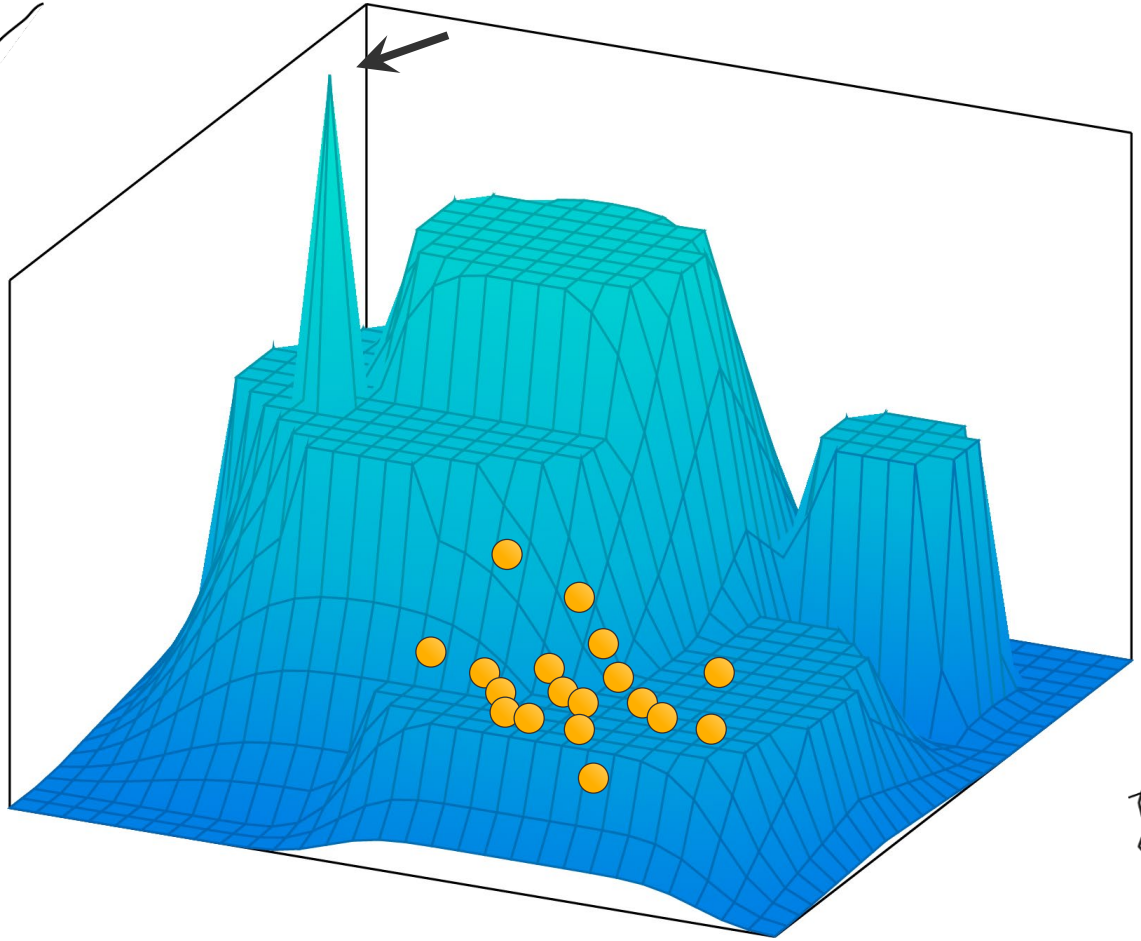


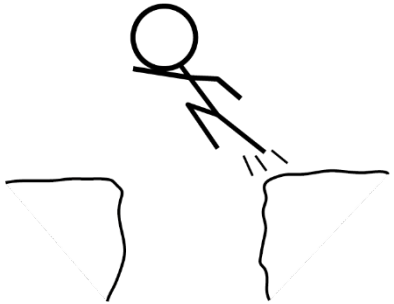


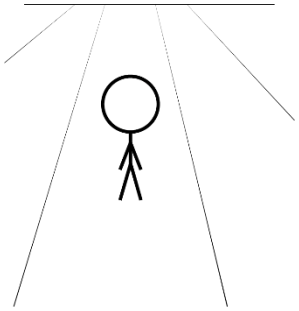


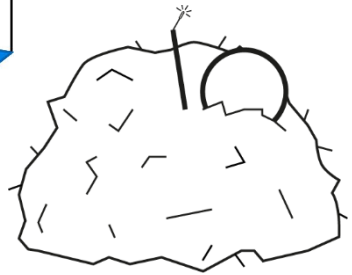




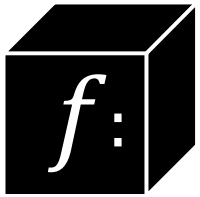




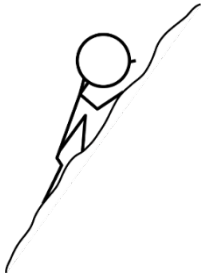




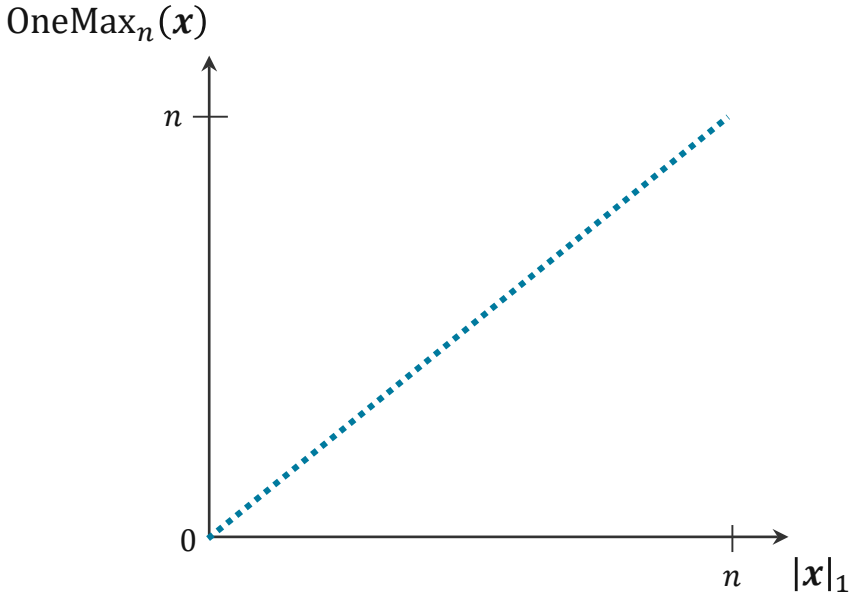
Theory Benchmarks



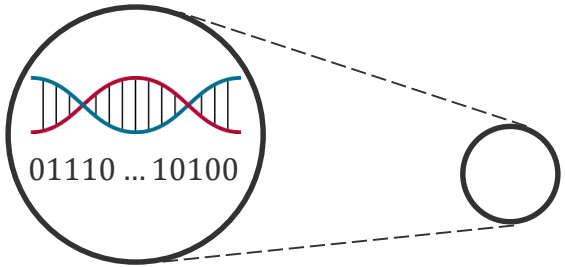
$$\{0,1\}^n \to \mathbf{R}$$



$$\text{OneMax}_n\colon \boldsymbol{x} \mapsto \sum_{i=1}^{n} \boldsymbol{x}_i$$



The (1+1) EA



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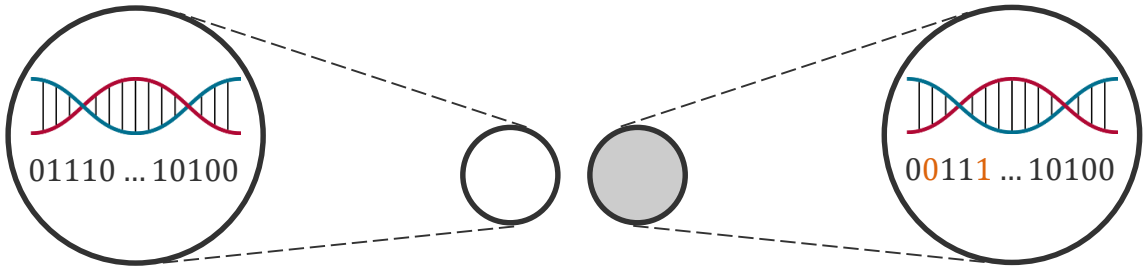

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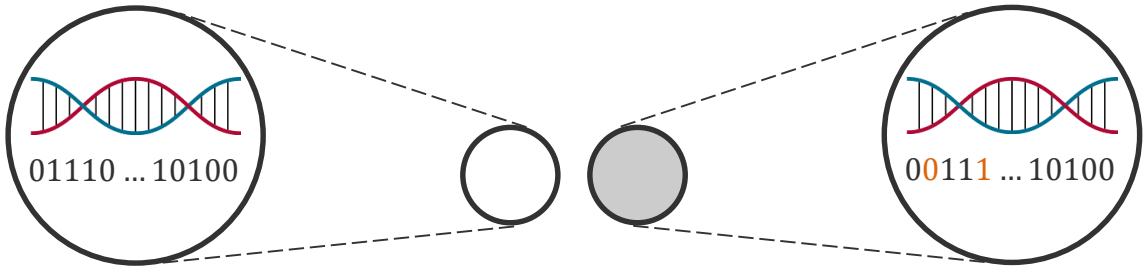
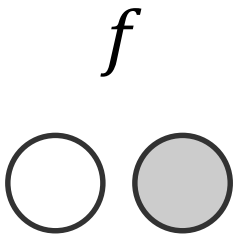


$$\Pr[\text{flipping exactly one bit}] = \left(1 - \frac{1}{n}\right)^{n-1} \geq \frac{1}{\mathrm{e}}$$

The (1+1) EA



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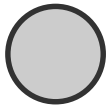

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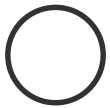
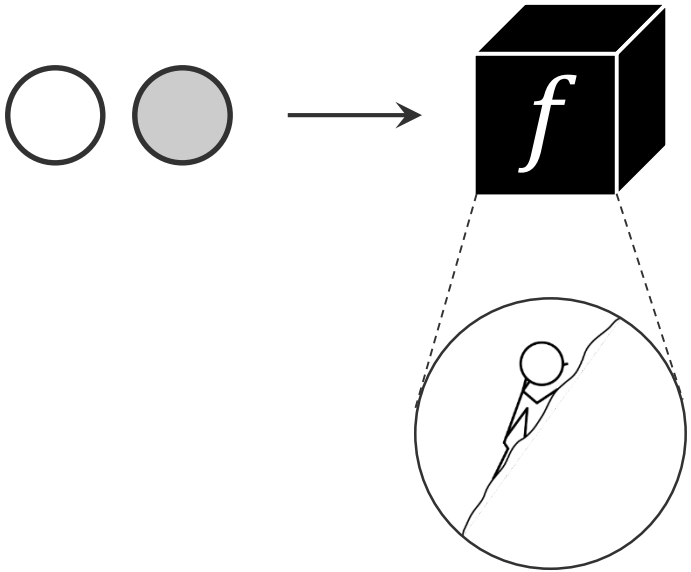


A Basic Result [Droste, Jansen, Wegener 2002]

Theorem: The (1+1) EA optimizes OneMax_n in $\Theta(n \log n)$ iterations in expectation.



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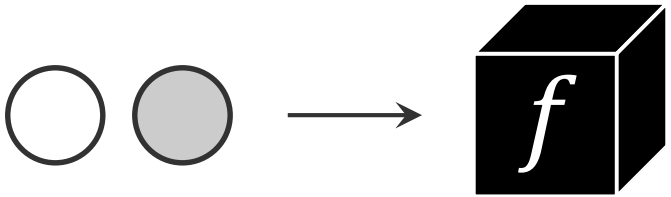

- Coupon collector
- Drift theory

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