Computational Learning Theory
1.2 Training Sequences

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Homogeneous Inputformat

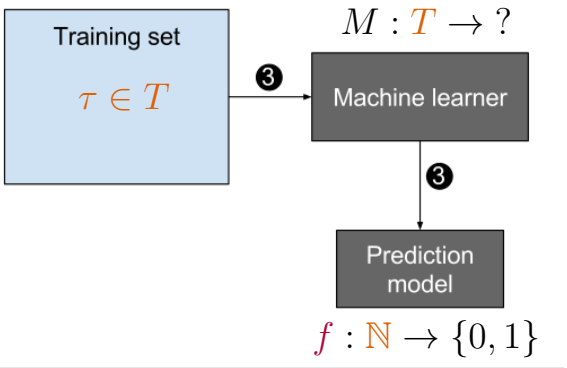


Image from: https://commons.wikimedia.org/wiki/File:Machine_learning_nutshell_--_Train_a_machine_learning_model.svg

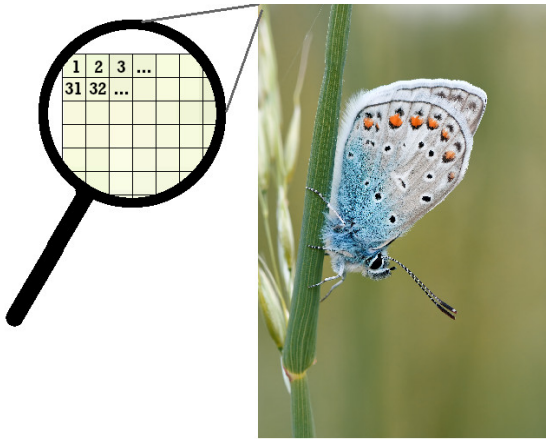
Definition (Prediction Model, Domain of M)

A *prediction model* f is a function $f : \mathbb{N} \to \{0, 1\}$.

The *set of all training data sequences* T is the set of all finite sequences $\tau = ((n_1, y_1), \ldots, (n_{|\tau|}, y_{|\tau|}))$ of *consistently* binary labeled natural numbers.

Representation of Images

- An image of 50×30 pixels yields a vector with 4500 color values (RGB).
- Knowing height and width helps to interpret the color vector correctly.
- An image is uniquely specified by its height, width and the color vector [f8f8e0, f8f8e0, ..., 6c7a47].



How can an image be encoded into a natural number such that it can be reconstructed?

From Images to Natural Numbers

50, 30, [f8f8e0, . . . , 6c7a47]	height, width, array of color values
110010, 011110, [111110001111100011100000, . . . , 110110001111010010001111]	respective binary representations
101101011100, 111110001111100011100000 . . . 110110001111010010001111	merge height and width bitstring; concatenate color representations to color bitstring
010101010100000001010101010000000 101010000000000 . . . 010100010100000 001010111001110001100101011010 1	merge size bitstring and color bitstring to obtain image bitstring $x = (x_{\|x\|-1}, \ldots, x_2, x_1, x_0)$
$n \in \mathbb{N}$	evaluate $\sum_{i=0}^{\|x\|-1} x_i \cdot 2^i$

From Natural Numbers to Images

167389379610311487033352502172731518702664890704680683002 53

$n \in \mathbb{N}$

1010101010101010101010101010101010101010101010101010101010101
0000000000000000000000000000000000000000000000000010101010101
0000000000000000000000000000000000000000000000000010101011111 0101

transform to binary system

1100, 11111111111111111111111111111110000000000000000000000000111
11111000000000000000000000000011111111

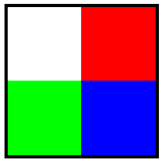
extract size string

10, 10, [11111111111111111111, 11111111000000000000000000,
000000001111111100000000, 000000000000000011111111]

extract height and width,
split color by length 24

2, 2, [ffffff, ff0000, 00ff00, 0000ff]

interpret width and height as numbers and
color string as color values



reconstruct image

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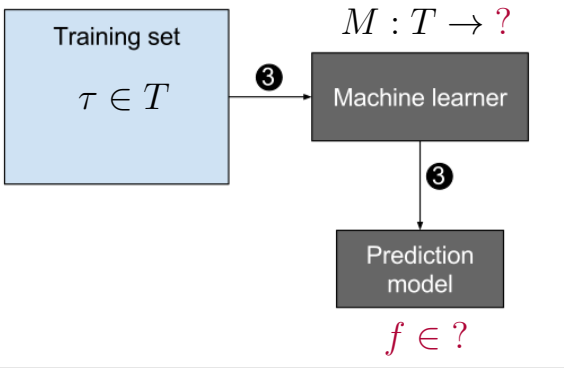
An image can be encoded into a natural number
such that it can be reconstructed, e.g. by a merge argument.

- The formal definitions match the binary classification of images.
- Feature vectors for images (and other virtual objects) correspond to natural numbers in a similar way.
- Encoding techniques for feature vectors in electronic devices differ.

Combinatorial Analysis of Learning



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Can a *machine learner* M *learn* a well performing $f : \mathbb{N} \to \{0, 1\}$ from sequences $\tau \in T$ of increasing length?