Hasso Plattner Institut
Digital Engineering • Universität Potsdam

Computational Learning Theory
2.1 Learnability of Hypothesis Spaces

Karen Seidel
Hasso-Plattner-Institute
University of Potsdam

Combinatorial Analysis of Binary Classification

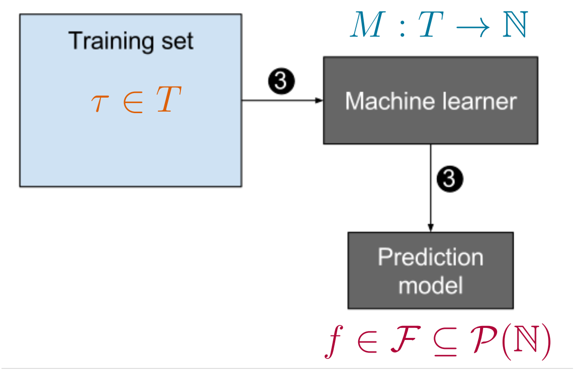
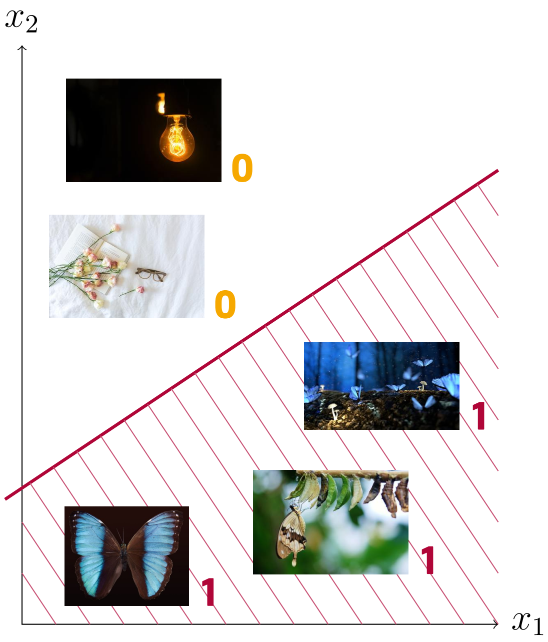


Image from: `https://commons.wikimedia.org/wiki/File:Machine_learning_nutshell_--_Train_a_machine_learning_model.svg`



Is there an M that *learns* $\mathcal{F} = \{f_i \mid i \in \mathbb{N}\}$?

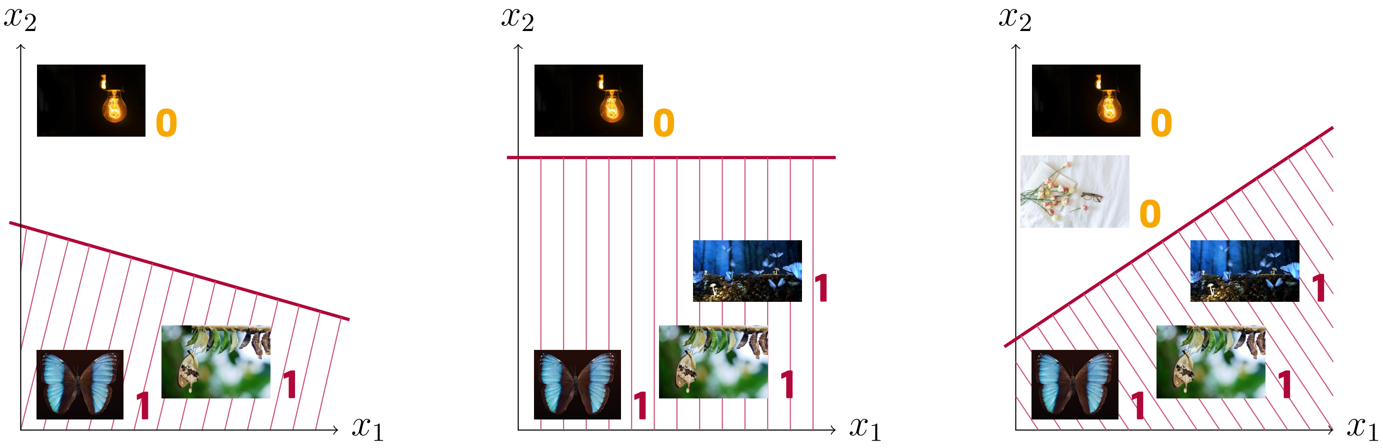
Learning by Enumeration (Occam's Razor)

Theorem (Learnability of Hypothesis Spaces)

Let $\mathcal{F} = \{f_i \mid i \in \mathbb{N}\}$ be a hypothesis space. Then there exists an M that learns $\mathcal{F}$.

Proof Idea.

Choose the minimal i such that f_i is consistent with the data. $\square$



Applying Machine Learning Paradigms revisited

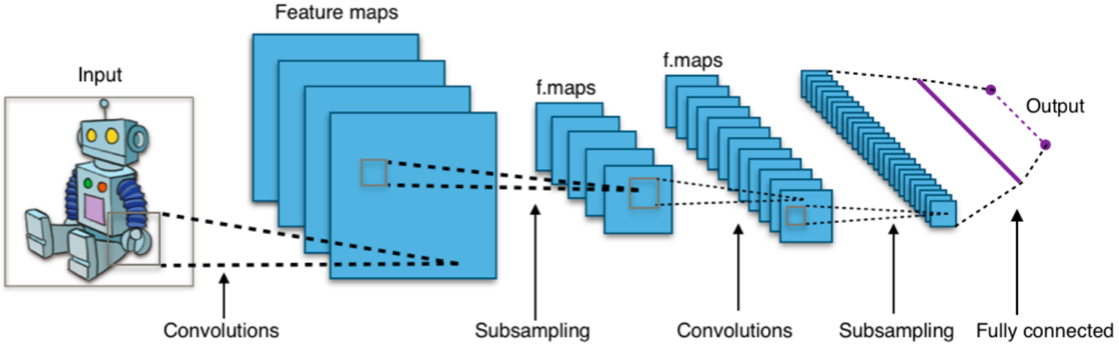


Image from: `https://en.wikipedia.org/wiki/File:Typical_cnn.png`

image	fixed learning algorithm with hypothesized parameters	label
$n \in \mathbb{N}$	$f \in \mathcal{F}$	1 or 0

- (Full) Batch: learner uses all training data $I[t]$ observed so far
- Minibatch: learner only uses k (batchsize) inputs and the prediction model before seeing them

Modelling Minibatch Learning

Definition (Iterative Learner)

A learner M is *k-iterative* if its output only depends on the last k inputs $I(t-k), \ldots,$ $I(t-1)$ and the corresponding prediction model $M(I[t-k])$.
If $k = 1$, we say that M is *iterative*.

Lemma ($\mathbf{It} \Rightarrow \mathbf{It}_k$)

If M is iterative, then it is k-iterative for every $k \in \mathbb{N}$.

Proof Idea.

The iterativeness of M is applied k times. $\square$

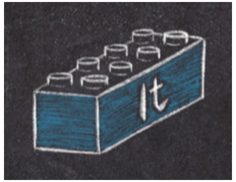
Lemma ($[\mathbf{It}_k\mathbf{InfEx}] \subseteq [\mathbf{ItInfEx}]$)

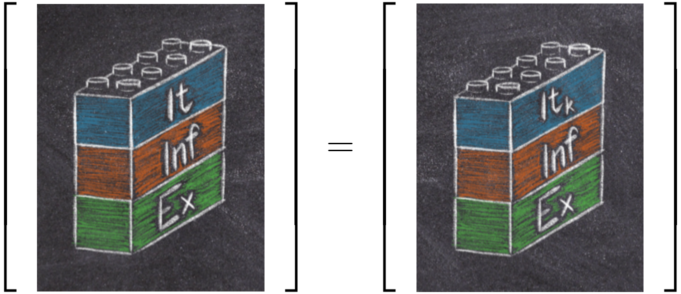
Let $k \in \mathbb{N}$. If a k-iterative M learns $\mathcal{F}$, then an iterative N learns $\mathcal{F}$.

Proof Idea.

N simulates M on the input data repeating every element k times. $\square$

Building Blocks for Models





$$[\mathbf{ItInfEx}] = [\mathbf{It}_k\mathbf{InfEx}]$$

Learnability of Hypothesis Spaces

Every hypothesis space is learnable by enumeration.

(Full) Batch Learner:  1  1  0  1  0 . . .

Minibatch Learner:  1  1  0  1  0 . . .

Learnability does not depend on the batch size.

Is there an iterative M that *learns* the family of halfspaces?