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OMEinsumContractionOrders: A Julia package for tensor network contraction order optimization

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DOI: 10.xxxxxx/draft

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Editor: Open Journals

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Submitted: 01 January 1970
Published: unpublished

Summary

`OMEinsumContractionOrders` (One More Einsum Contraction Orders, or OMECO) is a Julia package (Bezanson et al., 2012) that implements state-of-the-art algorithms for optimizing tensor network contraction orders. OMECO is designed to search for near-optimal contraction orders for exact tensor network contraction, and provides a comprehensive suite of optimization algorithms for tensor network contraction orders, including greedy heuristics, simulated annealing, and tree width solvers. In this paper, we present the key features of OMECO, its integration with the Julia ecosystem, and performance benchmarks.

Statement of need

A *tensor network* is a mathematical framework that represents multilinear algebra operations as graphical structures, where tensors are nodes and shared indices are edges. This diagrammatic approach transforms complex high-dimensional contractions into visual networks that expose underlying computational structure.

The framework has remarkable universality across diverse domains: *einsum* notation (Harris et al., 2020) in numerical computing, *factor graphs* (Bishop & Nasrabadi, 2006) in probabilistic inference, *sum-product networks* in machine learning, and *junction trees* (Villescas et al., 2023) in graphical models. Tensor networks have enabled breakthroughs in quantum circuit simulation (Markov & Shi, 2008), quantum error correction (Piveteau et al., 2024), neural network compression (Qing et al., 2024), strongly correlated quantum materials (Haegeman et al., 2016), and combinatorial optimization problems (J.-G. Liu et al., 2023).

The computational cost of tensor network contraction depends critically on the *contraction order*—the sequence in which pairwise tensor multiplications are performed. This order can be represented as a binary tree where leaves correspond to input tensors and internal nodes represent intermediate results. The optimization objective balances multiple complexity measures through the cost function:

$$\mathcal{L} = w_\mathrm{t} \cdot \mathsf{tc} + w_\mathrm{s} \cdot \max(0, \mathsf{sc} - \mathsf{sc}_{\mathrm{target}}) + w_\mathrm{rw} \cdot \mathsf{rwc},$$

where w_t, w_s, and w_rw represent weights for time complexity (tc), space complexity (sc), and read-write complexity (rwc), respectively. In practice, memory access costs typically dominate computational costs, motivating $w_\mathrm{rw} > w_\mathrm{t}$. The space complexity penalty activates only when $\mathsf{sc} > \mathsf{sc}_{\mathrm{target}}$, allowing unconstrained optimization when memory fits within available device capacity.

Finding the optimal contraction order—even when minimizing only time complexity—is NP-complete (Markov & Shi, 2008). Algorithms for finding near-optimal contraction orders have been developed and achieve impressive scalability (Gray & Kourtis, 2021; Roa-Villescas et al., 2024), handling tensor networks with over 10^4 tensors. However, an efficient and reliable implementation of these methods in Julia is still missing.

OMECO addresses this gap by offering a unified and extensible interface to a comprehensive suite of optimization algorithms for tensor network contraction orders, including greedy heuristics, simulated annealing, and tree-width-based solvers. OMECO has been integrated into the `OMEinsum` package and powers several downstream applications: `Yao` (Luo et al., 2020) for quantum circuit simulation, `GenericTensorNetworks` (J.-G. Liu et al., 2023) and `TensorBranching` for combinatorial optimization, `TensorInference` (Roa-Villescas & Liu, 2023) for probabilistic inference, and `TensorQEC` for quantum error correction. These applications are reflected in the ecosystem built around OMECO, as illustrated in Figure 1. This infrastructure is expected to benefit other applications requiring tree or path decomposition, such as polynomial optimization (Magron & Wang, 2021).

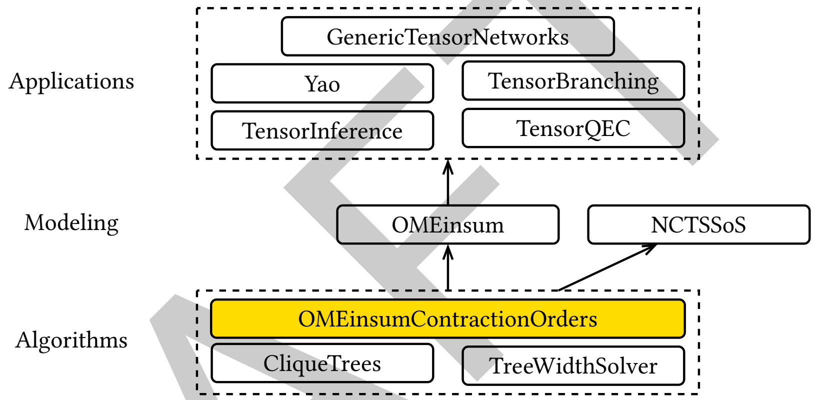

Figure 1: The ecosystem built around `OMEinsumContractionOrders` and its dependencies. OMECO serves as a core component of the tensor network contractor `OMEinsum`, which powers applications including `Yao` (quantum simulation), `TensorQEC` (quantum error correction), `TensorInference` (probabilistic inference), `GenericTensorNetworks` and `TensorBranching` (combinatorial optimization).

Features and benchmarks

The major feature of OMECO is contraction order optimization. OMECO provides several algorithms with complementary performance characteristics that can be simply called by the `optimize_code` function:

Optimizer	Description
`GreedyMethod`	Fast greedy heuristic with modest solution quality
`TreeSA`	Reliable simulated annealing optimizer (Kalachev et al., 2021) with high-quality solutions
`PathSA`	Simulated annealing optimizer for path decomposition
`HyperND`	Nested dissection algorithm for hypergraphs, requires `KaHyPar` or `Metis`
`KaHyParBipartite`	Graph bipartition method for large tensor networks (Gray & Kourtis, 2021), requires `KaHyPar`
`SABipartite`	Simulated annealing bipartition method, pure Julia implementation

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Optimizer	Description
`ExactTreewidth`	Exact algorithm with exponential runtime (Bouchitté & Todinca, 2001), based on `TreeWidthSolver`
`Treewidth`	Clique tree elimination methods from `CliqueTrees` package (Samuelson & Fairbanks, 2025)

The algorithms `HyperND`, `Treewidth`, and `ExactTreewidth` operate on the tensor network's line graph and utilize the `CliqueTrees` and `TreeWidthSolver` packages, as illustrated in Figure 1. Additionally, the `PathSA` optimizer implements path decomposition by constraining contraction orders to path graphs, serving as a variant of `TreeSA`.

These methods balance optimization time against solution quality. Figure 2 displays benchmark results for the Sycamore quantum supremacy circuit(Arute et al., 2019; Pan & Zhang, 2021), highlighting the Pareto front where contraction order quality is balanced with optimization runtime. Real-world examples demonstrating applications to quantum circuit simulation, combinatorial optimization, and probabilistic inference are available in the OMEinsumContractionOrdersBenchmark repository. Optimizer performance is highly problem-dependent, with no single algorithm dominating across all metrics and graph topologies.

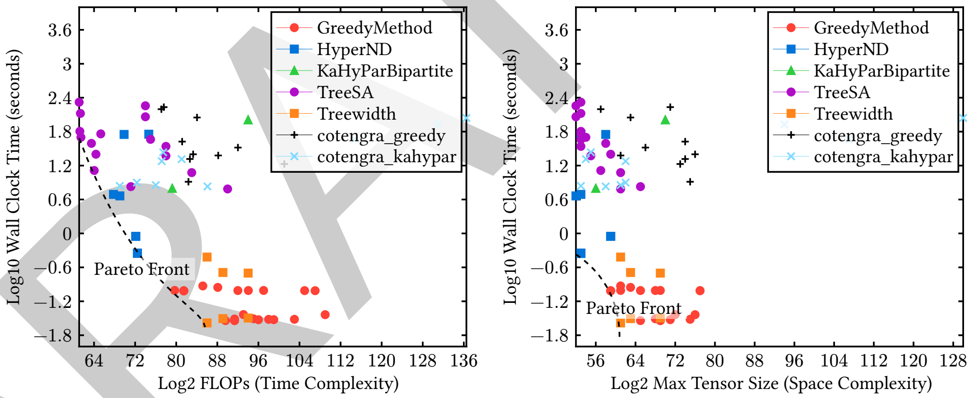

Figure 2: Time complexity (left) and space complexity (right) benchmark results for contraction order optimization on the Sycamore quantum circuit tensor network (Intel Xeon Gold 6226R CPU @ 2.90GHz, single-threaded). The x-axis shows contraction cost, y-axis shows optimization time. Each point represents a different optimizer configuration tested with varying parameters. `TreeSA` and `HyperND` achieve the lowest contraction costs, while `GreedyMethod` offers the fastest optimization time. The parameter setup for each optimizer is detailed in our benchmark repository `OMEinsumContractionOrdersBenchmark`.

Optimizers prefixed with `cotengra_` are from the Python package cotengra (Gray & Kourtis, 2021); all others are OMECO implementations. For both optimization objectives (minimizing time and space complexity), OMECO optimizers dominate the Pareto front. Given sufficient optimization time, `TreeSA` consistently achieves the lowest time and space complexity. `GreedyMethod` and `Treewidth` (backed by minimum fill (MF) (Ng & Peyton, 2014), multiple minimum degree (MMD) (J. W. Liu, 1985), and approximate minimum fill (AMF) (Rothberg & Eisenstat, 1998)) provides the fastest optimization but yields suboptimal contraction orders, while `HyperND` offers a favorable balance between optimization time and solution quality.

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Another key feature of OMECO is index slicing, a technique that trades time complexity for reduced space complexity by explicitly looping over a subset of tensor indices. OMECO provides the `slice_code` interface for this purpose, currently supporting the `TreeSASlicer` algorithm, which implements dynamic slicing based on the `TreeSA` optimizer. Figure 3 demonstrates this capability using the Sycamore quantum circuit, where slicing reduces the space complexity from 2^{52} to 2^{31}.

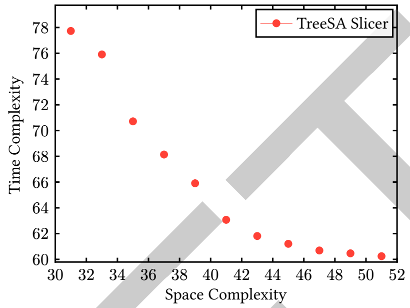

Figure 3: Trade-off between time complexity and target space complexity using `TreeSASlicer` on the Sycamore quantum circuit. The original network has a space complexity of 2^{52}.

The figure shows that moderate slicing increases time complexity only slightly, while aggressive slicing can induce significant overhead.

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