Modeling Hoek-Brown Rock using Mohr-Coulomb Failure Criterion

Background

Snail is a geotechnical engineering software developed and maintained by Geotechnical Services, Division of Engineering Services, California Department of Transportation (Caltrans). This software was developed to assist users in performing stability analyses of soil nail walls and analyses of structural facing of soil nail walls [1].

However, two problems arise when handling nails within rock layer. The first problem is the selection of appropriate properties in Snail as the excavation progresses into rock. This difficulty arises because there is no direct correlation to convert the Recovery, Rock quality Designation (RQD), and Uniaxial Compressive Strength (UC) acquired from field exploration program into Mohr-Coulomb failure criterion, which is the only available criterion in Snail. In practice, we typically assume the rock behaves like undrained soil and manually assigns undrained strength, often without a clear explanation.

The second problem is modeling tieback in Snail. In some cases, we design shafts with a combination of tieback and soil nails. Typically, we ignore the contribution of tiebacks to the Factor of Safety (FS) during modeling, as Snail has only provided the strength criterion for soil nails. This approach may be overly conservative.

Addressing the two problems above, this document provides a general procedure for shaft design in rock using Snail with two key topics:

1. Method of modeling rock with Mohr-Coulomb failure criterion in Snail
2. Method of modeling tieback in Snail

Modeling rock with Mohr-Coulomb (MC) failure criterion

Before jumping into such an abstract conversion, let's build a general understanding about rock and its classification systems. The detailed guidance of the following classification systems can be found in E. Hoek and E. T. Brown's textbook [2], a more practical specification in America is published by American Society for Testing and Materials [3].

CSIR Classification of Jointed Rock Masses (Bieniawski, 1974, 1976).

This classification system has been introduced by South African Council of Scientific and Industrial Research (CSIR). The council has summarized five basic classification parameters that are directly related with the strength of rock and has convert them into dimensionless rating numbers using an established lookup table. These rating numbers are then summarized and adjusted based on project types.

$$RMR = \sum_i RMR_i + RMR_{adjustment}$$

Where the five basic classification parameters are:

1. Strength of intact rock material (UC)
2. Rock quality designation (RQD)
3. Spacing of joints
4. Condition of joints (continuity, surface roughness, infilling material etc.)
5. Ground water conditions (inflow rate)

And the adjustment parameter:

6. Strike and dip orientations of joints and the project type (tunnel, foundation, slope)

NGI Tunnelling Quality Index (Barton, N., Lien, R., and Lude, J., 1974)

Unlike the summation used in CSIR, Norwegian Geotechnical Institute (NGI) has organized similar basic classification parameters through multiplication [4].

$$Q = \left(\frac{RQD}{J_n}\right) \times \left(\frac{J_r}{J_a}\right) \times \left(\frac{J_w}{SRF}\right)$$

Where: J_n is the joint set number.

J_r is the joint roughness number.

J_a is the joint alteration number.

J_w is the joint water reduction factor.

SRF is a stress reduction factor.

Bieniawski (1976) has introduced an equation that can adequately describe the relationship between these two indexes (RMR and Q):

$$RMR = 9 \times \log_e Q + 44$$

Support Recommendations based on NGI Tunnelling Quality Index

The two indexes can then be used for the support recommendations. By the time of writing, the writer has found only the instruction for NGI Tunnelling Index system in updating. Thus, it is with more confident to introduce the support recommendations based on NGI Tunnelling Index system here.

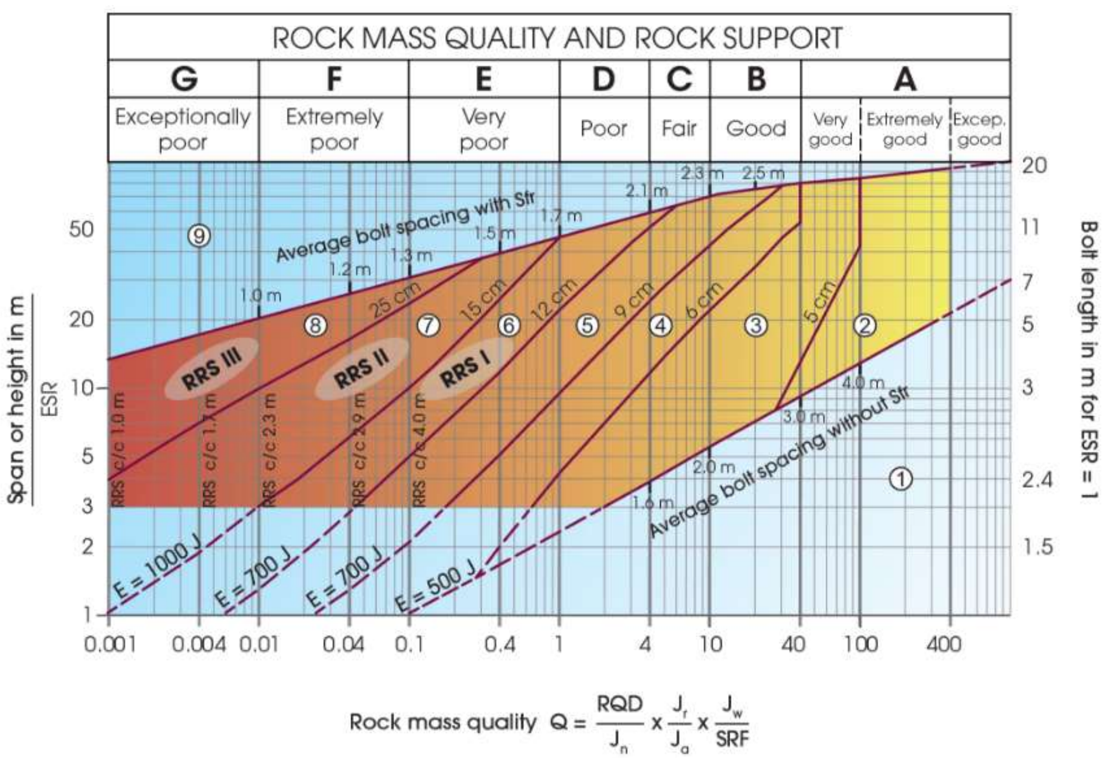

Figure 1. Rock Mass Quality and Rock Support [4]

Once Q value has been determined following [3 or 4], the support recommendation related to the size of the span and the rock mass quality can be determined based on NGI's guidance [4]. Please keep in mind that this recommendation is general and does not cover all the special situations. Further inspection would be necessary if such a situation occurs (i.e. concentration of joints).

Rock's Hoek-Brown (HB) Failure Criterion

The Hoek-Brown failure criterion is an empirical stress surface that is used in rock mechanics to predict the failure of rock. It was first developed in 1980 [2] and then be extended for applicability to slope stability and surface excavation problems in 1988 [3].

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$$

Where σ_1, σ_3 are the major and minor principal stress at failure.
σ_c is the uniaxial compressive strength of the intact rock material.
m, s are material constants.

Similar to MC which can be written in terms of $\sigma_1 = \sigma_3 + (A\sigma_3 + B)$, HB adds nonlinearity by adding square root over the $(A\sigma_3 + B)$ term. This has been proved to be a better fit of the test data [2].

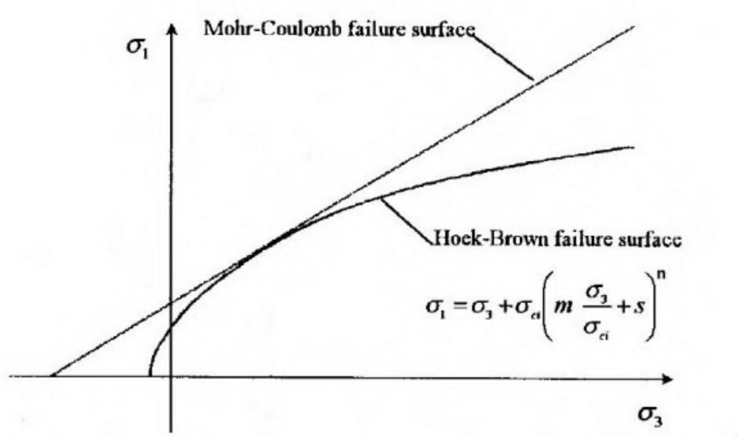

Figure 2. Comparison of MC and HB Models

Development of Hoek-Brown (HB) Failure Criterion

Recall HB:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$$

Once RMR or Q has been evaluated, the user can refer to [5] to acquire parameters m and s. **Please do note that Hoek-Brown model does not consider any structural discontinuities, which means the joint direction should not govern the design while using this model. Read the limitations in the paper for more details.**

The uniaxial compressive strength (UC) of rock specimen is generally provided in Geotechnical Report (GR). It is the reader's engineering judgement to specify a nominal $\sigma_{c,n}$ based on the available data, here we shall provide two ideas of selection:

1. $\sigma_{c,n} = \min\,(\boldsymbol{\sigma_c})$
2. $\sigma_{c,n} = \text{mean}(\boldsymbol{\sigma_c}) - n(X)\text{dev}(\boldsymbol{\sigma_c})$, $n(X)\text{dev}(\boldsymbol{\sigma_c})$ is the compressive strength with X percent confidence (i.e. $n(95) = 2$).

Conversion of Failure Criterion from Hoek-Brown (HB) to Mohr-Coulomb (MC)

The reason for converting the nonlinear HB to linear MC is the unknown normal stress distribution of the slides on their failure surface.

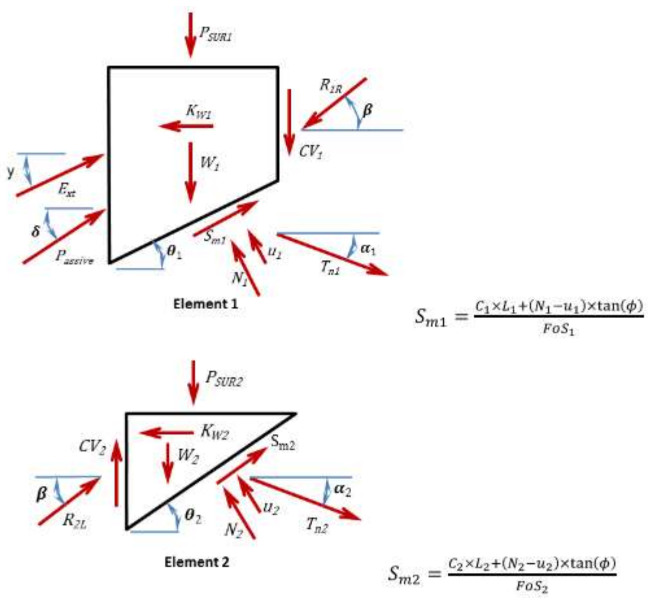

Figure T-1 Forces and Directions on Bilinear Wedge Elements

Figure 3. Snail Stability Algorithm [1]

Assume the normal stress distribution: $\begin{cases} \sigma_n = \sigma_n(s) \\ \int_S \sigma_n(s)\,ds = N \end{cases}$

If MC:

$$\tau_n(s) = c + \tan(\phi)\sigma_n(s)$$

We have:

$$\int_S \tau_n(s)\, ds = cS + \tan(\phi) = \int_S \sigma_n(s)\, ds = cS + \tan(\phi)\, N$$

Normal stress distribution does not affect the total shear resistance of the slides!

Now HB:

$$\tau_n(s) = HB\big(\sigma_n(s)\big)$$

If find:

$$\tau_n'(s) = c' + tan(\phi')\,\sigma_n(s) < HB\big(\sigma_n(s)\big) \text{ for } \forall \sigma_n(s) \in [0, \Sigma]$$

Then:

$$\int_S \tau_n{}'(s)\, ds = c'S + \tan(\phi')\, N > \int_S HB\big(\sigma_n(s)\big)\, ds = \int_S \tau_n(s)\, ds$$

Please note that it is hard to directly derive $\tau_n = HB(\sigma_n)$ from $\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}$. Instead, we construct the parametric equations $\begin{cases} \sigma_n = \sigma_n(\sigma_1, \sigma_3) \\ \tau_n = \tau_n(\sigma_1, \sigma_3) \\ \sigma_1 = \sigma_1(\sigma_3) \end{cases}$ and find the $\tau_n = HB(\sigma_n)$ numerically. These parametric equations can be found in textbook [2] Pg. 139. Here we shall show the alternative expressions:

$$\begin{cases} \sigma_n = \sigma_3 + \dfrac{2(\sigma_1 - \sigma_3)^2}{4(\sigma_1 - \sigma_3) + m\sigma_c} \\ \tau_n = \dfrac{\sigma_1 - \sigma_3}{2}\sqrt{1 - \left(\dfrac{m\sigma_c}{4(\sigma_1 - \sigma_3) + m\sigma_c}\right)^2} \\ \sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} \end{cases}$$

It can be further simplified as:

$$\begin{cases} \sigma_n = \sigma_3 + \dfrac{2(m\sigma_c\sigma_3 + s\sigma_c^2)}{4\sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} + m\sigma_c} \\ \tau_n = \dfrac{\sqrt{m\sigma_c\sigma_3 + s\sigma_c^2}}{2}\sqrt{1 - \left(\dfrac{m\sigma_c}{4\sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} + m\sigma_c}\right)^2} \end{cases}$$

The c' and $\tan(\phi')$ can be determined by drawing a secant line between $(0, HB(0))$ and $(\Sigma_n, HB(\Sigma_n))$ in σ_n, τ_n domain:

$$\begin{cases} c' = HB(0) \\ \tan(\phi') = \dfrac{HB(\Sigma) - HB(0)}{\Sigma} \end{cases}$$

If $\Sigma > \max(\sigma_n)$, The convex shape of $\tau_n'(s) = c' + tan(\phi')$ promises:

$$\int_S \tau_n'(s)\, ds = c'S + \tan(\phi')\, N > \int_S HB\big(\sigma_n(s)\big)\, ds = \int_S \tau_n(s)\, ds$$

Since the $\tan(\phi')$ would not vary too much when $\Sigma \ll \sigma_{c,n}$, it is typically useful to choose the maximum of all possible $\Sigma = \max\left(\frac{\mathrm{N}}{\mathrm{L}}\right)$, which is the effective vertical stress at the bottom of the excavation (BOE).

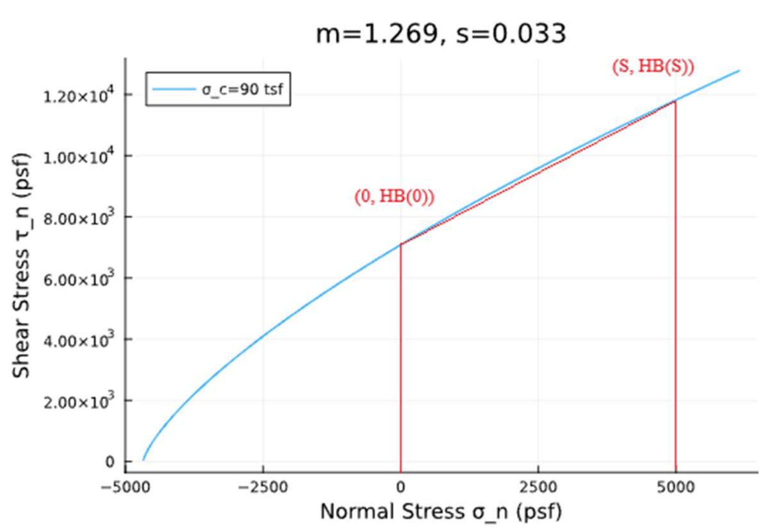

Figure 4. HB-MC Conversion

Table 1 : Approximate relationship between rock mass quality and material constants

Disturbed rock mass m and s values | *undisturbed rock mass m and s values*

EMPIRICAL FAILURE CRITERION $\sigma_1' = \sigma_3' + \sqrt{m\sigma_c\sigma_3' + s\sigma_c^2}$ σ_1' = major principal effective stress σ_3' = minor principal effective stress σ_c = uniaxial compressive strength of intact rock, and m and s are empirical constants.		CARBONATE ROCKS WITH WELL DEVELOPED CRYSTAL CLEAVAGE *dolomite, limestone and marble*	LITHIFIED ARGILLACEOUS ROCKS *mudstone, siltstone, shale and slate (normal to cleavage)*	ARENACEOUS ROCKS WITH STRONG CRYSTALS AND POORLY DEVELOPED CRYSTAL CLEAVAGE *sandstone and quartzite*	FINE GRAINED POLYMINERALLIC IGNEOUS CRYSTALLINE ROCKS *andesite, dolerite, diabase and rhyolite*	COARSE GRAINED POLYMINERALLIC IGNEOUS & METAMORPHIC CRYSTALLINE ROCKS – *amphibolite, gabbro gneiss, granite, norite, quartz-diorite*
INTACT ROCK SAMPLES						
Laboratory size specimens free	m	7.00	10.00	15.00	17.00	25.00
from discontinuities	s	1.00	1.00	1.00	1.00	1.00
CSIR rating: RMR = 100	*m*	*7.00*	*10.00*	*15.00*	*17.00*	*25.00*
NGI rating: Q = 500	*s*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*
VERY GOOD QUALITY ROCK MASS						
Tightly interlocking undisturbed rock	m	2.40	3.43	5.14	5.82	8.56
with unweathered joints at 1 to 3m.	s	0.082	0.082	0.082	0.082	0.082
CSIR rating: RMR = 85	*m*	*4.10*	*5.85*	*8.78*	*9.95*	*14.63*
NGI rating: Q = 100	*s*	*0.189*	*0.189*	*0.189*	*0.189*	*0.189*
GOOD QUALITY ROCK MASS						
Fresh to slightly weathered rock, slightly	m	0.575	0.821	1.231	1.395	2.052
disturbed with joints at 1 to 3m.	s	0.00293	0.00293	0.00293	0.00293	0.00293
CSIR rating: RMR = 65	*m*	*2.006*	*2.865*	*4.298*	*4.871*	*7.163*
NGI rating: Q = 10	*s*	*0.0205*	*0.0205*	*0.0205*	*0.0205*	*0.0205*
FAIR QUALITY ROCK MASS						
Several sets of moderately weathered	m	0.128	0.183	0.275	0.311	0.458
joints spaced at 0.3 to 1m.	s	0.00009	0.00009	0.00009	0.00009	0.00009
CSIR rating: RMR = 44	*m*	*0.947*	*1.353*	*2.030*	*2.301*	*3.383*
NGI rating: Q = 1	*s*	*0.00198*	*0.00198*	*0.00198*	*0.00198*	*0.00198*
POOR QUALITY ROCK MASS						
Numerous weathered joints at 30-500mm,	m	0.029	0.041	0.061	0.069	0.102
some gouge. Clean compacted waste rock	s	0.000003	0.000003	0.000003	0.000003	0.000003
CSIR rating: RMR = 23	*m*	*0.447*	*0.639*	*0.959*	*1.087*	*1.598*
NGI rating: Q = 0.1	*s*	*0.00019*	*0.00019*	*0.00019*	*0.00019*	*0.00019*
VERY POOR QUALITY ROCK MASS						
Numerous heavily weathered joints spaced	m	0.007	0.010	0.015	0.017	0.025
<50mm with gouge. Waste rock with fines.	s	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
CSIR rating: RMR = 3	*m*	*0.219*	*0.313*	*0.469*	*0.532*	*0.782*
NGI rating: Q = 0.01	*s*	*0.00002*	*0.00002*	*0.00002*	*0.00002*	*0.00002*

Figure5. HB Model Constant Selection [3]

Modeling Tieback in Snail

Snail user manual [1] has provided the description of soil nail model used in the software. It is possible to model tieback using nail model.

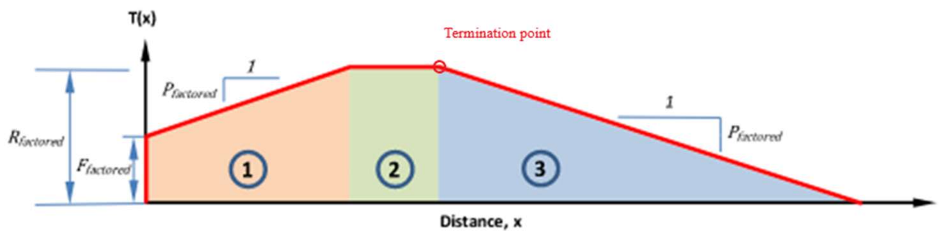

Figure T-3 Conceptual Factored Soil Nail Strength Envelope

Figure 6. Snail Nail Element Algorithm [1]

Recall the mechanism of tieback, if we have the soil nail model to develop its strength as the same ratio of tieback and terminate at the close end of the tieback fix length, we have successfully modeled tieback using nail model. This can be achieved by keeping all the dimensions of tieback and modifying only the yield strength (nominal yield strength).

Here the nominal yield strength:

$$f'_{y,nom} = \frac{R_{bond}}{A_{rod}} = \frac{q_n c_{grout} L_{fix}}{FS_{bond} A_{rod}}$$

Where: R_{bond} is the allowable bond strength of the tieback.

q_n is the nominal bond strength of the soil.

c_{grout} is the perimeter of the tieback grout.

L_{fix} is the fixed length of the tieback.

FS_{bond} is the factor of safety for bond strength.

A_{rod} is the nominal section area of the tieback.

The other important factor is facing resistance. This factor can typically be ignored while modeling tieback, as the failure wedge is likely to cross zone 2 and 3 of the tiebacks in Figure 5., which are typically located above soil nails. However, the reader should note that the facing resistance used in Snail is “artificial” for tiebacks and is unconservative if $F_{factored} > R_{bond}$.

Reference

[1] California Department of Transportation (2020), Snail User Guide

[2] E. Hoek & E.T. Brown (1980), Underground Excavation in Rock

[3] American Society for Testing and Materials (2019), D5878 – 19 Standard Guides for Using Rock-Mass Classification Systems for Engineering Purposes

[4] The Norwegian Geotechnical Institute (2022), Using the Q-System

[5] E. Hoek & E.T. Brown (1988), The Hoek-Brown Failure Criterion – A 1988 Update