Governing Equations

Assumption

Continuity, homogeneity, isotropy, linearity (material & deformation).

We shall review the more general solid mechanics without linearity later, but would keep this assumption for now as it will lead us to many practical theories (i.e. torsion, plate, etc.)

Governing Equations

Equilibrium $div(\boldsymbol{\sigma}) + \boldsymbol{f} = \boldsymbol{0}$

$$\begin{cases} \sigma_{x,x} + \tau_{yx,y} + \tau_{zx,z} + f_x = 0 \\ \tau_{xy,x} + \sigma_{y,y} + \tau_{zy,z} + f_y = 0 \\ \tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} + f_z = 0 \end{cases}$$

$$\begin{cases} \tau_{xy} = \tau_{yx} \\ \tau_{yz} = \tau_{zy} \\ \tau_{zx} = \tau_{xz} \end{cases}$$

Constitutive Law $\boldsymbol{\epsilon} = \boldsymbol{C}{:}\,\boldsymbol{\sigma}$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\mu & -\mu \\ -\mu & 1 & -\mu \\ -\mu & -\mu & 1 \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix}$$

$$\begin{bmatrix} \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} = \frac{1+\mu}{E} \begin{bmatrix} \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{bmatrix}$$

Deformation Continuity $\boldsymbol{\epsilon} = \frac{1}{2}(\boldsymbol{\nabla u} + \boldsymbol{u\nabla})$

$$\begin{cases} \epsilon_x = u_{,x} \\ \epsilon_y = v_{,y} \\ \epsilon_z = w_{,z} \end{cases}$$

$$\begin{cases} 2\gamma_{xy} = u_{,y} + v_{,x} \\ 2\gamma_{yz} = v_{,z} + w_{,y} \\ 2\gamma_{xz} = u_{,z} + w_{,x} \end{cases}$$