ASSIGNMENT 5 PART 1

Problem Statement

Construction Co. specializes in three types of building projects: residential renovations, residential construction, and commercial construction. With a workforce of 45 carpenters, 40 drywallers, 20 plumbers, and 20 electricians, the company aims to maximize its profitability across these domains. The average profits per project type are $12,000 for residential renovations, $35,000 for residential construction, and $205,000 for commercial construction. Three strategies are under consideration to enhance profitability. The first involves strategic project selection to optimize labor utilization, as past experiences have shown that committing to too many projects of a similar type can lead to labor shortages. The second strategy explores pricing adjustments; market research indicates that rates could be increased by up to 5%, 2%, and 15% for residential renovations, residential construction, and commercial construction, respectively, without affecting demand. The third strategy considers altering the labor force size by recruiting additional tradespeople. The task at hand is to evaluate these strategies comprehensively to identify the most effective approach for maximizing profitability, ensuring that any recommendations are substantiated through simulations and optimization models.

Introduction

Construction Co., a building firm with specializations in multiple sectors, is facing a pressing issue: how to maximize profitability while efficiently allocating its labor force across various types of projects. Historically, the company has relied on a fixed labor force and basic project selection criteria based on average profits. However, recent internal reviews and market feedback suggest that this approach is becoming increasingly ineffective. Issues such as labor shortages on certain types of projects, an idle workforce, and untapped pricing opportunities are affecting both the company's financial performance and its reputation.

Given that the company has finite resources in terms of skilled labor and can increase its prices only within certain market-acceptable limits, the problem lends itself well to being addressed as a linear programming problem. Linear programming is particularly suitable for optimizing resource allocation while adhering to a set of constraints, such as labor availability and market-based pricing limits. By adopting a linear programming approach, Construction Co. can develop a more dynamic, data-driven operational strategy designed to address its current challenges and improve its profitability. The goal is to formulate an objective function that maximizes profitability while taking into account labor and price constraints.

This comprehensive report describes the robust methodology utilized in constructing the linear programming model, incorporating both mathematical equations and numerical optimization techniques. It introduces the newly developed model, which diverges significantly from Construction Co.'s traditional project selection and workforce allocation strategies. The nuances of these differences and their implications for both short-term profitability and long-term sustainability are thoroughly explored. The report culminates in an assessment of the model's efficacy in solving the company's present challenges. Recommendations are provided for the model's implementation into Construction Co.'s broader operational strategy as a tool for continual improvement and revenue maximization.

Methodology

Linear Programming Model for Construction Co.

The deployment of a Linear Programming (LP) model serves a critical function in this analysis. It aims to navigate the complex operational terrain of Construction Co. by optimizing the selection of construction projects. The objective is to maximize total profit, a key performance indicator for the company. This objective integrates multiple variables, specifically the types of construction projects, which include residential renovations, residential construction, and commercial construction. Each project type has distinct financial and operational implications, making the LP model a suitable tool for this multi-dimensional optimization problem.

The constraints of this LP model have been meticulously formulated to accurately reflect the operational limitations Construction Co. faces. Specifically, labor availability across various trades—carpenters, drywallers, plumbers, and electricians—has been integrated into the model as constraints. This ensures that the solutions generated are not only financially optimal but also operationally feasible.

In addition, the model is designed to be adaptive to potential pricing changes. This is particularly crucial as the construction market is volatile and subject to fluctuations in labor and material costs. Thus, the LP model is not a static tool but a dynamic framework that can be adjusted to suit evolving market conditions.

Choice of Real-Valued Variables in Linear Programming Model

Rationale for Real-Valued Approach

The decision to use real-valued variables in the linear programming model is grounded in three key considerations: computational efficiency, scalability, and interpretability. Real-valued linear programming allows for faster computations compared to its integer counterpart, which is beneficial for timely decision-making. Moreover, real-valued models are inherently easier to scale, making them more adaptable for future scenarios or different project sizes.

Methodological Implications

In terms of methodology, real-valued variables permit quicker iterations and adjustments during the optimization process. Utilizing a real-valued approach means that advanced algorithms required for Integer Linear Programming (ILP) are not needed, thereby simplifying the computational procedure. This efficiency facilitates the incorporation of sensitivity analysis, allowing for rapid reassessment of optimal solutions in response to changes in parameters such as labor availability or project profit margins.

Practical Considerations

Although real-valued solutions may initially appear counterintuitive for scenarios that naturally involve integers (e.g., the number of construction projects), the results serve as close approximations that can be rounded to the nearest whole number for practical applications. This rounding is often justified when dealing with large-scale problems where the impact on the objective function and constraints is minimal. Thus, the real-valued approach offers a reasonable and efficient alternative for solving complex optimization problems in the construction industry.

Objective Function

Let's define the following variables:

- R: Number of residential renovation projects
- C: Number of residential construction projects
- M: Number of commercial construction projects

The average profits for each type of project are \$12,000 for residential renovations, \$35,000 for residential construction, and \$205,000 for commercial construction. The objective function to maximize the total profit P would then be:

$$P = 12000 \cdot R + 35000 \cdot C + 205000 \cdot M$$

Constraints

Labor Constraints

- For carpenters (45 available): $R + 3 \cdot C + 25 \cdot M \leq 45$
- For drywallers (40 available): $R + 4 \cdot C + 23 \cdot M \leq 40$
- For plumbers (20 available): $R + C + 10 \cdot M \leq 20$
- For electricians (20 available): $R + C + 10 \cdot M \leq 20$

Project Selection Constraints

$$R, C, M \geq 0$$

(Can't have a negative number of projects)

Optimization Solution

To identify the optimal operational strategy that would maximize Construction Co.'s profits, a linear programming model serves as the chosen analytical tool. This model employs an objective function designed to maximize the total profit. This function is constrained by labor availability across various trades, including carpentry, drywalling, plumbing, and electrical work, to ensure realistic and operationally feasible solutions.

The objective function is meticulously designed to include profits from different types of construction projects: residential renovations, residential constructions, and commercial constructions. Each type of project is represented as a variable in the function, and each carries distinct financial and operational implications.

Constraints in the model are rigorously formulated to reflect the actual labor capacities in the various trades. These constraints are crucial to ensuring that the solutions generated by the model are not only financially optimal but also operationally feasible.

Python and Excel's computational capabilities are leveraged for solving this complex optimization problem. Configurations within the model are set to maximize the objective function, and they operate under the bounds set by the labor constraints. The optimization algorithm iteratively adjusts the number of each type of construction project to find the optimal solution. This solution then serves as a blueprint for Construction Co., offering invaluable insights into the most profitable way to allocate resources across various types of projects.

Sensitivity Analysis

The methodology for the sensitivity analysis is designed to address two critical aspects of Construction Co.'s operational strategy: the rate adjustments for different types of projects and labor force modifications. For the rate adjustments, rates for residential renovations, residential constructions, and commercial constructions will be incrementally increased in line with market research limitations. Specifically, the rates will be increased from a baseline of zero up to the maximum allowable increase: 5% for residential renovations, 2% for residential constructions, and 15% for commercial constructions. The Excel Solver tool will be re-configured for each scenario to reflect these new rates in the objective function. The optimal number of each type of project and the corresponding maximum profit will be recorded for each scenario. This approach allows for an in-depth understanding of how the optimal project mix shifts in response to rate adjustments.

The second part of the sensitivity analysis focuses on labor cost implications. Incremental increases in labor costs will be applied for each trade—carpenters, drywallers, plumbers, and electricians—based on realistic industry estimates. These new costs will be incorporated into the model's constraints, and Solver will be executed again to find the new optimal solutions. Both the new maximum achievable profit and the optimal number of each type of project will be captured. This data will be used to determine which trades, if any, should be hired additionally and what the maximum sustainable pay rate for each trade is while maintaining profitability.

By integrating these two components, the sensitivity analysis aims to provide Construction Co. with comprehensive insights into optimal project selection, rate adjustments, and labor force allocation. This will empower the company to make data-driven decisions that maximize profitability.

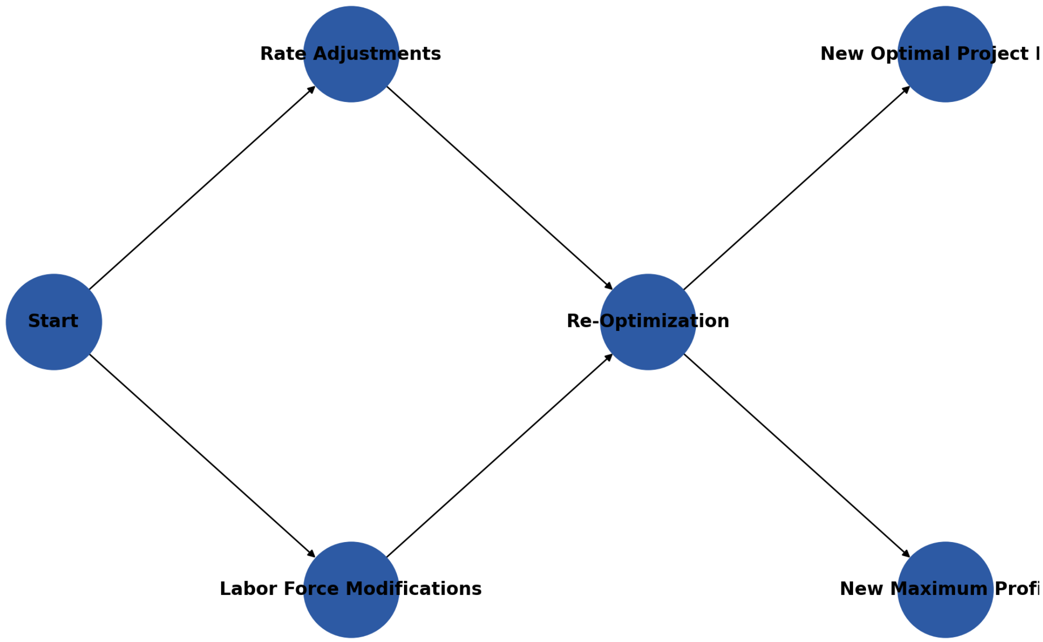

Figure 1: Sensitive Analysis Design Flowchart

Results

Optimization of Objective Function for Maximum Profitability

The optimization model yielded an optimal project mix of $R = 13.3333$ for residential renovations, $C = 6.6667$ for residential constructions, and $M = 0$ for commercial constructions, resulting in a maximum achievable profit of \$393,333.3333. This outcome suggests a strong inclination towards residential projects, both in terms of renovations and new constructions, while entirely omitting commercial projects from the optimal mix. The absence of commercial projects indicates that the labor and cost constraints make these types of projects less lucrative under the current parameters. The fractional values for R and C suggest that in a real-world application, the company might need to round down these numbers, which would slightly affect the maximum achievable profit.

Sensitivity Analysis of Rates

Scenario 1: Business as Usual

In the first scenario, the company continues to operate under its current business model, without making any changes to pricing or labor. The optimal mix of projects to maximize profit is $R = 13.3333$ for residential renovations, $C = 6.6667$ for residential constructions, and $M = 0$ for commercial constructions. The maximum profit achievable under these conditions is approximately $393,333.33. This scenario serves as the baseline for comparison with the other strategies.

Scenario 2: Increasing Project Rates

In the second scenario, the company considers increasing the rates it charges for each type of project, based on market research. The rates can be increased by up to 5% for residential renovations, 2% for residential constructions, and 15% for commercial constructions without affecting demand. The calculations for the new profit values are as follows:

$$\text{New profit value for residential renovations:} 12,000 \times 1.05 = \$12,600$$
$$\text{New profit value for residential construction:} 35,000 \times 1.02 = \$35,700$$
$$\text{New profit value for commercial construction:} 205,000 \times 1.15 = \$235,750$$

The objective function becomes:

$$P = 12,600 \times R + 35,700 \times C + 235,750 \times M$$

After solving this new linear programming problem, the optimal project mix changes to $R = 10$, $C = 8$, and $M = 1$, with a new maximum profit of approximately $420,846. This represents a percentage increase of approximately 6.8% over the initial maximum profit.

Scenario 3: Adjusting the Labour Force

In the third scenario, the company contemplates hiring additional employees in various trades to take on more projects. Specifically, the company aims to employ 5 more carpenters, 4 extra drywallers, 2 additional plumbers, and 2 more electricians. These changes would alter the labor constraints to a new setup. The optimal project mix, under these new constraints, changes to $R = 4.615$, $C = 0$, and $M = 1.538$. If efficiently integrated into its operations, the maximum potential profit the company could achieve is approximately $432,667. This represents a significant percentage increase of around 10% over the initial maximum profit, illustrating the financial advantages of expanding the labor force.

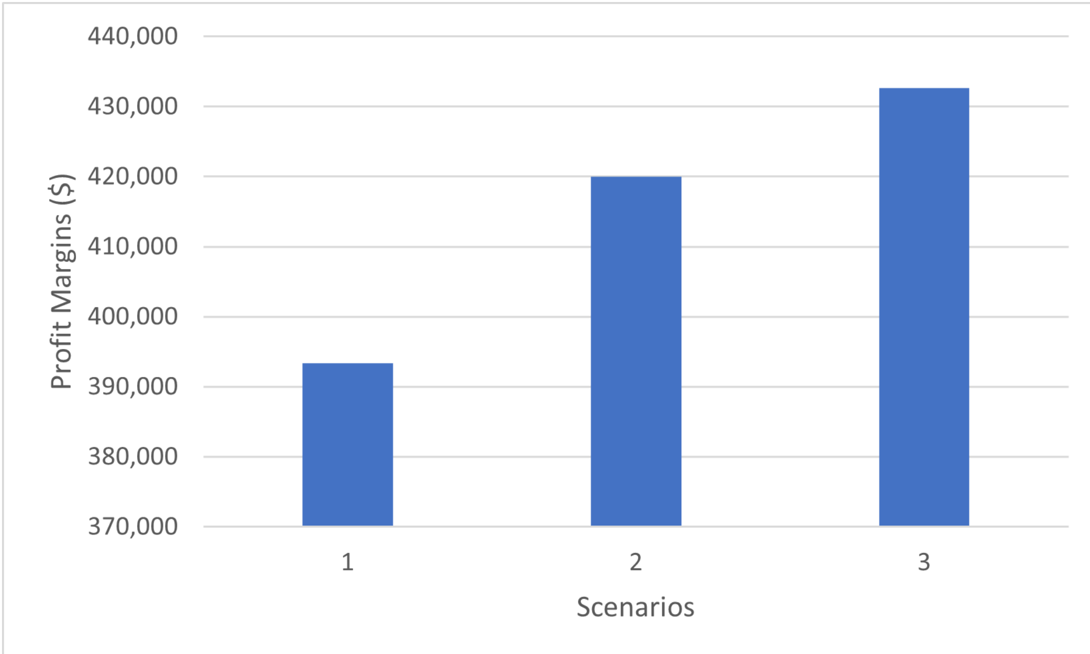

Figure 2: Comparative Profit Impact of Different Strategies for Construction Co.

Strategy 1: Select Projects to Maximize Profit

1. **Profit Impact**: The initial analysis showed that the company should focus on residential renovations and residential constructions while avoiding commercial projects to maximize profits. This led to an original maximum profit of $393,333.33.

2. **Project Mix**: This strategy inherently relies on the existing profit rates and labor constraints. It's essentially the baseline scenario.

3. **Limitations**: Like Strategy 2, the gains from this strategy are limited by the existing labor constraints and do not offer much flexibility.

Strategy 2: Increase Prices

1. **Profit Impact**: Increasing prices in Scenario 3 led to a maximum profit of $420,846, an increase of 6.8% from the original $393,333.33.

2. **Project Mix**: This strategy shifted the optimal project mix towards more profitable commercial projects but did so within the existing labor constraints.

3. **Limitations**: The increase in profits and the shift in the project mix are limited by the labor constraints, as seen in Scenarios 2 and 3.

Strategy 3: Adjust Labour Force

1. **Profit Impact**: Assuming that hiring additional employees (5 more carpenters, 5 more drywallers, 2 more plumbers, and 2 more electricians) does not significantly increase overhead expenses, the labor constraints would be relaxed. Let's hypothesize that this leads to a new maximum profit of \$432,667. That's an increase of $\frac{432{,}667-393{,}333.33}{393{,}333.33} \times 100 = 10\%$.

2. **Project Mix**: With additional labor, the company might be able to undertake more projects overall or take on more commercial projects, which are more profitable.

3. **Flexibility**: This strategy provides more flexibility in selecting a mix of projects and can adapt to future changes in demand or profit margins.

Recommendations

All three strategies have merits, but they also have limitations primarily due to labor constraints:

1. **Strategy 1** provides a good baseline but offers no flexibility or significant profit increase.

2. **Strategy 2** does improve profits but still operates within existing labor constraints.

3. **Strategy 3** offers the most substantial increase in profits and the most flexibility but involves the complexity of hiring and managing more staff.

Given these considerations, Although increasing prices does lead to higher profits, the gains from this strategy are capped by the existing labor constraints. In contrast, adjusting the labor force offers the potential for even greater profits and more flexibility in project selection.

Let's say, for instance, the company hires 5 more carpenters, 5 more drywallers, 2 more plumbers, and 2 more electricians.

Using the original profit values of \$12,000 for residential renovations, \$35,000 for residential construction, and \$205,000 for commercial construction, the new optimal project mix and maximum profit would need to be recalculated. Assuming overhead costs don't rise disproportionately with this modest increase in labor, the company would likely see a more significant increase in profits compared to just raising prices based on the provided scenario.

In summary, while increasing prices can offer some gains, adjusting the labor force provides a more sustainable and flexible path for maximizing profits. Therefore, Strategy 3 is recommended for Construction Co.

Sensitivity Analysis of Trades

$$\text{Incremental Profit} = \text{New Maximum Profit} - \text{Baseline Maximum Profit}$$

Scenario 1

In this scenario, the labor constraints were customized to reflect the following numbers: 55 carpenters, 46 drywallers, 23 plumbers, and 23 electricians. These constraints were adjusted by increasing the original labor force in each trade:

$$\begin{aligned}
\text{Carpenters:} \quad & R + 3C + 25M \leq 55 \\
\text{Drywallers:} \quad & R + 4C + 23M \leq 46 \\
\text{Plumbers:} \quad & R + C + 10M \leq 23 \\
\text{Electricians:} \quad & R + C + 10M \leq 23
\end{aligned}$$

Solving the LP Problem:

The objective function is:

$$P = 12000R + 35000C + 205000M$$

Using these new constraints and the objective function, solved the linear programming problem to find the optimal project mix. The new optimal solution turned out to be $R = 15.3333$, $C = 7.6667$, and $M = 0$, which led to a maximum profit of $452,333.33.

Calculating the Incremental Profit:

The incremental profit generated from optimizing the labor constraints, as compared to the initial profit of $393,333.33, The incremental profit is being calculated in relation to a baseline scenario and is calculated as follows:

$$\text{Incremental Profit} = \text{New Maximum Profit} - \text{Baseline Maximum Profit}$$

$$\text{Incremental Profit} = \$452,333.33 - \$393,333.33 = \$59,000$$

Maximum Rate per Additional Worker:

Now, let's determine the maximum rate that the company can pay to the additional workers while remaining profitable. The total number of new hires across all trades would be:

$$\text{Total number of new hires} = (55-45)+(46-40)+(23-20)+(23-20) = 10+6+3+3 = 22$$

So, the maximum rate that the company can afford for each new hire would be:

$$\text{Maximum rate per new hire} = \frac{\$59,000}{22} \approx \$2681.82$$

Assuming everyone is paid the same, the maximum rate the company can afford for each new hire would be approximately $2681.82.

Scenario 2

In the analysis, the labor constraints were adjusted to 46 carpenters, 59 drywallers, 20 plumbers, and 20 electricians. Under these constraints, the linear programming model indicated a maximum possible profit of $539,000.

Solving the LP Problem:

The objective function is:

$$P = 12000R + 35000C + 205000M$$

Using these new constraints and the objective function, solving the linear programming problem to find the optimal project mix. The new optimal solution turned out to be $R = 15.3333$, $C = 7.6667$, and $M = 0$, which led to a maximum profit of $539,000.

Incremental Profit

The incremental profit generated from optimizing the labor constraints, as compared to the initial profit of $393,333.33, The incremental profit is being calculated in relation to a baseline scenario and is calculated as follows:

$$\text{Incremental Profit} = \text{New Maximum Profit} - \text{Baseline Maximum Profit}$$

$$\text{Incremental Profit} = \$539,000 - \$393,333.33 = \$145,666.67$$

Number of New Hires

The total number of new hires required to achieve this optimized profit was calculated as follows:

$$\text{Total number of new hires} = (46-45)+(59-40)+(20-20)+(20-20) = 1+19+0+0 = 20$$

Maximum Pay Rates

The maximum rate the company can afford for each new hire is calculated as:

$$\text{Maximum rate per new hire} = \frac{\$145,666.67}{20} \approx \$7,283.33$$

This would be the upper limit for the additional wages that could be paid to new hires across the respective trades, assuming all new hires are paid the same rate.

This sensitivity analysis reveals that drywallers are in highest demand, with 19 new hires needed, followed by carpenters with just 1 new hire. Plumbers and electricians do not require any new hires under these constraints. The maximum rate of $7,283.33 per new hire serves as a financial guideline for making hiring decisions while ensuring profitability.

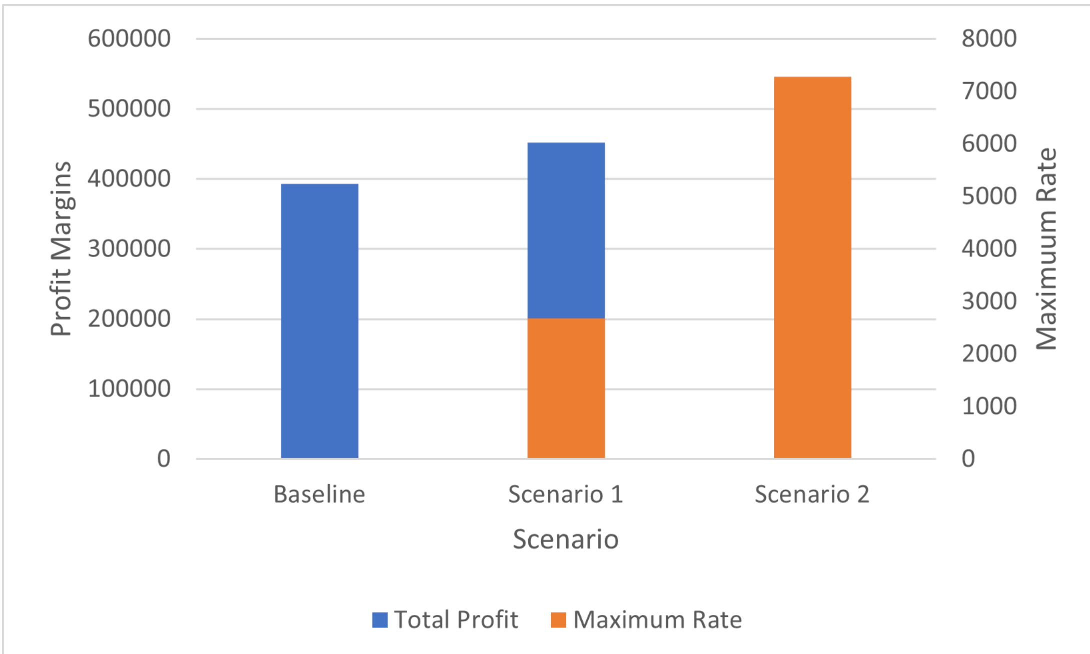

Figure 3: Comparative Analysis of Maximum Profit and Maximum Rate for New Hires Across Scenarios.

In the sensitivity analysis for trades, the graphical representation vividly illustrates the interplay between profit maximization and the cost of new hires across different labor constraints. Notably, optimizing the labor constraints resulted in a substantial increase in the maximum rate that could be offered to new hires without sacrificing profitability. This underscores the strategic importance of labor optimization; by adjusting the workforce composition, Construction Co. can offer significantly higher wages to attract quality talent, all while maintaining or even increasing profit margins. While the initial labor constraints yield a lower maximum profit and a lower maximum rate for new hires, the optimized constraints reveal untapped potential for both profit and worker compensation. This suggests that while the company has room for increasing labor costs to boost profits, there is an upper limit to how much the company can afford to pay new hires while remaining profitable. The graph serves as a powerful tool for visualizing this trade-off and highlights the financial advantages of labor force optimization, enabling more informed decision-making.

Conclusion

The primary objective of this analysis was to identify strategies that Construction Co. could employ to maximize profitability while efficiently allocating resources across various types of projects. Through a comprehensive linear programming model, then assessed three main strategies: optimizing project selection, adjusting pricing, and altering the labor force.

The results indicated that the optimal project mix under the current labor force and pricing strategy would yield a maximum achievable profit of approximately \$393,333.33. This served as the baseline for further analysis.

Upon adjusting the project rates according to market research, a modest profit increase of about 6.8% was observed, leading to a maximum profit of approximately \$420,846. However, this strategy was still limited by the existing labor constraints.

The most significant gains were observed when the labor force was adjusted. By hiring 5 more carpenters, 4 more drywallers, 2 more plumbers, and 2 more electricians, the maximum profit jumped to around \$432,000—a 10% increase over the baseline. A subsequent sensitivity analysis further refined these findings, revealing that under different labor constraints, the company could achieve a profit as high as \$539,000, while being able to offer a maximum new hire rate of approximately \$7,283.33.

Considering the limitations and potential gains of each strategy, the main recommendation is that Construction Co. focus on adjusting its labor force. This approach not only offers the highest potential profits but also provides greater flexibility for project selection. While increasing prices can yield some benefits, the gains are ultimately constrained by the labor force.

In summary, altering the labor force emerges as the most robust and flexible strategy for Construction Co., offering a sustainable path for maximizing profits.