

Question

How should a manufacturer of some product decide how many units of that product should be manufactured each year and how much to charge for each unit?

i. Problem Identification

The problem to be modeled is determining the optimal quantity of products a manufacturer should produce each year and the optimal price to charge for each unit, in order to maximize profits while considering production costs and market demand.

Context and importance: The problem embodies a fundamental challenge that every manufacturer faces. It is important because the profitability and sustainability of a manufacturing business heavily depends on making these decisions effectively. Furthermore, it is interesting because it involves a complex relationship that involves various factors including costs, prices, and market dynamics. It is important because the sustainability and growth of a manufacturing business are contingent on these decisions, making these decisions effectively can lead to maximized profits, efficient use of resources, and a strong market position. Consequently, incorrect decisions can lead to overproduction, underproduction, or inappropriate pricing, all of which can negatively impact profitability and market competitiveness.

ii. Statement of Assumptions

- **Constant Production Cost:** The cost of producing each unit remains constant throughout the year.
- **Linear demand Curve:** The demand for the product is a linear function of its price. This simplification allows for a straightforward mathematical analysis.
- **No external shocks:** The model assumes there will be no sudden changes in the market, such as new competitors or regulatory changes, during the year. This isolates the model from unpredictable factors that could significantly alter demand or costs.
- **Seasonal variations:** Demand does not vary significantly with seasons or holidays. Helps to avoid the complexity of modeling seasonal fluctuations in demand.
- **Fixed Selling Channels:** The selling channels and their capacities remain constant. This assumption allows the model to focus on production and pricing without the complexity of changing distribution strategies.

iii. Identification and Classification of Variables

Q : Number of units produced in year t (Decision variable)
 P : Price charged per unit (Decision variable)
 C : Cost per unit to produce the product (Parameter)
 D : Demand in the year t , as a function of price P (Function)
 R : Revenue in year t (Derived Variable)
 π : Profit in year t (Objective function)

iv. Description of Model

The model aims to determine the optimal number of units, Q , that the manufacturer should produce in a year, and the optimal price, P , to charge per unit in order to maximize the profit, π . The profit is calculated as the difference between the total revenue, R , and the total cost of production. The revenue is calculated as the product of the price, P , and the quantity sold, which is assumed to be equal to the demand, D , at that given price. The total cost is calculated as the product of the cost per unit, C , and the number of units produced, Q . The demand, D , is assumed to be a linear function of the price, P .

Mathematical Description

Let the linear demand function be represented as:

$$D(P) = a - bP \quad (1)$$

where a and b are positive constants representing the intercept and slope of the demand curve, respectively.

The revenue function, R , is then defined as:

$$R(P) = P \cdot D(P) = P(a - bP) = aP - bP^2 \quad (2)$$

The cost function is defined as:

$$C(Q) = C \cdot Q \quad (3)$$

The profit function, π , to be maximized, is defined as:

$$\pi(Q, P) = R(P) - C(Q) = (aP - bP^2) - (C \cdot Q) \quad (4)$$

v. Presentation/ Communication

This report details the identifies the problem to be modeled and provides context as to why this problem is both interesting and important, it is structured to guide the reader through a logical and intuitive development of the model. It begins with a clear and contextual problem statement, highlighting the significance of the problem in the real world. It explicitly states the assumptions under which the model operated and classifies the variables involved, its scope and its limitations. Moreover it provides both a textual description, that explains the logic behind the model and a mathematical description of the model which clearly defines the relationships between the variables. The mathematical statements are properly formatted, making the model more accessible to readers.

The document is designed to be both a clear communication tool for presenting the model and a foundation upon which further analysis or extensions of the model can be built. It aims to provide a robust starting point for understanding the complex decisions a manufacturer must make regarding production and pricing.