Interest Rates Term Structure

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Coupon Bearing Bond Price

$$P_0 = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t} + \frac{F}{(1+i)^n}$$

Zero-Coupon Bond Price

$$P_0 = \frac{F}{(1+i)^n}$$

Perpetuities Price

$$P_0 = C/i$$

Macaulay Duration, Time Weighted Present Values

i = yield/year compounded m times/year
n = number of periods, D measured in years

$$D = \sum_t tPV_t = \sum_t t \frac{c_t}{[1 + (i/m)]^t}$$

Modified Duration

$$PV_t = \frac{c_t}{[1 + (i/m)]^t}$$

$$\frac{\delta PV_t}{\delta i} = \frac{-(t/m)c_t}{[1 + (i/m)]^{t+1}} = -\frac{t/m}{1 + (i/m)} PV_t$$

$$P = \sum_t PV_t$$

$$D_{mod} = -\frac{1}{P}\frac{\delta P}{\delta i} = -\frac{D}{1 + (i/m)}$$

Convexity

$$C = \frac{1}{P}\frac{\delta^2 P}{\delta i^2} = \frac{1}{P}\sum_{k=1}^{n}\frac{\delta^2 PV_k}{\delta i^2}$$

$$C = \frac{1}{P[1+(i/m)]^2}\sum_{k=1}^{n}\frac{k(k+1)}{m^2}\frac{c_k}{[1+(i/m)]^k}$$

Bond Price Change, based on Duration and Convexity

$$\Delta P = -D_{mod}P\Delta i + \frac{PC}{2}(\Delta i)^2$$

Yield Curve

- Par Rates
  Example of Treasury Yield Curve
- Zero/Spot Rates s_t
- Discount Factors d_t
- Forward Rates $f_{t1,t2}$
  Arbitrage Theory

Term Structure Theories

- Expectations Theory
- Liquidity Preference
- Market Segmentation

Yield Curve II

- Short Rates $f_{t,t+1}$
- Zero from two coupon bonds (Excel)
- Bootstrap Zero from Par
- Running Present Value

R Session Info

> *toLatex(sessionInfo())*

- R version 3.6.1 (2019-07-05), `x86_64-w64-mingw32`
- Locale: `LC_COLLATE=English_United States.1252`, `LC_CTYPE=English_United States.1252`, `LC_MONETARY=English_United States.1252`, `LC_NUMERIC=C`, `LC_TIME=English_United States.1252`
- Running under: `Windows 10 x64 (build 17134)`
- Matrix products: default
- Base packages: base, datasets, graphics, grDevices, methods, stats, utils
- Other packages: FinCal 0.6.3, knitr 1.27, quantmod 0.4-15, rattle 5.3.0, TTR 0.23-6, xtable 1.8-4, xts 0.12-0, zoo 1.8-7
- Loaded via a namespace (and not attached):