

Interest Rates Term Structure

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- Review
- Yield Curves
 - Spot Rates
 - Forward Rates and Arbitrage
 - Zeros Curve
 - Zeros Curve from Coupon Bonds
 - Bootstrapping Zeros Curve

- Bond Risk
- Bond Pricing and Sensitivity
- Macaulay Duration and Sensitivity
- Macaulay Duration to Modified Duration
- Convexity

Coupon Bearing Bond Price

$$P_0 = \sum_{t=1}^n \frac{C_t}{(1+i)^t} + \frac{F}{(1+i)^n}$$

Zero-Coupon Bond Price

$$P_0 = \frac{F}{(1+i)^n}$$

Perpetuities Price

$$P_0 = C/i$$

Macaulay Duration, Time Weighted Present Values

i = yield/year compounded m times/year

n = number of periods, D measured in years

$$D = \sum_t tPV_t = \sum_t t \frac{C_t}{[1 + (i/m)]^t}$$

Modified Duration

$$PV_t = \frac{c_t}{[1 + (i/m)]^t}$$

$$\frac{\delta PV_t}{\delta i} = \frac{-(t/m)c_t}{[1 + (i/m)]^{t+1}} = -\frac{t/m}{1 + (i/m)} PV_t$$

$$P = \sum_t PV_t$$

$$D_{mod} = -\frac{1}{P} \frac{\delta P}{\delta i} = -\frac{D}{1 + (i/m)}$$

Convexity

$$C = \frac{1}{P} \frac{\delta^2 P}{\delta i^2} = \frac{1}{P} \sum_{k=1}^n \frac{\delta^2 PV_k}{\delta i^2}$$

$$C = \frac{1}{P[1 + (i/m)]^2} \sum_{k=1}^n \frac{k(k+1)}{m^2} \frac{c_k}{[1 + (i/m)]^k}$$

Bond Price Change, based on Duration and Convexity

$$\Delta P = -D_{mod}P\Delta i + \frac{PC}{2}(\Delta i)^2$$

Yield Curve

- Par Rates
Example of Treasury Yield Curve
- Zero/Spot Rates s_t
- Discount Factors d_t
- Forward Rates $f_{t1,t2}$
Arbitrage Theory

Term Structure Theories

- Expectations Theory
- Liquidity Preference
- Market Segmentation

Yield Curve II

- Short Rates $f_{t,t+1}$
- Zero from two coupon bonds (Excel)
- Bootstrap Zero from Par
- Running Present Value

R Session Info

- ```
> toLatex(sessionInfo())
```
- R version 3.6.1 (2019-07-05), x86\_64-w64-mingw32
  - Locale: LC\_COLLATE=English\_United States.1252,  
LC\_CTYPE=English\_United States.1252,  
LC\_MONETARY=English\_United States.1252,  
LC\_NUMERIC=C,  
LC\_TIME=English\_United States.1252
  - Running under: Windows 10 x64 (build 17134)
  - Matrix products: default
  - Base packages: base, datasets, graphics, grDevices,  
methods, stats, utils
  - Other packages: FinCal 0.6.3, knitr 1.27,  
quantmod 0.4-15, rattle 5.3.0, TTR 0.23-6, xtable 1.8-4,  
xts 0.12-0, zoo 1.8-7
  - Loaded via a namespace (and not attached):