

Algorithms and Programming

Lecture 9 – Search and sorting algorithms

Course content

Big image in programming

Detailed image in programming

- Introduction in the software development process
- Procedural programming
- Modular programming
- Abstract data types
- Software development principles
- Testing and debugging
- Recursion
- Complexity of algorithms
- Search and sorting algorithms
- Backtracking and other problem solving methods
- Recap

Last time

- Recursion
 - Basic concept
 - Mechanism
 - Recursive functions
- Computational complexity
 - Analyzing the efficiency of a program
 - Run time complexity
 - Classes of complexity

Today

- Search
 - Objective and problem specification
 - Types
 - Sequential seach
 - Binary search
- Sort
 - Objective and problem specification
 - Types
 - Selection sort
 - Insert sort
 - Bubble sort
 - Quick sort

Search methods: Objective

- For a set of data stored in memory as a list of elements (el1, el2, ..., eln)
 - The list may contain elements in any order
 - The list contains elements ordered by some criteria
- Look for
 - A certain element
 - Elements that satisfy different criteria
- Return
 - True or False if the element(s) exist in the list
 - The index of the element found

Search methods: problem specification

- Unorderd list of elements
 - Input data:
 - elem, n, list = (list_i) i=0,1, 2,...,n-1 (n natural number)
 - Results:
 - p, where $0 \le p \le n-1$, if elem = list[p] or -1, if elem is not in the list
- Ordered list of elements
 - Input data:
 - elem, n, list = (list_i), list[0]<list[1]<...list[n-1], i=0,1, 2,...,n-1 (n natural number)
 - Results:
 - p, where $0 \le p \le n-1$, if elem = list[p] or -1, if elem is not in the list

Search methods: implementation

Sequential search

- Basic idea: the elements of the list are examined one by one (the list can be ordered or not)
- Versions: simple and improved

Binary search

 Basic idea: the problem is divided in two similar but smaller subproblems (the list has to be ordered)

Python

Functions index and find

Sequential search: implementation Unordered list

```
def searchSeq(el, 1):
   Descr: search for an element in a list
   Data: an element and a list
   Res: the position of element in list or -1 if the elemnt is not in the list
   pos = -1
   for i in range(0, len(1)):
        if (el == l[i]):
            pos = i
   return pos
def test searchSeq():
    assert searchSeq(2, [3,2,4]) == 1
    assert searchSeq(2, [3,5,7,2]) == 3
    assert searchSeq(2, [2,5,4]) == 0
    assert searchSeq(2, [3,7,4]) == -1
    assert searchSeq(2, [3,2,4,2,7]) == 3
test_searchSeq()
```

| Case | T(n) |
|--------------|------------------------|
| Best case | $\sum_{i=1}^{n} 1 = n$ |
| Worst case | $\sum_{i=1}^{n} 1 = n$ |
| Average case | $\sum_{i=1}^{n} 1 = n$ |

O(n)

Sequential search: implementation Unordered list – Improved version

```
def searchSeq v2(e1, 1):
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or
    -1 if the elemnt is not in the list
    i = 0
    while ((i < len(l)) and (l[i] != el)):</pre>
        i = i + 1
    if (i < len(1)):</pre>
       return i
    else:
        return -1
def test searchSeq v2():
    assert searchSeq_v2(2, [3,2,4]) == 1
    assert searchSeq_v2(2, [3,5,7,2]) == 3
    assert searchSeq_v2(2, [2,5,4]) == 0
    assert searchSeq_v2(2, [3,7,4]) == -1
    assert searchSeq_v2(2, [3,2,4,2,7]) == 1
test searchSeq v2()
```

| Case | T(n) |
|--------------|------------------------|
| Best case | 1 |
| Worst case | $\sum_{i=1}^{n} 1 = n$ |
| Average case | (0+1+2++n-1)/n |

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Sequential search: implementation Ordered list

```
def searchSeqOrder(el, 1):
   Descr: search for an element in a list
   Data: an element and a list of ordered elements
   Res: the position of element in list or the position where the element can be inserted
   if (len(1) == 0): #1==[]
        return 0
   pos = -1
   for i in range(len(l) - 1, -1, -1):
        if (el <= l[i]):</pre>
            pos = i
   if (pos == -1):
        return len(1)
   return pos
def test searchSeqOrder():
    assert searchSeqOrder(2, [2,3,4]) == 0
    assert searchSeqOrder(4, [2,3,4,5]) == 2
    assert searchSeqOrder(2, [1,3,5,7]) == 1
    assert searchSeqOrder(9, [1,2,3]) == 3
```

test searchSeqOrder()

| Case | T(n) |
|--------------|------------------------|
| Best case | $\sum_{i=1}^{n} 1 = n$ |
| Worst case | $\sum_{i=1}^{n} 1 = n$ |
| Average case | $\sum_{i=1}^{n} 1 = n$ |

Sequential search: implementation Ordered list – Improved version

```
def searchSeqOrder v2(el, 1):
    Descr: search for an element in a list
    Data: an element and a list of ordered elements
    Res: the position of element in list or
    the position where the element can be inserted
    if (len(1) == 0): #1==[]
       return 0
    if (el <= 1[0]):
        return 0
    if (el > l[len(l)-1]):
        return len(1)
    i = 0
    while ((i < len(1)) \text{ and } (l[i] < el)):
        i = i + 1
    return i
def test searchSegOrder v2():
    assert searchSeqOrder_v2(2, [2,3,4]) == 0
    assert searchSeqOrder_v2(4, [2,3,4,5]) == 2
    assert searchSeqOrder_v2(2, [1,3,5,7]) == 1
    assert searchSeqOrder_v2(9, [1,2,3]) == 3
test searchSeqOrder v2()
```

| Case | T(n) |
|--------------|------------------------|
| Best case | 1 |
| Worst case | $\sum_{i=1}^{n} 1 = n$ |
| Average case | (0+1+2++n-1)/n |

Binary search: implementation Ordered list – recursive version

```
def binarySearch(el, 1, start, end):
    if (start > end):
        return -1
    middle = (start + end) // 2
    if (el < l[middle]):</pre>
       return binarySearch(el, l, start, middle)
    elif (el > l[middle]):
        return binarySearch(el, 1, middle + 1, end)
    else: #el == l[middle]
        return middle
def binarySearchRec(el, 1):
    #Descr: search for an element in a list
    #Data: an element and a list
    #Res: the position of element in list or
    # -1 if the element is not in the list
    if (len(1) == 0):
        return -1
    elif (el < 1[0]) or (el > 1[1en(1) -1]):
        return -1
    else:
        return binarySearch(el, 1, 0, len(1)-1)
```

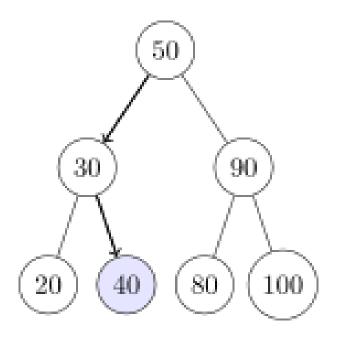
```
def test_binarySearchRec():
    assert binarySearchRec(4, [3,4,5]) == 1
    assert binarySearchRec(2, [-3,0,1,2]) == 3
    assert binarySearchRec(2, [2,5,6]) == 0
    assert binarySearchRec(2, [3,7,9]) == -1
    assert binarySearchRec(2, [1,2,2,5,6]) == 2
test_binarySearchRec()
```

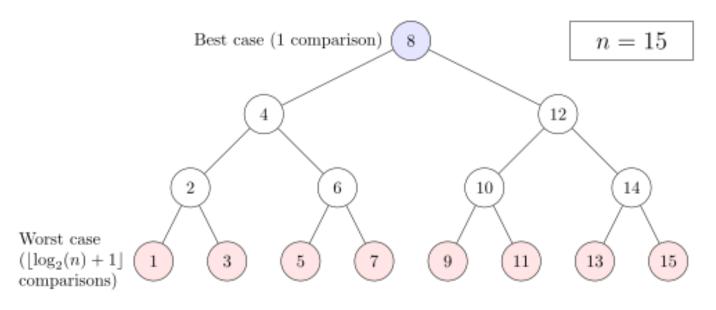
| Case | T(n) |
|--------------|------------|
| Best case | 1 |
| Worst case | $\log_2 n$ |
| Average case | $\log_2 n$ |

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Binary search: example

- List is [20, 30, 40, 50, 80, 90, 100]
- Search for element 40





https://en.wikipedia.org/wiki/Binary search algorithm

Binary search: implementation Ordered list – iterative version

```
def binarySearchIter(el, 1):
    Descr: search for an element in a list
    Data: an element and a list
    Res: the position of element in list or
    -1 if the elemnt is not in the list
    if (len(1) == 0):
        return -1
    elif (el < 1[0]) or (el > 1[len(1) -1]):
        return -1
    else:
        start = 0
        end = len(1) - 1
        while (start <= end):</pre>
            middle = (start + end) // 2
            if (el < l[middle]):</pre>
                end = middle
            elif (el > l[middle]):
                start = middle + 1
            else:
                return middle
```

```
def test_binarySearchIter():
    assert binarySearchIter(4, [3,4,5]) == 1
    assert binarySearchIter(2, [-3,0,1,2]) == 3
    assert binarySearchIter(2, [2,5,6]) == 0
    assert binarySearchIter(2, [3,7,9]) == -1
    assert binarySearchIter(2, [1,2,2,5,6]) == 2
test_binarySearchIter()

T(n) = \begin{cases} 1, if \ n = 1 \\ T(n/2) + 1, otherwise \end{cases}
if n = 2^k \rightarrow T(2^k) - T(2^{k-1}) + 1
```

$$I(n) = (T(n/2) + 1, otherwise)$$

$$if \ n = 2^k \to T(2^k) = T(2^{k-1}) + 1$$

$$T(2^k) = T(2^{k-1}) + 1$$

$$T(2^{k-1}) = T(2^{k-2}) + 1$$
...
$$T(2^1) = T(2^0) + 1$$

$$T(2^k) = k+1 \qquad k = \log_2 n \to T(n) = \log_2 n + 1$$

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Search methods: Python functions

- list.index(element)
 - Returns the index of the element in the list
 - If the element does not exist in the list, throws an exception
- list.count(element)
 - Returns the number of times the element appears in the list (if it exists)
 - Returns 0 if the element is not in the list

```
def test_index():
    1 = [7,2,13,4,1]
    assert l.index(2) == 1
    assert l.index(1) == 4
    try:
        l.index(3)
        assert False
    except ValueError as ex:
        print("elem not found")
        assert True
test_index()
```

```
def test_count():
    1 = [7,2,13,4,1]
    assert 1.count(2) == 1
    assert 1.count(1) == 1
    assert 1.count(3) == 0

test_count()
```

Sorting methods

- Objective
 - Rearrange the elements of a container such that they are in a certain relation of order
- Problem specification
 - Input data:
 - n, list = (list_i) i=0,1, 2,...,n-1 (n natural number)
 - Results:
 - n, list' = (list'_i) i=0,1, 2,...,n-1, orderRelation(list'_i, list_{i+1})=True for any i=0,1,...,n-2

Sorting methods: taxonomy

- Place where the elements are stored
 - Internal sort data to be sorted is available in the internal memory
 - External sort data available from files (external memory)
- The order relation
 - Ascending sort
 - Descending sort
- Keeping the initial order of the elements
 - Stable sort keep the initial order of equal elements
 - Instable sort the initial order of equal elements is not kept
- Space complexity
 - In-place sort the additional space (to that needed for the container) is small
 - Not-in-place / Out-of-place sort large additional space
- Mechanism
 - Selection sort
 - Insert sort
 - Bubble sort

Selection Sort

Basic idea

- Determine the smallest element from the collection and place it in first position (swap the smallest element with the first one)
- Repeat the first step for all elements different from the smallest element
- Algorithm
- Complexity
 - Time

$$T(n) = \sum_{i=0}^{n-2} (\sum_{j=i}^{n-1} 1 + 3) \approx n(n-1)/2 \rightarrow O(n^2)$$

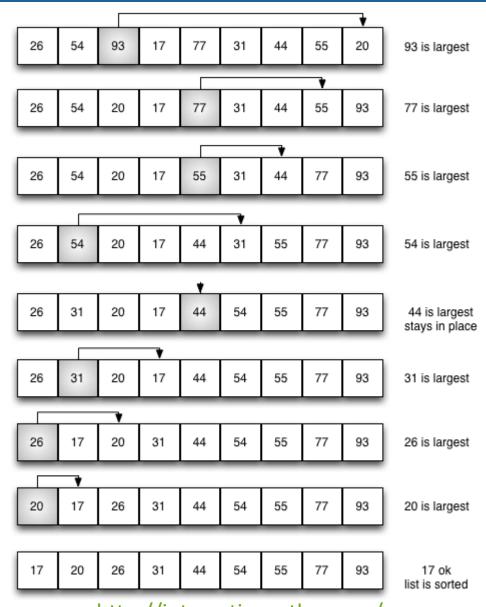
SpaceS(n)=n+1+1

```
def selectionSort(1):
    descr: sorts the leemnts of a list
    data: a list of elements
    res: the ordered list
    for i in range(0, len(1) - 1):
        min pos = i
        for j in range(i + 1, len(l)):
            if (1[j] < 1[min_pos]):</pre>
                min pos = j
        if (i < min_pos):</pre>
            aux = 1[i]
            l[i] = l[min pos]
            l[min_pos] = aux
   return 1
               1[i], 1[min_pos] = 1[min_pos], 1[i]
def test selectionSort():
    assert selectionSort([1,2,3]) == [1,2,3]
    assert selectionSort([3,2,1]) == [1,2,3]
    assert selectionSort([1,2,1]) == [1,1,2]
test_selectionSort()
```

Selection Sort: Example

• Each step, select the *largest* item and place it in the proper position

Alternative: each step, select the *smallest* item and place it in the proper position



http://interactivepython.org/

Insertion Sort

- Basic idea
 - Traverse the elements of the container and insert each element at the correct position in the subcontainer with the elements already sorted
 - At the end of the algorithm, the sub-container will have all the initial elements sorted
- Algorithm
- Complexity
 - Time

$$T(n) = \sum_{i=1}^{n-1} (1 + \sum_{j=0}^{i-1} 2 + 1) \approx n^2 + n - 2 \rightarrow O(n^2)$$

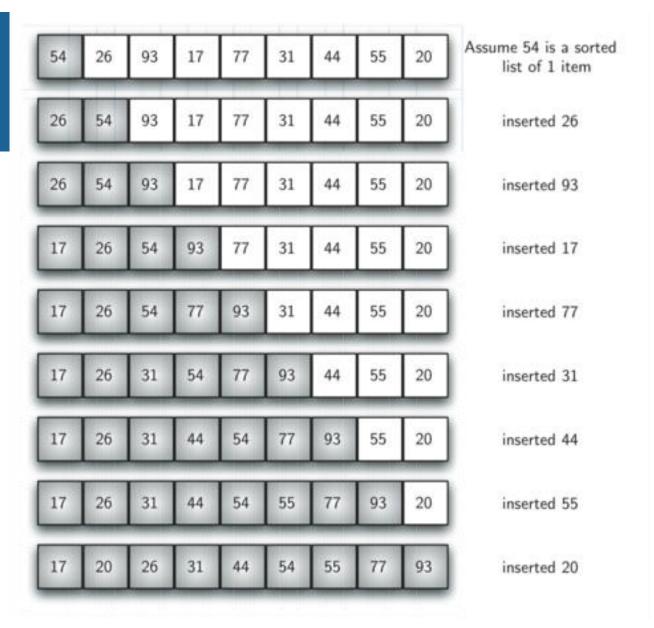
Space

$$S(n)=n+1+1+1$$

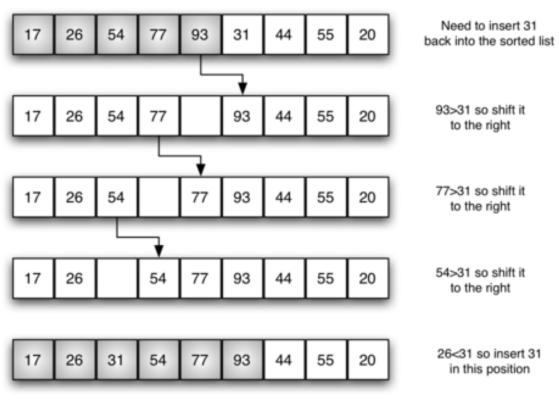
```
def insertSort(1):
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    for i in range(1, len(1)):
        noOfAlreadySort = i - 1
        crtElem = l[i]
        # insert crtElem in the right position
        # (<= noOfAlreadySort)</pre>
        j = noOfAlreadySort
        while ((j \ge 0)) and (crtElem < l[j]):
            1[i + 1] = 1[i]
            j = j - 1
        l[j + 1] = crtElem
    return 1
def test insertSort():
    assert insertSort([1,2,3]) == [1,2,3]
    assert insertSort([3,2,1]) == [1,2,3]
    assert insertSort([1,2,1]) == [1,1,2]
test_insertSort()
```

Insertion Sort: Example

- Maintain sorted sublists
- Insert each new item in the sorted sublist at the proper position



Insertion Sort: Example



http://interactivepython.org/

Bubble Sort

- Basic idea
 - Compare any 2 consecutive elements
 - If they are not in correct order, swap them
 - Until any 2 consecutive elements are in the correct order
- Algorithm
- Complexity
 - Time

```
T(n) \rightarrow O(n^2)
```

• Space

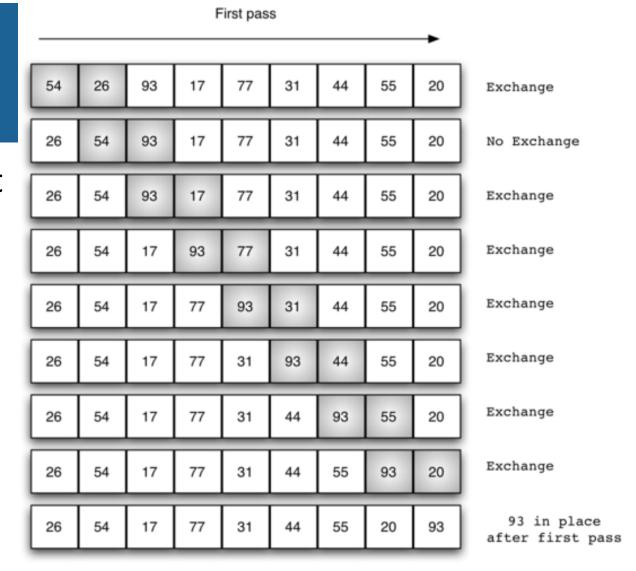
```
S(n)=n+1+1+1
```

```
def bubbleSort(1):
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    isSort = False
    while (not isSort):
        isSort = True
        for i in range(0, len(1) - 1):
            if (l[i] > l[i + 1]):
                aux = l[i]
                l[i] = l[i + 1]
                l[i + 1] = aux
                isSort = False
    return 1
def test bubbleSort():
    assert bubbleSort([1,2,3]) == [1,2,3]
    assert bubbleSort([3,2,1]) == [1,2,3]
    assert bubbleSort([1,2,1]) == [1,1,2]
test bubbleSort()
```

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Bubble Sort: Example

 Exchange adjacent items that are not in correct order



http://interactivepython.org/

Quick Sort

- Basic idea
 - Divide and conquer technique
 - 1. Divide: divide the container in 2 parts such that any element in the first sub-container ≤ any element in the second sub-container
 - Conquer: sort the two sub-containers (recursively)
- Algorithm

```
def test_quickSort():
    assert quickSort([1,2,3]) == [1,2,3]
    assert quickSort([3,2,1]) == [1,2,3]
    assert quickSort([1,2,1]) == [1,1,2]

test_quickSort()
```

```
def partition(l, start, end):
    pivot = l[start]
    i = start
    i = end
    while (i != j):
        while ((pivot <= l[j]) and (i < j)):
             j = j - 1
        l[i] = l[j]
        while ((1[i] \leftarrow pivot) \text{ and } (i \leftarrow j)):
             i = i + 1
        l[j] = l[i]
        l[i] = pivot
    return i
def quickSortRec(1, start, end):
    pivotPos = partition(1, start, end)
    if (start < pivotPos - 1):</pre>
        quickSortRec(1, start, pivotPos - 1)
    if (pivotPos + 1 < end):</pre>
        quickSortRec(l, pivotPos + 1, end)
def quickSort(1):
    descr: sorts the elemnts of a list
    data: a list of elements
    res: the ordered list
    111
    quickSortRec(1, 0, len(1) - 1)
    return 1
```

Quick Sort: Example

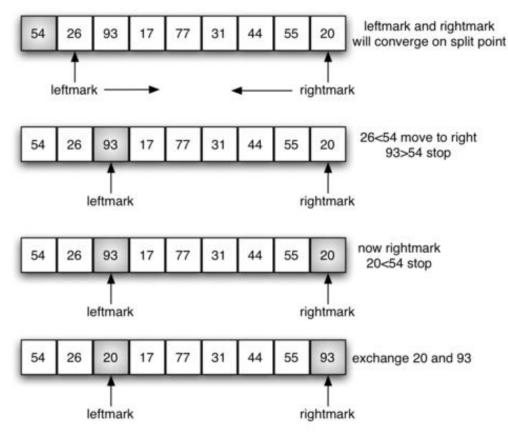
• First pivot value – first item

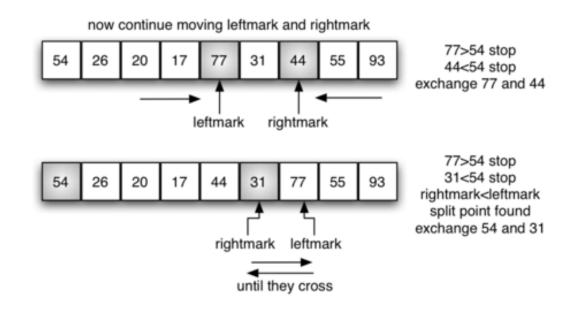


http://interactivepython.org/

Quick Sort: Example

Partition process

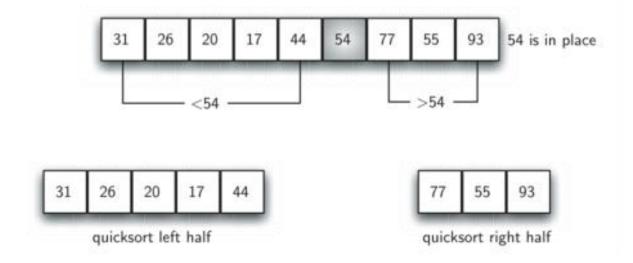




http://interactivepython.org/

Quick Sort: Example

Recursive call for the left half and right half



http://interactivepython.org/

Quick Sort: complexity

- The run time of quick-sort depends on the distribution of splits
 - The partitioning function requires linear time
 - Best case is when the partitioning function splits the array evenly

| Best case | $T(n)=2*T(n/2)+n \to O(n\log_2 n)$ |
|-----------------|--|
| Worst case | $T(n)=T(1)+T(n-1)+n=T(n-1)+n+1$ $T(n-1)=T(1)+T(n-2)+(n-1)=T(n-2)+n$ $T(2)=T(1)+T(1)+2=4$ $T(n)=(n+1)(n+2)/2-(1+2+3)=>O(n^2)$ |
| Average case | $L(n)=2*U(n/2)+n$ $U(n)=L(n-1)+n$ $L(n)=2*(L(n/2-1)+n/2)+n=2L(n/2-1_+2n=>O(n\log_2 n)$ |

Space complexity

• Average: $S(n) = \log_2 n$

• Worst: $S(n) = n^2$

Sorting in Python

- list.sort()
- sorted(lista)

```
>>> t = (5, 1, 17, 12)
>>> sorted(t)
[1, 5, 12, 17]
>>> t
(5, 1, 17, 12)
```

```
>>> a = [2, 1, 5, 7, 9]
>>> a
[2, 1, 5, 7, 9]
>>> sorted(a)
[1, 2, 5, 7, 9]
>>> a
[2, 1, 5, 7, 9]
>>> sorted(a, reverse=True)
[9, 7, 5, 2, 1]
>>> a
[2, 1, 5, 7, 9]
>>> a.sort()
>>> a
[1, 2, 5, 7, 9]
>>> a.sort(reverse=True)
>>> a
[9, 7, 5, 2, 1]
```

Sorting in Python: list of lists / tuples

Use the key argument in sorted/sort

key argument & lambda expressions

```
>>> sorted(my_list, key=lambda x: x[1])
[[2, 1, 3], [7, 2, 1], [1, 5, 7]]
>>> sorted(my_list, key=lambda x: x[2])
[[7, 2, 1], [2, 1, 3], [1, 5, 7]]
>>> sorted(my_list, key=lambda x: x[0])
[[1, 5, 7], [2, 1, 3], [7, 2, 1]]
```

Sorting in Python: list of custom objects

```
class Student:
    def __init__(self, name, grade):
        self.__name = name
        self.__grade = grade

def getName(self):
        return self.__name

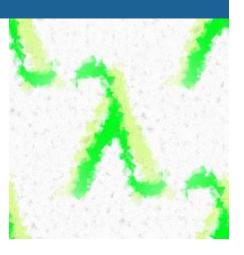
def getGrade(self):
    return self.__grade
```

Sorting in Python: list of custom objects

```
class Student:
   def __init__(self, name, grade):
       self. name = name
                                        >>> sorted(st list, key=lambda x: x.getGrade())
       self. grade = grade
                                        [Sara - 8, Emma - 9, Erin - 10]
                                        >>> sorted(st list, key=lambda x: x.getName())
   def getName(self):
                                        [Emma - 9, Erin - 10, Sara - 8]
       return self. name
                                        >>>
                                        >>> sorted(st list, key=lambda Student: Student.getName())
   def getGrade(self):
                                        [Emma - 9, Erin - 10, Sara - 8]
       return self. grade
   def repr (self):
       return self. name + " - " + str(self. grade)
```

Lambda expressions

- Small anonymous functions
- Defined and used in the same place
- ✓ Syntactically restricted to a single expression
- ✓ Can reference variables from the containing scope (just like nested functions)
- ✓ They are syntactic sugar for a function definition



Lambda expressions

Syntax

```
lambda arg1, arg2, ...argN : expression using arguments
```

- Lambda is an expression
 (def a function with name, statement)
- Body is simply an expression (not a block of stataments, no return statement)

```
>>> def f(x):
    return x**3

>>> f(2)
8
>>> g = lambda x: x**3
>>> g(2)
8
>>> (lambda x: x**3) (2)
8
```

Map and Lambda expressions

- Function map
 - r=map(function, sequence)

```
>>> m = map(f, [1,2,3])
>>> list(m)
[1, 8, 27]
>>> list(map(lambda x: x**3, [1,2,3]))
[1, 8, 27]
>>> list(map(lambda x,y: x**y, [1,2,3], [2, 3, 4]))
[1, 8, 81]
```

Filter and lambda expressions

- Function filter
 - filter(function, sequence)

```
>>> my_list = [1, 4, 5, 8, 9, 10]
>>> filter(lambda x: x%2 == 0, my_list)
<filter object at 0x02E4CFB0>
>>> list(filter(lambda x: x%2 == 0, my_list))
[4, 8, 10]
>>>
>>> fibonacci = [0,1,1,2,3,5,8,13,21,34,55]
>>> odd_numbers = list(filter(lambda x: x % 2, fibonacci))
>>> odd_numbers
[1, 1, 3, 5, 13, 21, 55]
>>>
>>> names = ["Zara", "Erin", "Carla", "Ana", "Nico"]
>>> filtered_names = list(filter(lambda x: x[-1] == "a", names))
>>> filtered_names
['Zara', 'Carla', 'Ana']
```

Sort and Lambda expressions

```
class Person:
    def __init__(self, n, a):
        self.name = n
        self.age = a

    def getName(self):
        return self.name

    def getAge(self):
        return self.age

    def __repr__(self):
        return self.name + "-" + str(self.age)
```

```
def sort python():
   11 = [4,2,3,1]
   11.sort()
   print(l1)
   l1s = sorted(l1)
   print(l1s)
   p1 = Person("nnnn", 20)
   p2 = Person("eeee", 21)
   p3 = Person("ttt", 10)
   12 = [p1, p2, p3]
   12s = sorted(12, key=lambda Person:Person.getName())
   print(12s)
   12s = sorted(12, key=lambda Person:Person.getAge())
   print(12s)
sort python()
```

Recap today

- Search
 - Sequential seach
 - Binary search
- Sort
 - Selection sort
 - Insert sort
 - Bubble sort
 - Quick sort

Next time

- Algorithms
 - Backtracking
 - Divide and conquer

Reading materials and useful links

- 1. The Python Programming Language https://www.python.org/
- 2. The Python Standard Library https://docs.python.org/3/library/index.html
- 3. The Python Tutorial https://docs.python.org/3/tutorial/
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