Algorithm Theory - Assignment 1

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February 4, 2020

Problem 1

In this problem, the objective is to maximize the profit of selling products X and Y, while satisfying production constraints on these products. The profit can be computed as

$$\begin{aligned} profit &= revenue - cost \\ &= revenue - (time_{machine} \times cost_{machine} + time_{craftsman} \times cost_{craftsman}) \end{aligned}$$

We can compute profits for each of the products

$$\begin{aligned} profit(X) &= 200 - (\frac{15}{60} \times 100 + \frac{20}{60} \times 20) = \frac{505}{3} \\ profit(Y) &= 300 - (\frac{20}{60} \times 100 + \frac{30}{60} \times 20) = \frac{770}{3} \end{aligned}$$

And formulate the problem as a Linear Programming problem. Let n_X and n_Y be the number of products X and Y produced.

$$\text{maximize } n_X \times profit(X) + n_Y \times profit(Y)$$
$$\text{subject to } \begin{cases} n_X \times time_{machine}(X) + n_Y \times time_{machine}(Y) \leq 40 \times 60 \\ n_X \times time_{craftsman}(X) + n_Y \times time_{craftsman}(Y) \leq 35 \times 60 \\ n_X \geq 10 \\ n_X, n_Y \geq 0 \end{cases}$$

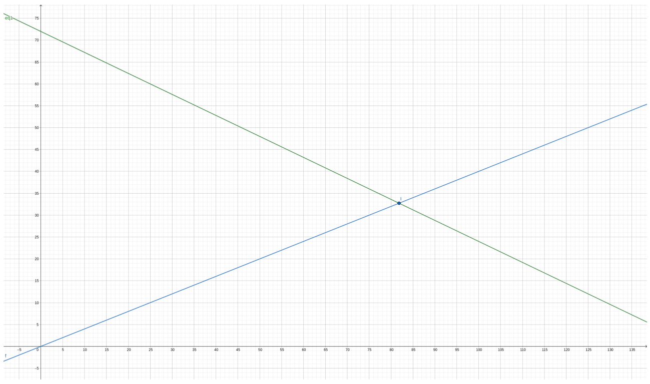
Which can be simplified as

$$\text{maximize } n_X \times \frac{505}{3} + n_Y \times \frac{770}{3}$$
$$\text{subject to } \begin{cases} 15 \times n_X + 20 \times n_Y \leq 2400 \\ 20 \times n_X + 30 \times n_Y \leq 2100 \\ n_X \geq 10 \\ n_Y \geq 0 \end{cases}$$

Problem 2

a) Let n_A and n_B be the number of products A and B produced.

$$\text{maximize } n_A \times 3 + n_B \times 5$$
$$\text{subject to } \begin{cases} 12 \times n_A + 25 \times n_B \leq 30 \times 60 \\ 2 \times n_B - 5 \times n_A \geq 0 \\ n_A, n_B \geq 0 \end{cases}$$



By graphing the feasible region, we see three candidate points. The first one $I_0 = (0, 0)$ can be trivially discarded as it leads to a profit of 0\$. Let's evaluate the objective function at the two other points. At $I_1 = (72, 0)$ the profit is 216\$. To compute the coordinates of the last point, we solve the following system.

$$\begin{cases} 12 \times n_A + 25 \times n_B = 1800 \\ -5 \times n_A + \ \ 2 \times n_B = 0 \end{cases} \implies \begin{cases} 22 \times n_A \qquad = 1800 \\ \qquad\qquad n_B = \frac{2}{5} \times n_A \end{cases} \implies \begin{cases} n_A \qquad = 81.8 \\ \qquad n_B = 32.7 \end{cases}$$

and at $I_2 = (81.8, 32.7)$ the profit is 408.9\$ which is the highest profit in this case. However, n_A and n_B are not integers. It is not specified in the exercise whether they should be or not, but as the problem is about a weekly production, we can assume that they do not need to be integer quantities as the weeks are continuous. If we wanted to optimal integer solution, we should pick $n_A = 81$ and $n_B = 33$, leading to a profit of 408\$.

b) By doubling the production capacity without modifying the other constraints, the resulting profit is doubled. The company should then pay less than 408\$ for renting an extra machine thath is profitable.