DAT600: Algorithm Theory
Assignment - 3: Greedy Algorithms

Submission Deadline:	

Problem-1: Finding the majority (Exam question, Dec 2012)

Let $E = \{e_1, e_2, \ldots, e_n\}$ be a sequence of integers. We say that an integer e_i forms a majority in E if it appears more than ceil(n/2) times in E. For instance, the integer 3 is a majority in sequence $E = \{2, 3, 3, 2, 3, 3\}$, whereas the sequence $E = \{6, 3, 2, 7, 3, 1\}$ has no majority.

It can be proved that if $e_i \neq e_j$ and E has value e_m as majority, then sequence $E - \{e_i , e_j\}$ also has e_m as majority.

a) Present a greedy algorithm to decide whether E has a majority.
b) Explain why we can or can't use Dynamic programming to solve this problem.

Problem-2: Maximizing Exam Marks

Maria is taking her algorithms exam. On the exam paper, the professor has clearly assigned points to each problem; the professor assigns points to the problems depending on the difficulty of the problem.

Being a clever student (just like all the students who take Reggie's famous course on Algorithms), Maria's opinion of the difficulty of each problem differs from the professor's. However, as an expert on greedy algorithms, Maria decides to apply a greedy algorithm to assign her own points to the problems; for each problem, she estimates how much time it will take her to solve the problem.

Maria's goal is to maximize the points she will receive given that:

- $\{p_1, p_2, \ldots, p_n\}$ are the number of points assigned by the professor to each of the n problems, where $\Sigma p_i = 100\%$.
- $\{t_1, t_2, \ldots, t_n\}$ are Maria's estimates of the time required to do each problem, where Σt_i ***need not be equal to*** T, T is the total time available for taking the exam, and
- It is possible to get partial points, if a problem is solved partially.

Describe a greedy algorithm that Maria could use to maximize her points.

Problem-3: Optimal Base station placement

Consider a straight road (a long line segment with an southern endpoint and a northern endpoint) with houses scattered very sparsely along it. We want to place cell phone base stations at certain points along the road, so that every house is within eight kilometers of one of the base stations.

Give an efficient algorithm to solve this problem using as few base stations as possible.

Problem-4: Finding the largest set

Given a set positive real numbers $X = \{x_1 \leq x_2 \leq \ldots \leq x_n\}$, there can be many subsets of X of unit-length closed intervals. E.g. if $X = \{1, 1.25, 1.25, 1.5, 2, 2.5, 2.75, 3, 4, 5, 7.5, 10\}$, there are five unit-length closed intervals: $X_1 = \{\underline{1}, 1.25, 1.25, 1.5, \underline{2}\}$, $X_2 = \{\underline{1.5}, 2, \underline{2.5}\}$, $X_3 = \{\underline{2}, 2.5, 2.75, \underline{3}\}$, $X_4 = \{\underline{3}, \underline{4}\}$, and $X_5 = \{\underline{4}, \underline{5}\}$. In other words, a unit-length closed interval includes x_m and all its successors x_i such that ($x_m \leq x_i \leq x_m+1$).
In the above example, the largest unit-length closed interval is X_1 as it contains five elements.

Using greedy algorithm, determine the largest set of unit-length closed interval; analyze the time complexity.