labsol

January 31, 2020

1 LAB 3

[1]:
```
%load_ext autoreload
%autoreload 2
```

[2]:
```
import numpy as np

from mayavi import mlab
from pdffuns import norm2D, plot_3d, plot_regions, parzen, knn, plot_knn
from matplotlib import pyplot as plt
from matplotlib import cm
```

[3]:
```
mlab.init_notebook('x3d', 800, 500)
plt.rcParams['figure.figsize'] = (14, 14)
```

```
Notebook initialized with x3d backend.
```

1.1 Scaled probability density functions

We first load the data samples from a file.

[4]:
```
samples_1, samples_2 = np.load('lab3.p', allow_pickle=True)
```

We then estimate μ and Σ for each class.

[5]:
```
mu_1 = samples_1.mean(axis=1).reshape(-1, 1)
mu_2 = samples_2.mean(axis=1).reshape(-1, 1)

sigma_1 = np.cov(samples_1)
sigma_2 = np.cov(samples_2)
```

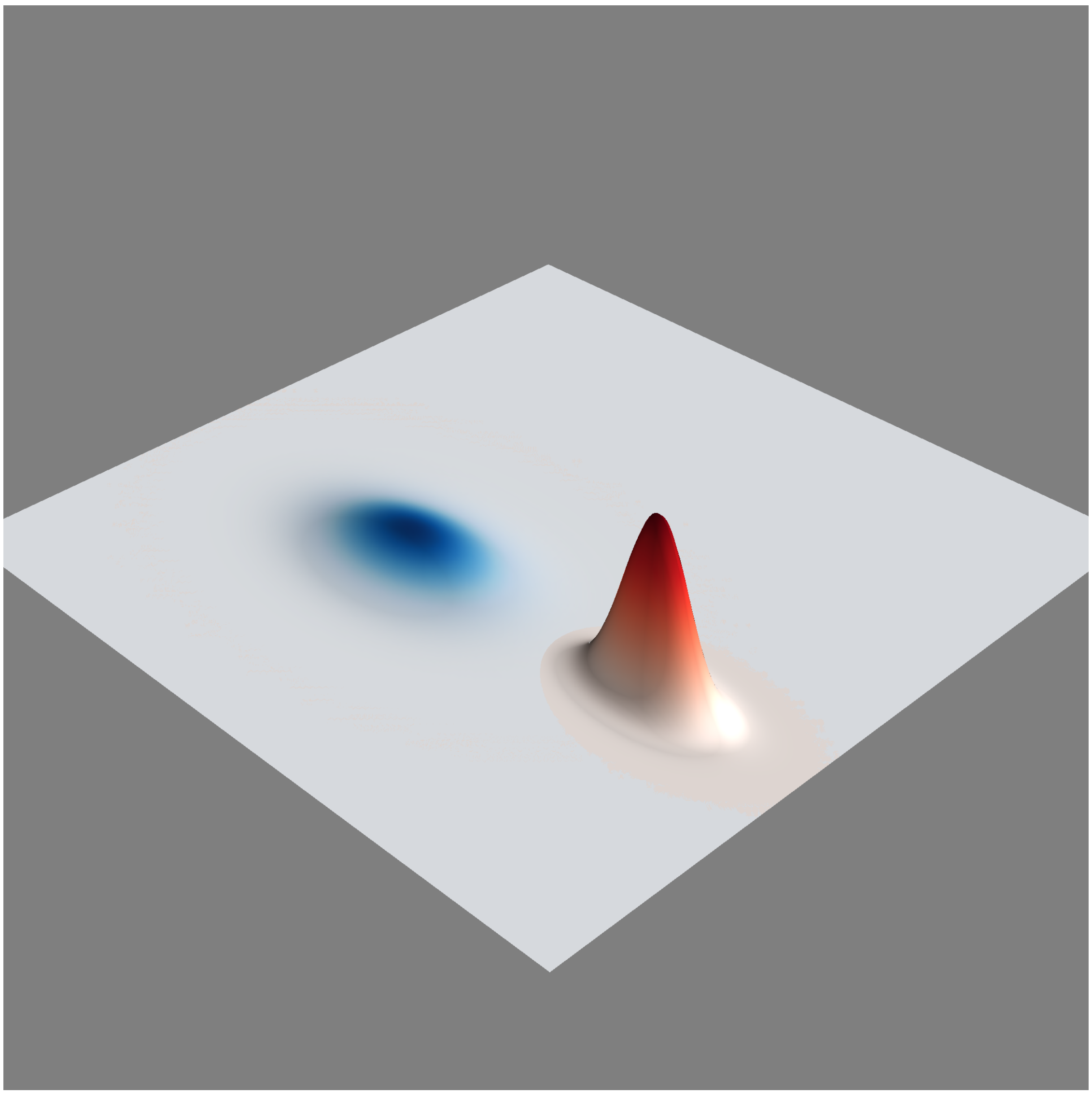
We can now compute the probability density functions over the the domain $x_1 \in [-10, 10]$ and $x_2 \in [-10, 10]$

[6]:
```
dist_1 = norm2D(mu_1, sigma_1)
dist_2 = norm2D(mu_2, sigma_2)
```

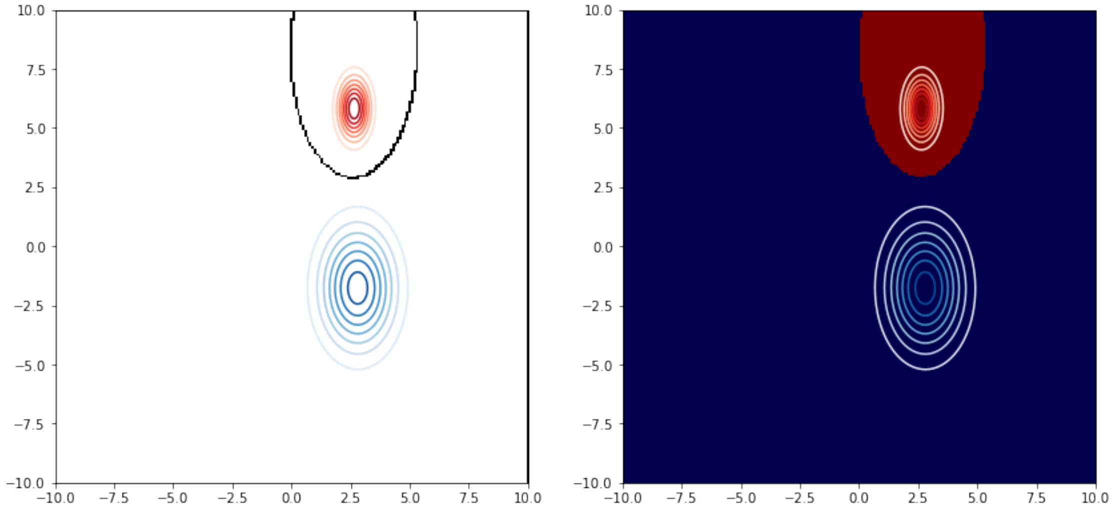
To display the scaled probability density functions, we use the prior probabilities as weights for the previously computed distributions.

[7]:
```
n_samples_total = samples_1.size + samples_2.size
w_1 = samples_1.size/n_samples_total
w_2 = samples_2.size/n_samples_total

w_dist_1 = [*dist_1[:2], w_1*dist_1[2]]
w_dist_2 = [*dist_2[:2], w_2*dist_2[2]]
```

[8]:
```
plot_3d(w_dist_1[2], w_dist_2[2], 'wpdf.png')
```



[9]:
```
plot_regions(w_dist_1, w_dist_2)
```

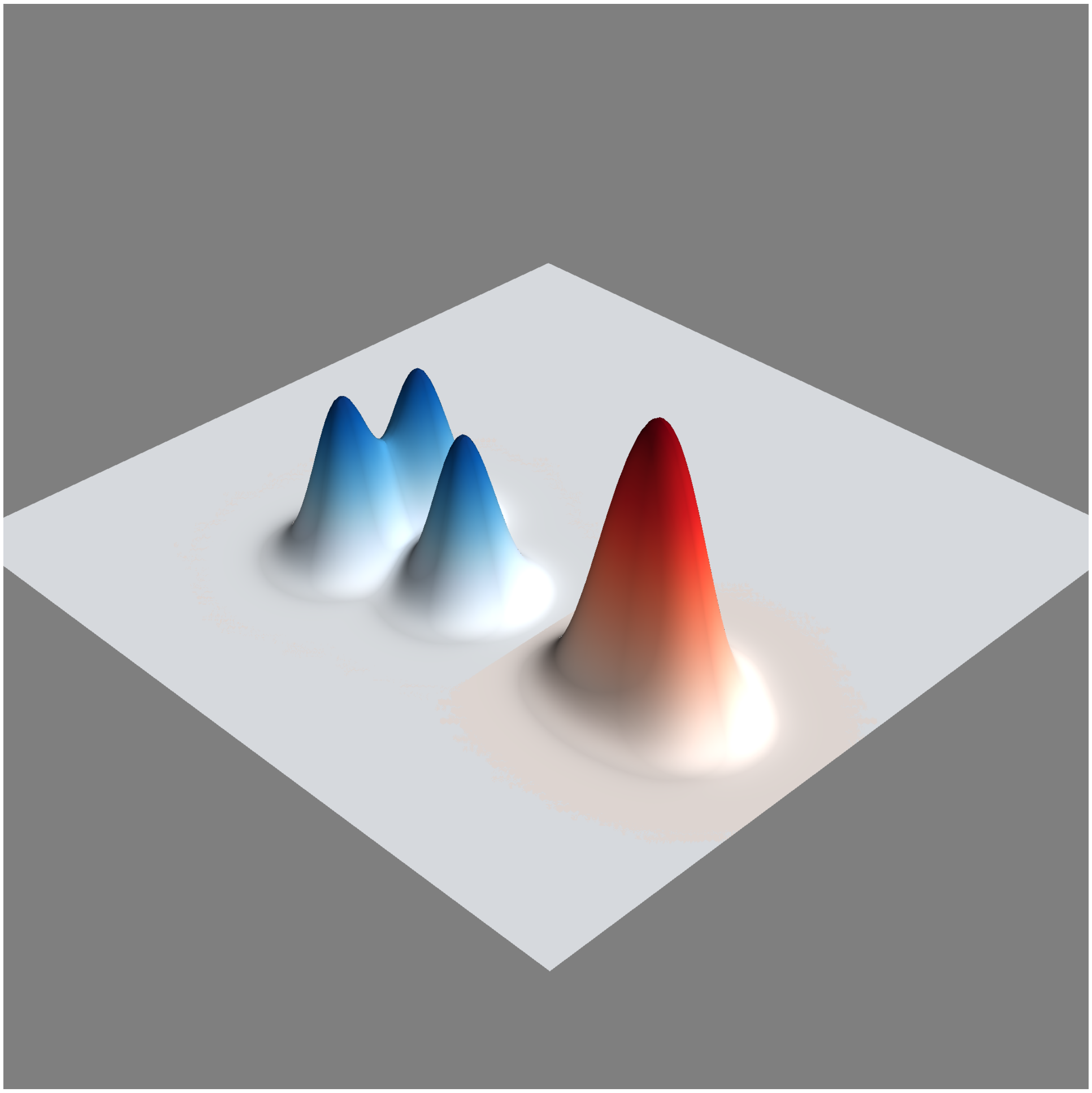


In this case, the decision boundary is an ellipse. We can note that in this case (ellipse), in lab 2 (parabola) and in exercise 3 (hyperbola), decision boundaries are conic sections.

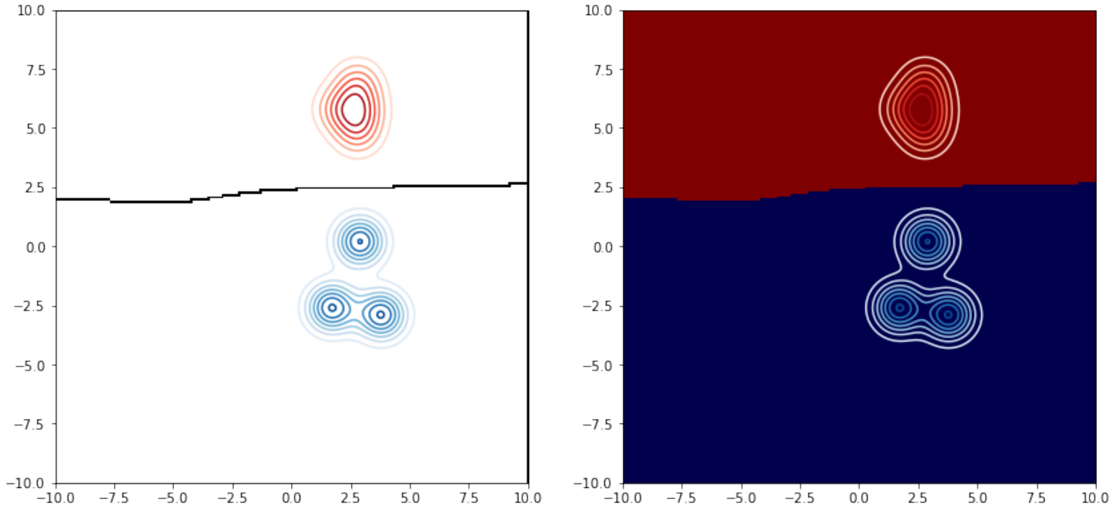
1.2 Parzen classifier

We are now going to estimate the underlying distributions with non-parametric techniques. The first one is a parzen classifier with a gaussian kernel.

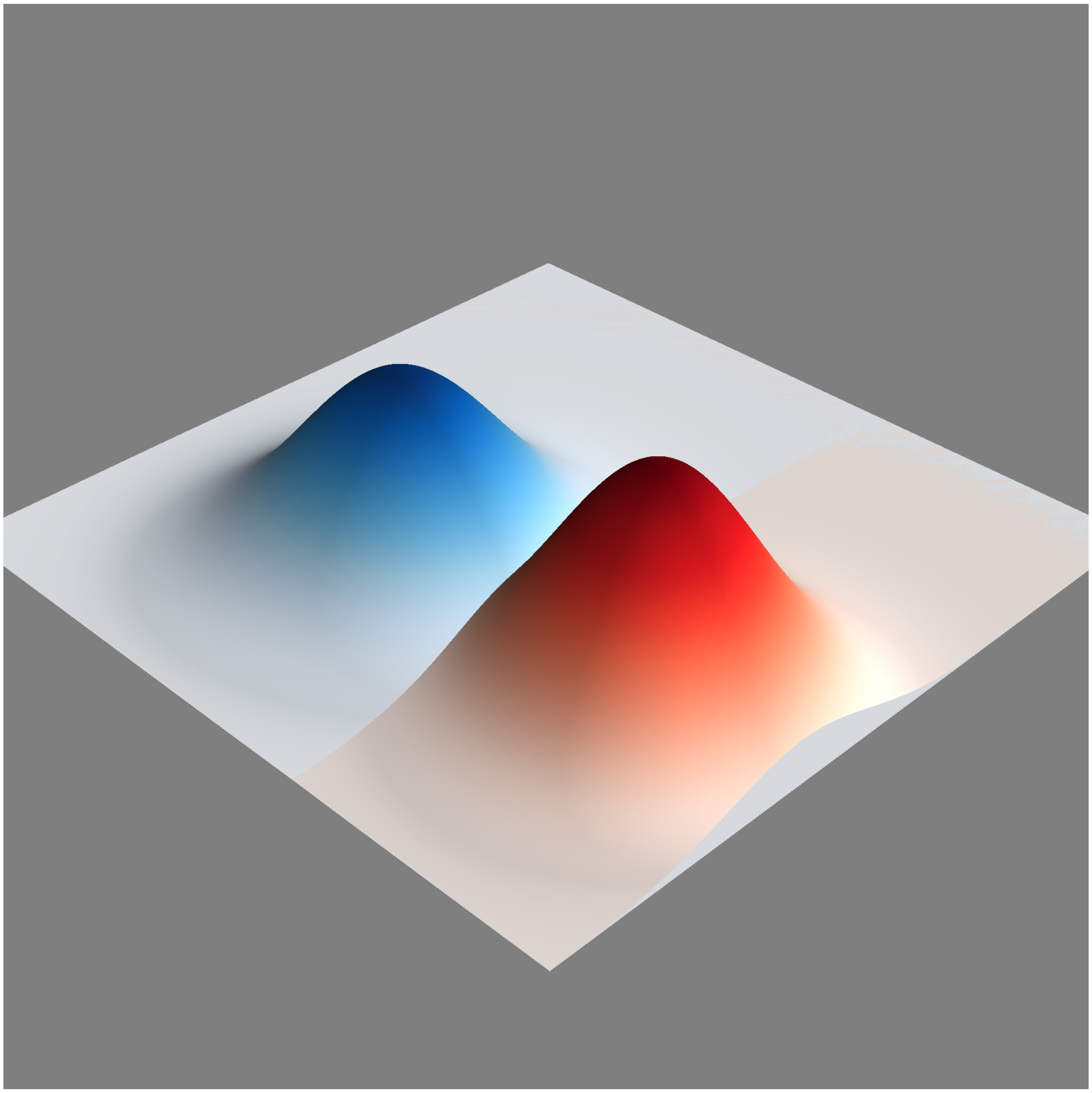
[10]:
```
dist1_parzen = parzen(samples_1.T, h1=0.5)
dist2_parzen = parzen(samples_2.T, h1=0.5)

plot_3d(dist1_parzen[2], dist2_parzen[2], 'parzen_3d_half.png', scale=4e2)
```



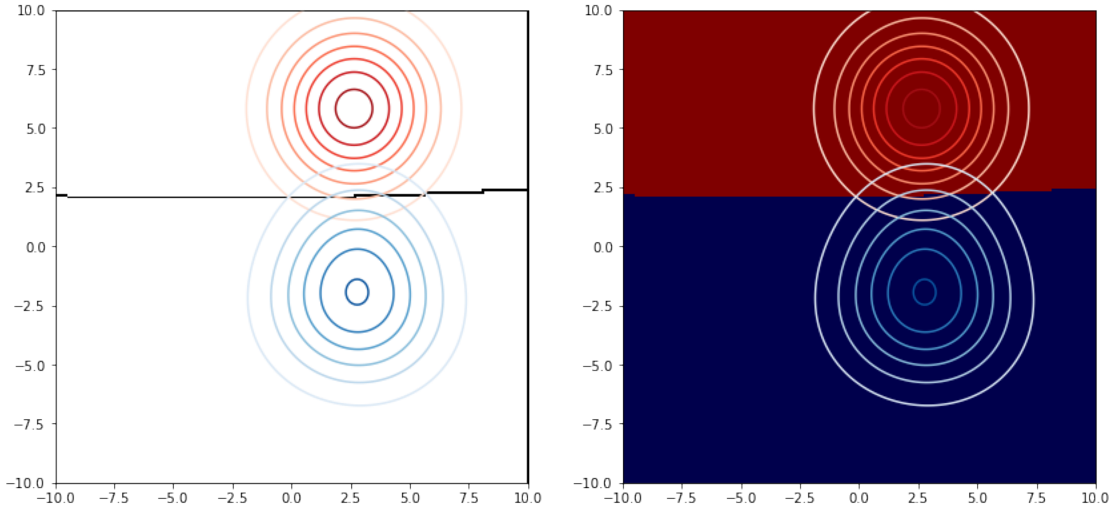
[11]:
```
plot_regions(dist1_parzen, dist2_parzen)
```




```
[12]: dist1_parzen = parzen(samples_1.T, h1=5)
      dist2_parzen = parzen(samples_2.T, h1=5)
      plot_3d(dist1_parzen[2], dist2_parzen[2], 'parzen_3d_5.png', scale=2e3)
```



[13]:
```
plot_regions(dist1_parzen, dist2_parzen)
```

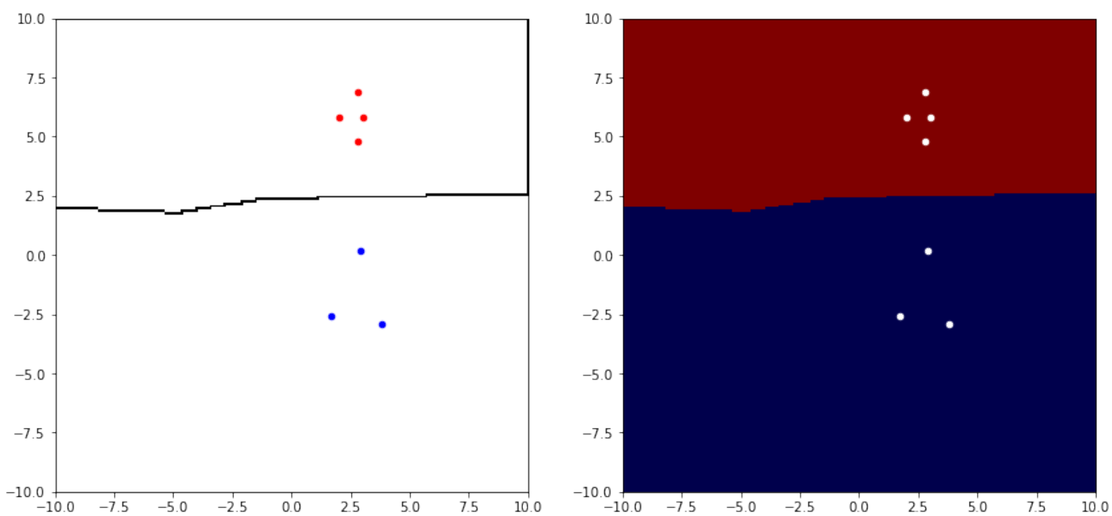


We see that in the first case, $h_1 = 0.5$ seems to be a too tight window to accurately describe the underlying distribution. In the second case however, $h_1 = 5$ seems to be a an appropriate window size. We can deduct that the kernel parameters play an important role when building a Parzen classifier.

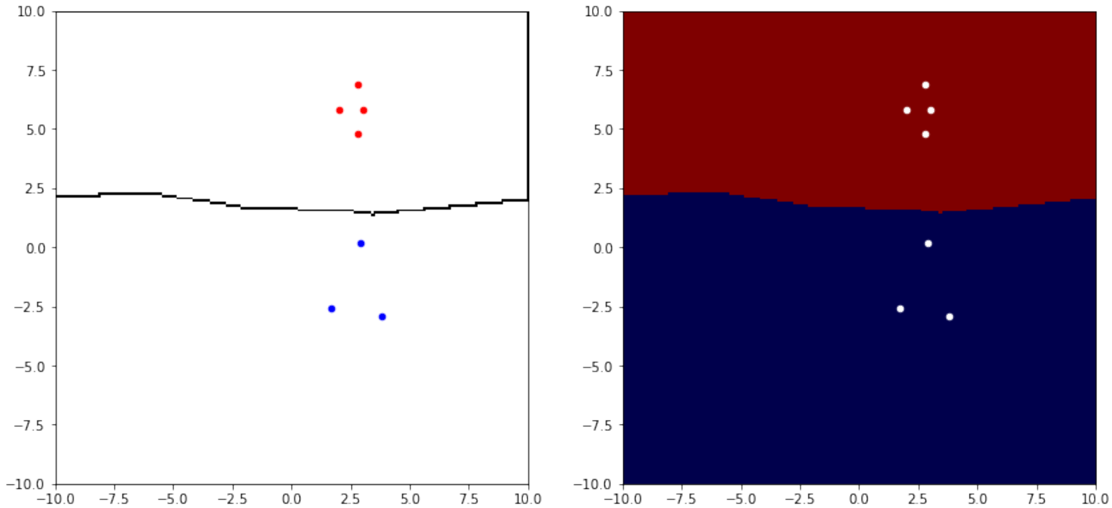
1.3 k-nearest neighbors

Another non-parametric technique in the k-nearest neighbour classifier. We display its decision boundary and decision regions for $k = 1, 3$ and show why it doesn't work for $k = 5$.

[14]:
```
knn_regions = knn(samples_1.T, samples_2.T, 1)

plot_knn(samples_1, samples_2, knn_regions)
```



```
[15]: knn_regions = knn(samples_1.T, samples_2.T, 3)

      plot_knn(samples_1, samples_2, knn_regions)
```




```
[16]: knn_regions = knn(samples_1.T, samples_2.T, 5)

      plot_knn(samples_1, samples_2, knn_regions)
```

```
     ␣
↪---------------------------------------------------------------------------

       ValueError                                Traceback (most recent call␣
↪last)

       <ipython-input-16-d7029c11de17> in <module>
   ----> 1 knn_regions = knn(samples_1.T, samples_2.T, 5)
         2
         3 plot_knn(samples_1, samples_2, knn_regions)


       ~/study/4-UiS/machine-learning/assignments/lab3/pdffuns.py in␣
↪knn(samples_1, samples_2, k, min_x1, max_x1, min_x2, max_x2)
        46             n_2 = 0
        47             while n_1+n_2 < k:
   ---> 48                 if min(dist_1) < min(dist_2):
        49                     n_1 += 1
```

```
     50                          dist_1.pop(0)


ValueError: min() arg is an empty sequence
```

In this case, there is not enough data samples in one of the classes to properly compute the decision boundary. When all data points from a certain class are identified as closest neighbors, there is no remaining sample from this class to compare the distance with the other class. We can deduce that k should be chosen to be at most the number of samples from the least represented class.