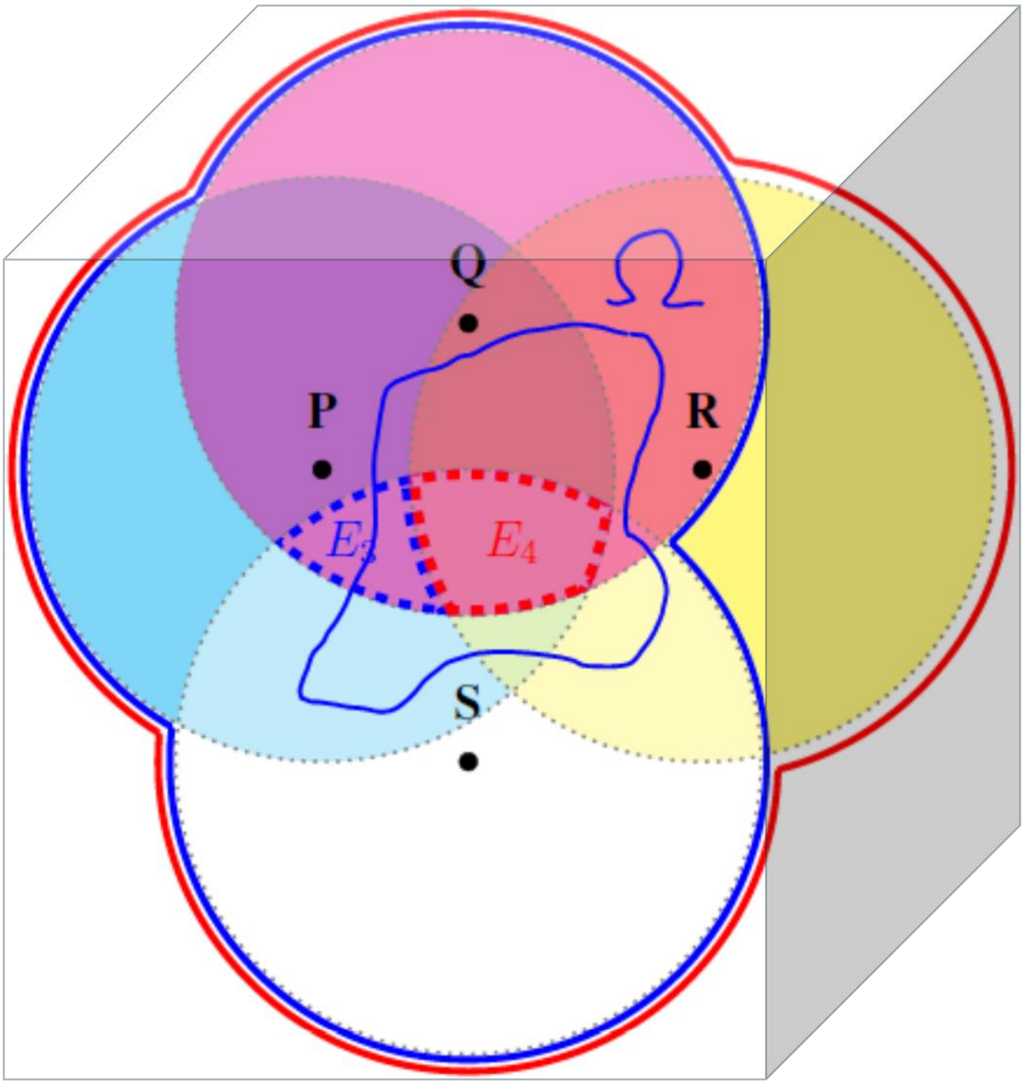
MULTI-GATEWAY LORA NETWORK STUDY

MODELLING AND PERFORMANCE EVALUTION OF A LORA NETWORK

25 janvier 2022



Informations relatives au document

INFORMATIONS GÉNÉRALES

Auteur(s)	Teo Geneau
Lieu	INSA Toulouse
Version	V3.0
Référence	EA1

HISTORIQUE DES MODIFICATIONS

Version	Date	Rédigé par	Modifications
V3.0	25/01/2022	Teo Geneau	Version Finale : ajouts de chapitres et corrections

TABLE OF CONTENTS

INTRODUCTION

The goal of this project is to get experience with performance evaluation of multi-gateway LoRa networks based on density models. LoRa networks use ALOHA and duty cycle to deal with channel access dilemma. During this exercise we will expand upon some calculations performance measures of interest using a Poisson distribution for the frames of the network considered, we will determine the throughput of that given network. Finally, we will then go through the simulation of our network that we will use to compute some features like throughput performance measures using a simulation tool written in Python. A comparison between the results obtained with the probability model and the simulation will be carried out to define the relevance of the model and validate it.

Theoretical exercise

For this study, we focus on a network consisting of three LoRaWAN gateways placed on each vertex of a right scalene triangle defined by sides being long 1, 4/3 and 5/3 with a transmission range equal to 1.
The goal is to determine what is the throughput, which is the term used for the rate of successful transmission in this case for at least one gateway, on the whole area covered by the three considered gateways.

Let's first take a closer look to the network geometry :

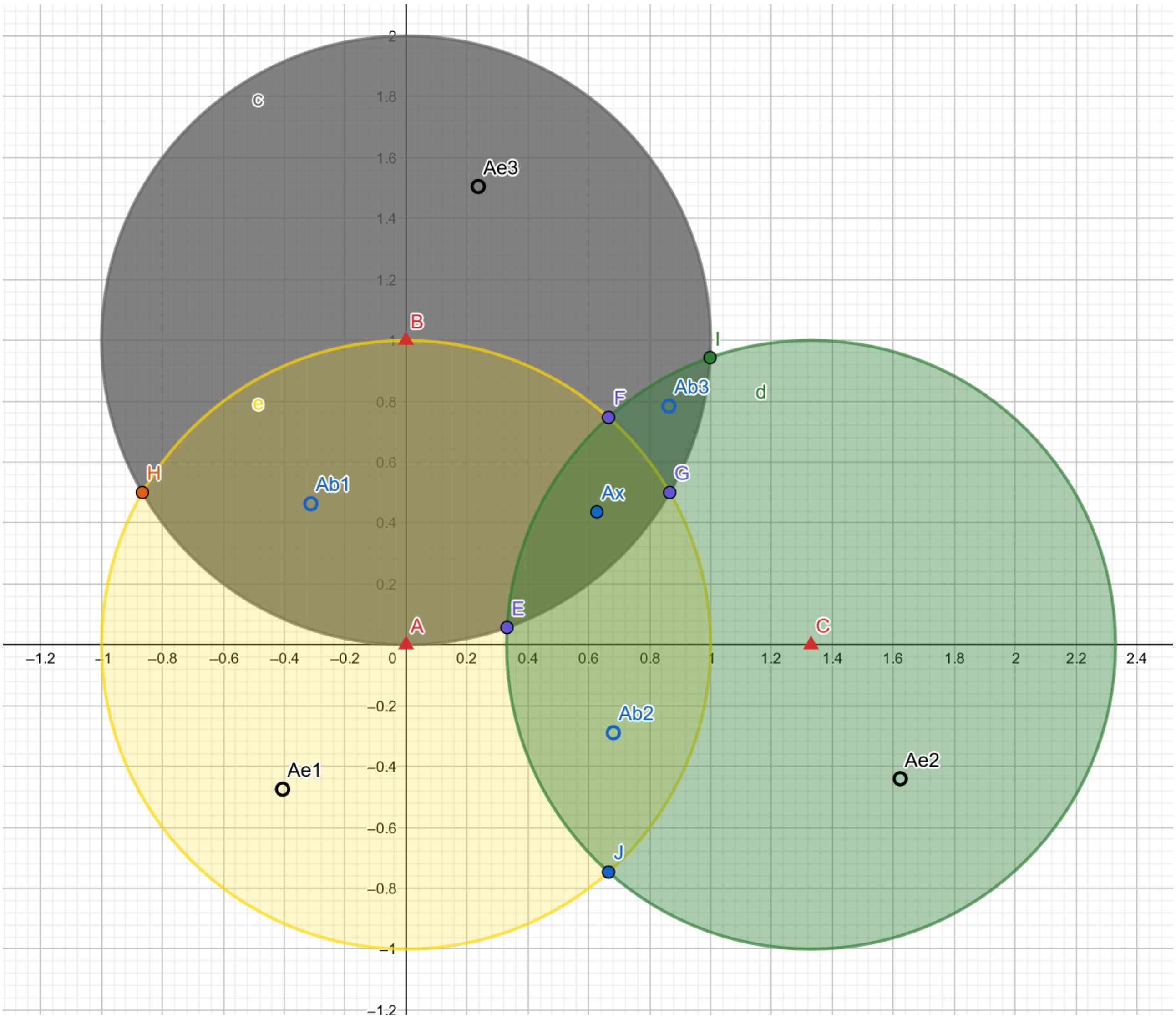


Figure 1 : The 3-gateway LoRa network studied

To obtain the throughput of at least one gateway for an end device positioned on anywhere on the covered area we must compute the throughput on the whole area which consists of the sum of each individual area throughput :

$$S_1 = S_1^{A_{e3}} + S_1^{A_{e2}} + S_1^{A_{e1}} + S_1^{A_{b3}} + S_1^{A_{b2}} + S_1^{A_{b1}} + S_1^{A_x}$$

Therefore, we have the average throughput for the whole area covered :

$$\widehat{S_L^{\Omega}} = \frac{\pi}{A(\Omega)} S_L(\Omega)$$

It is necessary to compute each term, nevertheless, we first have to determine each area A_i of the figure above, let's start with A_x.

A_x area corresponding to the surface of the intersection of the three circles : $A \cap B \cap C$ represented by the points F, G and E.

To proceed, Monte Carlo and Quadtree Approximation were first considered to solve this problem, however, the exact measure of the area can be obtained by considering that each intersection points is the vertex of a polygon, here a scalene triangle, and a convex circle arc. The area of the surface A_x corresponds to the sum of the area of the triangle, A_T, and the area of the three remaining surfaces $A_{s1,s2,s3}$ (corresponding to the surface between the segment of the triangle and the convex circle arc, in red in the figure below).

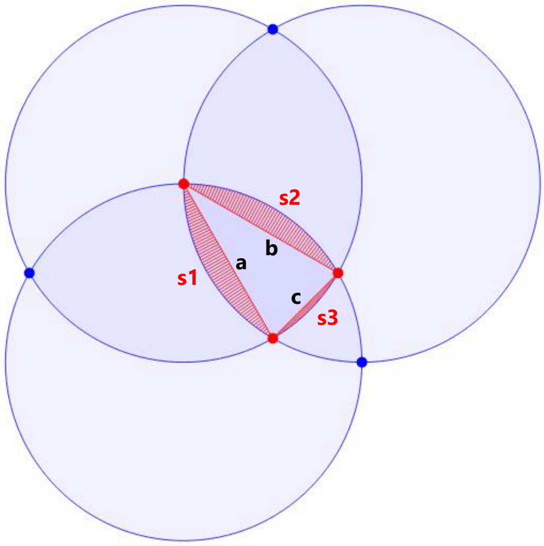


Figure 2 : surface of the intersection of 3 circles

Thus, we have : $A_x = A_T + A_{s1} + A_{s2} + A_{s3}$.

We will use Hero's formula to obtain A_T :

$$A_T = \frac{1}{4}\sqrt{4a^2b^2 - (a^2 + b^2 - c^2)^2}$$

That is why we need to compute each segment of the triangle a, b and c as we can see in the figure below.
For that, it is necessary to determine the coordinates of the interception points E, F and G.

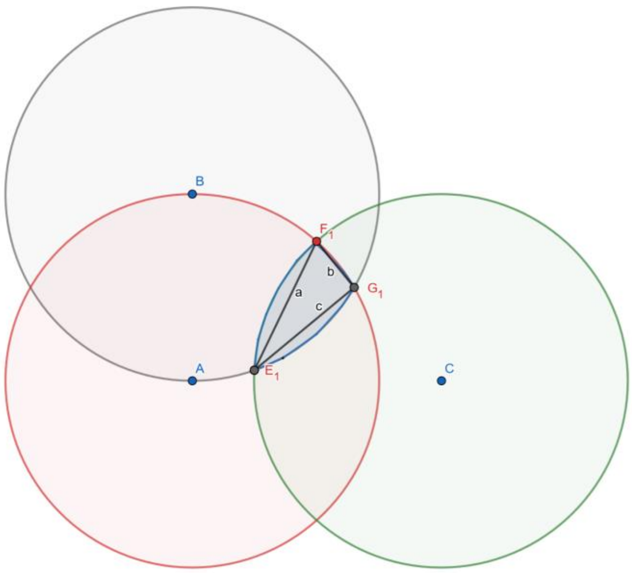


Figure 3 : Our specific case

The 3 circles are defined by :

$$(x-a)^2+(y-b)^2={r_A}^2 \quad (A)$$
$$(x-c)^2+(y-d)^2={r_B}^2 \quad (B)$$
$$(x-e)^2+(y-f)^2={r_C}^2 \quad (C)$$

For circle A : $(x-0)^2+(y-0)^2=1$

For circle B : $(x-0)^2+(y-1)^2=1$

For circle C : $(x-4/3)^2+(y-0)^2=1$

The interception point between the circle A and C, F($x_{A,C}, y_{A,C}$), is given by :

$$x_{A,C}=\frac{a+e}{2}+\frac{(e-a)({r_A}^2-{r_C}^2)}{2D^2}\pm 2\frac{b-f}{D^2}\delta$$

$$y_{A,C}=\frac{b+f}{2}+\frac{(f-b)({r_A}^2-{r_C}^2)}{2D^2}\pm 2\frac{a-e}{D^2}\delta$$

With D, the distance between the two centers of A and C :

$$D=\sqrt{(e-a)^2+(f-b)^2}$$
$$D=\sqrt{(4/3-0)^2+(0-0)^2}=4/3$$

And δ :

$$\delta=\frac{1}{4}\sqrt{(D+r_A+r_C)(D+r_A-r_C)(D-r_A+r_C)(-D+r_A+r_C)}$$
$$\delta=\frac{1}{4}\sqrt{(4/3+1+1)(4/3+1-1)(4/3-1+1)(-4/3+1+1)}$$
$$\delta=\frac{2\sqrt{5}}{9}$$

Thus :

$$x_{A,C}=\frac{0+4/3}{2}+\frac{(4/3-0)(1^2-1^2)}{2*(\frac{4}{3})^2}\pm 2\frac{0-0}{(\frac{4}{3})^2}*\frac{2\sqrt{5}}{9}$$
$$x_{A,C}=\frac{2}{3}=0.666666667$$

$$y_{A,C}=\frac{0+0}{2}+\frac{(0-0)(1^2-1^2)}{2*(\frac{4}{3})^2}\pm 2\frac{0-4/3}{(\frac{4}{3})^2}*\frac{2\sqrt{5}}{9}$$

$$y_{A,C}=\pm\frac{\sqrt{5}}{3}=\pm 0.74533$$

By visual identification we retain only $y_{A,C}=\frac{\sqrt{5}}{3}$, we have F($x_{A,C}, y_{A,C}$) = F($\frac{2}{3},\frac{\sqrt{5}}{3}$).

We use the same method to determine E and G and we finally obtain :

- E($\frac{20-3\sqrt{11}}{30},\frac{15-4\sqrt{11}}{30}$)
- F($\frac{2}{3},\frac{\sqrt{5}}{3}$)
- G($\frac{\sqrt{3}}{2}, 0.5$)

Now, it is possible to compute each segment a, b and c :

$$a=\sqrt{(x_F-x_E)^2+(y_F-y_E)^2}=0.7633846496$$

$$b = \sqrt{(x_G - x_F)^2 + (y_G - y_F)^2} = 0.3161383702$$

$$c = \sqrt{(x_G - x_E)^2 + (y_G - y_E)^2} = 0.6910420302$$

We can now apply Hero's formula to obtain A_T :

$$A_T = \frac{1}{4}\sqrt{4a^2b^2 - (a^2 + b^2 - c^2)^2}$$

$$A_T = 0.1092243$$

And A_{si} is obtained using the following formula :

$$A_{si} = r^2 sin^{-1}\left(\frac{a,b,c}{2r}\right) - \frac{a,b,c}{4} * \sqrt{4r^2 - a,b,c^2}$$

For example, for the surface between the segment a and the corresponding convex circle arc we have :

$$A_{s1} = r^2 sin^{-1}\left(\frac{a}{2r}\right) - \frac{a}{4} * \sqrt{4r^2 - a^2}$$

$$A_{s1} = sin^{-1}\left(\frac{a}{2}\right) - \frac{a}{4} * \sqrt{4 - a^2}$$

$$A_{s1} = 0.04809972701$$

We use the same method to determine A_{s2} and A_{s3} and we finally obtain :

- $A_{s1} = 0.03886877907$
- $A_{s2} = 0.00265300197$
- $A_{s3} = 0.02855321273$

And finally, we compute A_x :

$$A_x = A_T + A_{s1} + A_{s2} + A_{s3}$$

$$A_x = 0.1792992938$$

Henceforth, we can compute the area corresponding to each interception surfaces of two circles only, which are A_{b1}, A_{b2} and A_{b3} on the figure 1. Indeed, we just have to calculate the lens area between two circles and subtract A_x to obtain A_{bi} :

$$A_{bi} = A_{lens,i} - A_x$$

That is why we need to compute each lens of the figure 1, here, we are in the case where each circle shares the same radius r=1.

We can compute the area for the surface formed by the circle arc and the segment MM', called the segment surface, in the figure 4 below.

The total area of the interception lens between the two circles is equal to that area multiplied by two, indeed, we can observe a symmetry in the figure 1 for each interception lens.

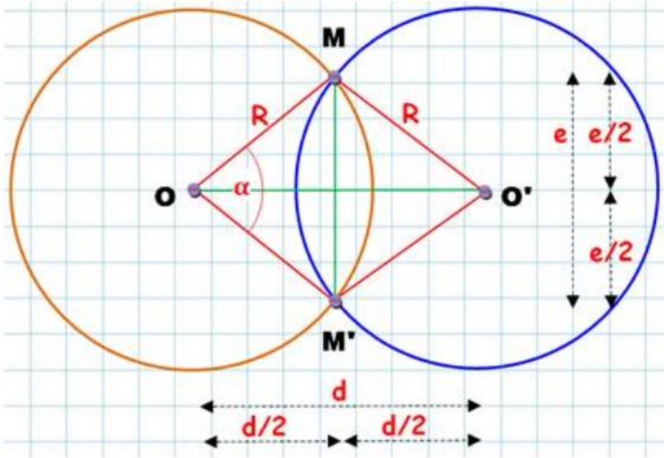


Figure 4 : Area of the intersection lens

With d the distance between the centre of each circle, we can write :

$$A_{segment} = 2r^2 \arccos\left(\frac{d}{2}\right) - d\sqrt{r^2 - \left(\frac{d}{2}\right)^2}$$

And :

$$A_{lens,i} = 2 * A_{segment}$$

For example, for the interception lens between the circle A and B correspond to the surface of $A \cap B$ we have r=1 and d=1, therefore :

$$A_{A\cap B} = 2 * r^2 \arccos\left(\frac{d}{2}\right) - d\sqrt{r^2 - \left(\frac{d}{2}\right)^2}$$

$$A_{A\cap B} = 2 * \arccos\left(\frac{1}{2}\right) - \sqrt{1 - \left(\frac{1}{2}\right)^2}$$

$$A_{A\cap B} = 1.228369699$$

We have for each lens :

- $A_{A\cap B} = 1.228369699$
- $A_{A\cap C} = 0.6883293511$
- $A_{C\cap B} = 0.2500864229$
-

Consequently, we obtain :

$$A_{b1} = A_{A\cap B} - A_x = 1.049070405$$
$$A_{b2} = A_{A\cap C} - A_x = 0.5090300573$$
$$A_{b3} = A_{C\cap B} - A_x = 0.0707871291$$

With these results, we can determine the area corresponding to the surface covered by only one circle (one gateway), indeed, it corresponds to the area of the whole circle (π) subtracted by A_x and the 2 corresponding intersection surfaces of 2 circles A_{bi} :

$$A_{e1} = \pi - A_x - A_{b1} - A_{b2} = 1.404192897$$
$$A_{e2} = \pi - A_x - A_{b3} - A_{b2} = 2.382476173$$
$$A_{e3} = \pi - A_x - A_{b1} - A_{b3} = 1.842435826$$

We also need to determine the area for each individual circle : A_1, and the area for each union of circles two by two $A \cup B$, $A \cup C$ and $C \cup B$, respectively named $A_{A\cup B}$, $A_{A\cup C}$ and $A_{C\cup B}$

Here, each circle is similar, thus $A_1 = \pi$.

The area for each union of circles two by two can be obtained with the following simple sum :

$$A_{A\cup B} = A_{e1} + A_{e3} + A_{b1} + A_{b2} + A_x + A_{b3}$$

$$A_{A\cup C} = A_{e1} + A_{e2} + A_{b1} + A_{b2} + A_x + A_{b3}$$

$$A_{C\cup B} = A_{e2} + A_{e3} + A_{b1} + A_{b2} + A_x + A_{b3}$$

And finally, we compute the area corresponding to the union of the 3 circles : $A \cup B \cup C$, called A_3 :

$$A_3 = A_{e1} + A_{e2} + A_{e3} + A_{b1} + A_{b2} + A_{b3} + A_x \sim 7.44$$

Now that we obtained each area for each surface of the figure 1, we are now able to compute the throughput of at least one gateway on the whole area covered by the three gateways :

$$S_1 = S_1^{A_{e3}} + S_1^{A_{e2}} + S_1^{A_{e1}} + S_1^{A_{b3}} + S_1^{A_{b2}} + S_1^{A_{b1}} + S_1^{A_x}$$

It is necessary to compute each term :

$$S_1^{Ax} = p\mu A_x * [3Q(A_1) - Q(A_{A\cup B}) - Q(A_{A\cup C}) - Q(A_{C\cup B}) + Q(A_3)]$$

$$S_1^{A_{e1}} = p\mu A_{e1} * Q(A_1)$$
$$S_1^{A_{e2}} = p\mu A_{e2} * Q(A_1)$$
$$S_1^{A_{e3}} = p\mu A_{e3} * Q(A_1)$$
$$S_1^{A_{b1}} = p\mu A_{b1} * [2Q(A_1) - Q(A_{A\cup B})]$$
$$S_1^{A_{b2}} = p\mu A_{b2} * [2Q(A_1) - Q(A_{A\cup C})]$$
$$S_1^{A_{b3}} = p\mu A_{b3} * [2Q(A_1) - Q(A_{C\cup B})]$$

$$S_1 = p\mu * [Q(A_1) * [3 * A_x + A_{e1} + A_{e2} + A_{e3} + 2 * (A_{b1} + A_{b2} + A_{b3})] + Q(A_{A\cup B}) * (-A_x - A_{b1}) + Q(A_{A\cup C}) * (-A_x - A_{b2}) + Q(A_{C\cup B}) * (-A_x - A_{b3}) + Q(A_3) * A_x]$$

Therefore, we have the average throughput for the whole area covered :

$$\widehat{S_L^{\Omega}} = \frac{\pi}{A(\Omega)} S_L(\Omega)$$

Here :

$$\widehat{S_1} = \frac{\pi\, p\mu}{A_3} * [S_1]$$

Validation with LoRaWAN-sim

In this part, we will simulate the behaviour of our network using a LoRaWAN simulation, we will use it for the evaluation of our network's performance in terms of throughput. The idea here is to use the results obtained with the simulation to compare them with the throughput obtained with the theoretical model. We will then discuss the difference between the two methods.

The LoRaWAN-sim is written in python and is suitable for our use-case, it is based on a Poisson distribution. It is divided in two main algorithms :

- The LoRaWAN-sim algorithm which allows us to simulate the behaviour of the network.
- The LoRaWAN-sim post processing algorithm which allows us to compare the simulation results and the theoretical model results to validate it or not.

It is essential to provide a relevant configuration for each algorithm file, indeed we have to set several features for each file, for the LoRaWAN-sim :

- A_{sim} corresponding to the area of simulation, which should contain the whole area of our network.
- T_{sim} the simulation time, the higher the better, at least 1 hour, we will target 24h.
- The coordinates of the gateways.
- The number of seed, the higher it is, the better the distribution.

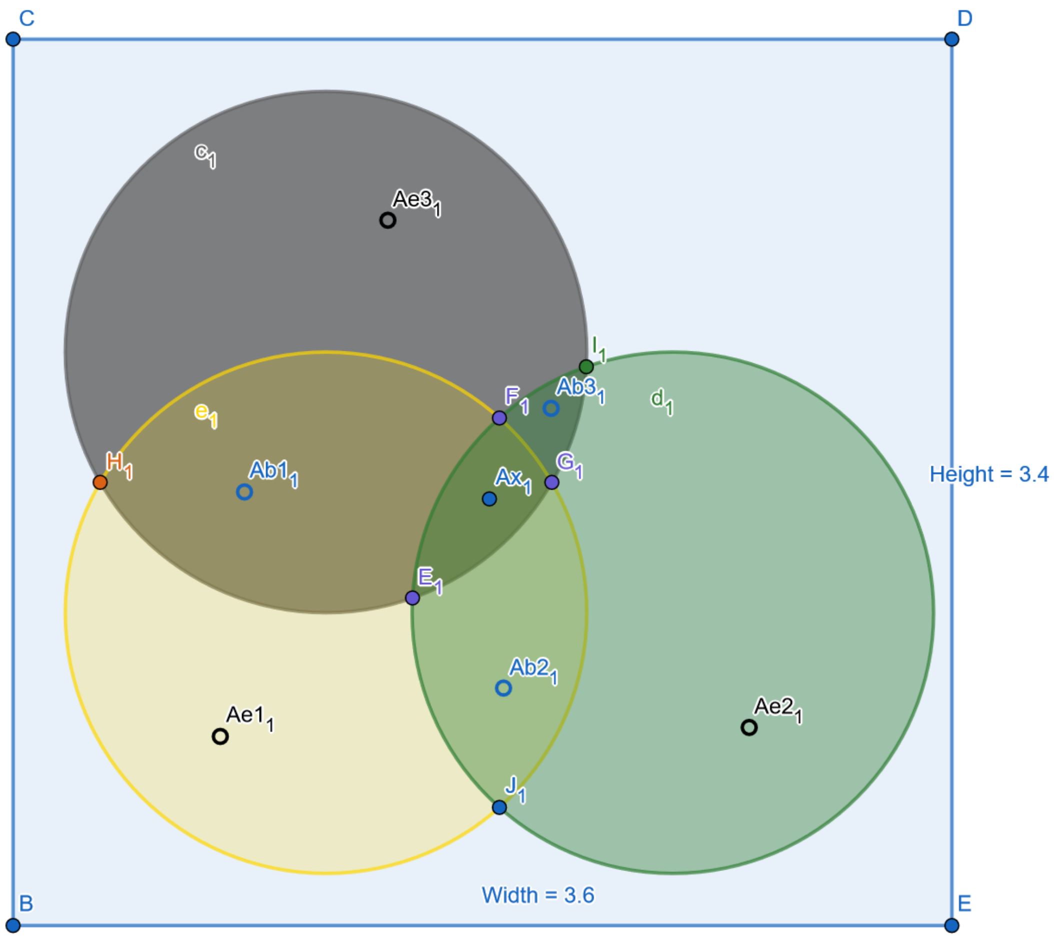


Figure 5 : Area of simulation

We set a rectangle as the area of simulation large enough to account for all coverage areas with a width of 3.6 and a height of 3.4 :

$$A_{sim} = 3.6 * 3.4 = 12.24$$

Indeed, the area of simulation is 1.64 times higher than the whole area of the considered network, which is enough to account it.

We have tried several simulation times, we decide to set $T_{sim}\ to\ 86400s$, corresponding to 24 hours for the simulation time to obtain relevant results and reduce the confidence interval.

To have the related disk of the gateway contained A_{sim}, we have to add an offset of 1.2 for the coordinates of each centre :

(1.2, 1.2) (A)
(1.2, 2.2) (B)
(2.53, 1.2) (C)

We set the seed number to 10, to enhance the distribution of the simulation.

```
    # modify from here
    seed_number = 10 # number of seeds to be used for statistical significance
    duration = 86400 # duration of each simulation in seconds
    deployment = Deployment.gateway_infrastructure(
                                                    width=3.6, # width of the simulation area (in units
of distance, i.e., the coverage radius, which is long 1) : it should be large enough to account for all
coverage areas
                                                    height=3.4, # height of the simulation area (in
units of distance, i.e., the coverage radius, which is long 1): it should be large enough to account
for all coverage areas
                                                    grid=[ # each line should contain the coordinates
(in units of distance, i.e., the coverage radius, which is long 1) of a gateway within the area: the
related disk should be into the coverage area
                                                            (1.2, 1.2),
                                                            (1.2, 2.2),
                                                            (2.53, 1.2)
                                                    ]
    )
    # to here
```

Figure 6 : configuration for the LoRaWAN-sim file

For the post-processing file, we just add the values that we computed during the theoretical part and we set the terms for the variables UNIONS and COEFFICENTS with areas and coefficients identified in the total throughput formula, S1.

```
### Constantes values computed from analysis part

Ax = 0.1792992938

Ab1 = 1.049070405
Ab2 = 0.5090300573
Ab3 = 0.0707871291

Ae1 = 1.404192897
Ae2 = 2.382476173
Ae3 = 1.842435826

A1 = math.pi

A2_A_B = Ae1 +  Ae3 + Ab1 + Ab2 + Ax + Ab3
A2_A_C = Ae1 +  Ae2 + Ab1 + Ab2 + Ax + Ab3
A2_C_B = Ae2 +  Ae3 + Ab1 + Ab2 + Ax + Ab3

A3 = Ae1 + Ae2 + Ae3 + Ab1 + Ab2 + Ab3 + Ax

CoefA1 = 3*Ax+Ae1+Ae2+Ae3+2*(Ab1+Ab2+Ab3)

CoefA2_A_B = -Ax-Ab1
CoefA2_A_C = -Ax-Ab2
CoefA2_C_B = -Ax-Ab3

CoefA3 = Ax

# modify from here
# the throughput is the sum of exonential functions
# T = p * mu * greek_PI / AREA * [ COEFFICIENT_1 * exp (-(1-q) * mu * UNION_1) + COEFFICIENT_2
* exp (-(1-q) * mu * UNION_2) + ...]


UNIONS = (A1, A2_A_B, A2_A_C, A2_C_B, A3) # (UNION_1, UNION_2, ...)
COEFFICIENTS = (CoefA1, CoefA2_A_B, CoefA2_A_C, CoefA2_C_B, CoefA3) # (COEFFICIENT_1,
COEFFICIENT_2, ...)
AREA = A3 # AREA

# to here
```

Figure 7 : configuration for the post processing file

After almost 6 hours of real time of simulation for 10 seeds and 86400 seconds for the simulation time, we obtained the corresponding figure after running the post processing python script :

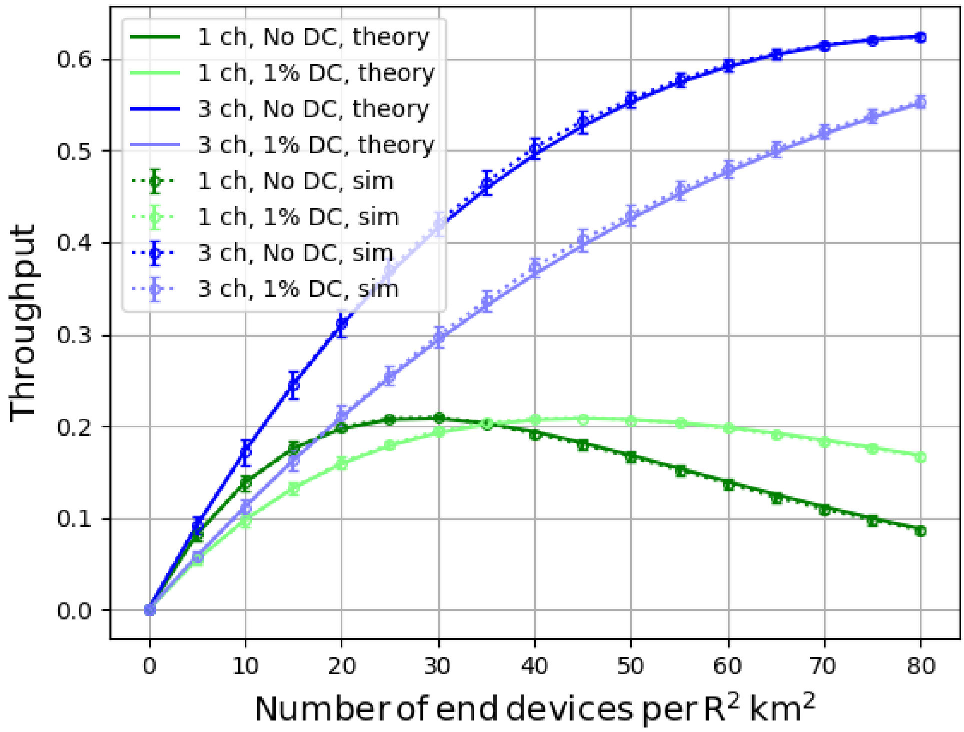


Figure 8 : simulation and theoretical results

Here we have a curve for 4 different features :

- In dark green : 1 channel considered with no duty cycle
- In light green : 1 channel considered with 1% duty cycle
- In dark blue : 3 channels considered with no duty cycle
- In light blue : 3 channels considered with 1% duty cycle

What is more, there are plotted the simulation curve, corresponding to the dotted line, and the theoretical curve with the continuous line.

At the beginning, the simulation was tested with $T_{sim} = 3600s$ and 2 seeds, nevertheless, it was not sufficient to have good enough confidence interval, that is why we increased both to 86400s and 10 seeds to enhance the confidence interval.

As we can observe in the figure 8, the average rate of successful transmissions to at least L=1 gateway, $\widehat{S_1}$, is plotted. The continuous and dotted line almost overlap and it is difficult to distinguish one from each other regardless of the parameters which means that our throughput model is validated by the simulation carried out with the LoRaWAN simulator.

We can observe that the higher the duty cycle and the channel number, the better the throughput.

The benefit of the theoretical model over the simulation is that we can compute quicker the throughput, however the drawback is that we need to study each network and determine the throughput formula for each different network geometry.

While the simulation is way slower, it allows to simulate a large size of scenarios without having to determine the exact throughput formula.

CONCLUSION

During this exercise, we were able to explore different performance evaluation methods and to understand their advantages and drawbacks to determine which methods would be the most appropriate according to the context of the study. We were able to obtain a theoretical model for the average rate of successful transmissions to at least L=1 gateway and validate it through the simulation which means that we could use the theoretical model and the simulation tool to study in a relevant way the behaviour of a real-life network deployment. Nevertheless, the theoretical model is based on certain assumptions, and it could be improved by considering them like taking account to buffers during transmission of the end devices.