Teofana-Ionela Moisi,
Group 934

Numerical Calculus Exam

Problem 1.

Because the nodes are double, we double them in the table too.

We form the divided differences table.

We compute the first derivative of the function for the nodes where the denominator is 0, namely z0, z2 and z4. The rest of them are computed using the divided differences table rule.

After we have completed the table, we use the Hermite Polynomial interpolation in order to find the polynomial.

$m = 2 \Rightarrow n = 2m + 1 = 5$

$f'(x) = -2x \cdot e^{-x^2}, \quad f(x) = e^{-x^2}$

	x	$f(x)$	$D^1 f$	$D^2 f$	$D^3 f$	$D^4 f$	$D^5 f$
z_0	-1	e^{-1}	$2 \cdot e^{-1}$	$1-3e^{-1}$	$4e^{-1}$	$\frac{-4e^{-1}}{2}$	$1/2$
z_1	-1	e^{-1}	$1-e^{-1}$	$e^{-1}-1$	0	$\frac{2-4e^{-1}}{2}$	
z_2	0	1	0	$e^{-1}-1$	$2-4e^{-1}$		
z_3	0	1	$e^{-1}-1$	$1-3e^{-1}$			
z_4	1	e^{-1}	$-2e^{-1}$				
z_5	1	e^{-1}					

The Hermite Polynomial Interpolation is:

$$(H_5 f)(x) = f(z_0) + \sum_{i=1}^{5} (x-z_0) \cdot \ldots \cdot (x-z_{i-1}) (D^i f)(z_0)$$

$$(H_5 f)(x) = f(z_0) + (x-z_0)(D^1 f)(z_0) + (x-z_0)(x-z_1)(D^2 f)(z_0) + (x-z_0)(x-z_1)(x-z_2)(D^3 f)(z_0) + (x-z_0)(x-z_1)(x-z_2)(x-z_3)(D^4 f)(z_0) + (x-z_0)(x-z_1)(x-z_2)(x-z_3)(x-z_4)(D^5 f)(z_0)$$

$$(H_5 f)(x) = f(-1) + (x+1) \cdot 2 \cdot e^{-1} + (x+1)^2 (1-3e^{-1}) + x(x+1)^2 \cdot 4 \cdot e^{-1} + x^2 \cdot (x+1)^2 \cdot (-2e^{-1}) + \frac{1}{2} x^2 (x+1)^2 (x-1)$$

$$(H_5 f)(x) = e^{-1} + 2xe^{-1} + 2e^{-1} + x^2 + 2x + 1 - 3x^2 e^{-1} - 6xe^{-1} - 3e^{-1} + (x^3 + 2x^2 + x) \cdot 4e^{-1} + (x^4 + 2x^3 + x^2) \cdot (-2e^{-1}) + \frac{1}{2} \cdot (x-1)(x^4 + 2x^3 + x^2)$$

$$(H_5 f)(x) = 3\cancel{e^{-1}} - 3\cancel{e^{-1}} + 2xe^{-1} - 6xe^{-1} - 3x^2e^{-1}$$
$$+ \cancel{4x^3e^{-1}} + 8x^2e^{-1} + 4xe^{-1} - 2x^4e^{-1} \cancel{-4x^3e^{-1}} - 2x^2e^{-1}$$
$$+ \frac{x^2}{2} \cdot (x-1)(x+1)^2 + (x+1)^2$$

$$(H_5 f)(x) = \cancel{-4xe^{-1}} - 3x^2e^{-1} + 8x^2e^{-1} + \cancel{4xe^{-1}} - 2x^4e^{-1}$$
$$-2x^2e^{-1} + \frac{x^2}{2}(x-1)(x+1)^2 + (x+1)^2$$

$$(H_5 f)(x) = 3x^2e^{-1} - 2x^4e^{-1} + (x+1)^2\left(1 + \frac{x^3 - x^2}{2}\right)$$

$$(H_5 f)(x) = x^2e^{-1}(3 - 2x^2) + \frac{1}{2}(x+1)^2 \cdot (2 + x^3 - x^2)$$