Example 10:

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Considering the following data:

x	0	2	3
$f(x)$	0	10	12
$f'(x)$	5	3	7

find the corresponding Hermite interpolation polynomial:

Solution:

We have the following double nodes:

$x_0 = 0$, $x_1 = 2$, $x_2 = 3$, where

$f(x_0) = 0$, $f(x_1) = 10$, $f(x_2) = 12$ and

$f'(x_0) = 5$, $f'(x_1) = 3$, $f'(x_2) = 7$.

$$\begin{cases} m = 2 \Rightarrow n = 2m+1 = 5 \\ r_0 = r_1 = r_2 = 1 \end{cases}$$

We construct the table:

		$D^1 f$	$D^2 f$	$D^3 f$	$D^4 f$	$D^5 f$
$z_0=0$	$f(0)=0$	$f'(0)=5$	$\frac{5-5}{z_2-z_0}=0$	$\frac{-1-0}{z_3-z_0}=-\frac{1}{2}$	$\frac{0-(-\frac{1}{2})}{z_4-z_0}=\frac{1}{6}$	$\frac{2-\frac{1}{6}}{z_5-z_0}=\frac{11}{18}$
$z_1=0$	$f(0)=0$	$\frac{f(2)-f(0)}{z_2-z_1}=5$	$\frac{f'(2)-5}{z_3-z_1}=-1$	$\frac{-1-(-1)}{z_4-z_1}=0$	$\frac{6-0}{z_5-z_1}=2$	
$z_2=2$	$f(2)=10$	$f'(2)=3$	$\frac{2-3}{z_4-z_2}=-1$	$\frac{5-(-1)}{z_5-z_2}=6$		
$z_3=2$	$f(2)=10$	$\frac{f(3)-f(2)}{z_4-z_3}=2$	$\frac{f'(3)-2}{z_5-z_3}=5$			
$z_4=3$	$f(3)=12$	$f'(3)=7$				
$z_5=3$	$f(3)=12$					

$$(H_5 f)(x) = f(z_0) + \sum_{i=1}^{5} (x-z_0)\cdots(x-z_{i-1})\,(D^i f)(z_0)$$

$$= f(z_0) + (x-z_0)(D^1 f)(z_0) + (x-z_0)(x-z_1)(D^2 f)(z_0) + (x-z_0)(x-z_1)(x-z_2)(D^3 f)(z_0)$$
$$+ (x-z_0)(x-z_1)(x-z_2)(x-z_3)(D^4 f)(z_0) + (x-z_0)(x-z_1)(x-z_2)(x-z_3)(x-z_4)(D^5 f)(z_0)$$

$$(H_5 f)(x) = f(0) + (x-0)\cdot 5 + (x-0)(x-0)\cdot 0 + x^2(x-2)\cdot\left(-\frac{1}{2}\right)$$
$$+ x^2\cdot(x-2)^2\cdot\frac{1}{6} + x^2\cdot(x-2)^2\cdot(x-3)\cdot\frac{11}{18}$$

$$(H_5 f)(x) = 5x - \frac{x^3-2x^2}{2} + \frac{x^4-4x^3+4x^2}{6} + \frac{11}{18}\left(x^5-7x^4+16x^3-12x^2\right)$$