

# Formulario per il progetto NAPDE

Matteo Calafà

24 aprile 2021

## 1 Monodominio

### 1.1 Modelli analitici

#### Modello del monodominio

$$\begin{cases} \chi_m C_m \frac{\partial V_m}{\partial t} - \nabla \cdot (\Sigma \nabla V_m) + \chi_m I_{ion}(V_m, w) = I^{ext} & \text{in } \Omega_{mus} \times (0, T] \\ \frac{\partial w}{\partial t} = g(V_m, w) & \text{in } \Omega_{mus} \times (0, T] \\ \Sigma \nabla V_m \cdot n = b & \text{on } \partial\Omega_{mus} \times (0, T] \end{cases} \quad (1)$$

dove le incognite sono:

- $V_m = \Phi_i - \Phi_e$  (differenza tra potenziale interno e esterno)
- $w$  ("gating variable")

e sono date le costanti :  $\chi_m, C_m, \Sigma$

#### Modello di FitzHugh-Nagumo

$$\begin{aligned} I_{ion}(V_m, w) &= -kV_m(V_m - a)(V_m - 1) - w \\ g(V_m, w) &= \epsilon(V_m - \gamma w) \end{aligned} \quad (2)$$

### 1.2 Modello numerico semi-discretizzato

$$\left. \begin{aligned} V_{ij} &= \int_{\Omega} \nabla \varphi_j \cdot \nabla \varphi_i \\ I_{i,j}^T &= \sum_{F \in F_h^I} \int_F \{ \{ \nabla \varphi_j \} \} \cdot [ [\varphi_i] ] \\ I_{i,j} &= \sum_{F \in F_h^I} \int_F [ [\varphi_j] ] \cdot \{ \{ \nabla \varphi_i \} \} \\ S_{i,j} &= \sum_{F \in F_h^I} \int_F \gamma [ [\varphi_j] ] \cdot [ [\varphi_i] ] \end{aligned} \right\} \quad A = \Sigma(V - I^T - \delta I + S) \quad (3)$$

$$M_{ij} = \sum_{K \in \tau_h} \int_K \varphi_j \varphi_i \quad (4)$$

$$C(u_h)_{ij} = - \sum_{K \in \tau_h} \int_K \chi_m k(u_h - 1)(u_h - a) \varphi_j \varphi_i \quad (5)$$

$$F_i = \int_{\Omega} f \varphi_i - \sum_{F \in F_h^B} \int_F b \varphi_i \quad (6)$$

### Problema semi-discretizzato

$$\{\varphi_j\}_{j=1}^{N_h} \text{ base di } V_h^p = \{v_h \in L^2 : v_h|_K \in \mathbb{P}^{p_k}(K) \quad p_k \leq p \quad \forall K \in \tau_h\}$$

$$u_h(t) = \sum_{j=1}^{N_h} u_j(t) \varphi_j, \quad w_h(t) = \sum_{j=1}^{N_h} w_j(t) \varphi_j$$

$$\Rightarrow \boxed{\chi_m C_m M \dot{u} + Au + C(u_h)u - \chi_m Mw = F} \quad (7)$$

### 1.3 Modello numerico totalmente discretizzato

**Forma implicita** ( $\theta \in [0, 1]$ )

1.

$$\begin{aligned} \chi_m C_m M \frac{u^{k+1} - u^k}{\Delta t} + A(\theta u^{k+1} + (1 - \theta)u^k) + C(u^k)(\theta u^{k+1} + (1 - \theta)u^k) + \\ - \chi_m Mw^{k+1} = \theta F^{k+1} + (1 - \theta)F^k \end{aligned} \quad (8)$$

2.

$$\frac{w^{k+1} - w^k}{\Delta t} = \epsilon(u^k - \gamma w^{k+1}) \quad (9)$$

**Forma esplicita** ( $\theta \in [0, 1]$ )

1.

$$\begin{aligned} [\chi_m C_m M + \theta \Delta t A + \theta \Delta t C(u^k)] \mathbf{u}^{k+1} = \theta \Delta t F^{k+1} + (1 - \theta) \Delta t F^k + \\ [\chi_m C_m M - (1 - \theta) \Delta t A - (1 - \theta) \Delta t C(u^k)] u^k + \chi_m \Delta t M w^{k+1} \end{aligned} \quad (10)$$

2.

$$[1 + \epsilon \gamma \Delta t] \mathbf{w}^{k+1} = w^k + (\epsilon \Delta t) u^k \quad (11)$$

## 2 Bidominio

### 2.1 Modelli analitici

#### Modello del bidominio

$$\begin{cases} \chi_m C_m \frac{\partial V_m}{\partial t} - \nabla \cdot (\Sigma_i \nabla \phi_i) + \chi_m I_{ion}(V_m, w) = I_i^{ext} & \text{in } \Omega_{mus} \times (0, T] \\ -\chi_m C_m \frac{\partial V_m}{\partial t} - \nabla \cdot (\Sigma_e \nabla \phi_e) - \chi_m I_{ion}(V_m, w) = -I_e^{ext} & \text{in } \Omega_{mus} \times (0, T] \\ \frac{\partial w}{\partial t} = g(V_m, w) & \text{in } \Omega_{mus} \times (0, T] \\ \Sigma_i \nabla \phi_i \cdot n = b_i & \text{on } \partial\Omega_{mus} \times (0, T] \\ \Sigma_e \nabla \phi_e \cdot n = b_e & \text{on } \partial\Omega_{mus} \times (0, T] \end{cases} \quad (12)$$

dove le incognite sono:

- $\phi_i, \phi_e$  ( $V_m = \phi_i - \phi_e$ )
- $w$  ("gating variable")

e sono date le costanti :  $\chi_m, C_m, \Sigma_i, \Sigma_e$

#### Modello di FitzHugh-Nagumo

$$\begin{aligned} I_{ion}(V_m, w) &= -kV_m(V_m - a)(V_m - 1) - w \\ g(V_m, w) &= \epsilon(V_m - \gamma w) \end{aligned} \quad (13)$$