Formulario per il progetto NAPDE

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1 Monodominio

1.1 Modelli analitici

Modello del monodominio

$$\begin{cases} \chi_m C_m \frac{\partial V_m}{\partial t} - \nabla \cdot (\Sigma \nabla V_m) + \chi_m I_{ion}(V_m, w) = I^{ext} & \text{in } \Omega_{mus} \times (0, T] \\ \frac{\partial w}{\partial t} = g(V_\mathrm{m}, w) & \text{in } \Omega_{mus} \times (0, T] \\ \Sigma \nabla V_\mathrm{m} \cdot n = b & \text{on } \partial\Omega_{mus} \times (0, T] \end{cases} \tag{1}$$

dove le incognite sono:

- $V_m = \Phi_i - \Phi_e$ (differenza tra potenziale interno e esterno)
- w ("*gating variable*")

e sono date le costanti : χ_m, C_m, Σ

Modello di FitzHugh-Nagumo

$$\begin{gathered} I_{ion}(V_m, w) = -kV_m(V_m - a)(V_m - 1) - w \\ g(V_m, w) = \epsilon(V_m - \gamma w) \end{gathered} \tag{2}$$

1.2 Modello numerico semi-discretizzato

$$\left.\begin{aligned} V_{ij} &= \int_\Omega \nabla\varphi_j \cdot \nabla\varphi_i \\ I^T_{i,j} &= \sum_{F \in F^I_h} \int_F \{\{\nabla\varphi_j\}\} \cdot [[\varphi_i]] \\ I_{i,j} &= \sum_{F \in F^I_h} \int_F [[\varphi_j]] \cdot \{\{\nabla\varphi_i\}\} \\ S_{i,j} &= \sum_{F \in F^I_h} \int_F \gamma[[\varphi_j]] \cdot [[\varphi_i]]] \end{aligned}\right\} \quad A = \Sigma(V - I^T - \delta I + S) \tag{3}$$

$$M_{ij} = \sum_{K \in \tau_h} \int_K \varphi_j \varphi_i \tag{4}$$

$$C(u_h)_{ij} = -\sum_{K \in \tau_h} \int_K \chi_m k(u_h - 1)(u_h - a)\varphi_j \varphi_i \tag{5}$$

$$F_i = \int_\Omega f\varphi_i - \sum_{F in F_h^B} \int_F b\varphi_i \tag{6}$$

Problema semi-discretizzato

$$\{\varphi_j\}_{j=1}^{N_h} \text{ base di } V_h^p = \{v_h \in L^2 : v_h|_K \in \mathbb{P}^{p_k}(K) \quad p_k \leq p \quad \forall K \in \tau_h\}$$

$$u_h(t) = \sum_{j=1}^{N_h} u_j(t)\varphi_j, \quad w_h(t) = \sum_{j=1}^{N_h} w_j(t)\varphi_j$$

$$\Rightarrow \quad \boxed{\chi_m C_m M\dot{u} + Au + C(u_h)u - \chi_m M w = F} \tag{7}$$

1.3 Modello numerico totalmente discretizzato

Forma implicita ($\theta \in [0, 1]$)

1.

$$\begin{aligned}\chi_m C_m M\frac{u^{k+1} - u^k}{\Delta t} + A\left(\theta u^{k+1} + (1-\theta)u^k\right) + C(u^k)\left(\theta u^{k+1} + (1-\theta)u^k\right) + \\ -\chi_m M w^{k+1} = \theta F^{k+1} + (1-\theta)F^k\end{aligned} \tag{8}$$

2.

$$\frac{w^{k+1} - w^k}{\Delta t} = \epsilon(u^k - \gamma w^{k+1}) \tag{9}$$

Forma esplicita ($\theta \in [0, 1]$)

1.

$$\begin{aligned}\left[\chi_m C_m M + \theta\Delta t A + \theta\Delta t C(u^k)\right] \boldsymbol{u^{k+1}} = \theta\Delta t F^{k+1} + (1-\theta)\Delta t F^k + \\ \left[\chi_m C_m M - (1-\theta)\Delta t A - (1-\theta)\Delta t C(u^k)\right] u^k + \chi_m \Delta t M w^{k+1}\end{aligned} \tag{10}$$

2.

$$[1 + \epsilon\gamma\Delta t]\, \boldsymbol{w^{k+1}} = w^k + (\epsilon\Delta t)u^k \tag{11}$$

2 Bidominio

2.1 Modelli analitici

Modello del bidominio

$$
\begin{cases}
\chi_m C_m \frac{\partial V_m}{\partial t} - \nabla \cdot (\Sigma_i \nabla \phi_i) + \chi_m I_{ion}(V_m, w) = I_i^{ext} & \text{in } \Omega_{mus} \times (0, T] \\
-\chi_m C_m \frac{\partial V_m}{\partial t} - \nabla \cdot (\Sigma_e \nabla \phi_e) - \chi_m I_{ion}(V_m, w) = -I_e^{ext} & \text{in } \Omega_{mus} \times (0, T] \\
\frac{\partial w}{\partial t} = g(V_\mathrm{m}, w) & \text{in } \Omega_{mus} \times (0, T] \\
\Sigma_i \nabla \phi_i \cdot n = b_i & \text{on } \partial\Omega_{mus} \times (0, T] \\
\Sigma_e \nabla \phi_e \cdot n = b_e & \text{on } \partial\Omega_{mus} \times (0, T]
\end{cases}
\tag{12}
$$

dove le incognite sono:

- $\phi_i, \phi_e \quad (V_m = \phi_i - \phi_e)$
- w ("*gating variable*")

e sono date le costanti : $\chi_m, C_m, \Sigma_i, \Sigma_e$

Modello di FitzHugh-Nagumo

$$
\begin{gathered}
I_{ion}(V_m, w) = -kV_m(V_m - a)(V_m - 1) - w \\
g(V_m, w) = \epsilon(V_m - \gamma w)
\end{gathered}
\tag{13}
$$