Exercițiul 2

$(f_n)_{n\geq 1}$ unde $f_n : [0,+\infty) \to \mathbb{R}$

$f_n(x) = x^4 e^{-nx+5}$, $x \in [0,+\infty)$

Calculăm $\lim\limits_{n\to\infty} f_n(x) = \lim\limits_{n\to\infty} x^4 e^{-nx+5} =$

$= \lim\limits_{n\to\infty} x^4 e^{-(nx-5)} = \lim\limits_{n\to\infty} x^4 \underbrace{\frac{1}{e^{nx-5}}}_{0} = 0$

deoarece $x \geq 0 \Rightarrow e^{nx-5} \to \infty$

Fie $f : [0,+\infty) \to \mathbb{R}$, $f(x) = \lim\limits_{n\to\infty} f_n(x) = 0 \Rightarrow$

$\Rightarrow f_n \xrightarrow{s} f$ converge simplu

Fixăm $n \in \mathbb{N}$: $\sup\limits_{x\in[0,+\infty)} |f_n(x) - f(x)| = \sup\limits_{x\in[0,+\infty)} g(x) \Rightarrow$

$\Rightarrow g(x) = f_n(x) - 0 = x^4 e^{-nx+5}$

$g'(x) = (x^4 e^{-nx+5})' = 4x^3 e^{-nx+5} + x^4 e^{-nx+5} \cdot (-n)$

$= x^3 \cdot e^{-nx+5} (4 - nx)$

$g'(x) = 0 \Rightarrow x^3 \cdot e^{-nx+5} (4-nx) = 0 \Rightarrow x = 0$

sau

$4 - nx = 0 \Rightarrow$

$\Rightarrow \underset{n\geq 1}{4 = nx} \Rightarrow x = \frac{4}{n} \in [0,+\infty)$

Facem tabelul de semne al lui $g(x)$:

$g(x) = x^4 e^{-nx+5}$

$g : [0,+\infty) \to \mathbb{R}$

	0		$\frac{4}{m}$		$+\infty$
$g'(x)$	0	$+ + +$	0	$- - -$	
$g(x)$	0	$\nearrow$	$\left(\frac{4}{m}\right)^4 \cdot e$	$\searrow$	0

$$g(0) = 0$$

$$g\left(\frac{4}{m}\right) = \left(\frac{4}{m}\right)^4 e^{-\cancel{m}\cdot\frac{4}{\cancel{m}} + 5} = \left(\frac{4}{m}\right)^4 \cdot e > 0 \Rightarrow$$

$\Rightarrow g(x), x \in \left[0, \frac{4}{m}\right]$ crescătoare

$$\lim_{x\to\infty} g(x) = \lim_{x\to\infty} x^4 e^{-mx+5} =$$

$$= \lim_{x\to\infty} \frac{x^4}{e^{mx-5}} \underset{l'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{4x^3}{m \cdot e^{mx-5}} \overset{l'H}{=}$$

$$= \lim_{x\to\infty} \frac{12x^2}{m^2 e^{mx-5}} \overset{l'H}{=} \lim_{x\to\infty} \frac{24x}{m^3 e^{mx-5}} \overset{l'H}{=}$$

$$= \lim_{x\to\infty} \underbrace{\frac{24}{m^4 e^{mx-5}}}_{0} = 0$$

deoarece $e^{mx-5} \cdot m^4 \to \infty$

$$\Rightarrow \sup_{m \geq 1} g(x) = \left(\frac{4}{m}\right)^4 \cdot e \in \mathbb{R} \Rightarrow$$

$\Rightarrow f_m \overset{u}{\to} f$ converge uniform