Exercițiul 1

$$\sum_{n\geq 1} \frac{(n+7)!\cdot x^n}{a(a+1)\dots(a+n)}, \quad a,x \in \mathbb{R}^+$$

aplicăm criteriul raportului:

notăm $a_n = \dfrac{(n+7)!\cdot x^n}{a(a+1)\dots(a+n)} \Rightarrow$

$$\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+8)!\; x^{\cancel{n+1}}}{(a)(a+1)\dots(a+n+1)} \cdot \frac{a(a+1)\dots(a+n)}{(n+7)!\;\cancel{x^n}}$$

$$= \lim_{n\to\infty} \frac{\overset{(n+8)}{(n+\cancel{8})!}\; x}{(a)(a+1)\dots(a+n+1)} \cdot \frac{a(a+1)\dots(a+n)}{(\cancel{n+7})!} =$$

$$= \lim_{n\to\infty} \frac{(n+8)\cancel{a(a+1)}\dots(a+\cancel{n})x}{\cancel{a(a+1)}\dots(a+\cancel{n})(a+n+1)} = \lim_{n\to\infty} \frac{x(n+8)}{a+n+1} =$$

$$= \lim_{n\to\infty} \frac{n(x+\frac{8x}{n})}{n(1+\frac{a}{n}+\frac{1}{n})} = x \overset{\text{not.}}{=} l$$

Dacă $x<1 \Rightarrow l<1 \Rightarrow \sum a_n$ convergentă

Dacă $x>1 \Rightarrow l>1 \Rightarrow \sum a_n$ divergentă

Dacă $x=1 \Rightarrow l=1 \Rightarrow$ nu funcționează criteriul raportului

$x=1 \Rightarrow a_n = \dfrac{(n+7)!}{a(a+1)\dots(a+n)}$

criteriul Raabe - Duhamel

$$\lim_{n\to\infty} n\left(\frac{a_n}{a_{n+1}}-1\right) = \lim_{n\to\infty} n\left(\frac{(\cancel{n+7})!}{\cancel{a}\dots(\cancel{a+n})}\cdot\frac{\cancel{a}\dots\cdot(a+n+1)}{(n+\cancel{8})!}-1\right) = \lim_{n\to\infty} n\left(\frac{a+n+1}{n+8}-1\right)$$

$$= \lim_{n\to\infty} n\left(\frac{a-7}{n+8}\right) = \lim \frac{n(a-7)}{n+8} = a-7$$

dacă $a-7<1 \Rightarrow a<8 \Rightarrow \sum a_n$ divergentă

$a-7>1 \Rightarrow a>8 \Rightarrow \sum a_n$ convergentă