

CPS305 Midterm Test Fall 2010

(24 marks, 5 pages)

First Name:

Last Name:

Student ID:

Section or Lab Day/time

Signature: _____

For questions 1-3, **what kind of recursion** does the given algorithm use? Circle the best answer for each. (1 mark each)

- (a) tail recursion →
- (b) edges-and-center recursion → ←
- (c) last-and-all-but-last recursion
- (d) division-in-halves recursion /2
- (e) none of the above

where

- **A** is an array, and **x, y** are valid indexes of **A**, and $x \leq y$
- **Join(K, M)** makes a list from item(s) in **K** followed by item(s) in **M**
- **C(x)** is the usual math ceiling function $\lceil x \rceil$, (if **x** is a whole number, it returns **x**; if **x** is a fraction it returns the next highest whole number.)

1. algorithm F: (a) (b) (c) (d) (e)

```
F(A, x, y)
  if (x == y) return 1
  if (x < y) return x * F(A, x+1, y)
```

2. algorithm G: (a) (b) (c) (d) (e)

```
G(A, x, y, n)
  if (x == y) return n //n is irrelevant to the type of recursion this is
  if (x < y) return G(A, x+1, y, n+1)
```

3. algorithm H: (a) (b) (c) (d) (e)

```
H(A, x, y)
  if (x == y) return A[x]
  if (x < y) return Join( H(A, x + C((y-x)/2), y), H(A, x, x + C((y-x)/2) - 1) )
```

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For questions 4-8, circle the best answer. (1 mark each)

✓ 4. Which of the following is a **primitive data type** in **C**:

- a. array
- ☒ b. char
- c. complex number
- d. more than one of a, b, c
- e. none of a, b, c

✓ 5. An **ADT**:

- a. is made up of operations and a data structure (object)
- b. uses the principle of information-hiding
- c. can have many lower-level representations
- ☒ d. more than one of a, b, c
- e. none of a, b, c

✓ 6. We used the technique of **unrolling** to:

- a. help us write the base case for a recursive function
- b. help us produce the annotations on call trees
- ☒ c. help us solve recurrence relations
- d. none of the above

X 7. An **iterative** algorithm to **multiply** together **two nxn matrices** will have **time complexity**

- a. $O(n)$
- ☒ b. $O(n^2)$
- c. $O(n^3)$
- d. $O(n^4)$
- e. none of the above

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & & & \\ b_{31} & & & \\ \vdots & & & \\ b_{n1} & \dots & b_{nn} \end{bmatrix}$$

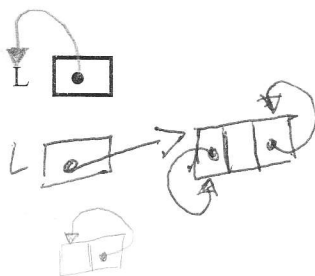
for $(i=1, i \leq n, i++) \{ n$
for $(j=1, j \leq n, j++) \{ n$
prod = $a_{ij} * b_{ij}$
 $n * n = n^2$

8. A List of n integers is implemented 2 ways: (1) an array of MAX items, (2) a linked list with standard nodes/pointers. Assume integers and pointers use equal storage, and that both implementations keep an integer "count" of the current number of items in the list.

For large MAX and n , it is more space-efficient to use the first (array) implementation:

- ☒ a. when $n > (1/3 * MAX)$, approximately
- ☒ b. when $n > (1/2 * MAX)$, approximately
- c. when $n > (3/4 * MAX)$, approximately
- d. when $n > (2 * MAX)$, approximately

9. (2 marks) List L is **circular, doubly-linked** (two-way), with **header**. The drawing below shows L after "L=NULL;". Change the drawing to show empty list, L, after "Initialize(&L);"



10. (3 marks) For each of the following Computer Science applications, indicate whether it tends to be associated **more** with Stacks, or Queues. **Put a checkmark** in the appropriate box:

		LIFO	FIFO
Application	Stack?	Queue?	
print spool		✓	
backtracking	✓		
evaluating postfix expressions	✓		
level-order tree traversal	✓		
answering server requests		✓	
implementing function calls	✓		

11. (3 marks)

- a. On the algorithm below, **circle** what you would count for **time complexity**
 b. Use a **recurrence relation** to find **time complexity**, in **Big-O** notation, for the following algorithm. Assume n is always an integer ≥ 1

Factorial (n)

if ($n \text{ } \textcircled{=}$ 1) return 1

else return ($n \text{ } \textcircled{*}$ Factorial ($n \text{ } \textcircled{-}$ 1))

$$T(0) = 1$$

$$T(1) = k$$

$$T(n) = k + T(n-1)$$

$$= 2k + T(n-2)$$

$$= 3k + T(n-3)$$

$$\vdots$$

$$= n + T(n-n)$$

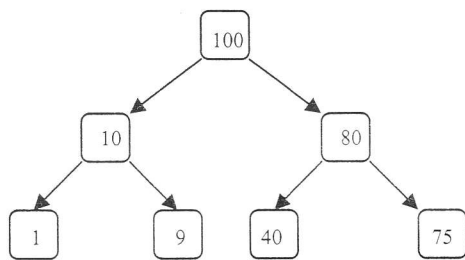
$$= n + T(0)$$

$$= n + 1$$

$$= O(n)$$

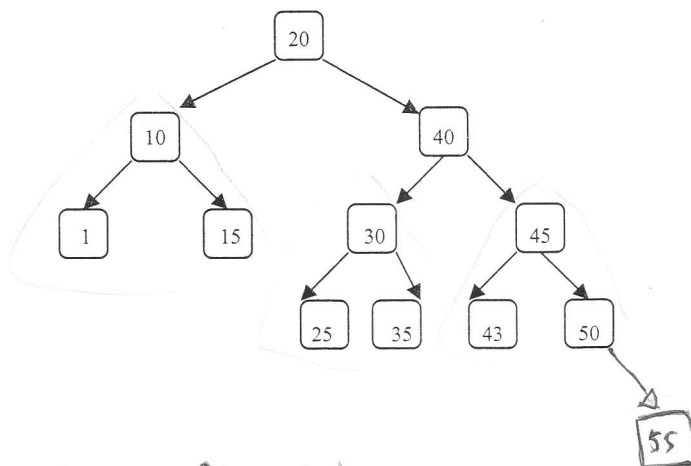
12. (2 marks) In the space below, give an in-order traversal of the following tree.

In-order Traversal: 1, 10, 9, 100, 40, 80, 75

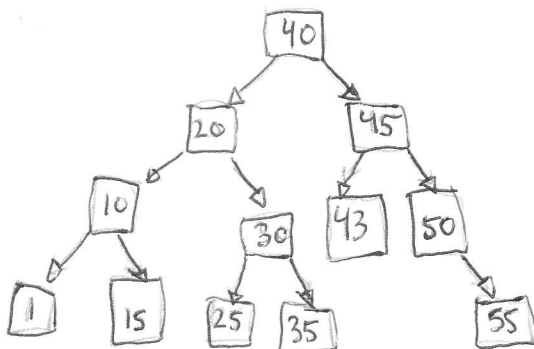


LRR

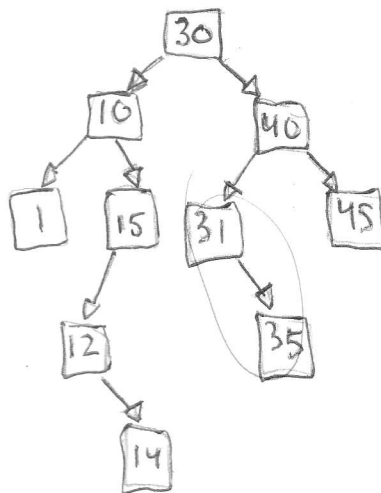
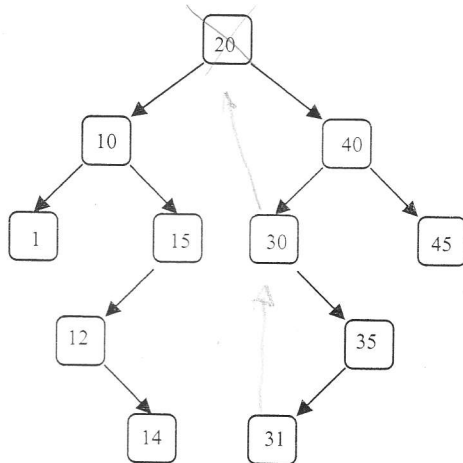
13. (2 marks) Insert key 55 into this AVL tree. Re-draw the new AVL tree.



Single left rotation.



14. (2 marks) Delete key 20 from the given BST. Re-draw the new BST.



15. (2 marks) Organize the following keys into a heap: 1, 2, 3, 4, 5, 6, 7, 8. Draw only the final tree.

