Constrained Portfolio Optimization and Risk Analysis

A Quantitative Asset Allocation Study

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1 Introduction

Modern portfolio construction requires a rigorous balance between return maximization, risk control, and practical investment constraints. Classical mean–variance theory provides a foundational framework, but real-world implementation demands additional considerations such as liquidity, leverage limits, and robustness under uncertainty.

This project develops a fully reproducible quantitative pipeline for portfolio optimization under realistic constraints. The study integrates data engineering, constrained optimization, historical backtesting, and stochastic simulation to evaluate both realized and probabilistic portfolio performance.

The objective is twofold:

- Construct an economically meaningful optimal portfolio under realistic constraints.
- Assess its robustness through empirical and probabilistic risk diagnostics.

2 Computational Environment

The project is implemented in Python, leveraging widely adopted libraries in quantitative finance and statistical computing:

- **pandas**, **numpy**: data manipulation and numerical computation.
- **scipy**: numerical optimization.
- **PyPortfolioOpt**: portfolio optimization routines.
- **matplotlib**: visualization.

All computations are performed in a fully reproducible environment, with intermediate datasets stored explicitly to ensure transparency and traceability.

3 Data Acquisition and Preprocessing

Daily adjusted closing prices and trading volumes are collected for a diversified universe of financial assets. Raw data are stored separately from processed datasets to preserve data integrity.

3.1 Price Cleaning and Alignment

Let $P_{i,t}$ denote the adjusted closing price of asset i at time t. The preprocessing pipeline enforces:

- Valid calendar alignment across assets.
- Conversion to numeric types.
- Removal of invalid or missing observations.

3.2 Return Computation

Daily simple returns are computed as:

$$r_{i,t} = \frac{P_{i,t}}{P_{i,t-1}} - 1$$

The resulting return matrix $R \in \mathbb{R}^{T \times N}$ constitutes the fundamental input for all subsequent analyses.

4 Portfolio Optimization Framework

4.1 Model Setup

Consider a portfolio of N assets with weight vector:

$$w = (w_1, \ldots, w_N)^\top$$

Let $\mu \in \mathbb{R}^N$ denote the vector of expected returns and $\Sigma \in \mathbb{R}^{N \times N}$ the covariance matrix. The portfolio's expected return and volatility are given by:

$$\mathbb{E}[R_p] = w^\top \mu, \quad \sigma_p = \sqrt{w^\top \Sigma w}$$

4.2 Sharpe Ratio Maximization

The optimization objective is the Sharpe ratio:

$$\text{Sharpe}(w) = \frac{w^\top \mu - R_f}{\sqrt{w^\top \Sigma w}}$$

where R_f denotes the risk-free rate, set to zero for simplicity.

4.3 Constraints

The portfolio is subject to:

$$\sum_{i=1}^{N} w_i = 1 \quad \text{(full investment)} \tag{1}$$

$$0 \leq w_i \leq 0.3 \quad \forall i \quad \text{(long-only, concentration control)} \tag{2}$$

4.4 Optimization Problem

The constrained optimization problem is:

$$\begin{aligned} \max_{w} \quad & \frac{w^\top \mu}{\sqrt{w^\top \Sigma w}} \\ \text{s.t.} \quad & \sum_{i=1}^{N} w_i = 1 \\ & 0 \leq w_i \leq 0.3 \end{aligned}$$

The problem is solved numerically using Sequential Least Squares Programming (SLSQP).

5 Empirical Optimization Results

The optimal portfolio exhibits the following annualized characteristics:

$$\mathbb{E}[R_p] = 34.9\%, \quad \sigma_p = 20.9\%, \quad \text{Sharpe} = 1.62$$

5.1 Optimal Allocation

The solution is sparse:

$$w^* = \begin{cases} 0.5789 & \text{GLD} \\ 0.3692 & \text{NVDA} \\ 0.0276 & \text{MSFT} \\ 0.0242 & \text{AMZN} \\ 0 & \text{otherwise} \end{cases}$$

This sparsity arises endogenously due to binding inequality constraints.

5.2 Theoretical Consistency

The solution satisfies the Karush–Kuhn–Tucker conditions:

$$\nabla_w \mathcal{L}(w^*, \lambda, \nu) = 0$$

confirming optimality under the imposed constraints.

6 Historical Backtesting

6.1 Portfolio Dynamics

Let $r_{p,t} = w^\top r_t$ denote the daily portfolio return. Portfolio value evolves as:

$$V_t = V_0 \prod_{\tau=1}^{t} (1 + r_{p,\tau})$$

6.2 Risk Metrics

Annualized Volatility

$$\sigma_p = \sqrt{252} \cdot \mathrm{Std}(r_{p,t})$$

Sharpe Ratio

$$\mathrm{Sharpe} = \frac{252 \cdot \mathbb{E}[r_{p,t}]}{\sigma_p}$$

Maximum Drawdown

$$\mathrm{MDD} = \max_t \left(\frac{\max_{\tau \le t} V_\tau - V_t}{\max_{\tau \le t} V_\tau} \right)$$

6.3 Empirical Results

- Annualized volatility: **20.86%**
- Sharpe ratio: **1.57**
- Maximum drawdown: **36.07%**

7 Monte Carlo Simulation

7.1 Stochastic Model

Portfolio returns are modeled as:

$$r_{p,t} \sim \mathcal{N}(\mu_p, \sigma_p^2)$$

where parameters are estimated from historical data.

7.2 Simulation Procedure

For each simulation j:

$$V_t^{(j)} = V_0 \prod_{\tau=1}^{t} (1 + r_{p,\tau}^{(j)})$$

A total of 10,000 independent paths are simulated over 252 trading days.

7.3 Risk Interpretation

Monte Carlo analysis provides:

- Distribution of terminal wealth
- Tail-risk assessment
- Probability of capital loss

This probabilistic layer complements historical backtesting and strengthens the robustness of the study.

8 Conclusion

This project demonstrates a complete quantitative asset allocation pipeline, integrating constrained optimization, empirical validation, and stochastic risk modeling. The results highlight the importance of combining theoretical rigor with practical constraints to obtain economically meaningful portfolios.