$\mathcal{L}^1_{\text{UCS}}$: A Monodic Logic for Bounded Model Checking of Unbounded Client-Server Systems

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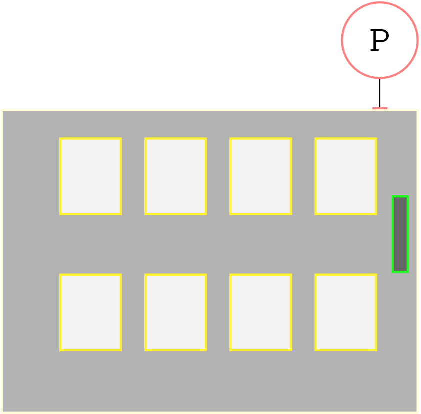
A toy example

An empty Parking Lot waits for vehicles



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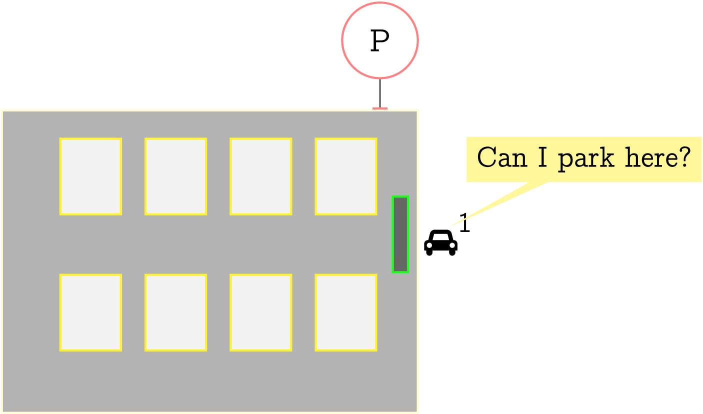

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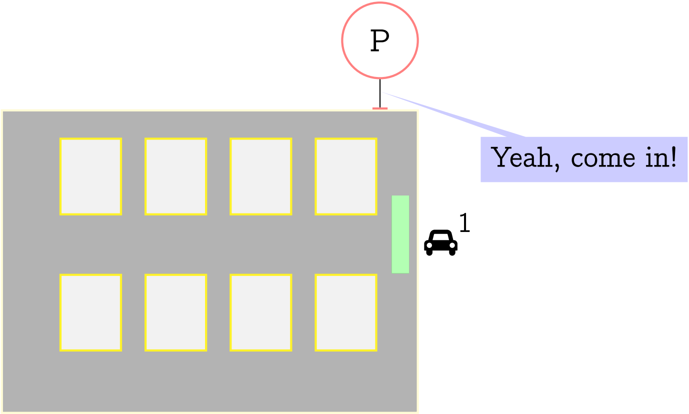

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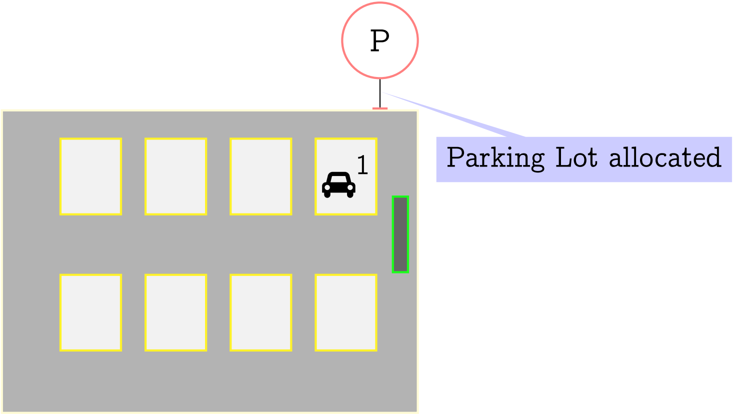
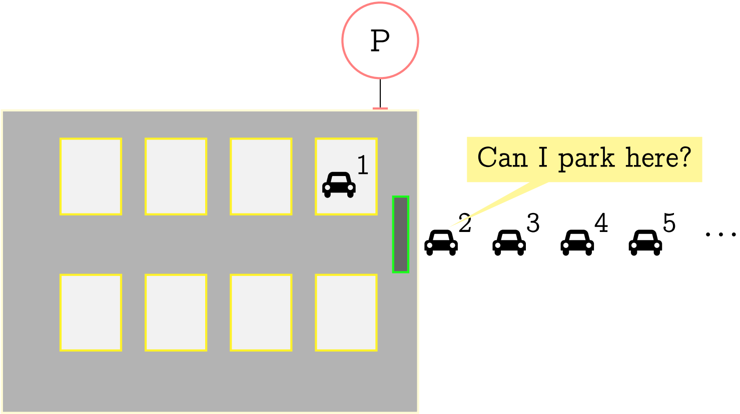


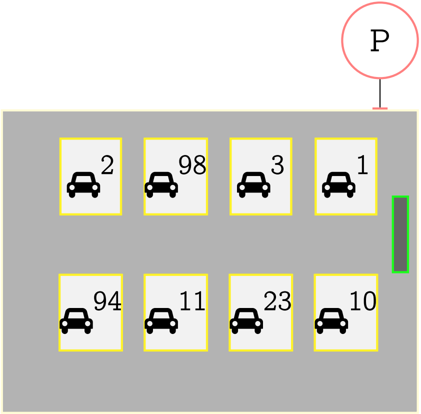
A toy example

Unbounded number of parking requests



A toy example

No parking space



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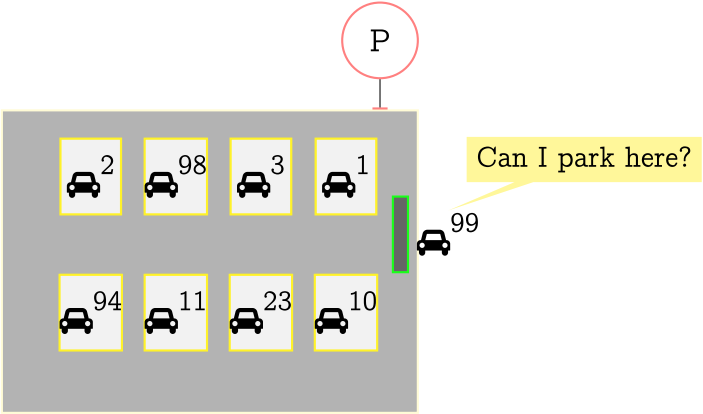

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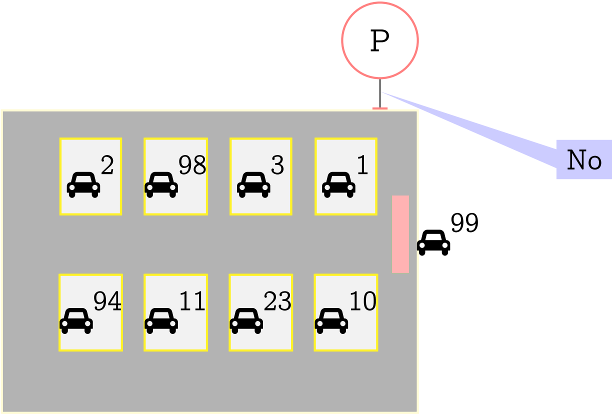

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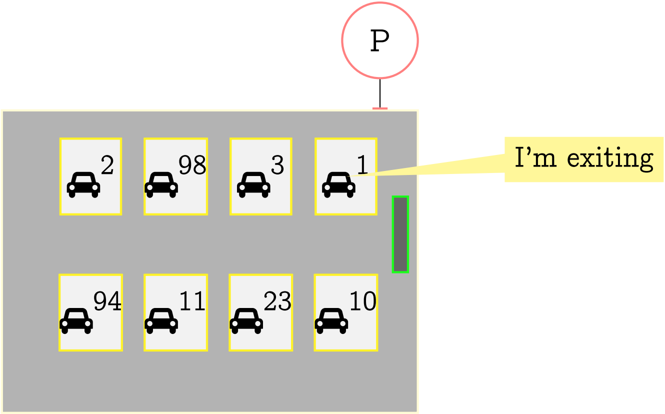

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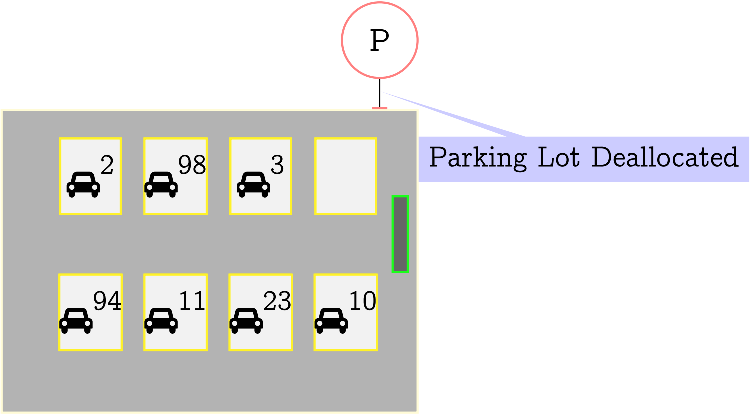
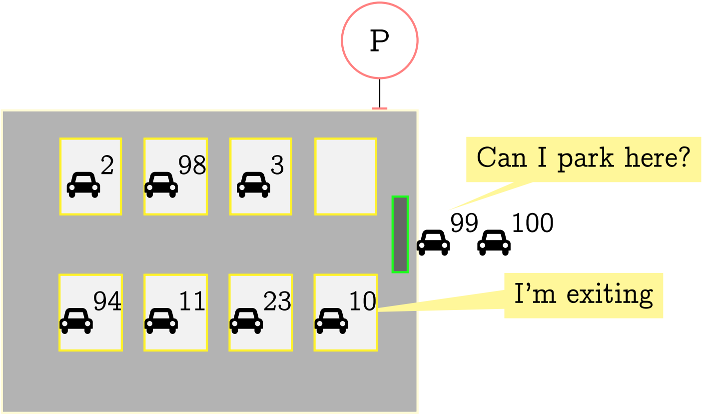
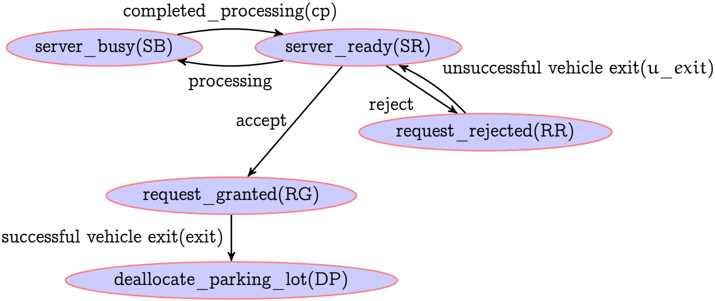


A toy example

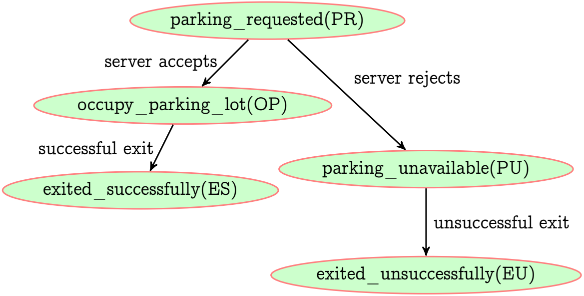
- single server-multiple client system
- unbounded, distinguishable clients of the same type



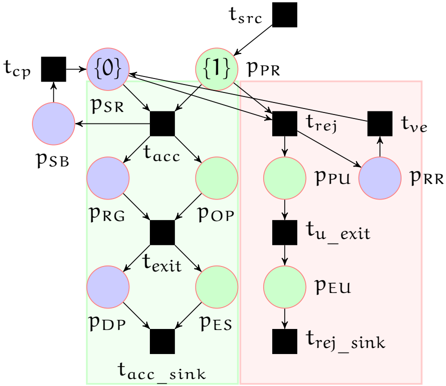
Case Study Parking System: Server behaviour



Case Study Parking System: Client behaviour



Modelling an Unbounded Client-Server (UCS)



Finite sets of atomic client propositions P_c and server propositions P_s:

$$
\begin{aligned}
P_c = \{ & parking\_requested(PR), \\
& occupy\_parking\_lot(OP), \\
& parking\_unavailable(PU), \\
& exit\_successfully(ES), \\
& exited\_unsuccessfully(EU)\} \\
P_s = \{ & server\_ready(SR), \\
& server\_busy(SB), \\
& request\_granted(RG), \\
& request\_rejected(RR), \\
& deallocate\_parking\_lot(DP)\}
\end{aligned}
$$

Question: What properties of the system are we interested in?

Properties

1. When a vehicle requests a parking space, it is always the case that for every vehicle, it eventually exits the system, either successfully after being granted a parking space, or unsuccessfully, when its request is denied.

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2. It is always the case that if the client occupies a parking lot, it will eventually exit the parking lot.

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2. It is always the case that if the client occupies a parking lot, it will eventually exit the parking lot.

$$G_s(\forall x)\Big(\mathit{occupy\_parking\_lot}(x) \Rightarrow F_c(\mathit{exit\_successfully}(x))\Big)$$

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3. There may be clients whose requests are rejected.

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The proposed logic: The Monodic* Logic $\mathcal{L}^1_{UCS}$

The set of client formulae Δ:

$$\Delta ::= p(x) \mid \neg\alpha \mid \alpha \vee \beta \mid \alpha \wedge \beta \mid X_c\alpha \mid F_c\alpha \mid G_c\alpha \mid \alpha\ U_c\ \beta$$

where $\alpha, \beta \in \Delta$ and $p \in P_c$, the set of atomic client propositions.

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The set of server formulae Ψ:

$$\Psi ::= q \mid \neg\psi \mid (\exists x)\alpha \mid (\forall x)\alpha \mid \psi_1 \vee \psi_2 \mid \psi_1 \wedge \psi_2 \mid X_s\psi \mid F_s\psi \mid G_s\psi \mid \psi_1 \; U_s \; \psi_2$$

where $\psi, \psi_1, \psi_2 \in \Psi$ and $q \in P_s$, the set of atomic server propositions, $\alpha \in \Delta$.

The Monodic Logic $\mathcal{L}^1_{\text{UCS}}$ (contd)

$$\psi = (\exists x)(\exists y)\Big(\big(PR(x) \wedge PR(y)\big) \wedge F_c\big(ES(x) \wedge ES(y)\big)\Big)$$

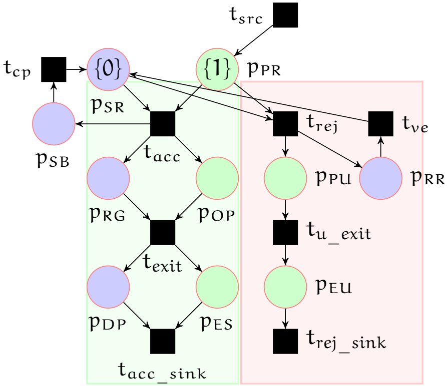
- Is $\psi \in \mathcal{L}^1_{\text{UCS}}$?

The Monodic Logic $\mathcal{L}^1_{\mathsf{UCS}}$ (contd)

$$\psi = (\exists x)(\exists y)\Big(\big(\mathsf{PR}(x) \wedge \mathsf{PR}(y)\big) \wedge \mathbf{F}_c\big(\mathsf{ES}(x) \wedge \mathsf{ES}(y)\big)\Big)$$

- Is $\psi \in \mathcal{L}^1_{\mathsf{UCS}}$?
- Answer: No. **not a well formed formula** in $\mathcal{L}^1_{\mathsf{UCS}}$

Recall: our model



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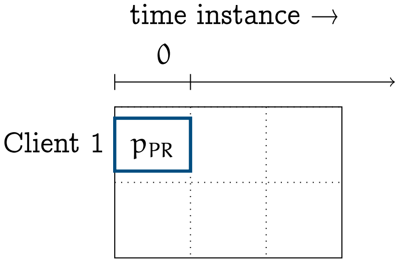
Snapshots of the system + live windows



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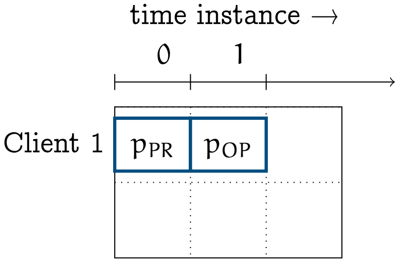

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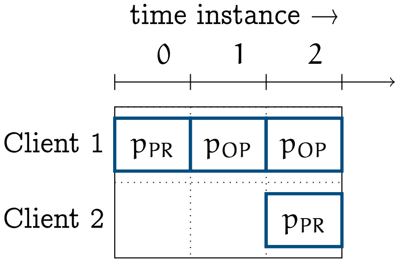
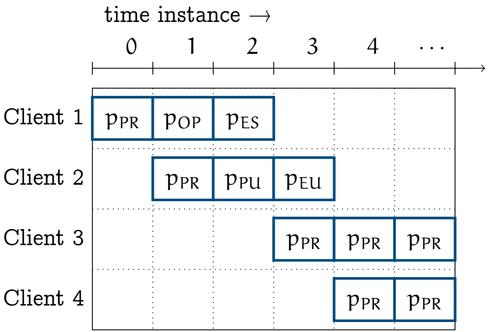


Snapshots of the system + live windows (contd)



- 4 distinguishable clients, with overlapping *live windows*, with the bound 5.
- client 1 is at p_{PR} at instance 0. i.e, $PR(x)$ is satisfied in client 1 at instance 0.
- For client 1, the *left boundary* is at instance 0 and its *right boundary* is at instance 2.

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- For instance, $\mathfrak{Z}(a, i) \in q$, means that the local state of each client a at instance i is state q, where $q \in Q$.

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A model is a triple $M = (\nu, V, \xi)$ where

- ν gives the local behaviour of the server as follows:
  $\nu = \nu_0 \nu_1 \nu_2 \ldots$, where for all $0 \leq i$, $\nu_i \subseteq P_s$

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- V gives the set of live agents (clients) at each instance.
  $V = V_0 V_1 V_2 \ldots$, where for all $0 \leq i$, V_i is a finite subset of CN, gives the set of live agents at the ith instance.
  For every $0 \leq i$, V_i and V_{i+1} satisfy the following properties:
  1. if $V_i \subseteq V_{i+1}$ then for every $a \in V_{i+1} - V_i$ such that $\mathfrak{Z}(a, i+1) \in I$.
  2. if $V_{i+1} \subseteq V_i$ then for every $a \in V_i - V_{i+1}$ such that $\mathfrak{Z}(a, i) \in F$.

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- $\xi = \xi_0 \xi_1 \xi_2 \cdots$, where for all $0 \leq i$, $\xi_i : V_i \to 2^{P_c}$ gives the properties satisfied by each live agent at ith instance.
  Also, $L(\mathfrak{Z}(a, i)) = \xi_i(a)$.

Semantics

$M, i \models (\exists x)\alpha$ iff $\exists a \in CN,\ a \in V_i$ and $M, [x \mapsto a], i \models_x \alpha$.

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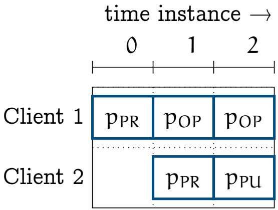

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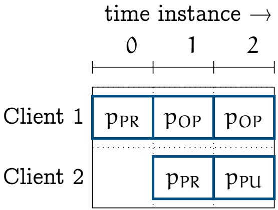

Given $\alpha = OP(x) \vee PR(x)$ and $CN = \{1, 2\}$.

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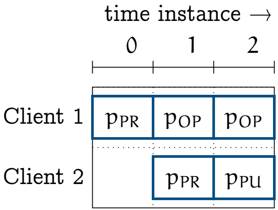
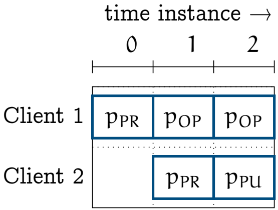


Given $\alpha = OP(x) \vee PR(x)$ and $CN = \{1, 2\}$.

- $M, 0 \models^k (\exists x)(OP(x) \vee PR(x))$.
- $M, 1 \models^k (\exists x)(OP(x) \vee PR(x))$.
- $M, 2 \models^k (\exists x)(OP(x) \vee PR(x))$.

Bounded Semantics

$M, i \models^k (\forall x)\alpha$ iff $\forall a \in CN$, if $a \in V_i$ then $M, [x \mapsto a], i \models^k_x \alpha$



Given $\alpha = ES(x)$ and $CN = \{1, 2\}$.

- $M, 0 \not\models^k (\exists x)(ES(x))$.
- $M, 1 \not\models^k (\exists x)(ES(x))$.
- $M, 2 \not\models^k (\exists x)(ES(x))$.

Bounded Semantics

$M, i \models^k F_s\psi$ iff $\exists j : i \leq j \leq k$, $M, j \models^k \psi$.
Given $\psi = (\exists x)ES(x)$.

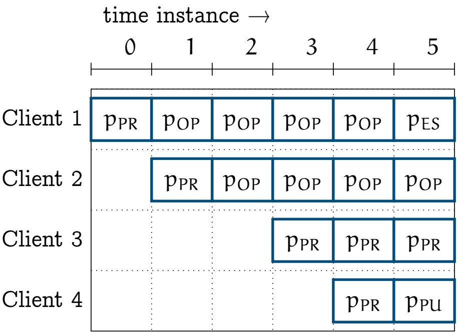


Figure: Snapshot with 4 clients

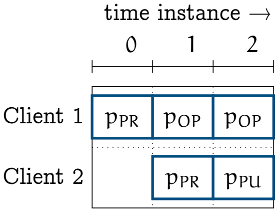


Figure: Snapshot with 2 clients

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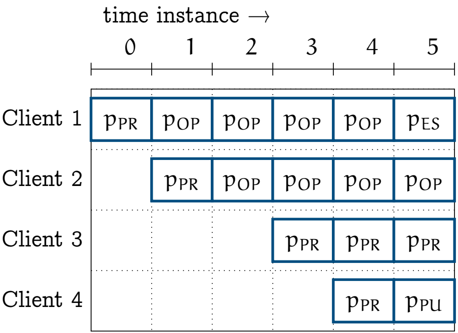

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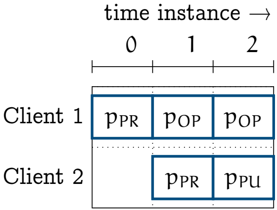

Figure: Snapshot with 2 clients

As shown above $M, 5 \models^k F_s(\exists x)ES(x)$.

Takeaway

- Proposed the logic $\mathcal{L}^1_{\mathrm{UCS}}$ for client-server specifications.
- Modeled the running example using nets.
- Detailed Semantics and encoding are in our paper!
- Currently working on building a model checker to verify $\mathcal{L}^1_{\mathrm{UCS}}$ properties.

Thank you for your attention

Appendix

Bounded Semantics

$M, [x \mapsto a], i \models_x^k F_c \alpha$ iff
$\exists j : i \leq j \leq k,\ a \in V_j$ and $M, [x \mapsto a], j \models_x^k \alpha$.

time instance →	0	1	2	3	4	5
Client 1	p_{PR}	p_{OP}	p_{OP}	p_{OP}	p_{OP}	p_{ES}
Client 2		p_{PR}	p_{OP}	p_{OP}	p_{OP}	p_{OP}
Client 3				p_{PR}	p_{PR}	p_{PR}
Client 4					p_{PR}	p_{PU}

Figure: Snapshot with 4 clients

time instance →	0	1	2
Client 1	p_{PR}	p_{OP}	p_{OP}
Client 2		p_{PR}	p_{PU}

Figure: Snapshot with 2 clients

Given $\alpha = PR(x)$. The above formula is satisfiable for all clients in both figures.