Quantum Programming Languages

Presentation

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Theoretical Foundation

Qbit

Qbit - Definition

A Qbit is a linear combination of the state vectors $|0\rangle$ and $|1\rangle$ multiplied with the probabilistic amplitudes α and β:

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,, \quad \alpha, \beta \in \mathbb{C}^2$$

$$\text{where} \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |\psi\rangle \in \mathbb{C}^2,$$

$$\text{and} \|\alpha\|^2 + \|\beta\|^2 = 1.$$

Superposition - Definition

α and β sufficiently describe a Qbit's state in superposition: $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$.

Upon measurement, the superposition collapses to one of the state vectors.

System of 2 Qbits

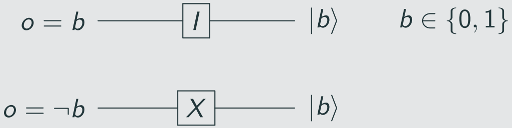
$$|\Psi\rangle = \alpha|00\rangle + \beta|10\rangle + \gamma|01\rangle + \delta|11\rangle \quad \text{with} \quad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \in \quad \mathbb{C}^4$$

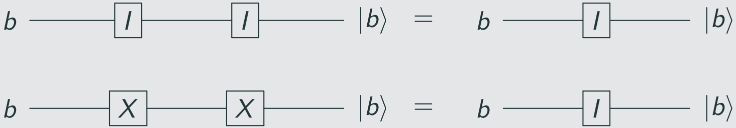
Identity & Bit Flip Gate

Read from right (input state) to left (output state o).

Note that both I and X are invertible by themselves (unitary).



Unitary



Hadamard Gate

Hadamard Gate

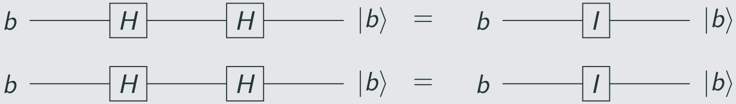
The Hadamard gate is more interesting. It produces a **probabilistic output** with $o \overset{unif}{\sim} \{0, 1\}$.

$$o = \begin{cases} 0 & \text{with } \mathbb{P}(0) = \frac{1}{2} \\ 1 & \text{with } \mathbb{P}(1) = \frac{1}{2} \end{cases}$$

$b \in \{0, 1\}$

Quantum Phenomenon

Strangly, it is also revertible by itself.



Sign Flip Gate

$$\begin{quantikz} \lstick{$o = b$} & \gate{Z} & \rstick{$|b\rangle$} \end{quantikz} \qquad b \in \{0, 1\}$$

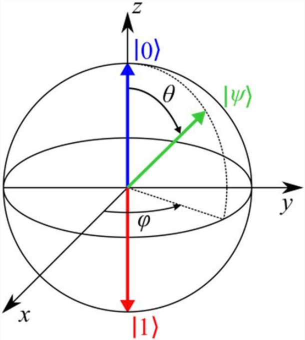
$$\begin{quantikz} \lstick{b} & \gate{Z} & \gate{Z} & \rstick{$|b\rangle$} \end{quantikz} = \begin{quantikz} \lstick{b} & \gate{I} & \rstick{$|b\rangle$} \end{quantikz}$$

But: Unintuitive Behavior in Superposition

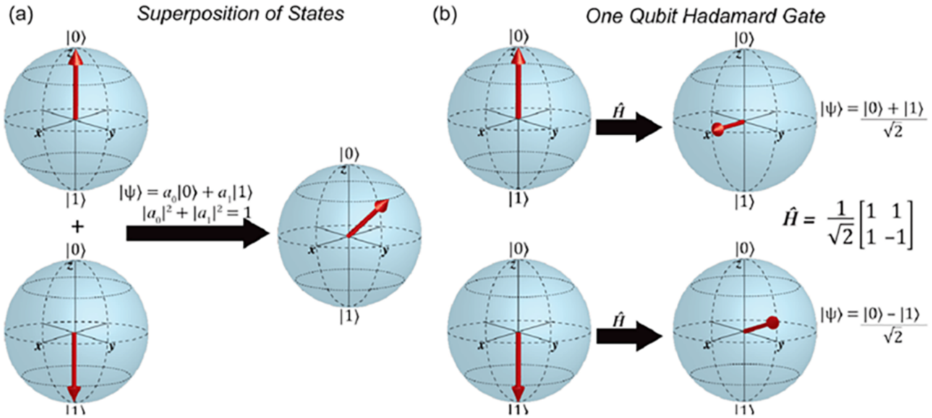
$$\begin{quantikz} \lstick{b} & \gate{H} & \gate{Z} & \gate{H} & \rstick{$|b\rangle$} \end{quantikz} = \begin{quantikz} \lstick{b} & \gate{X} & \rstick{$|b\rangle$} \end{quantikz}$$

Quantum Operations & Bloch Sphere

$$|\psi_{\text{final}}\rangle = \psi_{\text{final}} = \mathbf{U}\psi = \mathbf{U}\begin{pmatrix} U_{0,0} & U_{0,1} \\ U_{1,0} & U_{1,1} \end{pmatrix}\begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} = \begin{pmatrix} U_{0,0}\psi_0 + U_{0,1}\psi_1 \\ U_{1,0}\psi_0 + U_{1,1}\psi_1 \end{pmatrix} \in \mathbb{C}^2.$$



Bloch Sphere



Demo: Qiskit

Literature I

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[3] Bernhard Ömer.
Quantum Programming in QCL, 2000.