1 Nomenclature

H	set of time slot h
N	set of prosumers n
τ_h	Time of tariff at time slot h
$p_{h,n}^{inf}$	Load demand of prosumer n at time slot h
$p_{h,n}^{ev}$	EV (flexible) demand of prosumer n at time slot h
e_n^{ev}	EV battery state of charge of prosumer n
E_{min}^{ev}	EV minimum state of charge
P^{max}	Transfomer Thermal Limit
$p_{h,n}^{max}$	Power limit for prosumer n at time slot h

2 Lower Level

$$
\begin{aligned}
\min_{p_h^{ev}} \quad & \sum_{h \in H} \tau_h (p_h^{inf} + p_h^{ev}) \\
\text{subject to} \quad & \\
& E_{min}^{ev} - e^{ev} - \sum_{h \in H} p_h^{ev} \leq 0 \\
& p_h^{inf} + p_h^{ev} - p_h^{max} \leq 0 \quad \forall h \in H
\end{aligned}
$$

Lagrange:

$$
\begin{aligned}
\mathcal{L}(p_h^{ev}, \lambda) = & \sum_{h \in H} \tau_h (p_h^{inf} + p_h^{ev}) \\
& + \lambda^1 \left(E_{min}^{ev} - e^{ev} - \sum_{h \in H} p_h^{ev} \right) \\
& + \sum_{h \in H} \lambda_h^2 \left(p_h^{inf} + p_h^{ev} - p_h^{max} \right)
\end{aligned}
$$

$$
\text{Stationarity:} \quad \frac{\partial \mathcal{L}}{\partial p_h^{ev}} = \tau_h - \lambda^1 + \lambda_h^2 = 0 \quad \forall h \in H
$$

$$
\begin{aligned}
\text{Primal feasibility:} \quad & E_{min}^{ev} - e^{ev} - \sum_{h \in H} p_h^{ev} \leq 0 \\
& p_h^{inf} + p_h^{ev} - p_h^{max} \leq 0 \quad \forall h \in H
\end{aligned}
$$

$$
\begin{aligned}
\text{Dual feasibility:} \quad & \lambda^1 \geq 0 \\
& \lambda_h^2 \geq 0 \quad \forall h \in H
\end{aligned}
$$

$$
\begin{aligned}
\text{Complementary slackness:} \quad & \lambda^1 \left(E_{min}^{ev} - e^{ev} - \sum_{h \in H} p_h^{ev} \right) = 0 \\
& \lambda_h^2 \left(p_h^{inf} + p_h^{ev} - p_h^{max} \right) = 0 \quad \forall h \in H
\end{aligned}
$$

3 Bi-level Reformulation with KKT

$$\max_{p_{h,n}^{max}} \quad p_{h,n}^{max}$$

subject to

$$\sum_{n\in N} p_{h,n}^{max} - P^{max} \leq 0 \quad \forall h \in H$$

(Stationarity)

$$\tau_h - \lambda_n^1 + \lambda_{h,n}^2 = 0 \quad \forall h \in H, \forall n \in N$$

(Primal Feasibility)

$$E_{min}^{ev} - e_n^{ev} - \sum_{h\in H} p_{h,n}^{ev} \leq 0 \quad \forall n \in N$$

$$p_{h,n}^{inf} + p_{h,n}^{ev} - p_{h,n}^{max} \leq 0 \quad \forall h \in H, \forall n \in N$$

(Dual Feasibility)

$$\lambda_n^1 \geq 0 \quad \forall n \in N$$

$$\lambda_{h,n}^2 \geq 0 \quad \forall h \in H, \forall n \in N$$

(Complementary Slackness)

$$\lambda_n^1 \left(E_{min}^{ev} - e_n^{ev} - \sum_{h\in H} p_{h,n}^{ev} \right) = 0 \quad \forall n \in N$$

$$\lambda_{h,n}^2 \left(p_{h,n}^{inf} + p_{h,n}^{ev} - p_{h,n}^{max} \right) = 0 \quad \forall h \in H, \forall n \in N$$

(Big M reformulation for Complementary Slackness)

$$\lambda_n^1 \leq M z_n^1 \quad \forall n \in N$$

$$E_{min}^{ev} - e_n^{ev} - \sum_{h\in H} p_{h,n}^{ev} \leq M(1 - z_n^1) \quad \forall n \in N$$

$$\lambda_{h,n}^2 \leq M z_{h,n}^2 \quad \forall h \in H, \forall n \in N$$

$$p_{h,n}^{inf} + p_{h,n}^{ev} - p_{h,n}^{max} \leq M(1 - z_{h,n}^2) \quad \forall h \in H, \forall n \in N$$

$$z_n^1 \in \{0,1\}, \quad z_{h,n}^2 \in \{0,1\} \quad \forall h \in H, \forall n \in N$$