1

GEOMETRIA PROYECTIVA - HOJA 2

$r \equiv 3x - 2y = 5$

$s \equiv 3x - 2y = -2$

$$P = r \times s = \begin{vmatrix} i & j & k \\ a & b & c_1 \\ a & b & c_2 \end{vmatrix}$$

$r : \{ax + by + c_1 = 0\} \Rightarrow \{ax + by + c_1 z = 0\}; \quad r_1 = (a, b, c_1)$

$s : \{ax + by + c_2 = 0\}; \quad \{ax + by + c_2 z = 0\}; \quad s = (a, b, c_2)$

$P = (-2(2+5), -3(2+5), 0) = (-14, -21, 0); \quad \boxed{P = (-2, -3, 0)}$

2

$r \equiv 3x + 2y - 1 = 0 \qquad \mathbb{P}^2?$

$\Downarrow$

$3x + 2y = z \qquad$ Si $(a, b, c) \in r \Rightarrow (\lambda a, \lambda b, \lambda c) \in r$

$3\lambda a + 2\lambda b - \lambda c \Rightarrow \lambda(3a + 2b - c)$

*Para que sea un punto del infinito, debe igualarse a 0, por lo que $z = 0$ es una solución $\Rightarrow \quad \boxed{(2 : -3 : 0)}$

③ GEOMETRÍA PROYECTIVA - HOJA 2

$\pi_1 = (0,-1,1,-1)$ $\pi_2 = (0,-1,1,1)$

$L^* = \pi_1' \cdot \pi_2 - \pi_2' \circ \pi_1$; $\begin{pmatrix} 0 \\ -1 \\ 1 \\ -1 \end{pmatrix}(0\ -1\ 1\ 1) - \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix}(0\ -1\ 1\ -1) =$

$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & -1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & -2 & 0 \end{pmatrix}$; $L^* = (2,2,0,0,0,0)$

$P \in l \Rightarrow L^* P = 0$ por lo que:

$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 2 \\ 0 & 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ -2d \\ 2d \\ 2b-2c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

Si $d=0$, el vector con la solución es: $\boxed{(a,b,b,0)}$

Está en el infinito.

④

$\pi_1 = (2\ -1\ -1\ -1)$ π_2 (plano infinito) $= (0\ 0\ 0\ 1)$ $\pi_1 \cap \pi_2$?

$L^* = \pi_1' \pi_2 - \pi_2' \pi_1$

$\begin{pmatrix} 2 \\ -1 \\ -1 \\ -1 \end{pmatrix}(0\ 0\ 0\ 1) - \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}(2\ -1\ -1\ -1) = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & -1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 \\ -2 & 1 & 1 & 0 \end{pmatrix}$

La $\cap$ será: $\pi_1 \cap \pi_2$ = recta de Plucker $\rightarrow L = (0:0:2:0:1:-1)$

⑤

Octave

⑥

$P = (1,1,0) \qquad Q = (-1,0,1) \qquad \overrightarrow{PQ} = (-2,-1,1)$

Plucker

$P \times Q = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} i - \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} j + \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} k = \boxed{(1,-2,1)}$

$L = (\overrightarrow{PQ}, P \times Q) = (-2 : -1 : 1 : 1 : -2 : 1) \qquad \text{Matriz} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{pmatrix}$

⑦ Octave

⑧

Plano

ecs. recta $\begin{cases} x+y=1 \\ x-z=0 \end{cases}$; $\begin{cases} x = 1-\lambda \\ y = \lambda \\ z = 1-\lambda \end{cases}$ $r \equiv (-1,1,-1)$

Punto $P = (1,1,-3)$

$L = (-1,1,-1,0,1,1)$

$$\Pi = L^{*} \cdot P = \begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & -1 & -1 \\ -1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -1 \\ 2 \end{pmatrix}$$

$\Pi \cap \overline{PQ}$

$P = (1,0,-1) \qquad Q = (-3,-2,-3) \qquad \overrightarrow{PQ} = (-4,-2,-2)$

$P \times Q = \begin{vmatrix} 0 & -1 \\ -2 & -3 \end{vmatrix} i - \begin{vmatrix} 1 & -1 \\ -3 & -3 \end{vmatrix} j + \begin{vmatrix} 1 & 0 \\ -3 & -2 \end{vmatrix} k = (-2, 6, -2)$

$L = (\overrightarrow{PQ}, P \times Q) = (-4 : -2 : -2 : -2 : 6 : -2)$

$$P = L \cdot \Pi = \begin{pmatrix} 0 & -4 & -2 & -2 \\ 4 & 0 & -2 & -6 \\ 2 & 2 & 0 & -2 \\ 2 & 6 & 2 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 2 \\ 0 \end{pmatrix}$$

GEOMETRÍA PROYECTIVA. - HOJA 2

(9) ¿Qué parte del plano queda visible por la cámar?

$P_1 = (1,0,0,0) \qquad P_2 = (1,1,1,2)$

$P = L\pi \qquad L = PQ^t - QP^t$

$$L_1 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}(1\,1\,1\,2) - \begin{pmatrix}1\\1\\1\\2\end{pmatrix}(1\,0\,0\,0) = \begin{pmatrix}1&1&1&2\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} - \begin{pmatrix}1&0&0&0\\1&0&0&0\\1&0&0&0\\2&0&0&0\end{pmatrix} = \begin{pmatrix}0&1&1&2\\-1&0&0&0\\-1&0&0&0\\-2&0&0&0\end{pmatrix}$$

$$L_2 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}(1\,-1\,1\,2) - \begin{pmatrix}1\\-1\\1\\2\end{pmatrix}(1\,0\,0\,0) = \begin{pmatrix}1&-1&1&2\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} - \begin{pmatrix}1&0&0&0\\-1&0&0&0\\1&0&0&0\\2&0&0&0\end{pmatrix} = \begin{pmatrix}0&-1&1&2\\1&0&0&0\\-1&0&0&0\\-2&0&0&0\end{pmatrix}$$

$$L_3 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}(1\,-1\,-1\,2) - \begin{pmatrix}1\\-1\\-1\\2\end{pmatrix}(1\,0\,0\,0) = \begin{pmatrix}1&-1&-1&2\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} - \begin{pmatrix}1&0&0&0\\-1&0&0&0\\-1&0&0&0\\2&0&0&0\end{pmatrix} = \begin{pmatrix}0&-1&-1&2\\1&0&0&0\\1&0&0&0\\2&0&0&0\end{pmatrix}$$

$$L_4 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}(1\,1\,-1\,2) - \begin{pmatrix}1\\1\\-1\\2\end{pmatrix}(1\,0\,0\,0) = \begin{pmatrix}1&1&-1&2\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} - \begin{pmatrix}1&0&0&0\\1&0&0&0\\-1&0&0&0\\2&0&0&0\end{pmatrix} = \begin{pmatrix}0&1&-1&2\\-1&0&0&0\\1&0&0&0\\-2&0&0&0\end{pmatrix}$$

$L_1 = (1\,1\,2\,0\,0\,0)$; $L_2 = (-1\,1\,2\,0\,0\,0)$; $L_3(-1\,-1\,2\,0\,0\,0)$; $L_4 = (1\,-1\,2\,0\,0\,0)$

$$P_1 = \begin{pmatrix}0&1&1&2\\-1&0&0&0\\-1&0&0&0\\-2&0&0&0\end{pmatrix}\begin{pmatrix}1\\0\\-1\\7\end{pmatrix} = \begin{pmatrix}13\\-1\\-1\\-2\end{pmatrix} \qquad P_2 = \begin{pmatrix}0&-1&1&2\\1&0&0&0\\-1&0&0&0\\-2&0&0&0\end{pmatrix}\begin{pmatrix}1\\0\\-1\\7\end{pmatrix} = \begin{pmatrix}13\\1\\-1\\-2\end{pmatrix}$$

$$P_3 = \begin{pmatrix}0&-1&-1&2\\1&0&0&0\\1&0&0&0\\-2&0&0&0\end{pmatrix}\begin{pmatrix}15\\1\\1\\-2\end{pmatrix} \qquad P_4 = \begin{pmatrix}0&1&-1&2\\-1&0&0&0\\1&0&0&0\\2&0&0&0\end{pmatrix}\begin{pmatrix}1\\0\\-1\\7\end{pmatrix} = \begin{pmatrix}15\\-1\\1\\-2\end{pmatrix}$$