Efficient Surface Tension Simulation Through Focused Region Analysis

Zeyuan He

1. Introduction

Fluid simulation is consistently of interest in the field of physical simulation. Generally, models for fluid simulation utilize Eulerian and Lagrangian discretization, or more commonly in modern models, a mixture of both techniques. The level set method [1], [2], [3] is an example of Eulerian technique, which stores the relevant temporally variating data on fixed grids. One of the known drawbacks of this method is its numerical dissipation which refers to the unwanted viscosity-like effects and blurring of the high-frequency features of the modeled fluid. Semi-Lagrangian methods, developing from the initial PIC [4] to FLIP, effectively alleviates this issue by storing relevant information in particles instead of grids, avoiding the accumulated averaging effect in the process of Eulerian computations. It thus achieves better preservation of angular momentum and presentation of splashing effects. However, FLIP methods are generally more computationally expensive than pure grid-based methods.

In order to alleviate this, Narrow Band-FLIP (NB-FLIP) was first proposed by [5] and performs FLIP simulation only near the boundary of the liquid surface, i.e. a "narrow band". This model significantly reduced the number of simulated particles and improved the performance of simulation. Developing further on such idea to divide the simulated fluid into different regions, [6] proposed an extension of the NB-FLIP. In their proposed model, the simulator freely switches between FLIP methods and level-set methods within the volume of fluid instead of only doing so near the boundary. They have used a custom heat function which captures the intensity of fluid movement as the criterion for switching between models. Such modification allowed Extended NB-FLIP to simultaneously reconstruct calm surfaces with no high-frequency noise commonly seen in FLIP models as well as violently moving splashing effects which cannot be handled correctly with sharp details by level-set models.

Being aware of the good suitability of Extended NB-FLIP method for capturing the surface dynamics, and that such success depends on the utilization of a heat function with physical meaning, we may further extend the use of such function and allow us to model more complicated physical phenomenon that happen at specific detailed regions. Specifically, we plan to integrate the recent paper [7] on surface tension modeling with particle methods into the Extended NB-FLIP framework and improve its detail in highly dynamic regions.

2. Proposal

Heat Function Our work will be based on utilizing the heat function to determine whether to use detailed particle based simulation methods, and how much detail we shall expect which is related to some parameters for the model. Specifically, the spatial function $\boldsymbol{\gamma}(\boldsymbol{x})$ of heat is given by:

$$\boldsymbol{q}(\boldsymbol{x}) = \boldsymbol{u}(\boldsymbol{x}) - \int_{\Omega} \boldsymbol{G}(\boldsymbol{\sigma}, (\boldsymbol{x}' - \boldsymbol{x})/\Delta x)\boldsymbol{u}(\boldsymbol{x}')dV'$$

$$\boldsymbol{c}(x) = \phi(\boldsymbol{x}) - \int_{\Omega} \boldsymbol{G}(\boldsymbol{\sigma}, (\boldsymbol{x}' - \boldsymbol{x})/\Delta x)\phi(\boldsymbol{x}')dV'$$

$$\boldsymbol{\gamma}(\boldsymbol{x}) = \max(\max(0, A_u \min(1, ||\boldsymbol{q}(\boldsymbol{x})||) - T_u), max(0, A_g|c(\boldsymbol{x})|/\Delta x - T_g))$$

as in [6], where $\boldsymbol{G}(\boldsymbol{\sigma}, \boldsymbol{x})$ is the Gaussian function, A_u, A_g, T_u, T_g are respective amplification and threshold parameters for velocity and geometry, and $\boldsymbol{u}, \boldsymbol{v}$ are respective functions for velocity and Eulerian level sets. The heat function is calculated in the grids at the boundary of the fluid and propagated towards the interior.

The method we propose is to use FLIP in areas with positive heat value and incorporate the force by surface tension where it is above a threshold value T_s. We note that the modeling of surface tension is especially compatible with the current framework, as surface tension forces are minuscule in most of the surface area of a fluid volume, especially for static regions. However, it is costly to create the surface tension sampling points and apply respective calculations. This motivates our attempt to focus on the fast-moving parts of fluid in an analogous effort to narrow-band FLIP models which only applies FLIP modeling to a (small) fraction of the fluid.

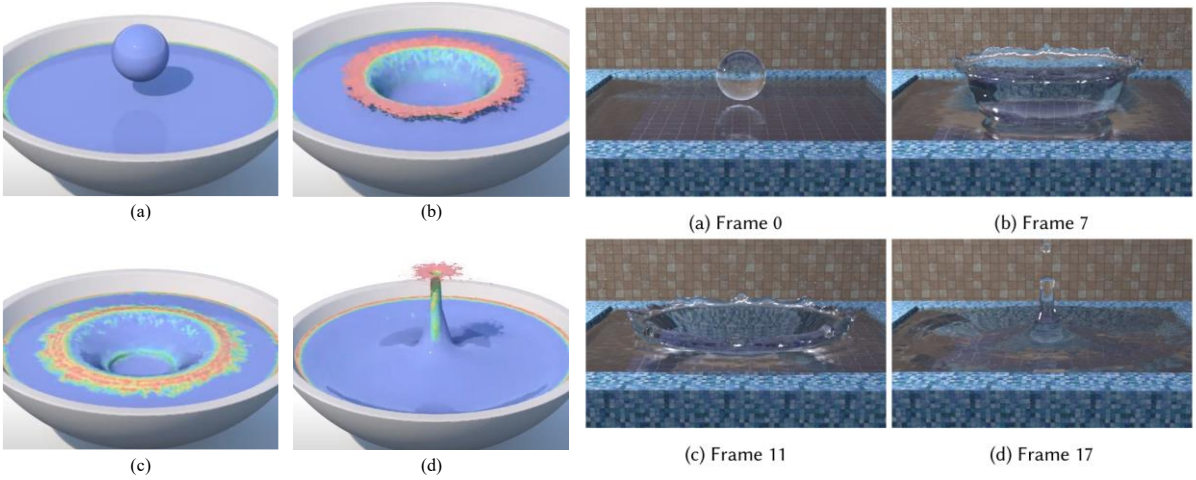

Figure 1 Comparison between models without and with surface tension forces. Note the significant difference between the "crown" forming in (c) and bead forming at the tip in (d). (i) Capture from video accompanying [6]; (ii) Simulation result from [7]

Surface Tension Modeling the surface tension of fluid is an important topic within the field of fluid simulation. It is responsible for many visually significant features including water beads forming, "bouncing" effect of water droplet, etc. Traditional approach to accurately simulate such forces is through surface meshes. Recently, [8] achieved great accuracy with a static mesh, but it cannot handle topological changes during fluid movement well. For more dynamic situations, [9] uses a particle-based Lagrangian mesh at the surface, but requires considerable effort in processing topological changes, and is computationally expensive due to its method for volume conservation. Hyde *et al.* [7] presents a pure FLIP-based model and computes surface tension forces by introducing some ghost particles at the reconstructed fluid surface, then running a slightly modified implicit pressure solver. They achieved accuracy comparable to mesh-based methods with performance comparable to common FLIP simulators.

However, the method proposed by [7] still have its drawbacks: the surface is reconstructed by simple level set isocontour calculated directly from particles, which could lead to significant loss of details, especially in regions with highly irregular surface. Also, the surface tension is computed throughout the entire surface including calm regions where no effect could be observed, while not posing a significant cost

in their model, it would be prohibitively expensive if we switch to finer methods. Furthermore, to avoid unwanted noises commonly seen in particle-based methods due to the imbalance of internal and gravitational forces, it introduces an additional hydrostatic initialization as such noises would be more significant with their modeling of surface tension. However, if Eulerian level set representation of surface is used, such noise would be automatically avoided.

In summary, it is possible to improve upon such method by using more accurate surface reconstruction methods, such as the method proposed in [10] for computing surface from particles with variable radii by computing the convex hull of their spherical level sets. Also, as the region for surface tension computing is reduced, we may increase the sampled points at the isocontour faces substantially and achieve better results in regions of interest. Furthermore, we may explore the possibility to perform surface mesh sampling and mesh-based computations based on Extended NB-FLIP.

3. Expected Contributions

- Improve upon the semi-Lagrangian surface tension model proposed by [7], achieving comparable detail and far better performance compared to mesh-based models.
- Switching to higher-cost but more accurate surface tracking frameworks, such as the method proposed in [10].
- Explore the possibility for smooth transition between high and low detailed regions within the fluid volume, demonstrated with the example of surface tension. We expect much more utilizations of the Extended NB-FLIP method to be found, as the heat function captures the saliency of the fluid movement well, and could be naturally used to find regions of interest of various physical effects.

4. Research Plan

1. Set up the infrastructure and implement the original Extended NB-FLIP model based on the Affine PIC method proposed by [11] in 3 months. The Igarashi laboratory has already done extensive works in this field, so I do not expect any difficulties.
2. Implement the surface tension model proposed by [7] on standard NB-FLIP in 3 months. Meanwhile, I should read other literatures extensively to see what other aspects of fluid simulation techniques could be coupled with the Extended NB-FLIP method to achieve great acceleration.
3. Modify the surface tension model to use finer reconstruction of surface, and dynamically switching with the heat function in 6 months. This would be the most important efforts during my research.
4. After that, keep refining the model and try to incorporate any additional improvements discovered in phase 2 into the model.

Reference

[1] M. Sussman, “A second order coupled level set and volume-of-fluid method for computing growth and collapse of vapor bubbles,” *J. Comput. Phys.*, vol. 187, no. 1, pp. 110–136, 2003.

[2] M. Sussman, P. Smereka, and S. Osher, “A level set approach for computing solutions to incompressible two-phase flow,” *J. Comput. Phys.*, vol. 114, no. 1, pp. 146–159, 1994.

[3] J. Sethian and P. Smereka, “Level set methods for fluid interfaces,” *Annu. Rev. Fluid Mech.*, vol. 35, no. 35, pp. 341–372, 2003.

[4] F. H. Harlow, “The particle-in-cell computing method for fluid dynamics,” *Methods Comput. Phys.*, vol. 3, pp. 319–343, 1964.

[5] F. Ferstl, R. Ando, C. Wojtan, R. Westermann, and N. Thuerey, “Narrow band FLIP for liquid simulations,” *Comput. Graph. forum*, vol. 35, no. 2, pp. 225–232, 2016.

[6] T. Sato, C. Wojtan, N. Thuerey, T. Igarashi, and R. Ando, “Extended narrow band FLIP for liquid simulations,” *Comput. Graph. Forum*, vol. 37, no. 2, pp. 169–177, 2018.

[7] D. A. B. Hyde, S. W. Gagniere, A. Marquez-Razon, and J. Teran, “An implicit updated lagrangian formulation for liquids with large surface energy,” *ACM Trans. Graph.*, vol. 39, no. 6, 2020.

[8] A. Jarauta, P. Ryzhakov, J. Pons-Prats, and M. Secanell, “An implicit surface tension model for the analysis of droplet dynamics,” *J. Comput. Phys.*, vol. 374, pp. 1196–1218, 2018.

[9] W. Zheng, B. Zhu, B. Kim, and R. Fedkiw, “A new incompressibility discretization for a hybrid particle MAC grid representation with surface tension,” *J. Comput. Phys.*, vol. 280, pp. 96–142, 2015.

[10] R. Ando, N. Thürey, and C. Wojtan, “Highly adaptive liquid simulations on tetrahedral meshes,” *ACM Trans. Graph.*, vol. 32, no. 4, 2013.

[11] C. Jiang, C. Schroeder, A. Selle, J. Teran, and A. Stomakhin, “The affine particle-in-cell method,” *Int. Conf. Comput. Graph. Interact. Tech.*, vol. 34, no. 4, pp. 1–10, 2015.