Matemáticas para las Ciencias Aplicadas III
Tarea 2

24 de septiembre de 2018

Anton-Bivens-Davis

Sección 14.2

15-18 Evaluate the double integral in two ways using iterated integrals: (a) viewing R as a type I region, and (b) viewing R as a type II region.
18.

$\iint\limits_R y \, \mathrm{d}A$; R is the region in the first quadrant enclosed between the circle $x^2 + y^2 = 25$ and the line $x + y = 5$.

19-24. Evaluate the double integral in two ways using iterated integrals:

22. $\iint\limits_R x \, \mathrm{d}A$; R is the region enclosed by $y = sin^{-1}x, x = \frac{1}{\sqrt{2}}$, and $y = 0$.

28.

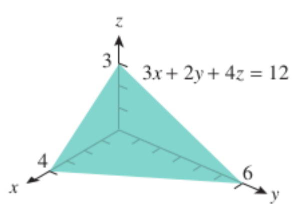
(a) By hand or with the help of a graphing utility, make a sketch of the region R enclosed between the curves $y = 4x^3 - x^4$ and $y = 3 - 4x + 4x^2$.

(b) Find the intersections of the curves in part (a).

(c) Find $\iint\limits_R x \, \mathrm{d}A$

37-38 Use double integration to find the volume of the solid.

37.

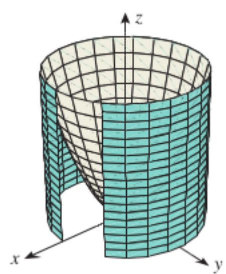


57. Try to evaluate the integral with a CAS using the stated order of integration, and then by reversing the order of integration.

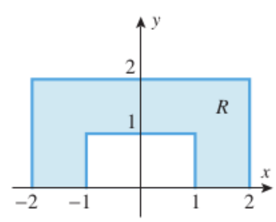
(a) $\int_{0}^{4}\int_{\sqrt{x}}^{2} \sin \pi y^3 \, \mathrm{d}x\mathrm{d}y$

(b) $\int_{0}^{1}\int_{sin^{-1}y}^{\frac{\pi}{2}} \sec \cos x^2 \, \mathrm{d}x\mathrm{d}y$

59. Evaluate $\iint_R xy^2 \, \mathrm{d}A$ over the region R shown in the accompanying figure.



▲ Figure Ex-58



▲ Figure Ex-59

63. Suppose that the temperature in degrees Celsius at a point (x, y) on a flat metal plate is $T(x, y) = 5xy + x^2$, where x and y are in meters. Find the average temperature of the diamond-shaped portion of the plate for which $|2x + y| \leq 4$ and $|2x - y| \leq 4$.

Sección 14.5

12. $\iiint_G \cos \frac{z}{y} \, \mathrm{d}V$, where G is the solid defined by the inequalities $\frac{\pi}{6} \leq y \leq \frac{\pi}{2}, y \leq x \leq \frac{\pi}{2}, 0 \leq z \leq xy$.

37. Let G be the tetrahedron in the first octant bounded by the coordinate planes and the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, (a > 0, b > 0, c > 0)$$

(a) List six different iterated integrals that represent the volume of G.

(b) Evaluate any one of the six to show that the volume of G is $\frac{1}{6}abc$.

38. Use a triple integral to derive the formula for the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Hughes-Hallet

Sección 16.2
35.

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{2 + x^3}\,\mathrm{d}x\,\mathrm{d}y$$

37.

$$\int_0^1 \int_{e^y}^e \frac{x}{\ln x}\,\mathrm{d}x\,\mathrm{d}y$$

60. Show that for a right triangle the average distance from any point in the triangle to one of the legs is one-third the length of the other leg. (The legs of a right triangle are the two sides that are not the hypotenuse.)

62. Find the area of the crescent-moon shape with circular arcs as edges and the dimensions shown in Figure 16.22.

Sección 16.3

In Problems 14–18, decide whether the integrals are positive,negative, or zero. Let S be the solid sphere $x^2 + y^2 + z^2 \leq 1$, and T be the top half of this sphere (with $z \geq 0$), and B be the bottom half (with $z \leq 0$), and R be the right half of the sphere (with $x \geq 0$), and L be the left half (with $x \leq 0$).

14.

$$\int_T e^z \, \mathrm{d}V$$

15.

$$\int_B e^z \, \mathrm{d}V$$

16.

$$\int_S \sin z \, \mathrm{d}V$$

17.

$$\int_T \sin z \, \mathrm{d}V$$

18.

$$\int_R \sin z \, \mathrm{d}V$$

31. A trough with triangular cross-section lies along the x-axis for $0 \leq x \leq 10$. The slanted sides are given by $z = y$ and $z = -y$ for $0 \leq z \leq 1$ and the ends by $x = 0$ and $x = 10$, where x, y, z are in meters. The trough contains a sludge whose density at the point (x, y, z) is $\delta = e^{-3x}$ kg per m^3.

a) Express the total mass of sludge in the trough in terms of triple integrals.

b) Express the total mass of sludge in the trough in terms of triple integrals.

55. E is the region bounded by $x = 0, y = 0, z = 0, z = 2$, and $2x + 4y + z = 4$.

57. Figure 16.28 shows part of a spherical ball of radius 5 cm. Write an iterated triple integral which represents the volume of this region.

66. Find the center of mass of the tetrahedron that is bounded by the xy, yz, xz planes and the plane $x + 2y + 3z = 1$. Assume the density is 1 gm/cm^3 and x, y, z are in centimeters.

Problems 67–69 concern a rotating solid body and its *moment of inertia* about an axis; this moment relates angular acceleration to torque (an analogue of force). For a body of constant density and mass m occupying a region W of volume V , the moments of inertia about the coordinate axes are

$$I_x = \frac{m}{V} \int_W (y^2 + z^2) \, \mathrm{d}V$$

$$I_y = \frac{m}{V} \int_W (x^2 + z^2) \, \mathrm{d}V$$

$$I_z = \frac{m}{V} \int_W (x^2 + z^2) \, \mathrm{d}V$$

67. Find the moment of inertia about the z-axis of the rectangular solid of mass m given by $0 \le x \le 1, 0 \le y \le 2, 0 \le z \le 3$.

Are the statements in Problems 74–83 true or false? Give reasons for your answer.

75. The region of integration of the triple iterated integral $\int_0^1 \int_0^1 \int_0^x f dzdydx$ lies above a square in the xy-plane and below a plane.

78. The iterated integrals $\int_{-1}^1 \int_0^1 \int_0^{1-x^2} f dzdydx$ and $\int_0^1 \int_0^1 \int_{-\sqrt{1-z}}^{\sqrt{1-z}} f dzdydx$ are equal.

80. If W is the unit cube $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ and $\int_W f \, \mathrm{d}V = 0$, then $f = 0$ everywhere in the unit.