

$$a) \tilde{X} = XA$$

$$H = X(X'X)^{-1}X'$$

After transformation:

$$\begin{aligned}\tilde{H} &= \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}' \\ &= XA(A'X'XA)^{-1}A'X' \\ &= XAA^{-1}(X'X)^{-1}(A')^{-1}A'X' \\ &= X(X'X)^{-1}X' = H\end{aligned}$$

$$\hat{\tilde{y}} = \tilde{H}y = Hy = \hat{y}$$

$$\tilde{e} = \tilde{M}y = (I - \tilde{H})y = (I - H)y = My = e$$

$$\tilde{s}^2 = \tilde{e}'\tilde{e}/(n-k) = e'e/(n-k) = s^2$$

$$\tilde{R}^2 = (\text{corr}(y, \hat{\tilde{y}}))^2 = (\text{corr}(y, \hat{y}))^2 = R^2$$

$$\begin{aligned}
 b) \quad \tilde{b} &= (\tilde{X}' \tilde{X})^{-1} \tilde{X}' y = (A' X' X A)^{-1} A' X' y \\
 &= A^{-1} (X' X)^{-1} (A')^{-1} A' X' y = A^{-1} (X' X)^{-1} X' y \\
 &= A^{-1} b
 \end{aligned}$$

$$y = X\beta + \varepsilon = X A A^{-1} \beta + \varepsilon = \tilde{X} \cdot (A^{-1} \beta) + \varepsilon$$

$$\beta \rightarrow A^{-1} \beta$$

$$b \rightarrow A^{-1} b$$