

(a)

$$AIC = \log(S^2) + \frac{2k}{n}$$

If smallest model is preferred.

$$\log(S_0^2) + \frac{2P_0}{n} < \log(S_1^2) + \frac{2P_1}{n}$$

$$\log(S_0^2) - \log(S_1^2) < \frac{2}{n}(P_1 - P_0)$$

$$\log\left(\frac{S_0^2}{S_1^2}\right) < \frac{2}{n}(P_1 - P_0)$$

$$\frac{S_0^2}{S_1^2} < e^{\frac{2}{n}(P_1 - P_0)}$$

(b) For large values of  $n$ ,  $\frac{2}{n}(P_1 - P_0)$  becomes very small

$$\text{So, } e^{\frac{2}{n}(P_1 - P_0)} \approx 1 + \frac{2}{n}(P_1 - P_0)$$

$$\frac{S_0^2}{S_1^2} < e^{\frac{2}{n}(P_1 - P_0)} \approx 1 + \frac{2}{n}(P_1 - P_0)$$

$$\frac{S_0^2}{S_1^2} - 1 = \frac{S_0^2 - S_1^2}{S_1^2} < \frac{2}{n}(P_1 - P_0)$$

(c)

$$\text{As } e_R' e_R = S_0, \quad e_U' e_U = S_1$$

$$\frac{e_R' e_R - e_U' e_U}{e_U' e_U} = \frac{S_0^2 - S_1^2}{S_1^2} < \frac{2}{n}(P_1 - P_0)$$

(d)

$$\text{We know that } F = \frac{(e_R' e_R - e_U' e_U)/g}{e_U' e_U / (n - k)}$$

Here  $g = P_1 - P_0$ ,  $k = P_1$ . As  $n$  is very large compared to  $P_1$

$$F = \frac{(e_R' e_R - e_U' e_U) / (P_1 - P_0)}{e_U' e_U / (n - P_1)} \approx \frac{(e_R' e_R - e_U' e_U) / (P_1 - P_0)}{e_U' e_U / n} < 2$$

Proven.