Problem Set 4.2

1

(a)

Minimize $100y_1 + 90y_2 + 500y_3$

Subject to

$$5y_1 - y_2 \geq 20$$

$$-4y_1 + 12y_2 + y_3 \geq 30$$

$$y_1, y_2, y_3 \geq 0$$

(b)

Put the problem into min form

Minimize $4x_1 - 3x_2$

Subject to

$$6x_1 + 11x_2 \geq -30$$

$$-2x_1 + 7x_2 \geq -50$$

$$-x_2 \geq -80$$

$$x_1, x_2 \geq 0$$

The dual problem:

Maximize $-30y_1 - 50y_2 - 80y_3$

Subject to

$$6y_1 - 2y_2 \leq 4$$

$$11y_1 + 7y_2 - y_3 \leq -3$$

$$y_1, y_2, y_3 \geq 0$$

(C) Put the problem into max form

Maximize $-x_1 + 2x_2$

Subject to

$$5x_1 + x_2 \leqslant 60$$

$$-3x_1 + 8x_2 \leqslant -10$$

$$x_1 + 7x_2 \leqslant 20$$

$$-x_1 - 7x_2 \leqslant -20$$

$$x_1,\ x_2 \geqslant 0$$

The dual problem:

Minimize $60y_1 - 10y_2 + 20(y_3 - y_4)$

Subject to

$$5y_1 - 3y_2 + (y_3 - y_4) \geqslant -1$$

$$y_1 + 8y_2 + 7(y_3 - y_4) \geqslant 2$$

$$y_1,\ y_2,\ y_3,\ y_4 \geqslant 0$$

Let $y_3' = y_3 - y_4$, the dual problem becomes

Minimizing $60y_1 - 10y_2 + 20y_3'$

Subject to

$$5y_1 - 3y_2 + y_3' \geqslant -1$$

$$y_1 + 8y_2 + 7y_3' \geqslant 2$$

$y_1, y_2 \geqslant 0$, y_3' unrestricted

or

Minimizing $60y_1 - 10y_2 + 20y_3$

Subject to

$$5y_1 - 3y_2 + y_3 \geqslant -1$$

$$y_1 + 8y_2 + 7y_3 \geqslant 2$$

$y_1, y_2 \geqslant 0$, y_3 unrestricted

When you are familiar with duality, you may write the dual problem more quickly.

Original problem:

$$\begin{array}{lll} \text{Maximize} & -x_1 + 2x_2 & \\ \text{Subject to} & 5x_1 + x_2 \leq 60 & (y_1) \\ & -3x_1 + 8x_2 \leq -10 & (y_2) \\ & x_1 + 7x_2 = 20 & (y_3) \\ & x_1,\ x_2 \geq 0 & \end{array}$$

The dual problem is

$$\begin{array}{ll} \text{Minimize} & 60y_1 - 10y_2 + 20y_3 \\ \text{Subject to} & 5y_1 - 3y_2 + y_3 \geq -1 \\ & y_1 + 8y_2 + 7y_3 \geq 2 \\ & y_1,\ y_2 \geq 0,\ y_3 \text{ unrestricted} \end{array}$$

(f) Let $y_3' = -y_3$ and put the problem into min form:

$$\begin{array}{lllll} \text{Minimize} & 2y_1 - 3y_2 - 4y_3' & & & \\ \text{subject to} & 8y_1 \qquad + y_3' & = 50 & & (x_1) \\ & \qquad -6y_2 + y_3' & \geq -70 & & (x_2) \\ & \qquad\qquad -y_3' & \geq -15 & & (x_3) \end{array}$$

$$y_1, y_2, y_3' \geq 0$$

The dual problem is

$$\begin{array}{lll} \text{Maximize} & 50x_1 - 70x_2 - 15x_3 & \\ \text{subject to} & 8x_1 & \leq 2 \\ & \qquad -6x_2 & \leq -3 \\ & x_1 + x_2 - x_3 & \leq -4 \end{array}$$

x_1 unrestricted, $x_2, x_3 \geq 0$

or

$$\begin{array}{lll} \text{Maximize} & 50x_1 - 70x_2 - 15x_3 & \\ \text{subject to} & 4x_1 & \leq 1 \\ & \qquad 2x_2 & \geq 1 \\ & -x_1 - x_2 + x_3 & \geq 4 \end{array}$$

x_1 unrestricted, $x_2, x_3 \geq 0$

3.

(b) The dual problem:

Minimize $15y_1 + 20y_2$

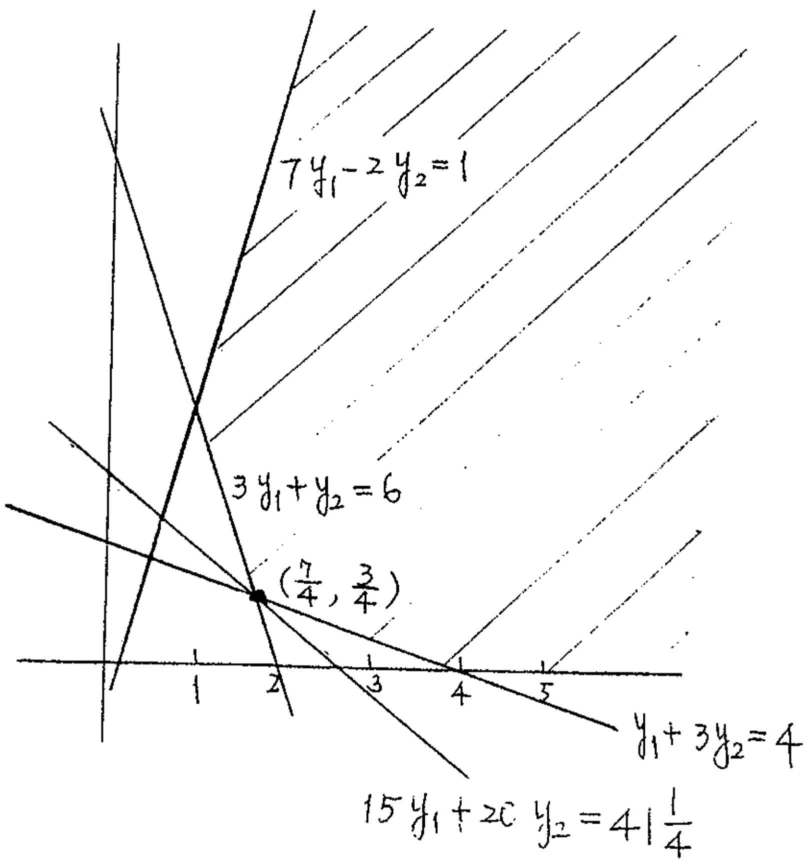
Subject to

$3y_1 + y_2 \geq 6$

$7y_1 - 2y_2 \geq 1$

$y_1 + 3y_2 \geq 4$

$y_1, y_2 \geq 0$



The isocost lines are

$15y_1 + 20y_2 = C$

where C is any constant. The farthest line to left which intersects the feasible region is the line through $(\frac{7}{4}, \frac{3}{4})$, i.e.,

$15y_1 + 20y_2 = 41\frac{1}{4}$

$\Rightarrow$ Minimal value of $b \cdot Y = 41\frac{1}{4}$

attained at $(\frac{7}{4}, \frac{3}{4})$

(c) Add slack variables x_4 and x_5 and put the problem into a minimization problem in canonical form:

$$\begin{aligned}
&\text{Minimize } -6x_1 - x_2 - 4x_3\\
&\text{Subject to}\\
&3x_1 + 7x_2 + x_3 + x_4 = 15\\
&x_1 - 2x_2 + 3x_3 + x_5 = 20\\
&x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	
x_4	3	7	1	1	0	15
x_5	1	-2	3	0	1	20
	-6	-1	-4	0	0	0
x_1	1	7/3	1/3	1/3	0	5
x_5	0	-13/3	8/3	-1/3	1	15
	0	13	-2	2	0	30
x_1	1	23/8	0	3/8	-1/8	25/8
x_3	0	-13/8	1	-1/8	3/8	45/8
	0	39/4	0	7/4	3/4	165/4

dual optimal solution

Maximal value of $c \cdot x = -(-\frac{165}{4}) = 41\frac{1}{4}$

attained at $(\frac{25}{8}, 0, \frac{45}{8})$