1. (a) canonical form:

$$\min \ -10x_1 - 6x_2 + 8x_3$$

$$\text{s.t.} \quad 5x_1 - 2x_2 + 6x_3 + x_4 = 20$$

$$10x_1 + 4x_2 - 6x_3 + x_5 = 30$$

$$x_i \geq 0 \quad \text{for } i = 1, 2 \dots 5$$

simplex:

	x_1	x_2	x_3	x_4	x_5	
x_4	5	-2	6	1	0	20
x_5	10	4	-6	0	1	30
	$\boxed{-10}$	-6	8	0	0	0
x_4	0	-4	9	1	$-1/2$	5
x_1	1	$2/5$	$-3/5$	0	$1/10$	3
	0	$\boxed{-2}$	2	0	1	30
x_4	10	0	3	1	$1/2$	35
x_2	$5/2$	1	$-3/2$	0	$1/4$	$15/2$
	5	0	$\boxed{-1}$	0	$3/2$	45
x_3	$10/3$	0	1	$1/3$	$1/6$	$35/3$
x_2	$15/2$	1	0	$1/2$	$1/2$	25
	$25/3$	0	0	$1/3$	$5/3$	$170/3$.

at $(0, 25, 35/3)$ we can obtain the maximum objective function value $170/3$.

1. (b). canonical form:

$$\min \quad -5x_2 - x_4 + 10x_6$$

$$\text{s.t.} \quad x_1 - 3x_2 - 4x_4 + 2x_6 = 60$$

$$-2x_2 + x_4 + x_5 - 4x_6 = 20$$

$$x_2 + x_3 + 3x_6 = 10$$

$x_i \geq 0$ for $i = 1, \dots, 6$

basic variables are: x_1, x_3 & x_5

we need to substitude x_3 & x_5 with x_2, x_4 & x_6 for objective function:

$$x_3 = 10 - x_2 - 3x_6$$

$$x_5 = 20 + 2x_2 - x_4 + 4x_6.$$

$$z = 9x_2 + 2x_3 - x_5$$

$$= 9x_2 + 2(10 - x_2 - 3x_6) - (20 + 2x_2 - x_4 + 4x_6)$$

$$= -5x_2 - x_4 + 10x_6.$$

simplex:

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	-3	0	-4	0	2	60
x_5	0	-2	0	1	1	-4	20
x_3	0	1	1	0	0	3	10
	0	$\boxed{-5}$	0	-1	0	10	0
x_1	1	0	3	-4	0	11	90
x_5	0	0	2	1	1	2	40
x_2	0	1	1	0	0	3	10
	0	0	5	$\boxed{-1}$	0	25	50
x_1	1	0	11	0	4	19	250
x_4	0	0	2	1	1	2	40
x_2	0	1	1	0	0	3	10
	0	0	7	0	1	27	90

at (250, 10, 0, 40, 0) we can obtain the maximum objective function value 90.

1 (c). canonical form:

$$\min \ -12x_1 + 2x_4 + 35$$

s.t.
$$x_1 - 2x_2 + x_3 + x_4 = 5$$
$$-x_1 - 3x_2 + 5x_4 + x_5 = 10$$
$$2x_1 + 2x_2 - 4x_4 + x_6 = 10$$
$$x_i \geq 0 \text{ for } i = 1, 2, \ldots, 6.$$

Substitude x_3 with x_1, x_2 & x_4.

$$x_3 = 5 - x_1 + 2x_2 - x_4$$

$$z = -5x_1 - 14x_2 + 7x_3 + 9x_4$$
$$= -5x_1 - 14x_2 + 7(5 - x_1 + 2x_2 - x_4) + 9x_4$$
$$= -12x_1 + 2x_4 + 35$$

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	1	-2	1	1	0	0	5
x_5	-1	-3	0	5	1	0	10
x_6	2	2	0	-4	0	1	10
	$\boxed{-12}$	0	0	2	0	0	-35
x_1	1	-2	1	1	0	0	5
x_5	0	-5	1	6	1	0	15
x_6	0	6	-2	-6	0	1	0
	0	$\boxed{-24}$	12	14	0	0	25
x_1	1	0	1/3	-1	0	1/3	5
x_5	0	0	-2/3	1	1	5/6	15
x_2	0	1	-1/3	-1	0	1/6	0
	0	0	4	$\boxed{-10}$	0	4	25
x_1	1	0	-1/3	0	1	7/6	20
x_4	0	0	-2/3	1	1	5/6	15
x_2	0	1	-1	0	1	1	15
	0	0	-8/3	0	10	37/3	175

redundand

or

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	1	-2	1	1	0	0	5
x_5	-1	-3	0	5	1	0	10
x_6	2	2	0	-4	0	1	10
	$\boxed{-12}$	0	0	2	0	0	-35
x_3	0	-3	1	3	0	-1/2	0
x_5	0	-2	0	3	1	1/2	15
x_1	1	1	0	-2	0	1/2	5
	0	12	0	$\boxed{-22}$	0	6	25
x_4	0	-1	1/3	1	0	-1/6	0
x_5	0	1	-1	0	1	1	15
x_1	1	-1	2/3	0	0	1/6	5
	0	$\boxed{-10}$	22/3	0	0	7/3	25
x_4	0	0	-2/3	1	1	5/6	15
x_2	0	1	-1	0	1	1	15
x_1	1	0	-1/3	0	1	7/6	20
	0	0	-8/3	0	10	37/3	175

the objective function is unbounded below

there is no optimum solution

1 (d) canonical form

$$\min \; 2x_1 + 2x_2 - 5x_3$$

$$\text{s.t.} \quad 3x_1 + 2x_2 - 4x_3 + x_4 = 7$$

$$x_1 - x_2 + 3x_3 + x_5 = 2.$$

$$x_i \geq 0 \text{ for } i = 1, 2, \ldots 5.$$

simplex :

		x_1	x_2	x_3	x_4	x_5		
	x_4	3	2	-4	1	0	7	
	x_5	1	-1	3	0	1	2	
stage 1 :		$\boxed{-4}$	-1	1			-9	
		2	2	-5			0	
	x_4	0	$+5$	-13			1	
	x_1	1	-1	3			2	
		0	$\boxed{-5}$	13			-1	
		0	4	-11			-4	
	x_2	0	1	$-13/5$			$1/5$	
	x_1	1	0	$2/5$			$11/5$	
		0	0	0			0	← min $\omega = 0$. end of stage 1.
stage 2 :		0	0	$\boxed{-3/5}$			$-24/5$	
	x_2	$13/2$	1	0			$29/2$	
	x_3	$5/2$	0	1			$11/2$	
		$3/2$	0	0			$-3/2$	

at $(0, 29/2, 11/2)$ we can obtain ~~maximum~~ the minimum objective function value of $3/2$.

1. (e) canonical form

$$\min \; x_1 - 3x_3$$

$$\text{s.t.} \quad x_1 + 2x_2 - x_3 + x_4 = 6$$

$$x_1 - x_2 + 3x_3 + x_5 = 3$$

$$x_i \geq 0 \text{ for } i = 1, \dots, 5.$$

simplex:

		x_1	x_2	x_3	x_4	x_5		
	x_4	1	2	-1	1	0	6	
stage 1:	x_5	1	-1	3	0	1	3	only x_5 is artificial variable.
		-1	1	$\boxed{-3}$	0	0	-3	
		1	0	-3	0	0	0	
	x_4	4/3	5/3	0	1		7	
	x_3	1/3	-1/3	1	0		1	
		0	0	0	0		0	← min $w = 0$. end of stage 1.
		2	$\boxed{-1}$	0	0		3	
stage 2:	x_2	4/5	1	0	3/5		21/5	
	x_3	3/5	0	1	1/5		12/5	
		14/5	0	0	3/5		36/5.	

at $(0, 21/5, 12/5)$ we can obtain the minimum objective function value $-36/5$

2. LP for the question: let x_1, X_2 & X_3 denote the # hours spent using Process 1, 2 & 3, respectively.

$$\min\ 160x_1 + 400X_2 + 300X_3$$

s.t.
$$3X_1 + 6X_2 + 6X_3 \geq 36$$
$$4X_1 + 6X_2 + 3X_3 \geq 20$$
$$2X_1 + 8X_2 + 4X_3 \geq 30$$
$$X_1, X_2, X_3 \geq 0.$$

canonical form:

$$\min\ 160X_1 + 400X_2 + 300X_3$$

s.t.
$$3X_1 + 6X_2 + 6X_3 - X_4 + X_7 = 36$$
$$4X_1 + 6X_2 + 3X_3 - X_5 + X_8 = 20$$
$$2X_1 + 8X_2 + 4X_3 - X_6 + X_9 = 30$$
$$X_i \geq 0 \text{ for } i = 1, 2, \dots, 9.$$

	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	
X_7	3	6	6	-1	0	0	1	0	0	36
X_8	4	6	3	0	-1	0	0	1	0	20
X_9	2	8	4	0	0	-1	0	0	1	30
	-9	$\boxed{-20}$	-13	1	1	1				-86
	160	400	300	0	0	0				0
X_7	-1	0	3	-1	1	0				16
X_2	2/3	1	1/2	0	-1/6	0				20/6
X_9	-10/3	0	0	0	4/3	-1				10/3
	13/3	0	$\boxed{-3}$	1	-7/3	1				-58/3
	-320/3	0	100	0	200/3	0				-400/3
X_3	-1/3	0	1	-1/3	1/3	0				16/3
X_2	5/6	1	0	1/6	-1/3	0				2/3
X_9	-10/3	0	0	0	4/3	-1				10/3
	10/3	0	0	0	$\boxed{-4/3}$	1				-10/3
	-220/3	0	0	100/3	100/3	0				-5600/3
X_3	1/2	0	1	-1/3	0	1/4				4.5
X_2	0	1	0	1/6	0	-1/4				1.5
X_5	-5/2	0	0	0	1	-3/4				2.5
	0	0	0	0	0	0				0
	10	0	0	100/3	0	25				-1950

end of stage 1 & stage 2.

when using process 1 for 0 hr, Process 2 for 1.5 hours & Process 3 for 4.5 hours. the total cost is minimized at $1950.