Assignment 3

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Answer on Question One

(a) Since the prior distribution is Dirichlet distribution, the posterior distribution is also the Dirichlet distribution: $p(\theta|y) =$ Dirichelet $(y_1+a_1+\cdots+y_n+a_n)$. From the properties of of the Dirichlet distribution, the marginal posterior distribution of $(\theta_1, \theta_2, 1-\theta_1-\theta_2)$ is also Dirichlet:

$p(\theta_1, \theta_2|y) theta_1^{y_1+a_2-1}\theta_2^{y_2+a_2-1}(1-\theta_1-\theta_2)^{y_{rest}+a_{rest}-1}$, where $y_{rest} = y_3+\cdots+y_J, a_{rest} = a_3+\cdots+a_J$.

We can do a change of the varialbes to $(\alpha, \beta) = (\frac{\theta_1}{\theta_1+\theta_2}, \theta_1+\theta_2$. The Jacobian of this tranformation is $\left[\frac{1}{\beta}\right]$, so the transformed density is :

Since the posterior density divides into separate factors for α and β, they are independent, so the posterior distribution is $y\ textBeta(y_1+a_1, y_2, a_2)$.

(b) The Beta(y_1+a_1, y_2+a_2) posterior distribution can also be derived from a Beta$(a_1, a_2$ prior distribution and a binomial observation y_1 with sample size y_1+y_2.

Answer on Question Two

Asume independent uniform prior distributions on he multinomial parameters. Then the posterior distributions are independent multinomial:

and $\alpha_1\frac{\pi_1}{\pi_1+\pi_2}$, $\alpha_2 = \frac{\pi_1^*}{\pi_1^*+\pi_2^*}$. From the properties of the Dirichlet distribution:

The histrogram of 2000 draws from the posterior density of $2-\alpha_1$ is attached, Based on this histogram, the posterior probability that there was a shift toward Bush is 19%

```
alpha.1 <- rbeta (2000, 295, 308)
alpha.2 <- rbeta (2000, 289, 333)
dif <- alpha.2 - alpha.1
hist (dif, xlab="alpha_2 - alpha_1", yaxt="n",
  breaks=seq(-.12,.08,.01), cex=2)
print (mean(dif>0))
```

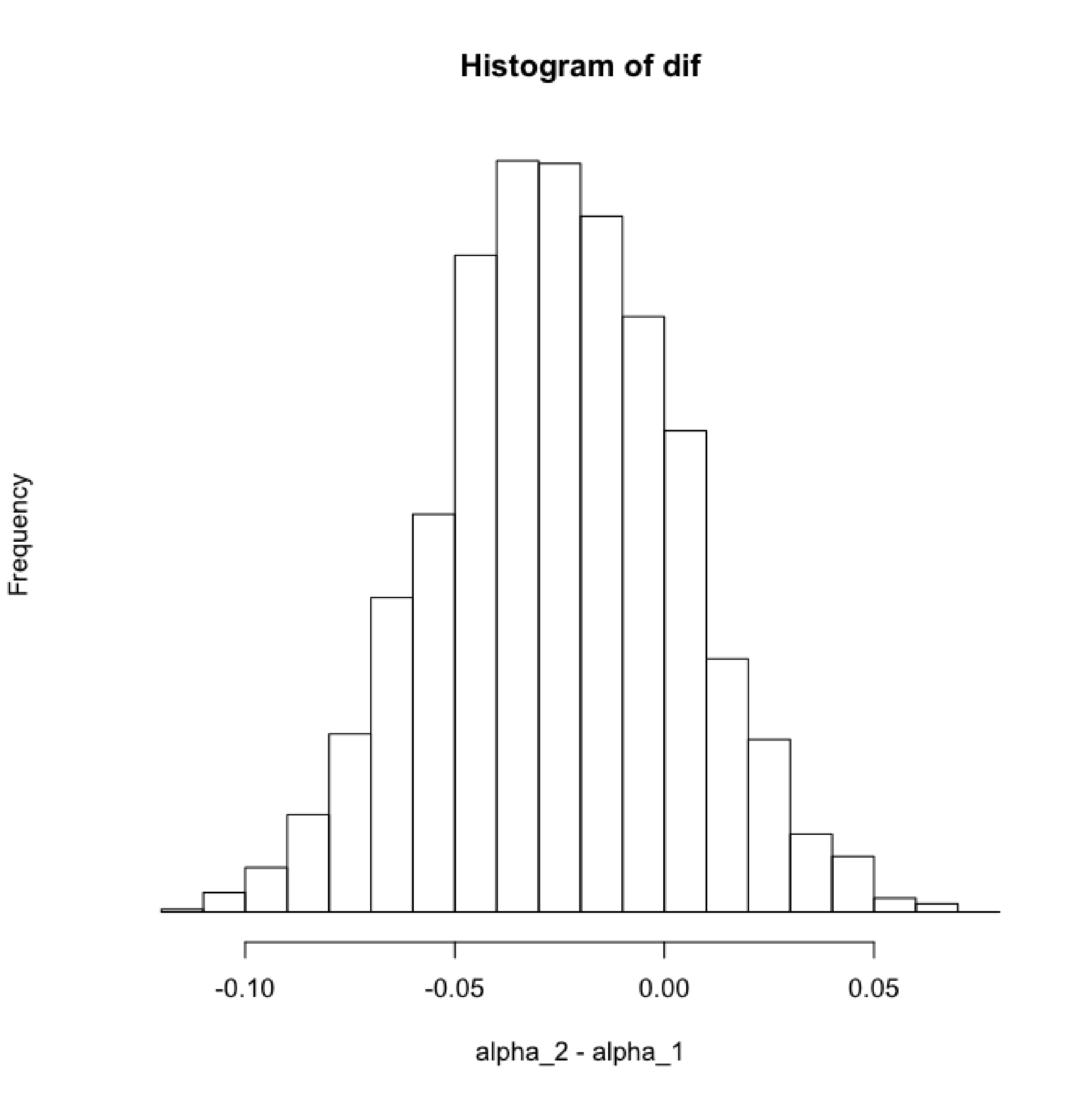

Figure 1:

Answer of Question Three

Data distribution is $p(y|\mu_c, \mu_t, \sigma_c, \sigma_t) = \prod_{i=1}^{3} 2N(y_{ci}\mu_c, \sigma_c^2) \prod_{i=1}^{3} 6N(y_{yi}|\mu_t, \sigma_t^2)$. Posterior distribution is $p(y|\mu_c, \mu_t, log\sigma_c, log\sigma_t) = p(y|\mu_c, \mu_t, log\sigma_c, log\sigma_t)p(y|\mu_c, \mu_t, log\sigma_c, log\sigma_t))$

THe posteriror density fators, so (μ_c, σ_c) are independent of (μ_t, σ_t) in the posterior distribution. So, under this model, we can analyze the two experiments separately. We notice that the posterior distributions for μ_c and μ_t are:

We can use R to print the plot:

```
mu.c <- 1.013 + (0.24/sqrt(32))*rt(1000,31)
mu.t <- 1.173 + (0.20/sqrt(36))*rt(1000,35)
dif <- mu.t - mu.c
hist (dif, xlab="mu_t - mu_c", yaxt="n", breaks=seq(-.1,.4,.02), cex=2)
print (sort(dif)[c(25,976)])
```

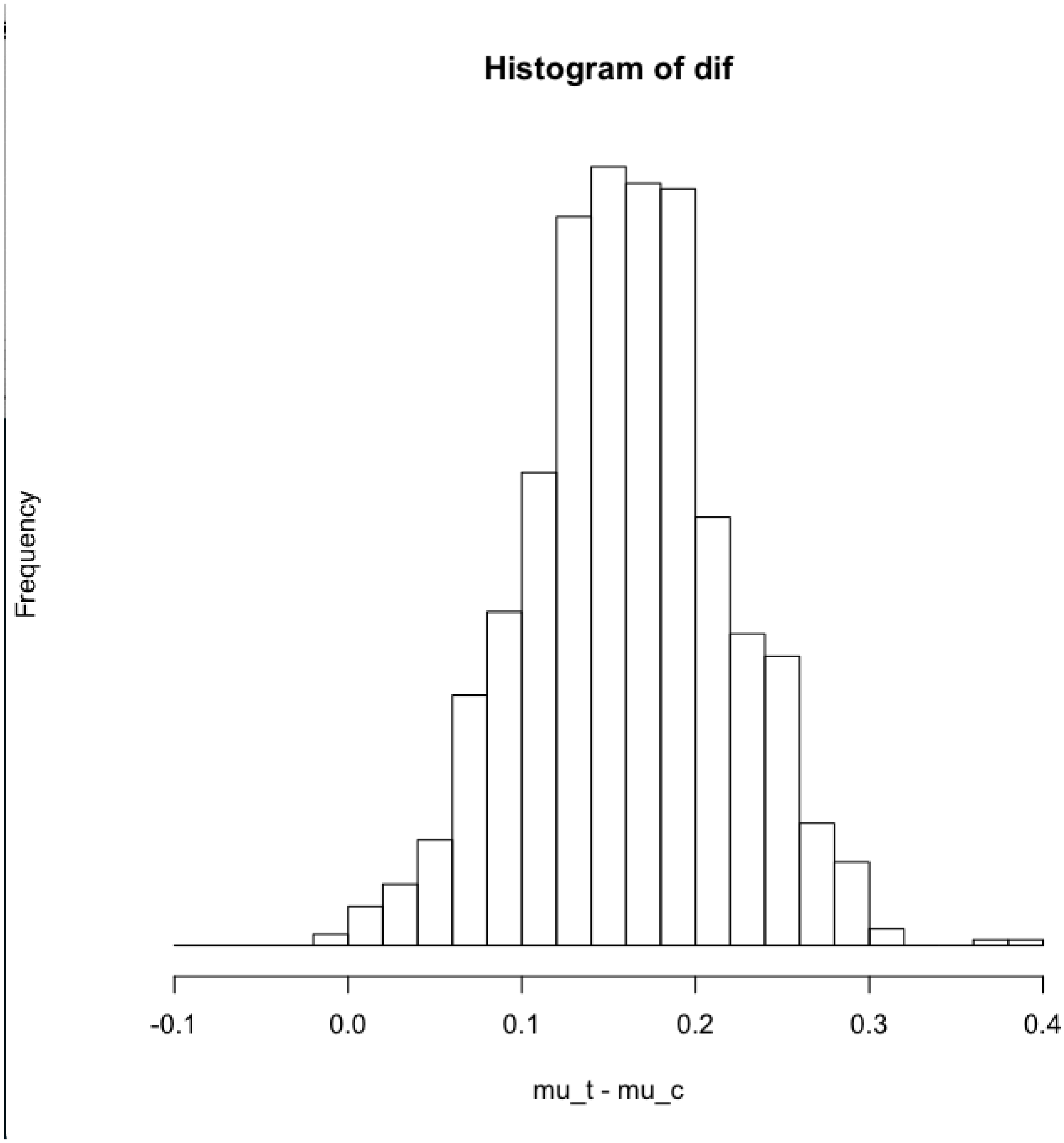

Figure 2: