

Regression Assignment Two

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2018 年 10 月 8 日

Question One Answer: For $\sigma = 2$, we can notice that:

$$\begin{aligned} p(y) &= Pr(\theta = 1)p(y|\theta = 1) + Pr(\theta = 2)p(y|\theta = 2) \\ &= 0.5N(y|1, 2^2) + 0.5N(y|2, 2^2) \end{aligned}$$

The graph can be drawn by R:

```
y <- seq(-7,10,.02)
density <- 0.5*dnorm(y,1,2) + 0.5*dnorm(y,2,2)
plot (y, density, ylim=c(0,1.1*max(density)))
```

The plot is :

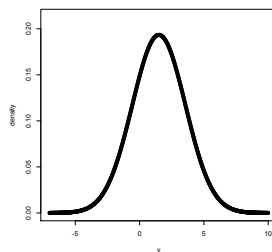


图 : Marginal probability density of y

(b)

$$\begin{aligned}
 Pr(\theta = 1|y = 1) &= \frac{p(\theta = 1, y = 1)}{p(\theta = 1, y = 1) + p(\theta = 2, y = 1)} \\
 &= \frac{Pr(\theta = 1)p(y = 1|\theta = 1)}{Pr(\theta = 1)p(y = 1|\theta = 1) + Pr(\theta = 2)p(y = 1|\theta = 2)} \\
 &= \frac{0.5N(1|1, 2^2)}{0.5N(1|1, 2^2) + 0.5N(1|2, 2^2)} \approx 0.62
 \end{aligned}$$

(c) If $\sigma \rightarrow \infty$, the posterior density for θ will approach the prior density and the probability will be $\frac{1}{2}$; if $\sigma \rightarrow 0$ the $0.5N(1|2, 2^2) \approx 0$ so the $Pr(\theta = 1|y = 1) \rightarrow 1$.

Question Two Answer: From the question, we can notice that: $Pr(fraternal\ twins) = \frac{1}{125}$ and $Pr(identical\ twins) = \frac{1}{300}$. Since the fraternal twins are in different perm cell. the $Pr(both\ boys) = \frac{1}{4}$

The conditional probability that Elvis was an identical twins is :

$$\begin{aligned}
 Pr(identical\ twins|twins\ brother) &= \frac{Pr(identical\ twins, twins\ brother)}{Pr(twins\ brother)} \\
 &= \frac{\frac{1}{2} \times \frac{1}{300}}{\frac{1}{2} \times \frac{1}{300} + \frac{1}{4} \times \frac{1}{125}} \\
 &= \frac{5}{11} \approx 45.5\%
 \end{aligned}$$

Question Three Answer:

Question Four Answer: (a) The prior predictive distribution for y :

$$\begin{aligned}
 Pr(y = k) &= \int_0^1 Pr(y = k|\theta) d\theta \\
 &= \int_0^1 \binom{n}{k} \theta^k (1 - \theta)^{n-k} d\theta \\
 &= \binom{n}{k} \frac{\Gamma(k+1)\Gamma(n-k+1)}{\Gamma(n+2)} = \frac{1}{n+1}
 \end{aligned}$$

(b) We can notice that the posterior mean is $\frac{\alpha+y}{\alpha+\beta+n}$. We assume that $\lambda \in (0, 1)$ and $\frac{\alpha+y}{\alpha+\beta+n} = \lambda \frac{\alpha}{\alpha+\beta} + (1-\lambda) \frac{y}{n}$

$$\begin{aligned}
 \frac{\alpha+y}{\alpha+\beta+n} &= \lambda \frac{\alpha}{\alpha+\beta} + (1-\lambda) \frac{y}{n} \\
 \frac{\alpha+y}{\alpha+\beta+n} - \frac{y}{n} &= \lambda \left(\frac{\alpha}{\alpha+\beta} - \frac{y}{n} \right) \\
 \lambda &= \frac{\alpha+\beta}{\alpha+\beta+n}
 \end{aligned}$$

(c) When we consider the uniform distribution, we can notice that $\alpha = \beta = 1$, so the prior variance is $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{1}{12}$. In this situation, we can calculate the posterior variance:

$$\begin{aligned}
 \text{Posterior Variance} &= \frac{(1+y)(1+n-y)}{(2+n)^2(3+n)} \\
 &= \left(\frac{1+y}{2+n} \right) \left(\frac{1+n-y}{2+n} \right) \left(\frac{1}{3+n} \right)
 \end{aligned}$$

The sum of $\frac{1+y}{2+n}$ and $\frac{1+n-y}{2+n}$ is not greater than $\frac{1}{4}$. And $n \geq 1$, so $\frac{1}{3+n}$ is less than $\frac{1}{4}$. So Posterior variance $\neq \frac{1}{16}$

(d) If $n = 2, y = 1, \alpha = 1$ and $\beta = 1$ The prior variance = $\frac{1}{12}$ and the posterior variance is 0.025