OR4030 OPTIMIZATION ASSIGNMENT - Ch1

Terry Liu l630005038

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For each of the following problems. formulate a suitable model, but you do not need to solve them

Question One: Suppose we have collected the following data:

t_i	1	2	4	5	8
y_i	3	4	6	11	20

for $i = 1, 2, 3, 4, 5$; and we expect varialbe y and t to roughly meet the relationship $y = x_1 e^{x_2 t}$, $i = 1, 2, \ldots, 5$. Formulate the problem using the **method of least squares** to determine the values of x_1 and x_2

Answer: We can use R language to help us estimate the equation:
first, we can do some preprocessing:

$$ln(y) = ln(x_1 e^{x_2 t}) \Rightarrow y = lnx_1 + x_2 t$$

In this form, we can calculate more easily.

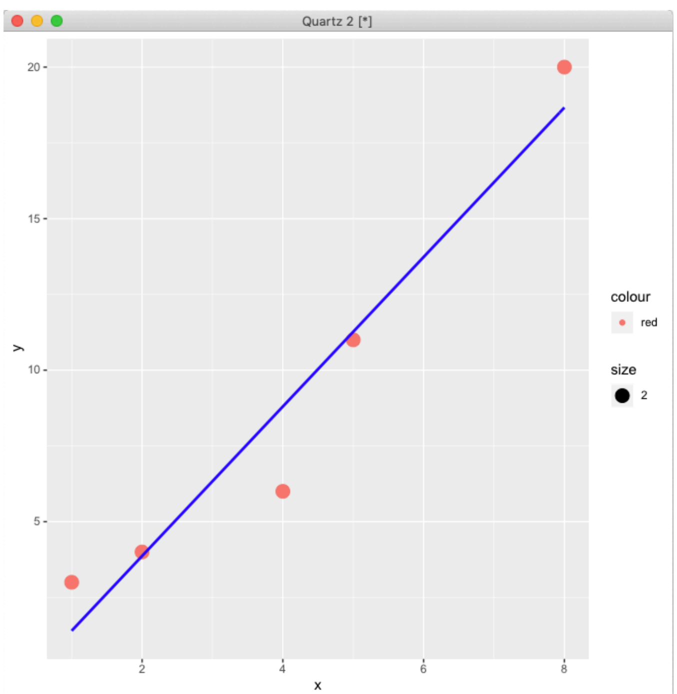
```
library(ggplot2)
Leastsquares <- function(x,y) {
  #x,y
  rm(list=ls())
  lenx <- length(x)
  leny <- length(y)
  s <- 0
  t<-0
  if (lenx != leny)
    stop("length(x) != length(y)")
```

```
  avgx <- mean(x)
  avgy <- mean(y)

  for(i in 1:lenx) {
    s <- s + (x[i]-avgx)*(y[i]-avgy)
    t <- t + (x[i] - avgx)^2
  }

  s <- s/t
  t <- avgy - s*avgx
  f<-function(a) {s*a+t}

  base<-qplot(x,y,colour="red",size=2)
  base + stat_function(fun=f,colour="blue",size=1)
  }
x <- c(1,2,4,5,8)
y <- c(3,4,6,11,20)
Leastsquares(x,y)
```



Question Two A rectangular heat storage unit of length L, width W and height H will be used to store heat energy temporarily. Heat of the unit shall be lost for two reasons, by convection and by radiation. The rates of heat loss hc due to convection and heat loss hr due to radiation are given respectively by:

$$h_c = k_c A(T - T_a)$$
$$h_r = k_r A(T^4 - T_a^4)$$

when k_c and k_r are constants,T is the temperature of the heat storage unit, A is the surface area of the unit, and assume that T and T_a are constants. The heat energy stored in the unit is given by:

$$Q = kV(T - T_a)$$

where k is a constant and V is the volume of the storage unit. The storage unit should have the ability to store at least Q' unit of heat energy initially. Furthermore, suppose that space availability restricts the dimensions of the storage unit to:

$$0 \le L \le L',\ 0 \le W \le W',\ 0 \le H \le H'$$

Formulate the problem of finding the dimensions L,W and H to minimize the heat loss in per unit of time.

Answer: In this question, we have to minimize A with those restricts:

$$\begin{aligned}
&minimize: && A \\
&s.t. && kV(T - T_a) \ge Q' \\
& && V = LWH \\
& && A = \pi DH + \frac{1}{2}\pi D^2 \\
& && 0 \le L \le L',\ 0 \le W \le W',\ 0 \le H \le H'
\end{aligned}$$

Question Three Formulate the model in the last exercise if the storage unit is a cylinder of diameter D and height H.

Answer: Since we have to minimize h_c+h_r and K, T and T_a are constants, so we can get this equation:

$$\begin{aligned}
&minimize: && A \\
&s.t. && kV(T-T_a) \geq Q' \\
& && V = LWH \\
& && A = \pi DH + \frac{1}{2}\pi D^2 \\
& && 0 \leq L \leq L',\ 0 \leq W \leq W',\ 0 \leq H \leq H'
\end{aligned}$$

Question Four An office room of length 60 meters and width 35 meters is to be illuminated by 16 light bulbs of wattage $W_i, (i \in (1, \ldots, 16))$. THe bulbs are to be located 2 meters above the working surface. Let (x_i, y_i) denote the x and y coordinates of the ith bulb. To ensure lighting, illmination is checked at the working surface level at grid points of the form (α, β), where:

$$\alpha = 10p,\ p = 0, 1, 2, \ldots, 6$$
$$\beta = 5q,\ q = 0, 1, \ldots, 7$$

The illumination at (α, β) resulting from a bulb of wattage W_i located at (x_i, y_i) is given by:

$$E_i(\alpha, \beta) = k\frac{W_i \parallel (\alpha, \beta) - (x_i, y_i) \parallel}{\parallel (\alpha, \beta, 2) - (x_i, y_i, 0) \parallel^3}$$

when k is a constant reflecting the efficiency of the bulb. The total illumination at (α, β) is equal to $\sum_{i=1}^{6} E_i(\alpha, \beta)$. At each of the points checked, an illumination of between 2.6 and 3.2 units is required. All bulbs have same wattage, say w, and assume that the wattage w may be chosen as any value between $40W, 200W$. Formulate a model to decide a minimum wattage w for the 16 bulbs so that the lighting requirement is met.

Answer: According to the question, we need to minimize the w:

$$\begin{aligned}
minimize: & & w \\
s.t. & & 2.6 \leq k\frac{W_i \parallel (\alpha,\beta)-(x_i,y_i)\parallel}{\parallel(\alpha,\beta,2)-(x_i,y_i,0)\parallel^3} \leq 3.2 \\
& & 40 \leq w \leq 200, where i \in (1,2,\ldots,6)
\end{aligned}$$

Question Five For the above Problem 4, now assume that the wattage w for the 16 bulbs has only four possible choices: $40W, 60W, 100W$ and $200W$. How would you revise the model?

Answer: According to the question, we need to minimize the w:

$$\begin{aligned}
minimize: & & w \\
s.t. & & 2.6 \leq k\frac{W_i \parallel (\alpha,\beta)-(x_i,y_i)\parallel}{\parallel(\alpha,\beta,2)-(x_i,y_i,0)\parallel^3} \leq 3.2 \\
& & w = 40k_1 + 60k_2 + 100k_3 + 200k_4 \\
& & k_1 + k_2 + k_3 + k_4 = 1,\ where i \in (1,2,\ldots,16) \\
& & k_1, k_2, k_3 \text{ and } k_4 \text{ are } 0-1 \text{ variables}
\end{aligned}$$

Question Six Return to Problem 4 Assume it is decided that all bulbs used have the wattage $100W$, but we want to use the least number of bulbs to meet the lighting request. It is known that 16 such bulbs are enough, but we wish to use as few bulbs as possible. Give a model to determine how many bulbs are required and where should they be located.

Answer: we can notice that:

$$\begin{aligned}
x_i &= ap',\ p = 0,1,\ldots\frac{60}{a} \\
y_i &= bq',\ q = 0,1,\ldots\frac{35}{b}, \\
where\ i &= 1,2,\ldots,\frac{(60+a)(35+b)}{ab}
\end{aligned}$$

a is the interval of the bulbs along the length of the office room while **b** is the width of the room. So, we can get the optimal equation:

$$
\begin{aligned}
&minimize: && \frac{(60+a)(35+b)}{ab} \\
&s.t. && 2.6 \leq k\frac{W_i \parallel (\alpha,\beta)-(x_i,y_i) \parallel}{\parallel (\alpha,\beta,2)-(x_i,y_i,0) \parallel^3} \leq 3.2 \\
& && w = 100
\end{aligned}
$$