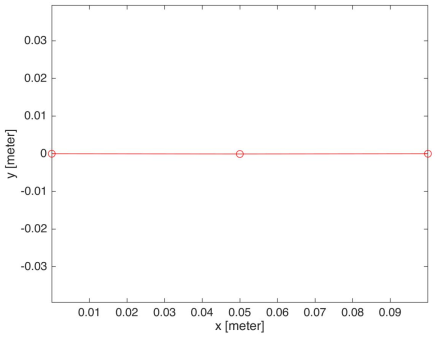
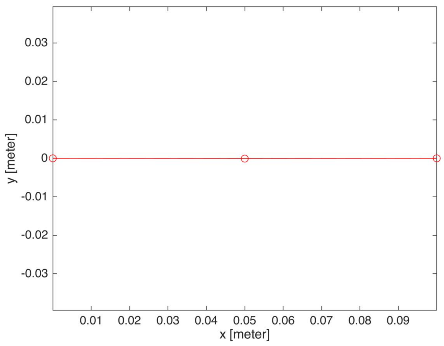
Assignment 1:

1.1

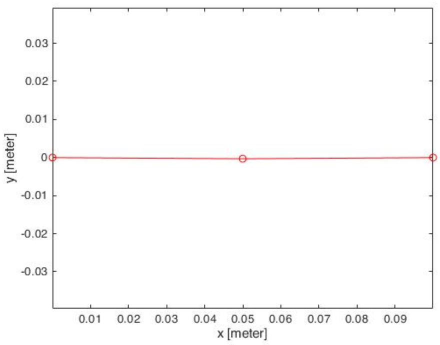
Shape of structure at t=0s:



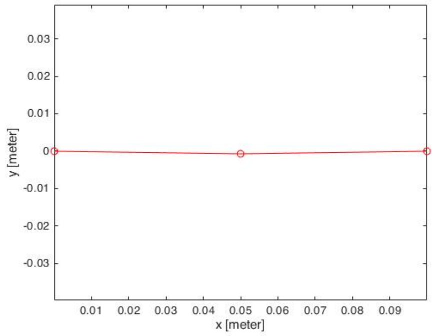
Shape of structure at t=0.01s:



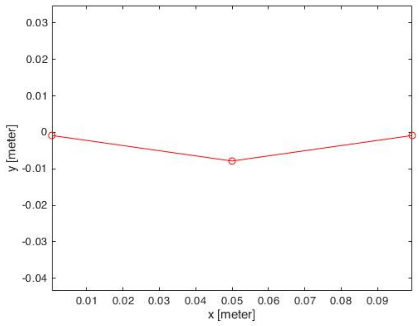
Shape of structure at t=0.05s:



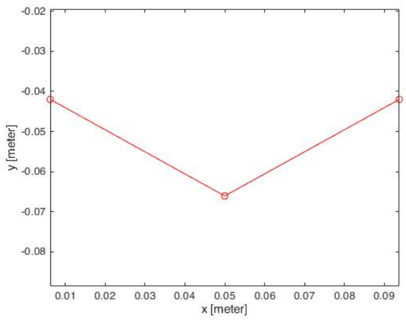
Shape of structure at t=0.1s:



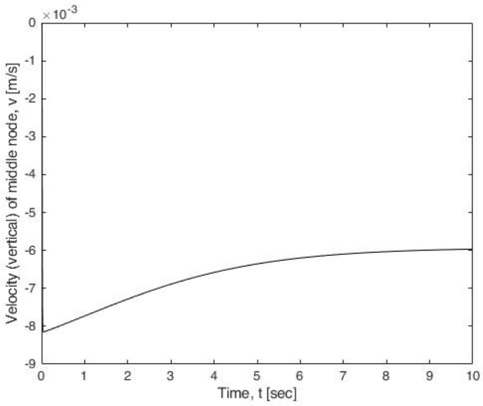
Shape of structure at t=1s:



Shape of structure at t=10s:

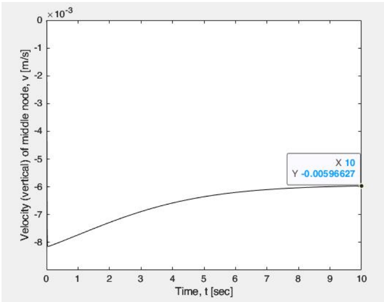


Velocity of R2(center) across time:



1.2

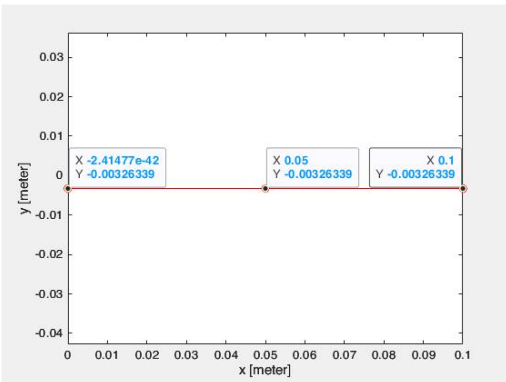
The velocity of the system center is -0.005966m/s in the y-direction after 10 seconds. Since the growth trend is asymptotic, the terminal velocity should be close to -0.00596m/s along y-direction.



1.3

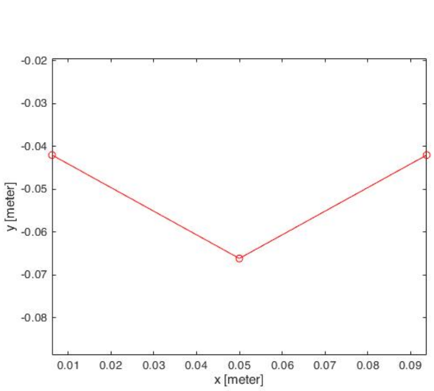
If the radius of all balls is the same, then the forces acted on the balls will be the same, meaning there will be no change of relative position and therefore no turning angle inside the system given zero initial condition.

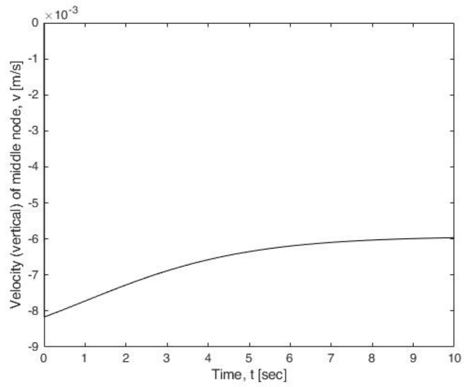
The simulation result agrees with my argument. All balls have the same y position after 10s.



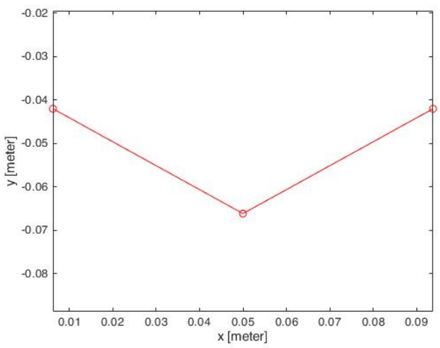
1.4

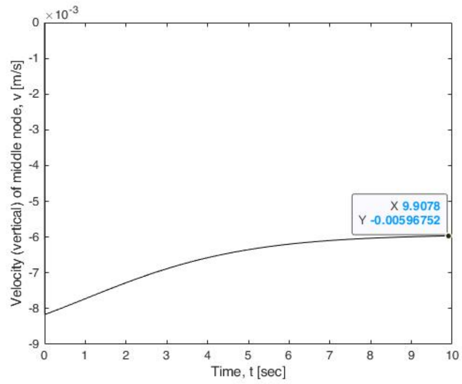
At the suggested 1e-5 step size for explicit method, the simulation result is about the same as the implicit method.



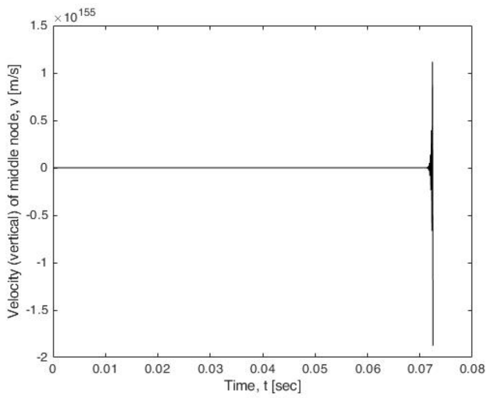


If we increase the step size to 5e-5, the run time of the explicit simulation will decrease from 36 seconds to 6.4 seconds with little impact to the accuracy of the simulation.





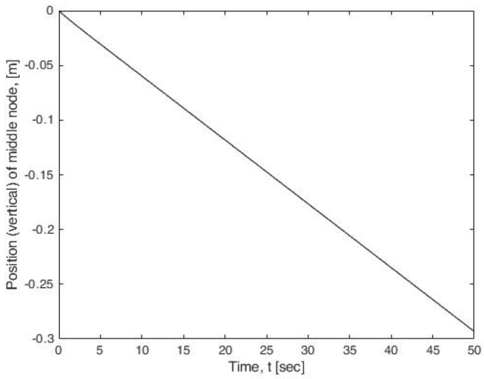
If we further increase the step size to 1e-4, the run time will be cut to 3.34 seconds, but the solution will not converge at this step size.(solution explodes)

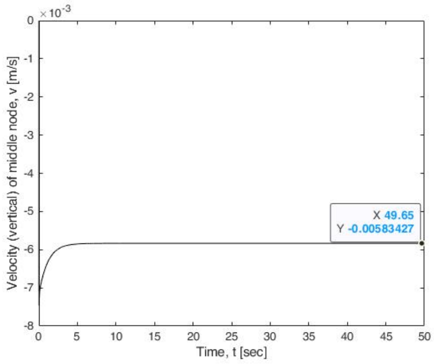


Compares to implicit method(which has a run time of 0.457 second without plotting at each step), the explicit method is much slower due to the high demand of step size. While the implicit method works well with 1e-2 step size, the explicit method explodes at 1e-4 step size. Even though the implicit method requires an additional loop for Newton Raphson method, the difference in step size is large enough for us to ignore the extra time brought by the loop.

Still, the explicit method is more intuitive and can be implemented with less lines of code. If the run time difference is no matter to you, then explicit method can still be used with proper selection of step size.

2.1

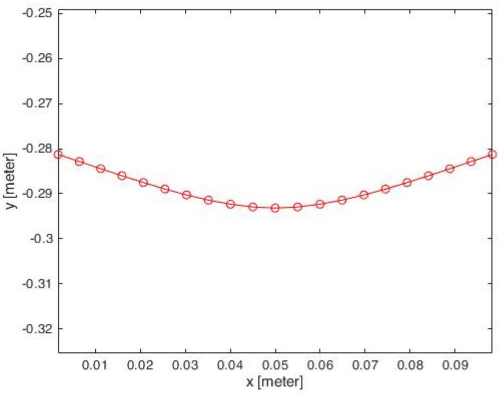




Y position and velocity of the middle node from t=0 to 50s respectively. Terminal velocity of the system is about -0.00583m/s in y-direction.

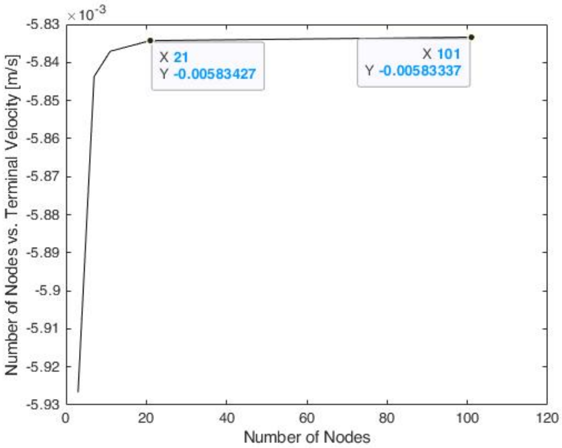
2.2

Final shape of the 21 ball system at t=50s:

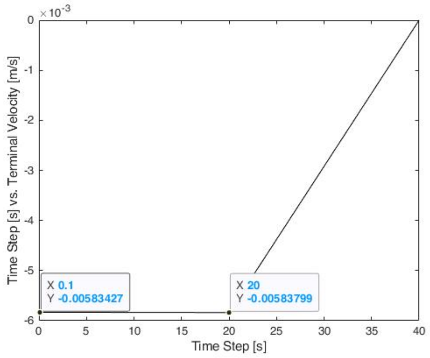


2.3

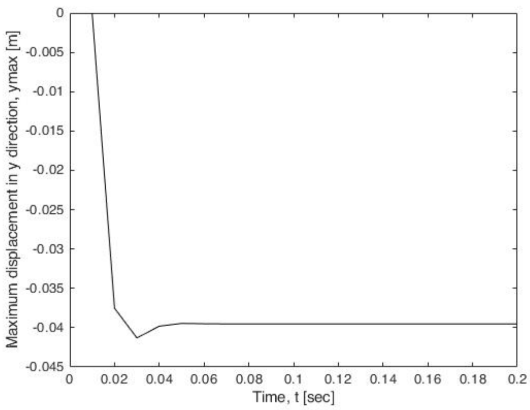
Changing the number of nodes from 3 to 101 nodes, the terminal velocity stabilized after N=21.

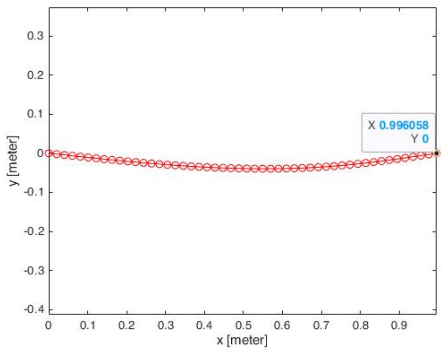


In terms of time step, the implicit method demonstrated amazing stability. The terminal velocity stabilized after dt <20s for implicit method.



3.1





Above are the max deflection in y direction, and the final shape of the beam. The max deflection of the beam fluctuate for a little bit but quickly reached the steady state value of -0.039m.

Using the Euler equation for beam:

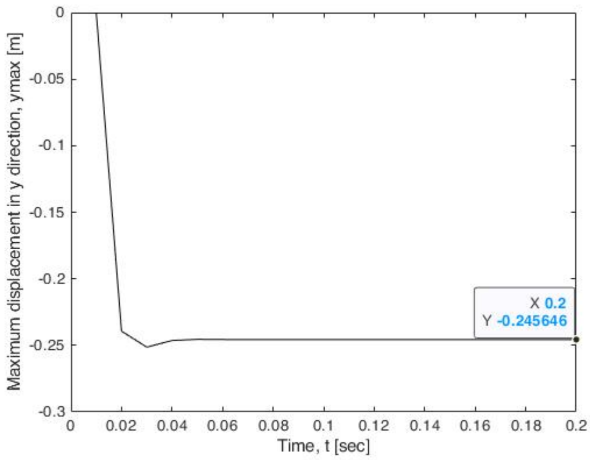
$$y_{\max} = \frac{Pc\left(L^2 - c^2\right)^{1.5}}{9\sqrt{3}EI\,l} \qquad \text{where} \qquad c = \min(d, l-d)$$

Plug in P=2000N, c = -0.75m, l = 1m, E=7e10, I = (pi/4)*(R^4-r^4), we get y_max = -0.0364m. Compares to our result with N=50, there is ~7% difference between simulation and Euler prediction.

3.2

From what I have learned during the undergrad, Euler-Bernoulli beam theory is only applicable when deformation is small. So, when deformation/external load is large, simulation is the way to go for beam deformation prediction.

When P = 20000N, the Euler prediction yields a y_max of -0.3639m assuming other conditions unchanged. Meanwhile, our simulation yields a y_max of -0.2456m.



Compares to the previous ~7%, the value difference between Euler & Simulation increased 4 times(to ~33%) when we change P from 2000N to 20000N.

In conclusion, even though simulation is harder to set up, it is more versatile than Euler beam theory alone.