the line joining you—"here and now"— to any event that you could possibly experience, is time-like. On the other hand, if you insist that the distinction between past and future is absolute, you can replace the exterior-hyperbolic space by its orientable covering manifold (which is representable on the whole one-sheeted hyper-sphere in Minkowskian space of $1+4$ dimensions). But do you not find it disturbing to envisage an exact replica of yourself at the "antipodes," living backwards?

**9. Conclusion.** Apart from all these difficulties, we have ignored the embarrassing subject of the essential emptiness of de Sitter's world, which has led cosmologists to propose a modification in the direction of an earlier hypothesis: Einstein's "cylindrical" world. ([4], p. 160.) But the fact remains that de Sitter's is the theoretical world of greatest interest to pure geometers, as it alone has an interesting group. ([3], p. 657.) Since the points of exterior-hyperbolic space are the absolute poles of the hyperplanes of ordinary hyperbolic space (of four dimensions), the above remarks show that this group is the hyperbolic metric group: its elements are the collineations that preserve an oval quadric in real projective four-space.

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## FOLDING THE CONICS

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The idea of applying the process of paper folding to the construction of the conics originated with Row.* The methods he gives, however, are quite involved in a structural sense. Improvements (at least for the central conics) are offered by Lotka† but the usefulness and charm of his methods are somewhat obscured by an accompanying analytical justification.

The purpose of this note is to offer the best of the methods of Lotka and Row and to present simple proofs to establish the processes involved. It is hoped

* T. Sundara Row, Geometric Exercises in Paper Folding (trans. by Beman and Smith) Chicago, 1901.

† A. J. Lotka, School Science and Mathematics, VII, 1907, 595–597; Scientific American Supplement, 1912, 112.

that this formation of the conics by paper folding will somehow find its way into the clasroom where it will undoubtedly be received with enthusiasm.

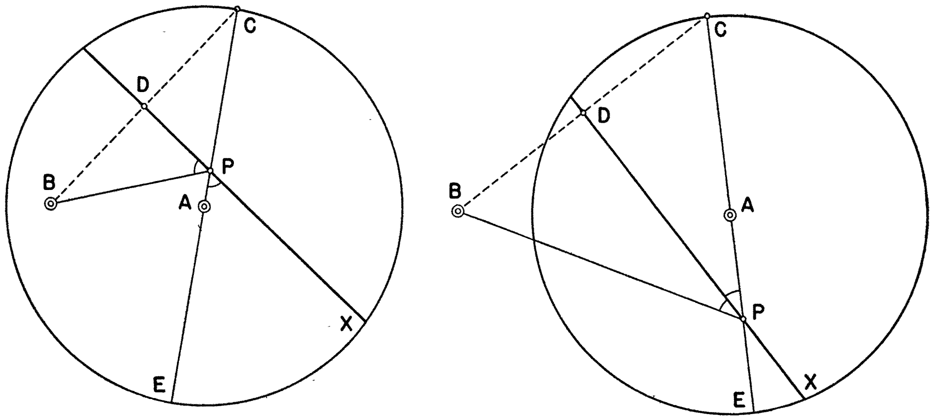


Fig. 1

A fixed point $B$ is selected within a circle of radius $r$ and center $A$. The point $B$ is folded over upon the circle, as at $C$, forming the crease $DX$ (the perpendicular bisector of $BC$) which meets the diameter $EC$ in the point $P$. As $B$ takes positions along the circle, the path of $P$ is the *ellipse* having foci at $A$ and $B$, major axis $r$, and the creases as tangents. For, since $P$ lies on the perpendicular bisector of $BC$,

$$AP + BP = AP + PC = r$$

and

$$\angle BPD = \angle DPC = \angle APX.$$

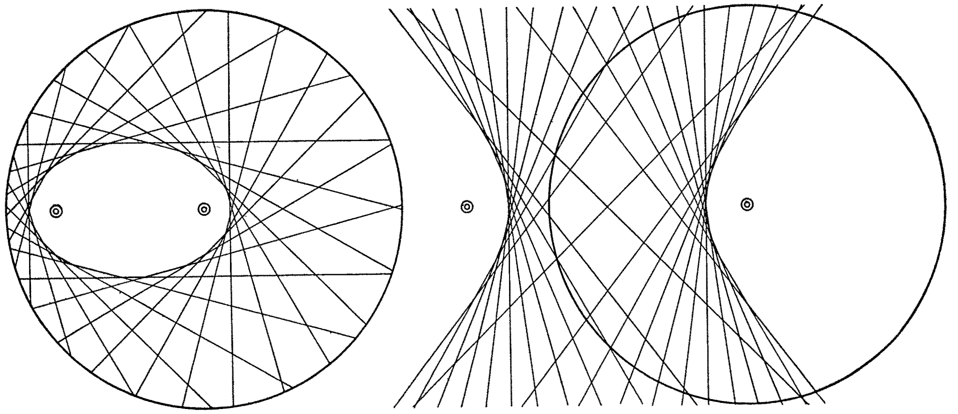

Fig. 2

(If $B$ is taken at $A$, the locus of $P$ is the circle with center at $A$ and radius $r/2$.)

If $B$ is selected outside of the circle, the locus of $P$ (the intersection of crease and corresponding diameter) is the *hyperbola* having $A$ and $B$ as foci, real axis $r$, and the creases as tangents. For, since $P$ lies on the perpendicular bisector of $BC$,

$$BP - AP = CP - AP = r$$

and

$$\angle\ BPD = \angle\ DPA.$$

The rectangular hyperbola is formed if $AB = r\sqrt{2}$. If $B$ is taken on the circle, the locus of $P$ is the point $A$.

The *parabola* (see Figure 3) presents a special case in which a line $L$ replaces the circle of the central conics. The point $B$ is folded over upon $L$, as at $C$, producing the crease $DX$. The perpendicular to $L$ at $C$ meets the crease in $P$. Points $P$ form the parabola having $B$ as focus, $L$ as directrix, and the creases as tangents. For, since $P$ lies on the perpendicular bisector of $BC$,

$$BP = PC$$

and

$$\angle\ BPD = \angle\ DPC.$$

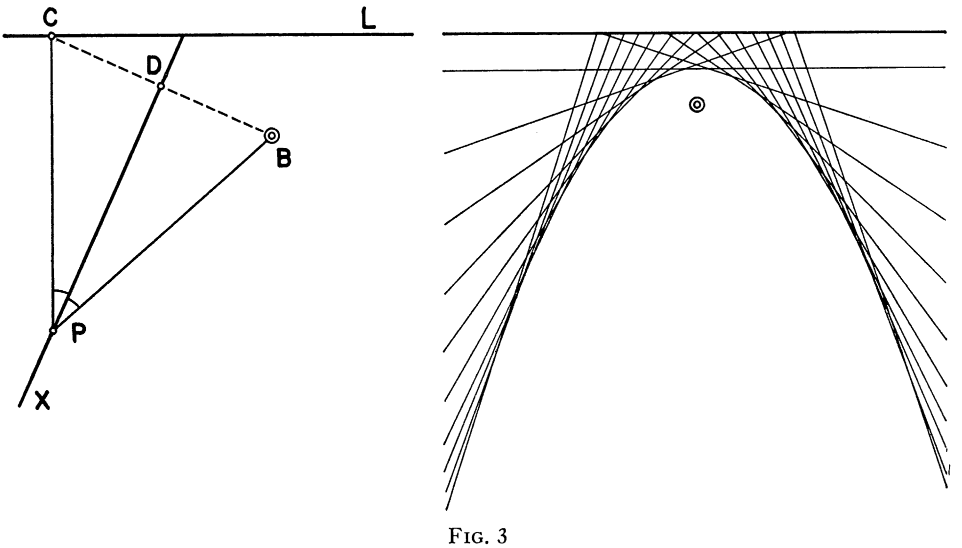


FIG. 3

This fascinating art of paper folding, which seems unfortunately relegated to the limbo, may be brought to full expression through the medium of ordinary wax paper found in every kitchen cabinet.