Розрахункова робота

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Варіант №5

Дано:	
$\rho(r) = \rho_0 \cos\dfrac{\pi x}{2d}$	$\oint \vec{E}\vec{n}\mathbf{d}s = \dfrac{Q}{\varepsilon_0}$
$\sigma = 0,5$ нКл/м^2	
$\rho_0 = 50$ нКл/м^3	
$d = 5$см$= 0.05$м	
$E_x(x),\ \varphi(x)-?$	

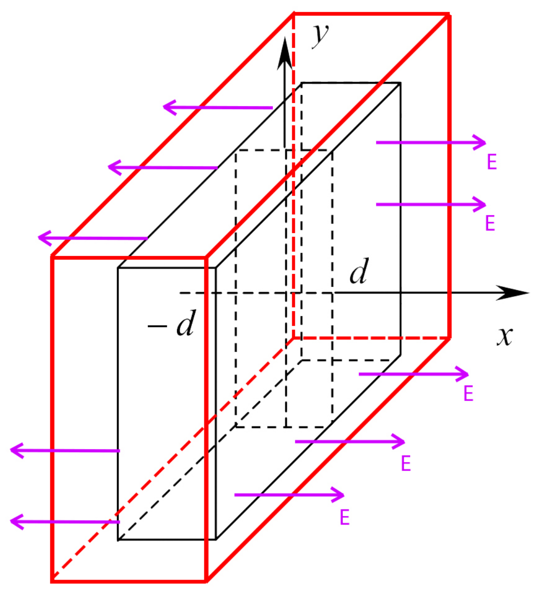

Рис. 1: Зовнішня математична поверхня.

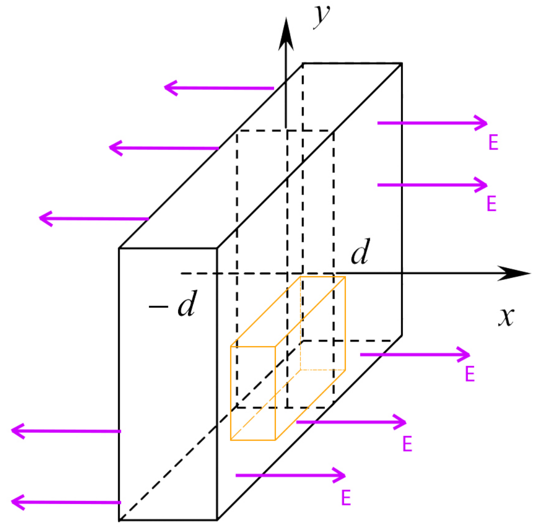

Рис. 2: Внутрішня математична поверхня.

Розв'язання

$$\boxed{\oint \vec{E}\mathbf{d}s = \vec{E}\oint \mathbf{d}s = \vec{E}\cdot S = \frac{Q}{\varepsilon_0} \Rightarrow \vec{E} = \frac{Q}{\varepsilon_0 \cdot S};\quad Q_{ex} = \int\limits_{-d}^{d} \rho \mathbf{d}v + 2\sigma S;\quad \mathbf{d}v = S\mathbf{d}x;\quad \varphi = -\int \vec{E}\mathbf{d}r + C}$$

Заряд Q

$$\text{I)}\ Q_{ex} = \int\limits_{-d}^{d} \rho_0 S \cos\frac{\pi x}{2d}\mathbf{d}x + 2\sigma S = \frac{2d\rho_0 S}{\pi}\cdot\left(\sin\frac{\pi x}{2d}\right)\Bigg|_{-d}^{d} + 2\sigma S = \frac{4d\rho_0 S}{\pi} + 2\sigma S$$

$$\text{II)}\ Q_{in} = \int\limits_{-x}^{x} \rho_0 S \cos\frac{\pi x}{2d}\mathbf{d}x = \frac{2d\rho_0 S}{\pi}\cdot\left(\sin\frac{\pi x}{2d}\right)\Bigg|_{-x}^{x} = \frac{4d\rho_0 S\sin(\frac{\pi x}{2d})}{\pi}$$

Напруженість електричного поля E

$$\text{I)}\ E_{ex} = \frac{4d\rho_0 S + 2\pi\sigma S}{\pi\varepsilon_0 S} = \frac{4d\rho_0 + 2\pi\sigma}{\pi\varepsilon_0}\cdot\frac{x}{|x|}$$

$$\text{II)}\ E_{in} = \frac{4d\rho_0 \cancel{S}\sin(\frac{\pi x}{2d})}{\pi\varepsilon_0\cancel{S}} = \frac{4d\rho_0\sin(\frac{\pi x}{2d})}{\pi\varepsilon_0}$$

Потенціал поля φ

$$\text{I)}\ \varphi_{ex} = -\int\left(\frac{4d\rho_0 + 2\pi\sigma}{\pi\varepsilon_0}\right)\mathbf{d}x = -\frac{x(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} + C$$

$$\text{II)}\ \varphi_{in} = -\frac{4d\rho_0}{\pi\varepsilon_0}\cdot\int \sin\frac{\pi x}{2d}\mathbf{d}x = \frac{8d^2\rho_0\cos(\frac{\pi x}{2d})}{\pi^2\varepsilon_0} + C$$

a) $\varphi_{in}(0) = 0$

$$\frac{8d^2\rho_0\cdot 1}{\pi^2\varepsilon_0} + C = 0 \Rightarrow C = -\frac{8d^2\rho_0}{\pi^2\varepsilon_0}$$

b) $\varphi_{in}(d) = \varphi_{ex}(d)$

$$\frac{8d^2\rho_0\cos(\frac{\pi\cancel{d}}{2\cancel{d}})}{\pi^2\varepsilon_0} - \frac{8d^2\rho_0}{\pi^2\varepsilon_0} = -\frac{d(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} + C \Rightarrow C = \frac{d(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} - \frac{8d^2\rho_0}{\pi^2\varepsilon_0}$$

$$C = \frac{4d^2\pi\rho_0 + 2\pi^2 d\sigma - 8d^2\rho_0}{\pi^2\varepsilon_0} = \frac{2d(2d\pi\rho_0 + \pi^2\sigma - 4d\rho_0)}{\pi^2\varepsilon_0} \Rightarrow$$

III) $\varphi_{ex} = -\frac{x(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} + \frac{2d(2d\pi\rho_0 + \pi^2\sigma - 4d\rho_0)}{\pi^2\varepsilon_0}$

IV) $\varphi_{in} = \frac{8d^2\rho_0 \cos(\frac{\pi x}{2d})}{\pi^2\varepsilon_0} - \frac{8d^2\rho_0}{\pi^2\varepsilon_0}$