

# РОЗРАХУНКОВА РОБОТА

ФІ-12 ЗАВАЛІЙ ОЛЕКСАНДР

ВАРІАНТ №5

Дано:

$$\rho(r) = \rho_0 \cos \frac{\pi x}{2d}$$

$$\sigma = 0,5 \text{ нКл/м}^2$$

$$\rho_0 = 50 \text{ нКл/м}^3$$

$$d = 5 \text{ см} = 0.05 \text{ м}$$

$$E_x(x), \varphi(x) - ?$$

$$\oint \vec{E} \vec{n} dS = \frac{Q}{\varepsilon_0}$$

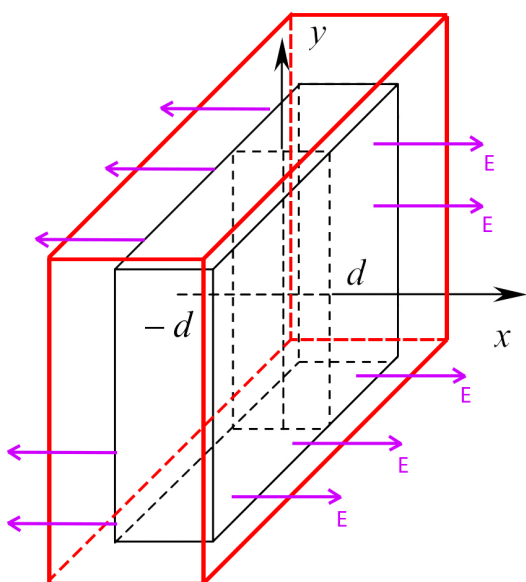


Рис. 1: Зовнішня математична поверхня.

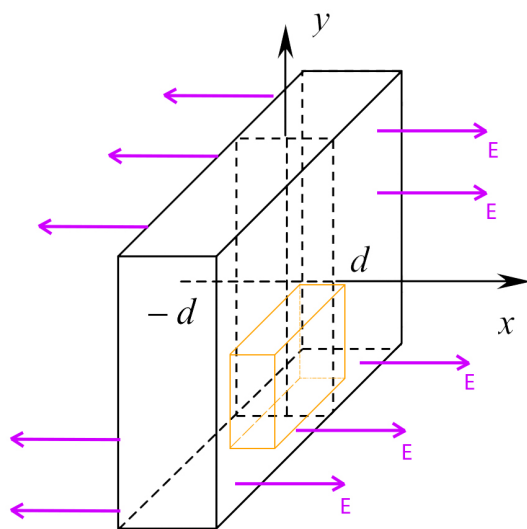


Рис. 2: Внутрішня математична поверхня.

## Розв'язання

$$\oint \vec{E} d\mathbf{s} = \vec{E} \oint d\mathbf{s} = \vec{E} \cdot \mathbf{S} = \frac{Q}{\varepsilon_0} \Rightarrow \vec{E} = \frac{Q}{\varepsilon_0 \cdot S}; \quad Q_{ex} = \int_{-d}^d \rho d\mathbf{v} + 2\sigma S; \quad d\mathbf{v} = S d\mathbf{x}; \quad \varphi = - \int \vec{E} d\mathbf{r} + C$$

### Заряд Q

$$\text{I) } Q_{ex} = \int_{-d}^d \rho_0 S \cos \frac{\pi x}{2d} d\mathbf{x} + 2\sigma S = \frac{2d\rho_0 S}{\pi} \cdot \left( \sin \frac{\pi x}{2d} \right) \Big|_{-d}^d + 2\sigma S = \frac{4d\rho_0 S}{\pi} + 2\sigma S$$

$$\text{II) } Q_{in} = \int_{-x}^x \rho_0 S \cos \frac{\pi x}{2d} d\mathbf{x} = \frac{2d\rho_0 S}{\pi} \cdot \left( \sin \frac{\pi x}{2d} \right) \Big|_{-x}^x = \frac{4d\rho_0 S \sin(\frac{\pi x}{2d})}{\pi}$$

### Напруженість електричного поля E

$$\text{I) } E_{ex} = \frac{4d\rho_0 S + 2\pi\sigma S}{\pi\varepsilon_0 S} = \frac{4d\rho_0 + 2\pi\sigma}{\pi\varepsilon_0} \cdot \frac{x}{|x|}$$

$$\text{II) } E_{in} = \frac{4d\rho_0 \sin(\frac{\pi x}{2d})}{\pi\varepsilon_0} = \frac{4d\rho_0 \sin(\frac{\pi x}{2d})}{\pi\varepsilon_0}$$

### Потенціал поля $\varphi$

$$\text{I) } \varphi_{ex} = - \int \left( \frac{4d\rho_0 + 2\pi\sigma}{\pi\varepsilon_0} \right) d\mathbf{x} = - \frac{x(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} + C$$

$$\text{II) } \varphi_{in} = - \frac{4d\rho_0}{\pi\varepsilon_0} \cdot \int \sin \frac{\pi x}{2d} d\mathbf{x} = \frac{8d^2\rho_0 \cos(\frac{\pi x}{2d})}{\pi^2\varepsilon_0} + C$$

$$\text{a) } \varphi_{in}(0) = 0$$

$$\frac{8d^2\rho_0 \cdot 1}{\pi^2\varepsilon_0} + C = 0 \Rightarrow C = - \frac{8d^2\rho_0}{\pi^2\varepsilon_0}$$

$$\text{b) } \varphi_{in}(d) = \varphi_{ex}(d)$$

$$\frac{8d^2\rho_0 \cos(\frac{\pi d}{2d})}{\pi^2\varepsilon_0} - \frac{8d^2\rho_0}{\pi^2\varepsilon_0} = - \frac{d(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} + C \Rightarrow C = \frac{d(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} - \frac{8d^2\rho_0}{\pi^2\varepsilon_0}$$

$$C = \frac{4d^2\pi\rho_0 + 2\pi^2d\sigma - 8d^2\rho_0}{\pi^2\varepsilon_0} = \frac{2d(2d\pi\rho_0 + \pi^2\sigma - 4d\rho_0)}{\pi^2\varepsilon_0} \Rightarrow$$

$$\text{III)} \quad \varphi_{ex} = -\frac{x(4d\rho_0 + 2\pi\sigma)}{\pi\varepsilon_0} + \frac{2d(2d\pi\rho_0 + \pi^2\sigma - 4d\rho_0)}{\pi^2\varepsilon_0}$$

$$\text{IV)} \quad \varphi_{in} = \frac{8d^2\rho_0 \cos(\frac{\pi x}{2d})}{\pi^2\varepsilon_0} - \frac{8d^2\rho_0}{\pi^2\varepsilon_0}$$

