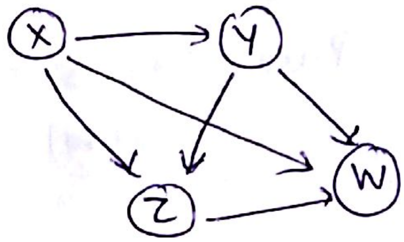
AKSHAY JAIN

Graphical Models

HW 1

1) Assuming W, X, Y, Z are binary discrete Random Variable

(a)



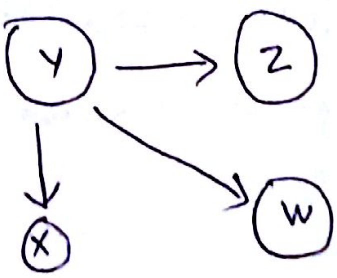
Parameters $= 1 + 2 + 4 + 8 = 15$

$1 \to P(X)$, $2 \to P(Y|X)$, $4 \to P(Z|X,Y)$, $8 \to P(W|X,Y,Z)$

(b)

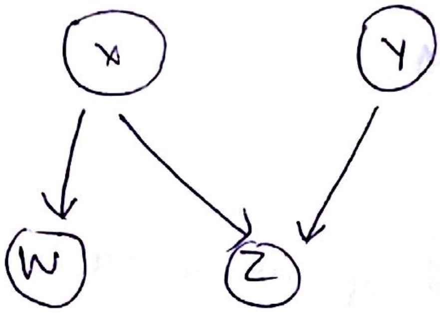
Parameters $= \underset{P(X)}{1} + \underset{P(Y)}{1} + \underset{P(Z)}{1} + \underset{P(W)}{1} = 4$

(c)



Parameters $= \underset{P(Y)}{1} + \underset{P(X|Y)}{2} + \underset{P(Z|Y)}{2} + \underset{P(W|Y)}{2} = 7$

d)



Parameters = 1 + 1 + 2 + 4 = 8

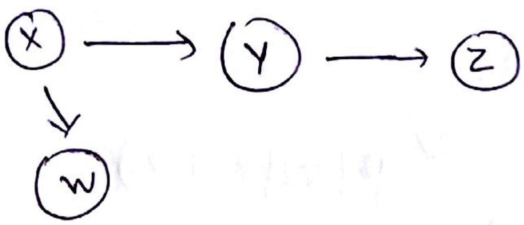
$P(x)$ $P(y)$ $P(w|x)$ $P(z|x,y)$

(e)

Parameters = 1 + 2 + 2 + 2

$P(z)$ $P(y|z)$ $P(x|y)$ $P(w|x)$

f)



Parameters = 1 + 2 + 2 + 2

$P(x)$ $P(y|x)$ $P(z|y)$ $P(w|x)$

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2)

(a) "Projector_plugged_in" and "sam_reading_book" are conditionally independent as there are no active path between them. Hence the knowledge of "projector -plugged-in" doesnot affect the belief in "Sam_reading-book".

(b) "Screen-lit-up" and "sam-reading-book" are not independent as there is an "split" between them which is active. Hence the knowledge of "Screen-lit-up" affects "sam-reading-book".

"power_in_building" and "projector_plugged_in" are conditionally dependent on observing "screen_lit_up". which in turn affects "Sam_reading_book". Hence "~~project~~ p it affects the belief of "sam_reading_book".

(d) "Projector_lamp_on", "screen_lit_up", "ray_says_"screen_is dark".

(e) All the probabilities except "light_switch_on", "mirror_working" & "ray_is_awake".

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3) a)

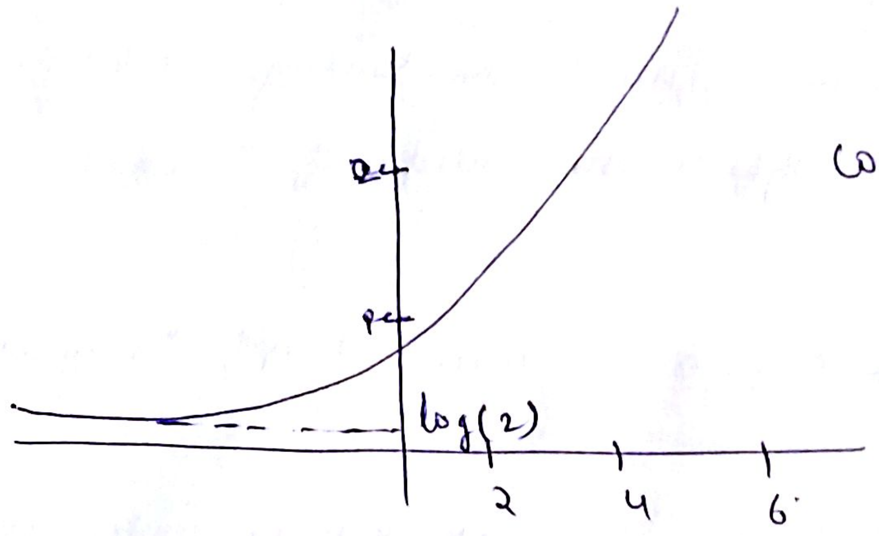
$$\sum_y \sum_x p(x,y;\theta) = 1$$

$$\Rightarrow \frac{\sum_y \sum_x \exp\left[\theta_x x + \theta_{xy} xy\right]}{e^{A(\theta)}} = 1$$

$$\Rightarrow e^{A(\theta)} = e^0 + e^0 + e^{\theta_x} + e^{(\theta_x + \theta_{xy})}$$

$$A(\theta) = \log\left(2 + e^{\theta_x} + e^{(\theta_x + \theta_{xy})}\right).$$

(b) For $\theta_{xy} = 1$ $A(\theta) = \log(2 + (1+e).e^{\theta_x})$ --



convex

(c) $\nabla A(\theta) = \left[\frac{\partial A(\theta)}{\partial \theta_x} \quad \frac{\partial A(\theta)}{\partial \theta_{xy}} \right]$

$= \frac{\cancel{\theta_x . e[\theta_x]} + \theta_x}{\cancel{\log}(2 + \exp(\theta_x) + \exp(\theta_x + \theta_{xy}))}$

$= \left[\frac{\theta_x . e^{\theta_x}}{2 + e^{\theta_x} + e^{\theta_x + \theta_{xy}}} , \frac{\theta_{xy} . e^{\theta_x + \theta_{xy}}}{2 + e^{\theta_x} + e^{\theta_x + \theta_{xy}}} \right]$

At $\theta = [1, 2]$ $\nabla A(\theta) = \left[\frac{e + e^3}{2 + e + e^3} , \frac{2 . e^3}{2 + e + e^3} \right]$

Gradient is monotonic, ever increasing