ECE 214 - Probability and Statistics, Spring 2024

Homework #1

Due: 02/12/24, 11:59 pm

1. I take a standard deck of 52 cards, consisting of 13 spades, 13 hearts, 13 diamonds, and 13 clubs. I am interesting in seeing how many non-heart cards I can draw before picking a heart. After each draw, I will put the card back in the deck, so there is a 1/4 chance I get a heart with each draw, and a 3/4 chance I do not get a heart. I think about this a little bit and come up with the following probability model for X, the number of non-hearts drawn before I get a heart.

$$P(X = n) = c\left(\frac{3}{4}\right)^n, \qquad n = 0, 1, 2, \ldots$$

(a) Using the fact that the probability space must have a total measure of one, find the value of c. (Yes, I know there is another way to find c directly from the experiment description, but I want you to use/remember this important infinite series. Of course, the answer you get should be consistent with what you would expect from the experiment.)

(b) Find $P(X \geq 2)$.

(c) Find $P(X \in \{0, 2, 4, 6, \ldots\})$.

2. Suppose that A, B, and C are events such that $P[A] = P[B] = 0.3, P[C] = 0.55, P[A \bigcap B] = 0, P[\overline{A} \bigcap \overline{B} \bigcap \overline{C}] = 0.1$, and $P[A \bigcap \overline{C}] = 0.2$.

For each of the events given below in parts (a)-(d), do the following:

(i) Write a set expression for the event. (Note that there are multiple ways to write this in many cases.)

(ii) Evaluate the probability of the event.

(Hint: Draw the Venn Diagram. You may then want to identify the probabilities of each of the disjoint regions in the diagram before starting the problem.)

(a) At least one of the events A, B, or C occurs.

(b) Exactly one of A, B, or C occurs.

(c) At most one of A, B, or C occurs.

(d) C occurs, but neither A nor B occurs.

3. Prove the three-set version of the inclusion-exclusion principle:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$