ECE 241
Fall 2023 – **Practice** Exam (Final)
Solution

Prof. M. Zink

Name: _____

ID Number: _____

	Maximum	Achieved
Question 1 Question 2 Question 3 Question 4 Question 5		
Total	100	

This exam is closed book, closed notes. No electronic devices (including calculators) are allowed. Be concise, but show your work. Write legibly. When writing code, please indent appropriately and give your variables meaningful names.

Time: 120 minutes.

Question 1 State Machines

This problem focuses on state machines and their implementation

a) For the state machine shown in Figure 1, which of the following expression will result in a match?

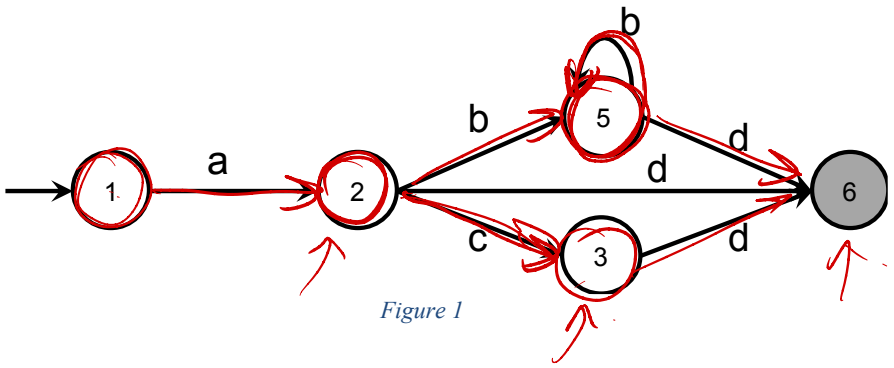


Figure 1

Expression	True	False
aacd		
ad		
abbbbd		
dabbbd		
a(b+|c)d		
bbacd		

b) List **4 valid expressions** for the state machine shown in Figure 1! E.g., abbbbbd is a valid expression.

acd

c) Create a state machine (similar to the one shown in Figure 1) that matches the regular expression ba*b.

d) Write a version for the class Edge that represents an edge in a finite state machine. (4 points)

```
class Edge:
    def __init__(self, ch):
        self.char = c
```

self.dest = d

e) Complete the missing lines in the code for the class Vertex that represents a vertex in a finite state machine. (6 points)

```
class Vertex:
def __init__(self, n):
        self.number = n
        self.edgeList = []
    self.isAcceptingState = ... False
def setAcceptingState(self):
    ..self.isAcceptingState = True
def addEdge(self,e):
    ..self.edgeList.append(e)
def followEdge(self,c):
    ... for edge in self.edgeList:
            ... if edge.char == c:
                ...return edge.dest
        return ... None
```

Question 2: Fast Fourier Transform (FFT)

We have learned that the FFT is an essential algorithm for Shazam (an App that identifies a song by playing a recording to a smartphone).

a) The Fast Fourier Transform makes use of a divide and conquer algorithm to simplify the number of operations, breaking a big FFT into smaller FFTs, which are easier to solve).

List the four steps the divide and conquer approach of the Fast Fourier Transform. What is the complexity O(…) of the algorithm?

nlogn

b) Complete the code below to implement a recursive FFT function.

```
# Recursive FFT function

import numpy as np


def FFT(x):
    N  =  len(x)


N = 1024
x = np.random.random(N)
t = Timer(lambda: FFT(x))
print('Elapsed time: {} s'.format(str(t.timeit(number=1))))
```

Question 3: Linear Programming

a) Consider a company that manufactures only two types of lemonade – A and B. Both lemonades require carbonated water and either lemons. To manufacture each unit of A and B, following quantities are required:

- Each unit of A requires 1 unit of carbonated water and 3 units of lemons

- Each unit of B requires 1 unit of carbonated water and 5 units of lemons

The company has a total of 12 units of carbonated water and 16 units of lemons. On each sale, the company makes a profit of $4 per unit A sold and $6 per unit B sold.

Let the total number of units produced of A = x, and the total number of units produced of B = y

Now, the company wishes to maximize its profit. How many units of A and B should it produce respectively?

Solve this optimization problem by formulating it as a linear program. Start by translating it in tabular form (use the table below for that task).

	Water	Lemons	Profit per Unit
A	1	3	$4
B	1	5	$6
Total	12	16	

maximize: $4x + 6y$

s.t.: $x + y \leq 12$

$3x + 5y \leq 16$

$x, y \geq 0$

b) Figure 2 shows the graphs for the linear program shown below. Briefly explain how you can determine the optimal result for x_1 and x_2 based on the graph shown in Figure 3. Identify all vertexes that define the area of all valid solutions and explain why only one of them represents the optimal solution!

- Maximize $x_1 + x_2$
- Subject to
  - $3x_1 - x_2 \leq 10$
  - $2x_1 + x_2 \leq 10$
  - $5x_1 - 2x_2 \geq -2$
  - $x_1, x_2 \geq 0$

$2x_1 + x_2 + s_1 = 10$

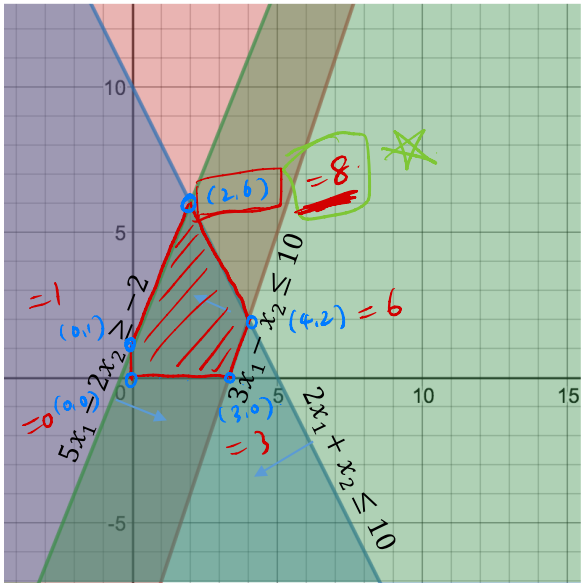


Figure 3

c) Given the linear program below, determine the maximum values for x_3, for which the constraints for x_1, x_4, x_5 are still valid.

$$z = 27 + \frac{x_2}{4} + \frac{x_3}{2} - \frac{3x_6}{4}$$

$$x_1 = 9 - \frac{x_2}{4} - \frac{x_3}{2} - \frac{x_6}{4}$$

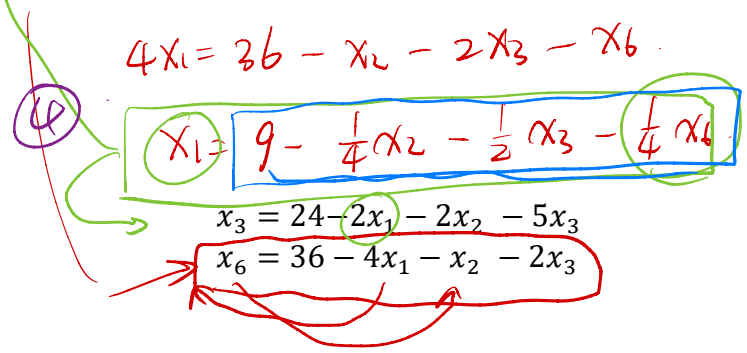
$$x_4 = 21 - \frac{3x_2}{4} - \frac{5x_3}{2} + \frac{x_6}{4}$$

$$x_5 = 6 - \frac{3x_2}{4} - 4x_3 + \frac{x_6}{2}$$

d) One major component of the simplex algorithm is pivoting. In this problem you are tasked to perform pivoting for the linear program shown below and $x_1 = 9$. Start by solving the 3rd constraint for x_1 and substitute it in the remaining equations

$$z = 3x_1 + x_2 + 2x_3$$

$$x_4 = 30 - x_1 - x_2 - 3x_3$$



$x_4 = 30 - (9 - \frac{1}{4}x_2 - \frac{1}{2}x_3 - \frac{1}{4}x_6) - x_2 - 3x_3$

$= 30 - 9 + \frac{1}{4}x_2 + \frac{1}{2}x_3 + \frac{1}{4}x_6 - x_2 - 3x_3$

② $x_4 = 21 - \frac{3}{4}x_2 - \frac{5}{2}x_3 + \frac{1}{4}x_6$

$x_3 = 24 - 2(9 - \frac{1}{4}x_2 - \frac{1}{2}x_3 - \frac{1}{4}x_6) - x_2 - 3x_3$

$= 24 - 18 + \frac{1}{2}x_2 + x_3 + \frac{1}{2}x_6 - x_2 - 3x_3$

③ $x_3 = 6 - \frac{1}{2}x_2 - 2x_3 + \frac{1}{2}x_6$

$z = 3x_1 + x_2 + 2x_3$

$= 3(9 - \frac{1}{4}x_2 - \frac{1}{2}x_3 - \frac{1}{4}x_6) + x_2 + 2x_3$

$= 27 - \frac{3}{4}x_2 - \frac{3}{2}x_3 - \frac{3}{4}x_6 + x_2 + 2x_3$

① $z = 27 + \frac{1}{4}x_2 + \frac{1}{2}x_3 - \frac{3}{4}x_6$

Question 4: Machine Learning

We have seen that Linear Regression is one of the many approaches to build a model in Machine Learning, which can be used to predict labels based on a certain set of features.

a) A Linear Regression example is shown in Figure 4. First specify the general equation that defines the Mean Square Error. Based on that equation calculate the Mean Square Error. To allow us to give you partial credit, calculate the error value for each data point and use these values for your Mean Square Error Calculation.

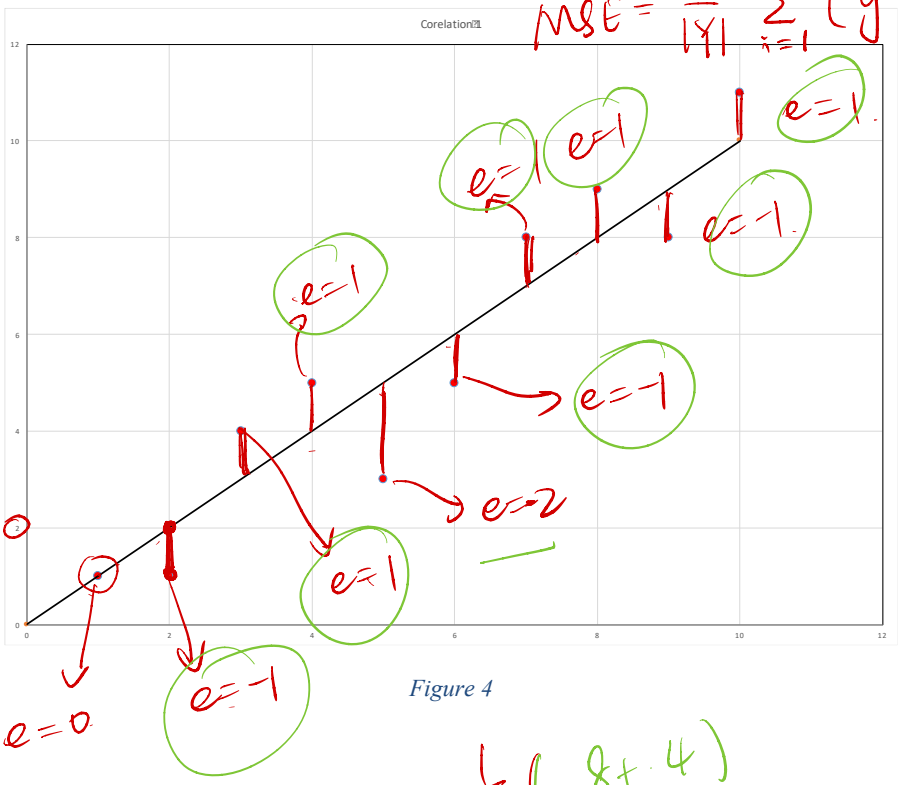


Figure 4

b) The example in Figure 5 shows on potential model for the data points (shown as red dots). Can this model be improved? If so, draw a new line that represents the improved model in the figure and explain why it is a better model.

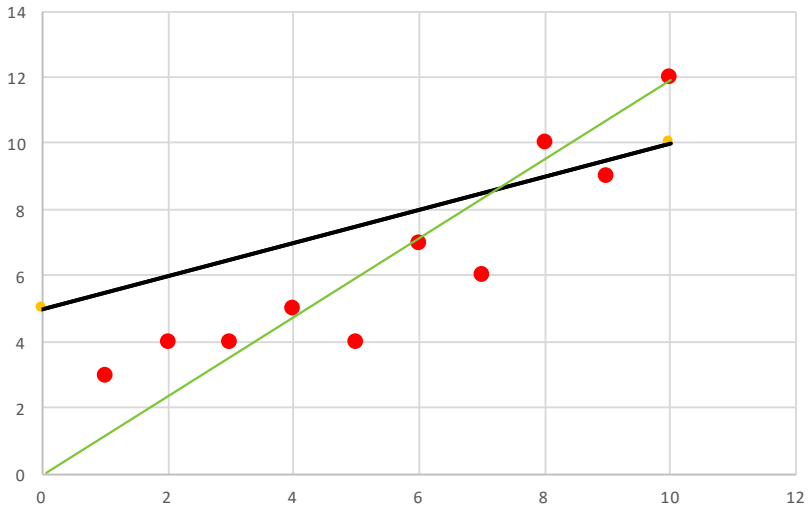


Figure 5

c) Assume you are given a dataset that contains the information shown below. Your goal is to develop a Machine Learning approach that predicts class of an Iris flower. Based on the given information in the table below, which columns identify potential features, which identify labels?

Sepal length	Sepal width	Petal length	Petal width	Class

Question 5: FFT in Action

We have learned that the FFT is an essential algorithm for Shazam (an App that identifies a song by playing a recording to a smartphone).

a) (10 Points) Briefly explain how the Shazam App uses FTT to identify a song that is recorded on a smartphone by identifying steps 1. through 5. shown in Figure 6. What other algorithm is used in the fingerprinting process?

→ hashing.

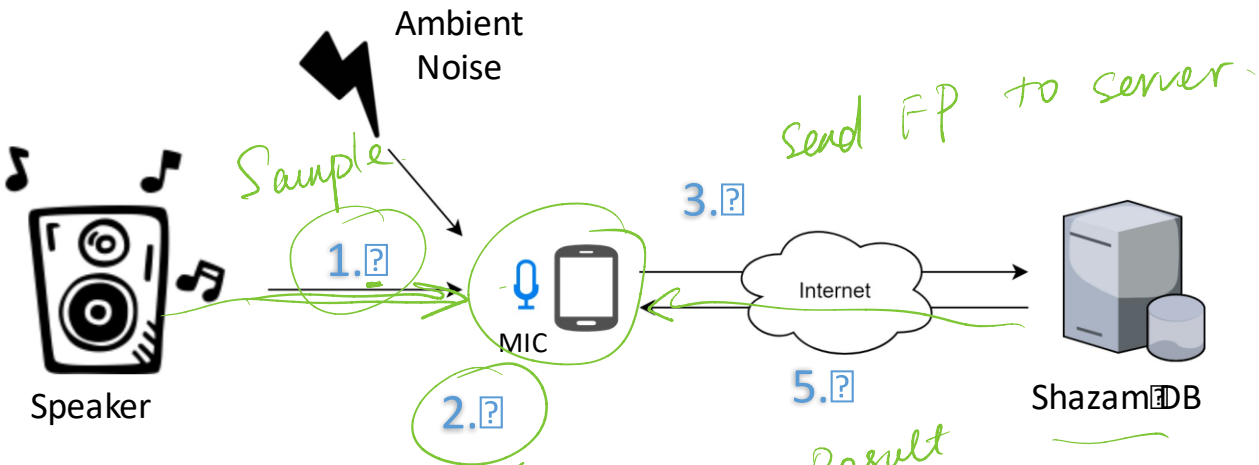


Figure 7