1.1)

$$\det\begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{pmatrix}$$

$$\begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2-x_1 & x_2^2-x_1^2 \\ 0 & x_3-x_1 & x_3^2-x_1^2 \end{bmatrix}$$

$$\downarrow$$

$$(b-a)(c-b)\begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & x_2-x_1 \\ 0 & 1 & x_3-x_1 \end{bmatrix}$$

$$(b-a)(c-b)\begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & b+a \\ 0 & 0 & c-a \end{bmatrix}$$

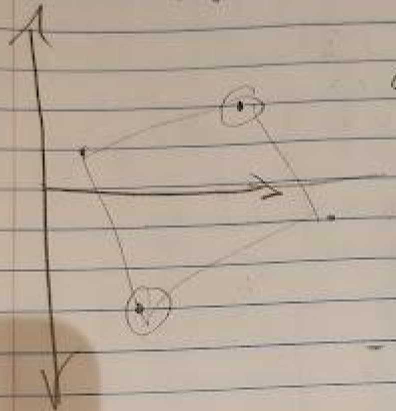
$$(b-a)(c-b)(c-a)\ \det\begin{bmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & b+a \\ 0 & 0 & 1 \end{bmatrix}$$

$$(b-a)(c-b)(c-a)\cdot 1$$

$$(bc-ba-ac+a^2)(c-b)$$

$$bc^2-b^2c+b^2a-ac^2+a^2c-ba^2$$

1.2 a.)



$\det\begin{pmatrix} 6 & 8 \\ 2 & 2 \end{pmatrix}$ $-12-16 = |-28|$

b.) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 6 \end{bmatrix}$

$\det\begin{pmatrix} 2 & 3 & 1 \\ -1 & 1 & -1 \\ 0 & 1 & 5 \end{pmatrix}$

$\begin{matrix} 2 & 3 & 1 \\ 0 & 5/2 & .5 \\ 0 & 0 & 5\frac{1}{5} \end{matrix}$

$\frac{2}{5} \cdot .5 = \frac{1}{5}$

$2 \cdot \frac{5}{2} = 5 \cdot \frac{26}{5}$

26

Vol of $\mathbb{R}^3 = 26$

1.2) C

$$\det\begin{pmatrix} 2 & 3 & 1a \\ -1 & 1 & -1b \\ 0 & 1 & 5c \end{pmatrix} \quad \begin{matrix} 2 & 3 & 1a \\ 0 & \frac{5}{2} & -1b-\frac{a}{2} \\ 0 & 0 & 5c-\frac{2}{5}(-1b-\frac{a}{2}) \end{matrix}$$

$$2 \cdot 5c - \tfrac{2}{5}(-1b-\tfrac{a}{2})$$

$$\tfrac{5}{2} \cdot 10c - \tfrac{4}{5}(-1b-\tfrac{a}{2})$$

$$25c - 2(-1b-\tfrac{a}{2})$$

$$\boxed{25c+2b+a}$$

2.)

$V =$ Polynomials degree ≤ 2

a.) $V_1, V_2, V_3 =$ Basis (V)

$V_1 = 1 \quad V_2 = X \quad V_3 = X^2$

Linear independence

$C_1 + C_2 X + C_3 X^2 = 0$ thus must be

$C_1 = C_2 = C_3 = 0$ linearly independent

Spanning set

V in the form $a_0 + a_1 X + a_2 X^2$

can simply see $1 + x + x^2$ in so it

spans V

proves $1, X, X^2$ is basis (V)

b.) set smooth functions $\mathbb{R} \to \mathbb{R}$ show

V is kernel of $\frac{d}{dx} : S \to S$

Show $f(x) = 1$ is in V

$V = \text{Kernel}\left(\frac{d}{dx}\right)$

$f \in V$ then $\frac{df}{dx} = 0 \rightarrow$ shows f is a constant

$V = \text{kernel}\left(\frac{d}{dx}\right) = \{C : C \in \mathbb{R}\}$

$1 \neq 0$ and each element of V is

multiple of 1 so $f(x) = \{1\}$ is a basis(V)

$T(f) = f'' + f = \left[\frac{d}{dx} \circ \frac{d}{dx} + Id\right]$

2.c $f(x) = ecx$

$c^2 ecx + ecx = 0 = c^2 + 1 = 0$

$c = \pm i$

So $f(x) = c_1 \cos x + c_2 \sin x$

$T(f) = 0$ any linear combo of kernel T

$f''(x) = (-c_1 \cos x - c_2 \sin x) + (c_1 \cos x + c_2 \sin x)$

$= \varnothing$

Which proves $f(x) \in \text{Kernel}(T)$