ECE124: Discussion

Discussion #7

Yeonsik Noh, PhD

Minimization via K-Maps

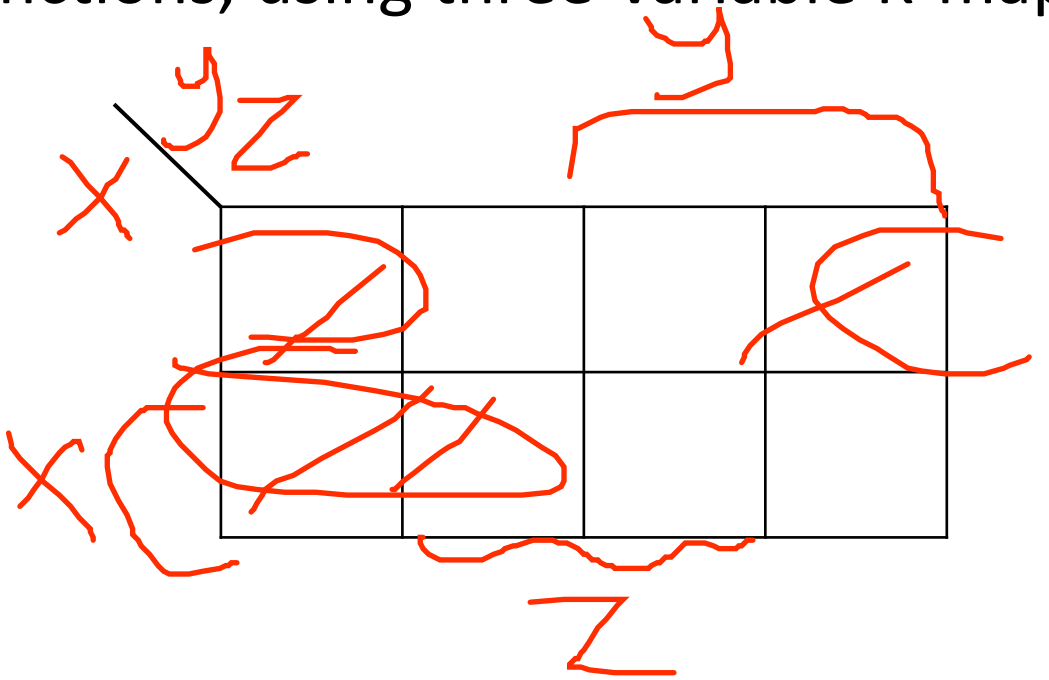
To obtain minimized SOP expression for a Boolean function F…

- Step1: Translate truth table to K-map
- Step2: Cover all 1's in the smallest number of largest blocks
- Step3: Extract one product term for each block
- Step4: Sum of extracted product terms=minimized SOP expression!

3.1 Simplify the following Boolean functions, using three-variable K-maps:

(a) F (x, y, z) = ∑ (0, 2, 4, 5)

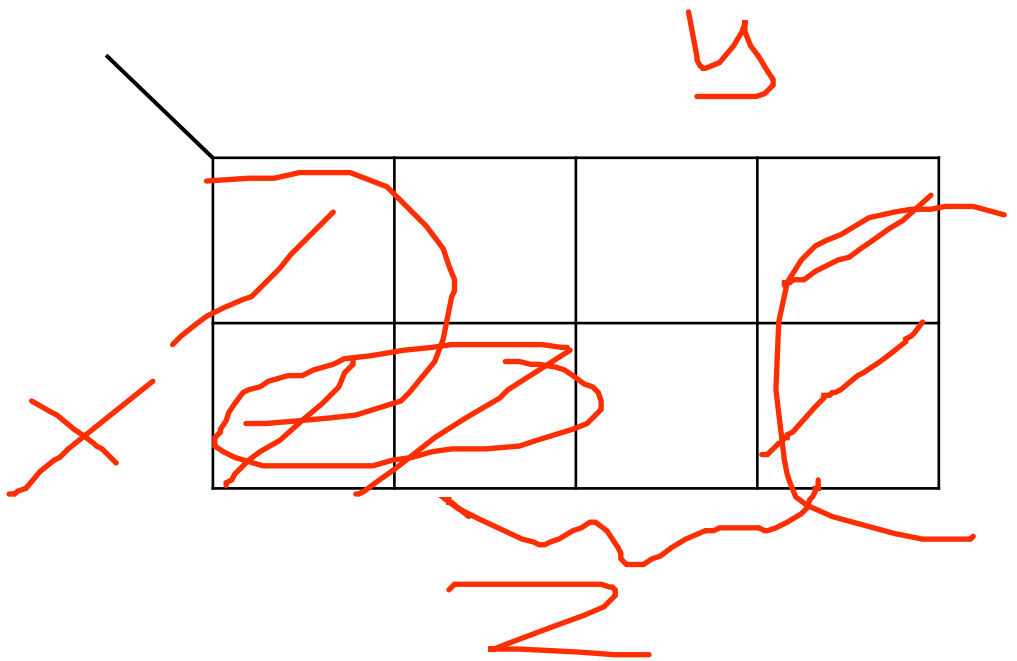
$xy' + xz'$



3.1 Simplify the following Boolean functions, using three-variable K-maps:

(b) F (x, y, z) = ∑ (0, 2, 4, 5, 6)

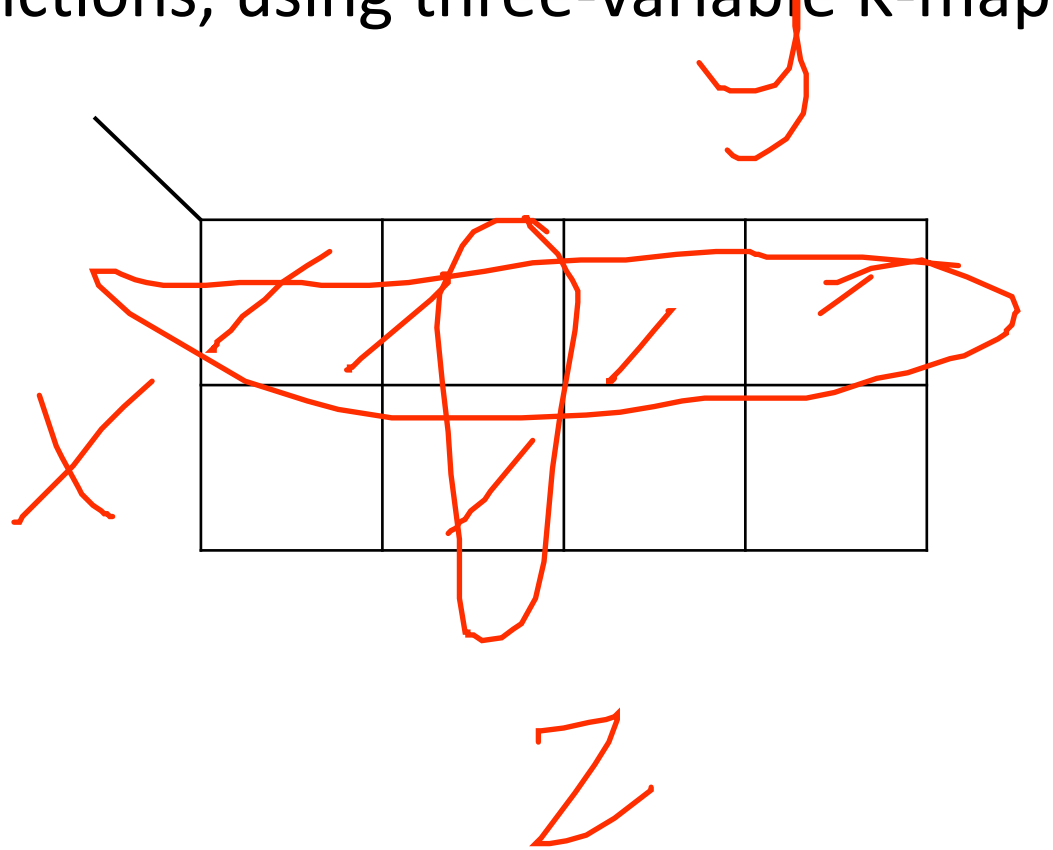
$F = z' + xy'$



3.1 Simplify the following Boolean functions, using three-variable K-maps:

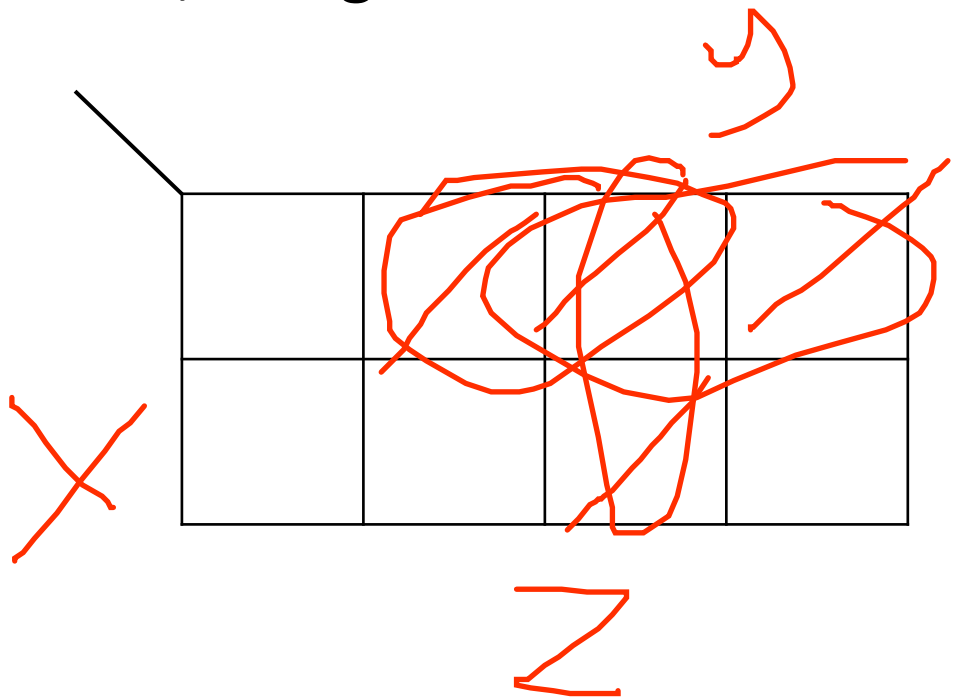
(c) F (x, y, z) = ∑ (0, 1, 2, 3, 5)

$F = x' + y'z$



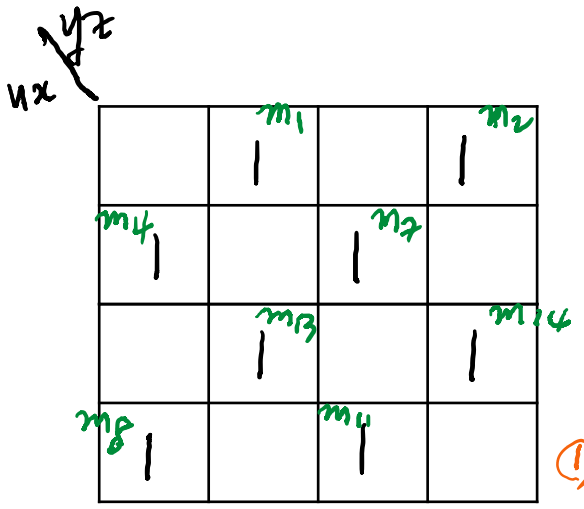
3.1 Simplify the following Boolean functions, using three-variable K-maps:

(d) F (x, y, z) = ∑ (1, 2, 3, 7)



f =x'z+yz+x'z

- 1(b). Boolean expression for F.



✱ All minterms are exclusive.

$F = m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}$

$= u'x'y'z + u'x'yz' + u'xy'z' + u'xyz + ux'y'z' + ux'yz + uxy'z + uxyz'$

$0001_{(2)} = 1_{(10)}$ $0010_{(2)} = 2_{(10)}$ $0100_{(2)} = 4_{(10)}$ $0111_{(2)} = 7_{(10)}$ $1000_{(2)} = 8_{(10)}$ $1011_{(2)} = 11_{(10)}$ $1101_{(2)} = 13_{(10)}$ $1110_{(2)} = 14_{(10)}$

① $= u'x'(y'z + yz') + u'x(y'z' + yz) + ux'(y'z' + yz) + ux(y'z + yz')$

② $= (u'x' + ux)(y'z + yz') + (u'x + ux')(y'z' + yz)$

③ $= (u \oplus x)'(y \oplus z) + (u \oplus x)(y \oplus z)'$

($u \oplus x$: A', $y \oplus z$: B, $u \oplus x$: A, $(y \oplus z)'$: B')

$= (u \oplus x) \oplus (y \oplus z)$

$= u \oplus x \oplus y \oplus z$

✱ $A \oplus B = A'B + AB'$

$(A \oplus B)' = A'B' + AB$

$= A \odot B$

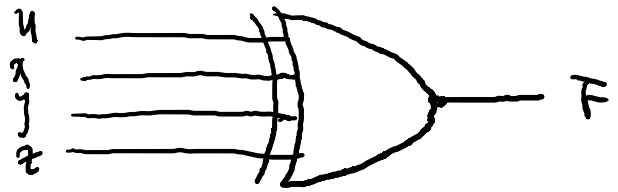
✱ $A'B' + AB \neq 1$

if, $A = B \rightarrow A'B' + AB = 1$

- 1(c). Draw a logic diagram for F.

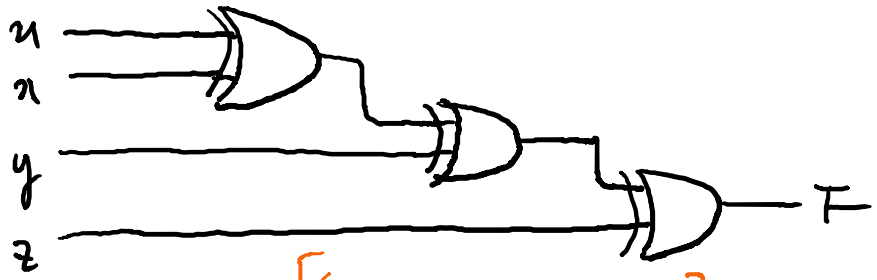
$F = u \oplus x \oplus y \oplus z$

i)



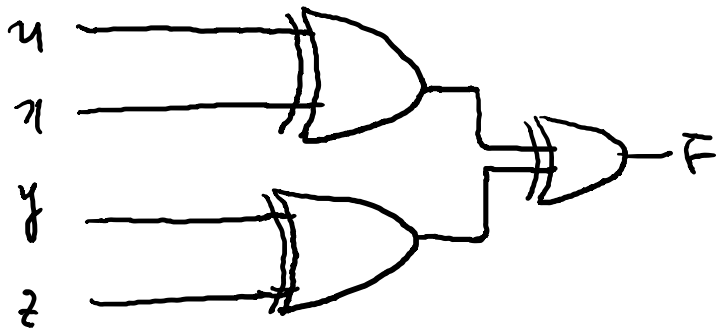
$F = u \oplus x \oplus y \oplus z$

ii)



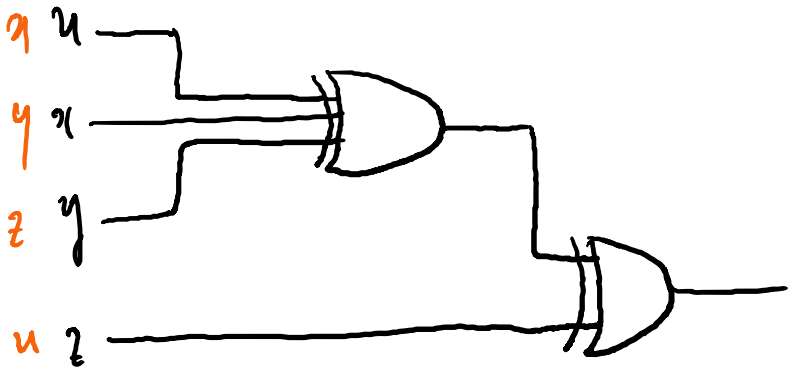
$F = [(u \oplus x) \oplus y) \oplus z]$

iii)



$F = (u \oplus x) \oplus (y \oplus z)$

iv)



$F = (u \oplus x \oplus y) \oplus z$

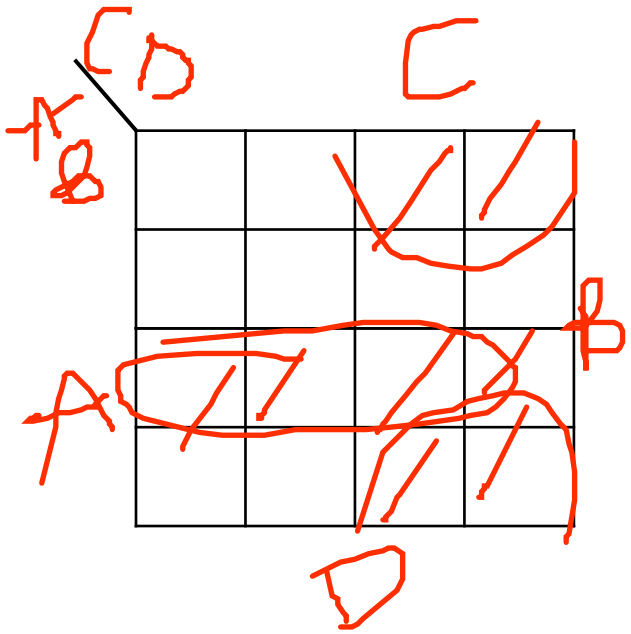
$= u \oplus (x \oplus y \oplus z)$

- 1) Consider the Boolean function: F (A, B, C, D) = AB + B’C + ACD + ABD’ + ACD’
- 2) Extract a minimized Boolean expression for F using the Karnaugh map of 1).
- 3) Simplify the Boolean expression F and Compare it with the minimized Boolean expression of 2)

F = AB+B'C+ACD+ABD'+ACD'

AB(1+D')+AC(D+D')+B'C

AB+AC+B'C



ABCD	abcd	AB'CD
1100	1010	0010
1101	1011	0011
1110	1110	1010
1111	1111	1011

consensus

AB+AC+B'C

(A+B)(A+C)(B'+C)

x'y+yz+xz

A'B+(B+B')AC+B'C

AB+ABC+AB'C+B'C

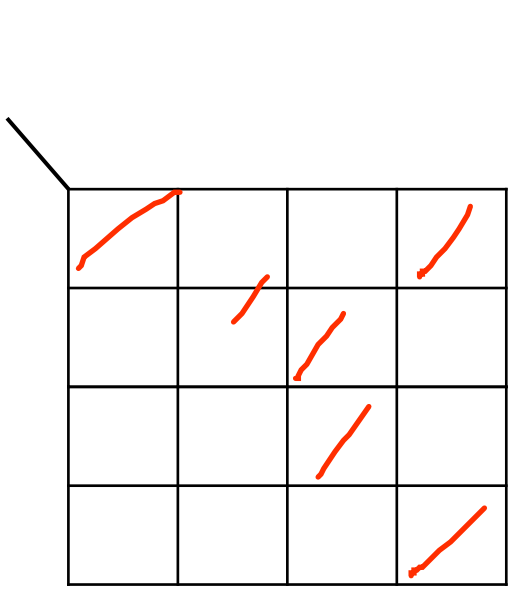
AB(1+C)+B'C(A+1)

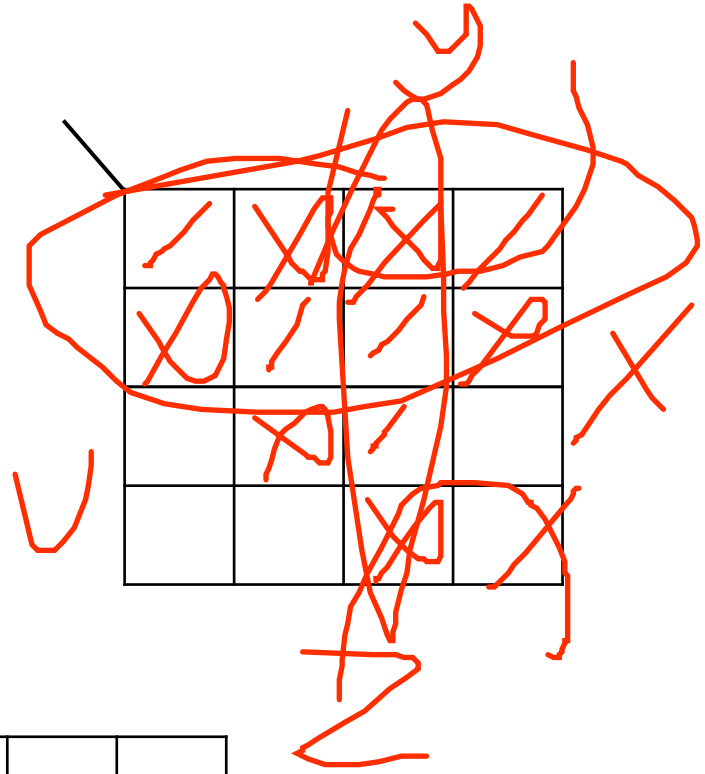
F=AB+B'C

- 2(a). Consider the Karnaugh map for F together with the don't care conditions, d. Show the Karnaugh map for G.

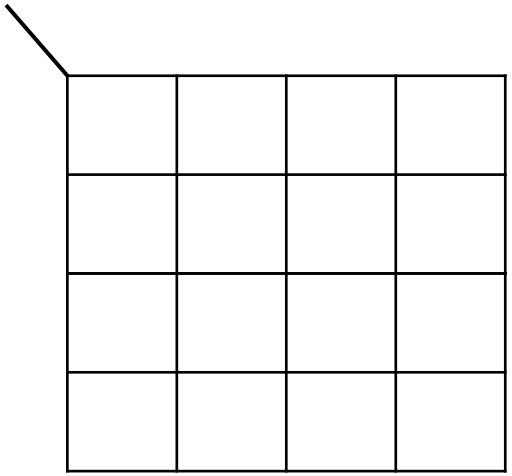
$F(u, x, y, z) = \sum(0, 2, 5, 7, 10, 15)$ $d(u, x, y, z) = \sum(1, 3, 4, 6, 11, 13)$

$G(u, x, y, z) = F + d$





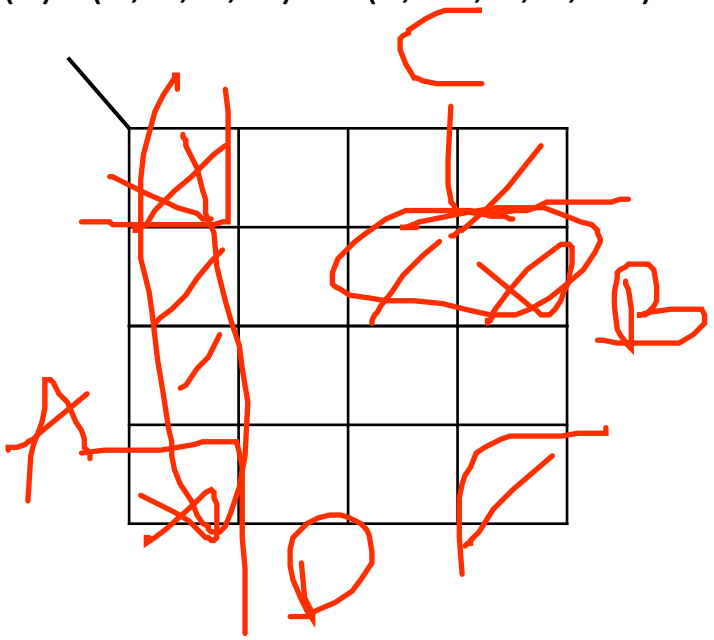
F=u'+yz+x'y



3.15 Simplify the following Boolean function F, together with the don’t care conditions d, and then express the simplified function in sum-of-minterms form:

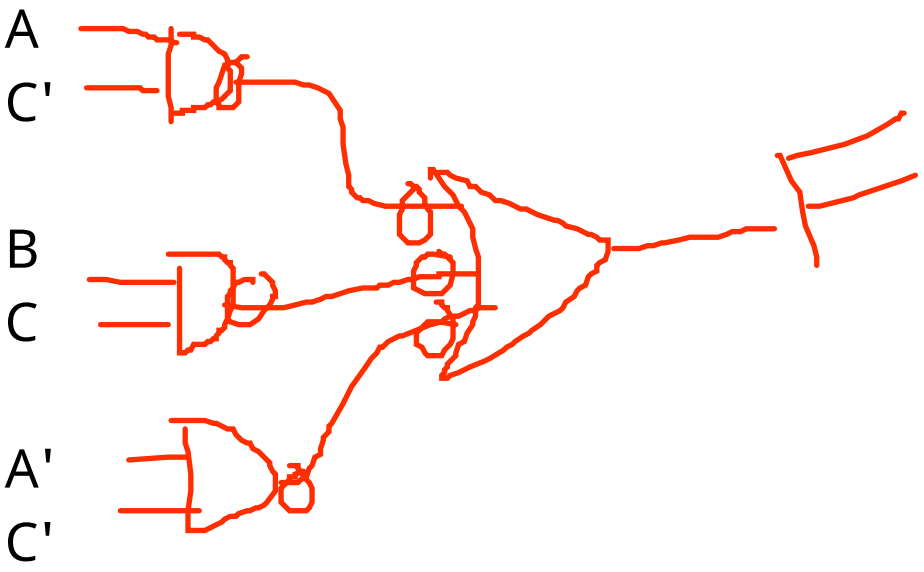
(d) F(A, B, C, D) = Σ(4, 12, 7, 2, 10)

d (A, B, C, D) = Σ(0, 6, 8)



F=B'D'+C'D'+A'BC

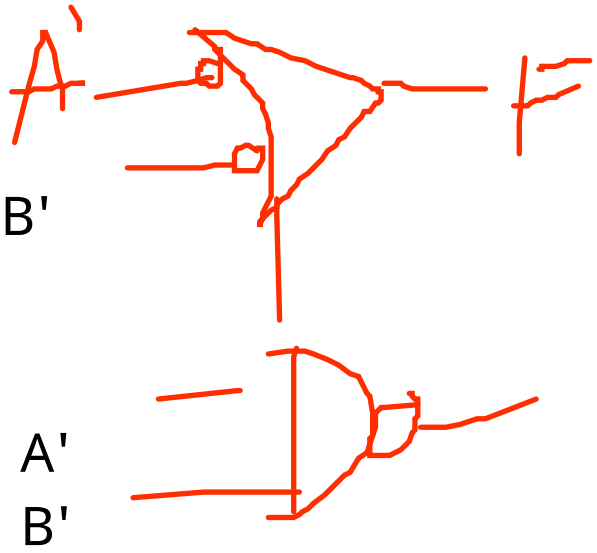
- Convert the Boolean expression F=AC’+BC+A’C’ to only NAND gates.



F=AC'+BC+A'C'

F = (F')' = {(AC'+BC+A'C'}'= {(AC')'(BC)'(A'C')'}'

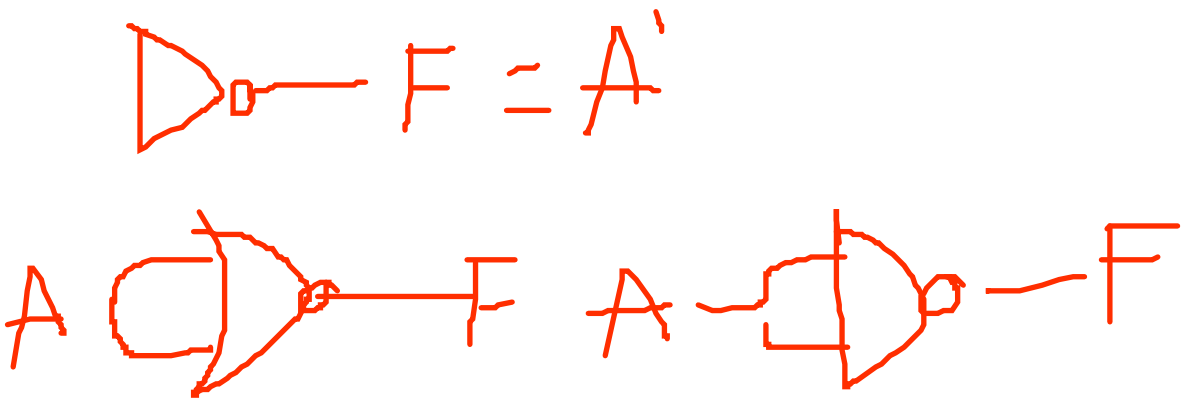
- Draw a logic diagram for F=A+B using only NAND gates.



F'=(A+B)'

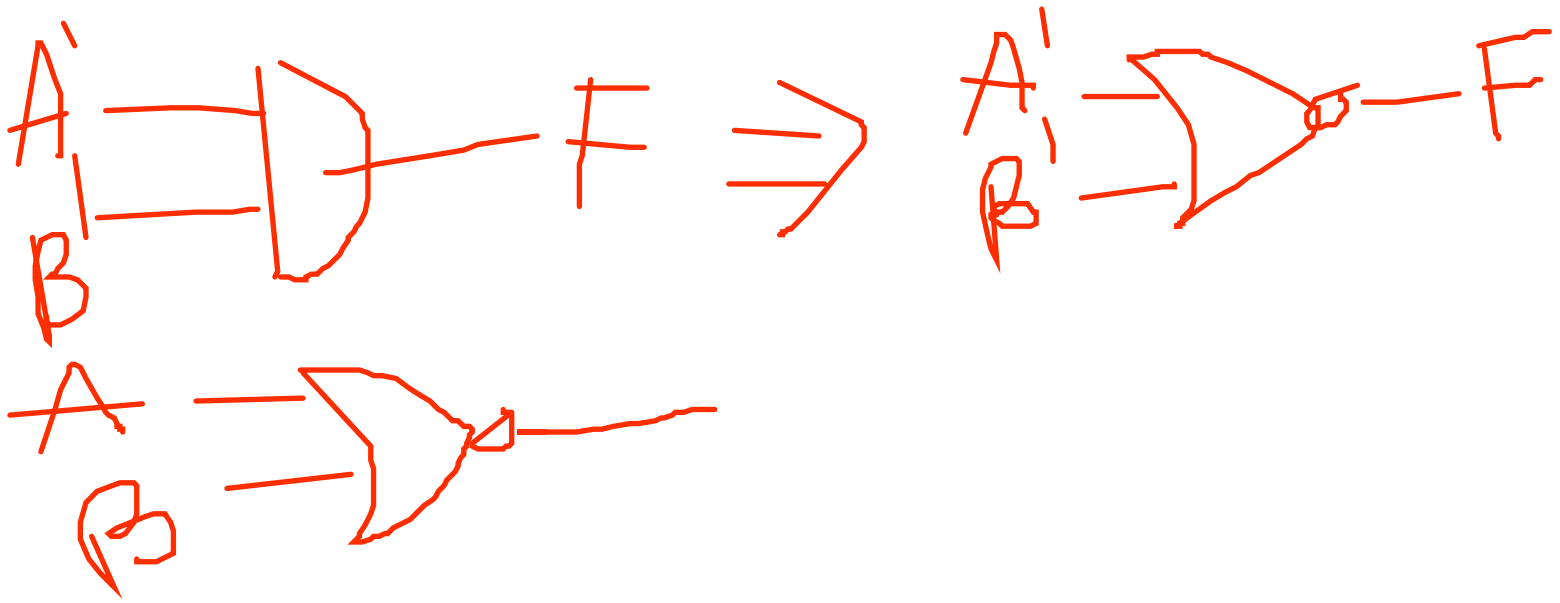
F = (A'B')'

- Draw a logic diagram for F=A’ using only NOR gates.



X	Y	NOR	NAND
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	0

- Draw a logic diagram for F=AB using only NOR gates.



A'B' = (A+B)'

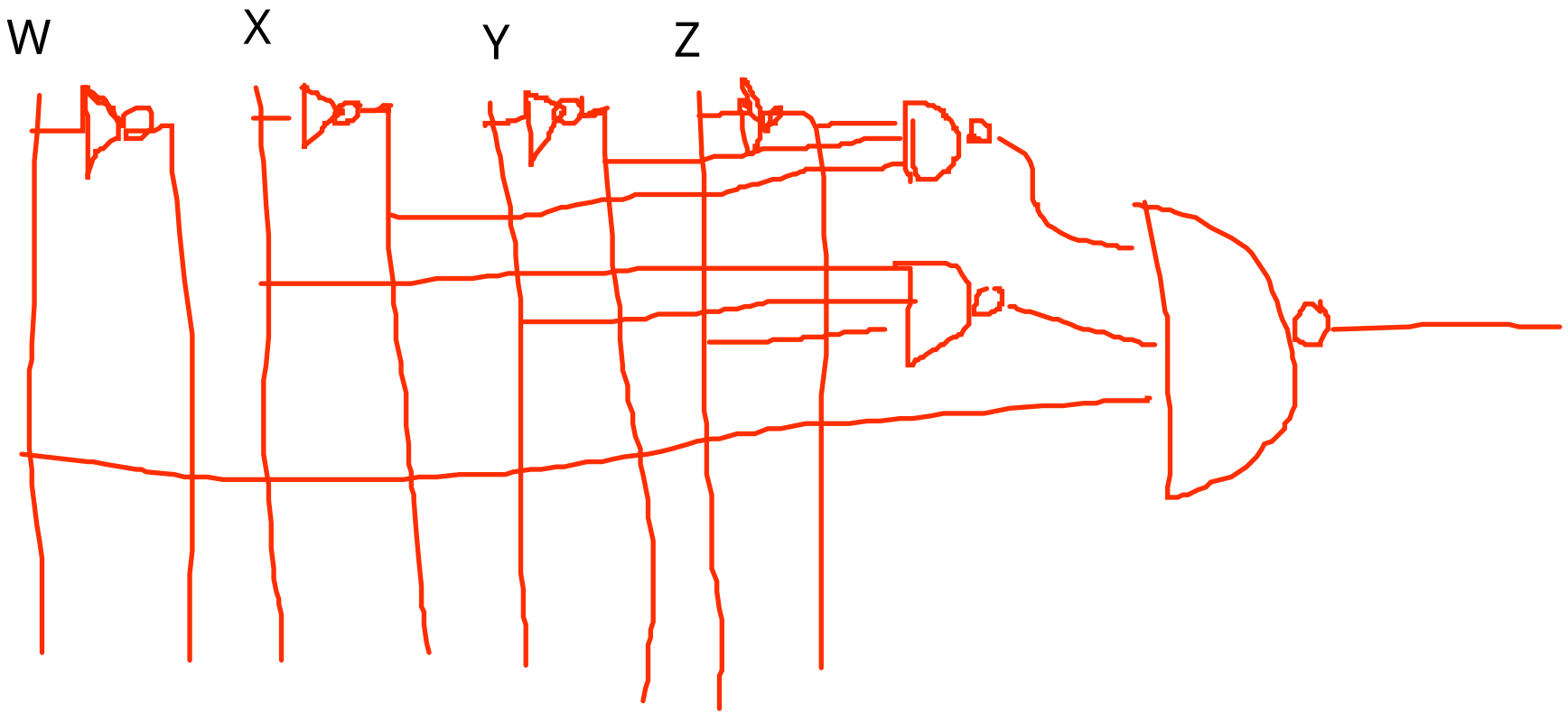
F = (A'+B')'

3.21 Draw (a) the multiple-level NAND circuit for the following expression and (b) repeat (a) for a NOR circuit.

$$w(x + y + z) + xyz$$

X+Y+Z = (X'Y'Z')'

W(X'Y'Z')' + xyz = (W(X'Y'Z')'(XYZ)')'



3.21 (b) repeat (a) for a NOR circuit.

$$w(x + y + z) + xyz$$

W(X+Y+Z) = (W'+(X+Y+Z)')'

xyz = (X'+y'+z')'

(W'+(X+Y+Z)')' + (X'+y'+z')'