C11 Derivation

$z = Ae^{j\varphi}$

$z^{1/n} = (Ae^{j\varphi})^{1/n} = A^{1/n}(e^{j\varphi})^{1/n} = A^{1/n} e^{j\frac{\varphi}{n}} \rightarrow Re^{jB}$

$\boxed{R = A^{1/n} \qquad B = \frac{\varphi}{n}}$

Euler's Formula

$Re^{jB} = x + jy$

$R(\cos B + j \sin B) = x + jy$

$R\cos B + jR\sin B = x + jy$

$\boxed{x = R\cos B \qquad y = R\sin B}$

Derive dB

principal root:

$(Ae^{j\varphi})^{1/n} = A^{1/n}(e^{j\varphi})^{1/n} = A^{1/n} e^{j\frac{\varphi}{n}}$

"first root":

$(Ae^{j(\varphi+360°)})^{1/n} = A^{1/n}(e^{j(\varphi+360°)})^{1/n} = A^{1/n} e^{j\frac{(\varphi+360°)}{n}}$

$B_1 = \frac{\varphi}{n} \qquad B_2 = \frac{\varphi+360°}{n}$

$dB = B_2 - B_1 = \frac{\varphi+360°}{n} - \frac{\varphi}{n} = \frac{\varphi+360°-\varphi}{n} = \boxed{\frac{360°}{n}}$

```matlab
% Adrian Nelson
% 1/12/2024
% Getting Started with C11 in ECE 296C, Complex Numbers
% This script finds and plots the roots of complex numbers
% given in rectangular form

clf   % clear all figures
clear   % remove all variables from the workspace

% needed to draw axes
ax = [-100,100]; ze = [0,0];

hold on
% plot x- and y-axes as black lines (draw the axes before using QUIVER)
plot(ax,ze, 'k', 'LineWidth', 1)
plot(ze,ax, 'k', 'LineWidth', 1)


% ----- input z = a + jb, then convert to exponential form -----

z = input('Input a complex number z as a+bj: ');
n = input('Input the power of the root, i.e., n of z^1/n: ');
a = real(z); b = imag(z);

% z = a+jb = A e^jP

A = norm(z);   % amplitude of z
P = rad2deg(angle(z));   % phase of z in degrees, -180deg < phi <= 180deg


% ----- set up parameters needed to find n roots of z -----

% z^1/n = R e^jB

R = A^(1/n);   % amplitude R of z^1/n
B = P/n;  % phase in deg, associated with principal value of z^1/n
dB = 360/n;   % difference between phase angles, in degrees


% R e^jB = X + jY

X = R*cosd(B); Y = R*sind(B);


% ----- plot roots and compute checks -----

% plot an arrow to represent the principal value of z^1/n, in red
quiver(0, 0, X, Y, 0, 'r', 'LineWidth', 3)

check = zeros(n,1);   % initialize n checks as a column vector

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```matlab
check(1) = z-(X+j*Y)^n;   % first check, output at the end

for i = 2:n   % cycle through the other values to make the rest of the arrows

    B = B + dB;   % add dB to find the next nth root of z
    X = R*cosd(B); Y = R*sind(B);

    % plot an arrow for the next root, in blue
    quiver(0, 0, X, Y, 0, 'b', 'LineWidth', 3)

check(i) = z-(X+j*Y)^n;   % the rest of the checks

end

check   % output the n checks; each should be close to 0+j0


% ----- make the figure look nicer -----

grid on; axis equal   % make the frame square (emphasize symmetry of the roots)

ac = gca;
ac.FontSize = 16; ac.GridAlpha = 0.5;   % change fonts to 16pt; make grid darker

xlabel('Re({\textbf{z}})', 'FontSize', 20, 'Interpreter', 'latex');
ylabel('Im({\textbf{z}})', 'FontSize', 20, 'Interpreter', 'latex')

% determine the sign of b, so that we can include z in the title
bSgn='+';
if b<0
    bSgn='-';
end
bMag = norm(b);   % magnitude of b

title({'ECE 296C Exercise C11', ...
    sprintf('Finding and drawing the %g values of $(%g%sj%g)^{1/%d}$', ...
    n,a,bSgn,bMag,n)}, 'FontSize', 24, 'Interpreter', 'latex')

max = ceil(R+0.1);   % round up R to the next integer
axis([-max max -max max])   % set the upper and lower limits of the axes

hold off

% e) -1 in exponential form is e^(j180°). It can then be determined that
% (-1)^(1/3)=(e^(j180°))^(1/3)=e^(j180°/3)=e^(j60°).
% The principal cube root of -1 is therefore e^(j60°), not -1=e^(j180°).

% f) d3 is the same as the complex number in e): (-1)^(1/3)=e^(j60°)
% Coverting the complex number d4 to exponential form:
% d4 = (-1-j0.0001)^(1/3) = (e^(-j180°))^(1/3) (approximation)
% d4 simplified: (e^(-j180°))^(1/3)= e^(-j180°/3)= e^(-j60°)
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% Using the result from e), e^(-j60°) is not equal to e^(j60°), however,
% these complex numbers are conjugates.

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```
>> ECE296CMATLABC11
Input a complex number z as a+bj: 3+4j
Input the power of the root, i.e., n of z^1/n: 5

check =

   1.0e-14 *

  -0.1776 - 0.2665i
  -0.0444 + 0.1332i
  -0.6661 + 0.3553i
  -0.7550 + 0.1776i
  -0.7550 + 0.2665i

>> ECE296CMATLABC11
Input a complex number z as a+bj: 2-5j
Input the power of the root, i.e., n of z^1/n: 4

check =

   1.0e-14 *

  -0.0444 - 0.0888i
  -0.0444 - 0.0888i
   0.3775 + 0.1776i
   0.3775 + 0.1776i

>> ECE296CMATLABC11
Input a complex number z as a+bj: -1+0j
Input the power of the root, i.e., n of z^1/n: 3

check =

   1.0e-15 *

  -0.2220 - 0.1110i
   0.0000 + 0.0000i
  -0.2220 + 0.1110i

>> ECE296CMATLABC11
Input a complex number z as a+bj: -1-0.0001j
Input the power of the root, i.e., n of z^1/n: 3

check =

   1.0e-15 *
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  -0.3331 + 0.0445i
  -0.2220 + 0.2110i
  -0.4441 - 0.4091i

>> ECE296CMATLABC11
Input a complex number z as a+bj: 0-1j
Input the power of the root, i.e., n of z^1/n: 9

check =

   1.0e-15 *

  -0.0833 - 0.5551i
  -0.2776 - 0.5551i
  -0.0555 + 0.4441i
   0.0555 + 0.4441i
   0.2776 - 0.5551i
   0.0833 - 0.5551i
  -0.3331 - 0.5551i
   0.0000 + 0.0000i
   0.3331 - 0.5551i

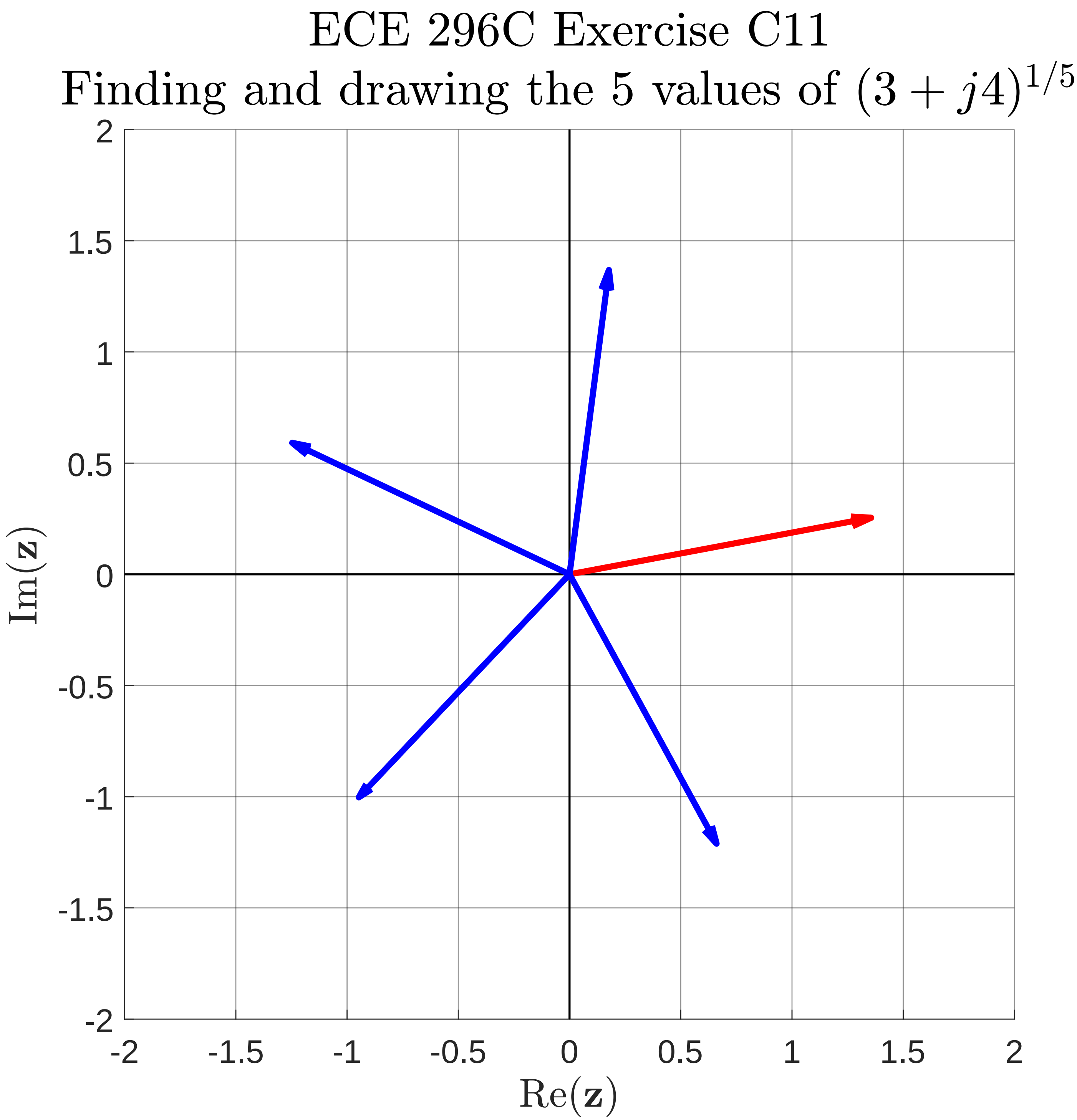
>> ECE296CMATLABC11
Input a complex number z as a+bj: 5+15j
Input the power of the root, i.e., n of z^1/n: 7

check =

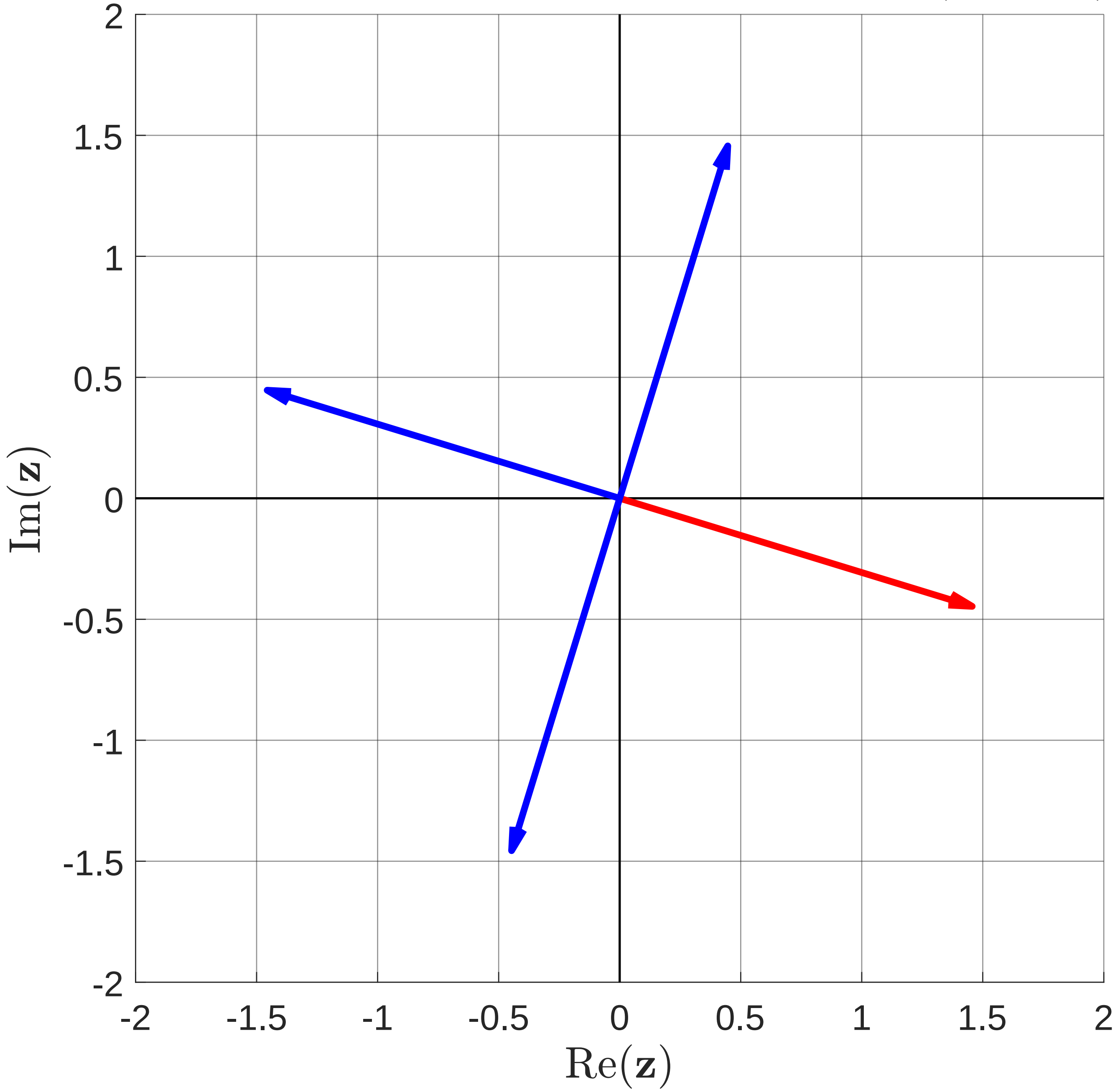
   1.0e-12 *

  -0.0036 + 0.0000i
   0.0044 - 0.0089i
  -0.0053 + 0.0053i
   0.0284 + 0.0000i
   0.0622 - 0.0178i
   0.0817 - 0.0409i
   0.1030 - 0.0355i

>>
```

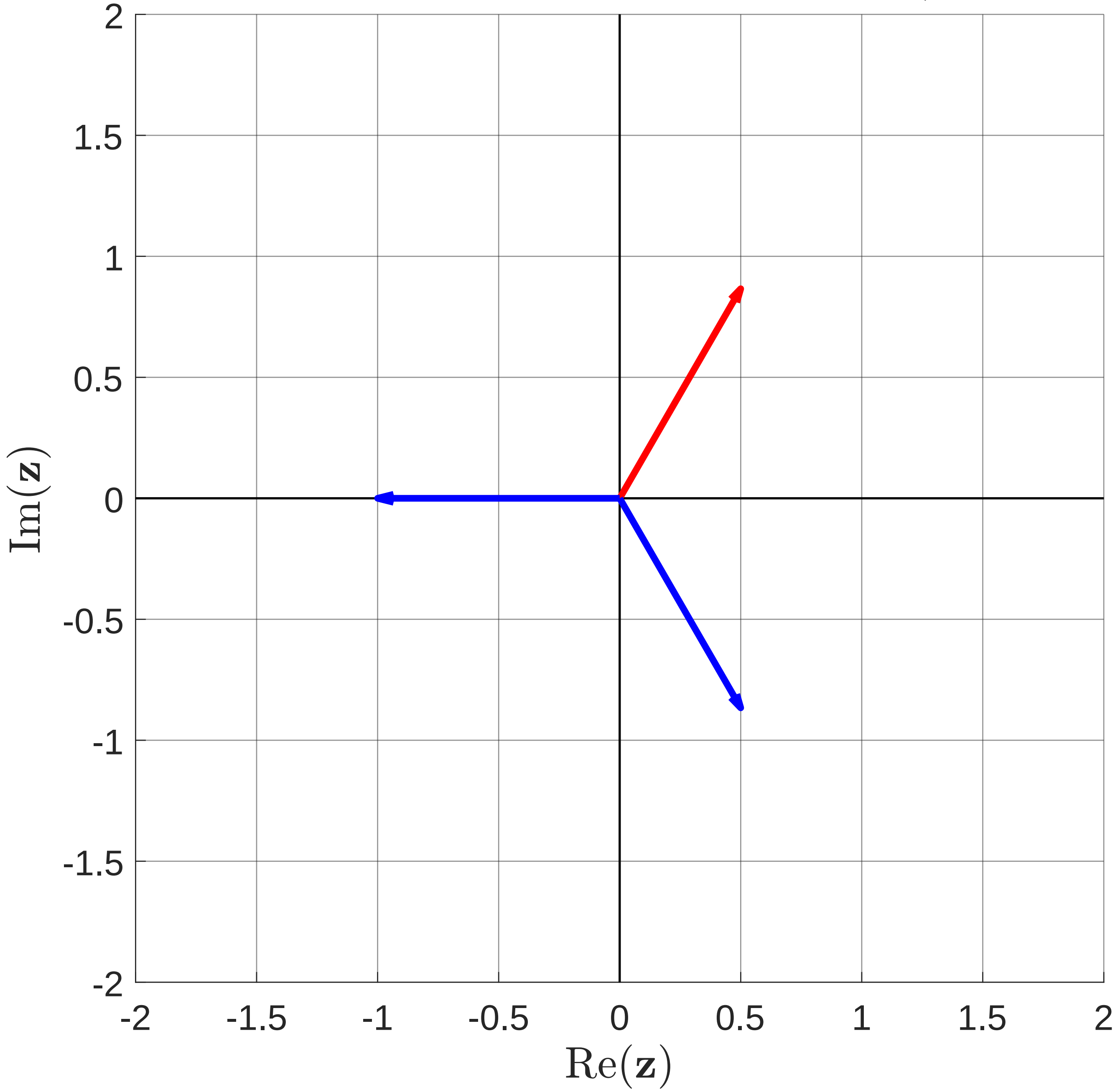

ECE 296C Exercise C11
Finding and drawing the 5 values of $(3+j4)^{1/5}$
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$\mathrm{Im}(\mathbf{z})$
$\mathrm{Re}(\mathbf{z})$

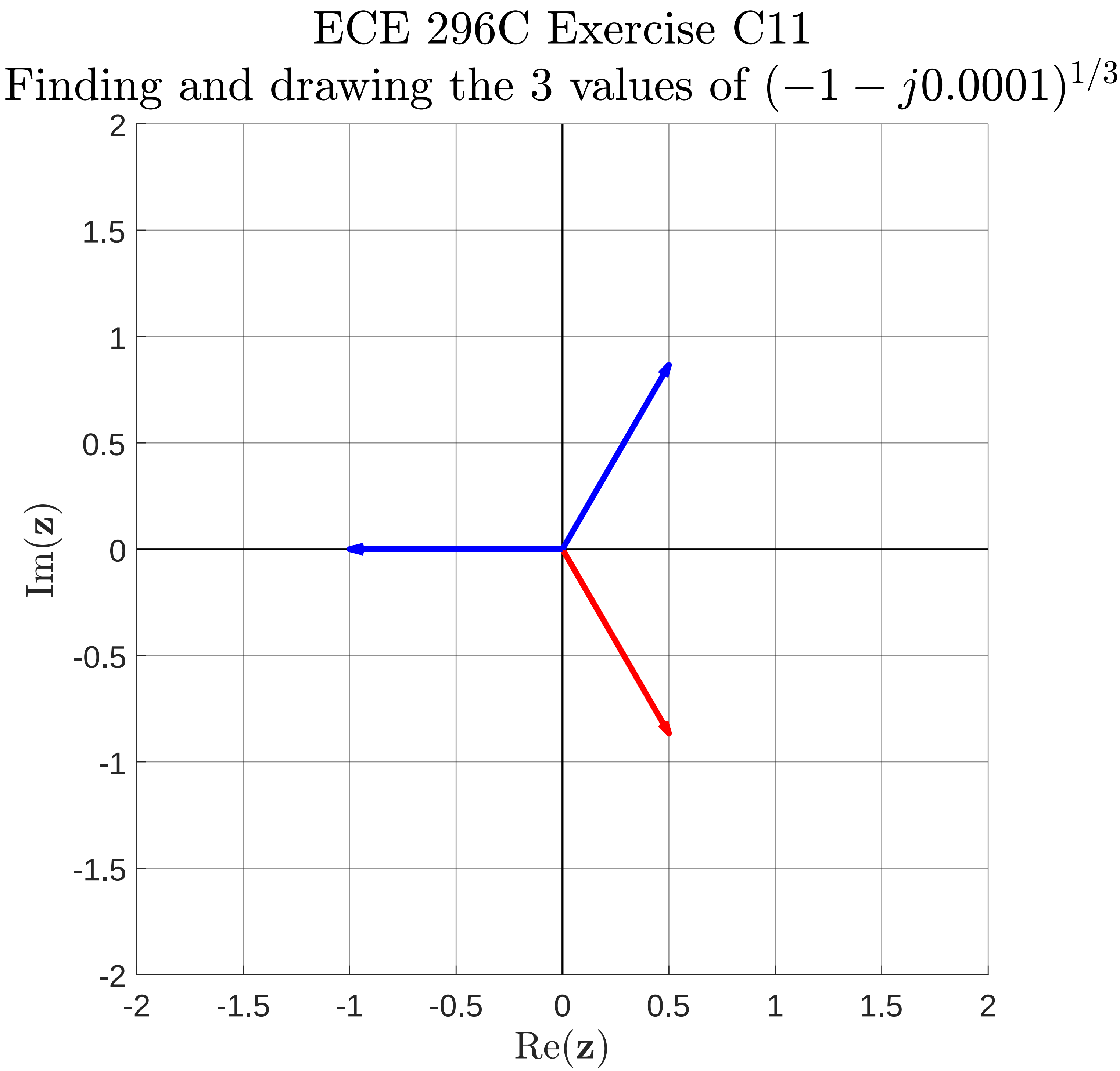
ECE 296C Exercise C11
Finding and drawing the 4 values of $(2 - j5)^{1/4}$



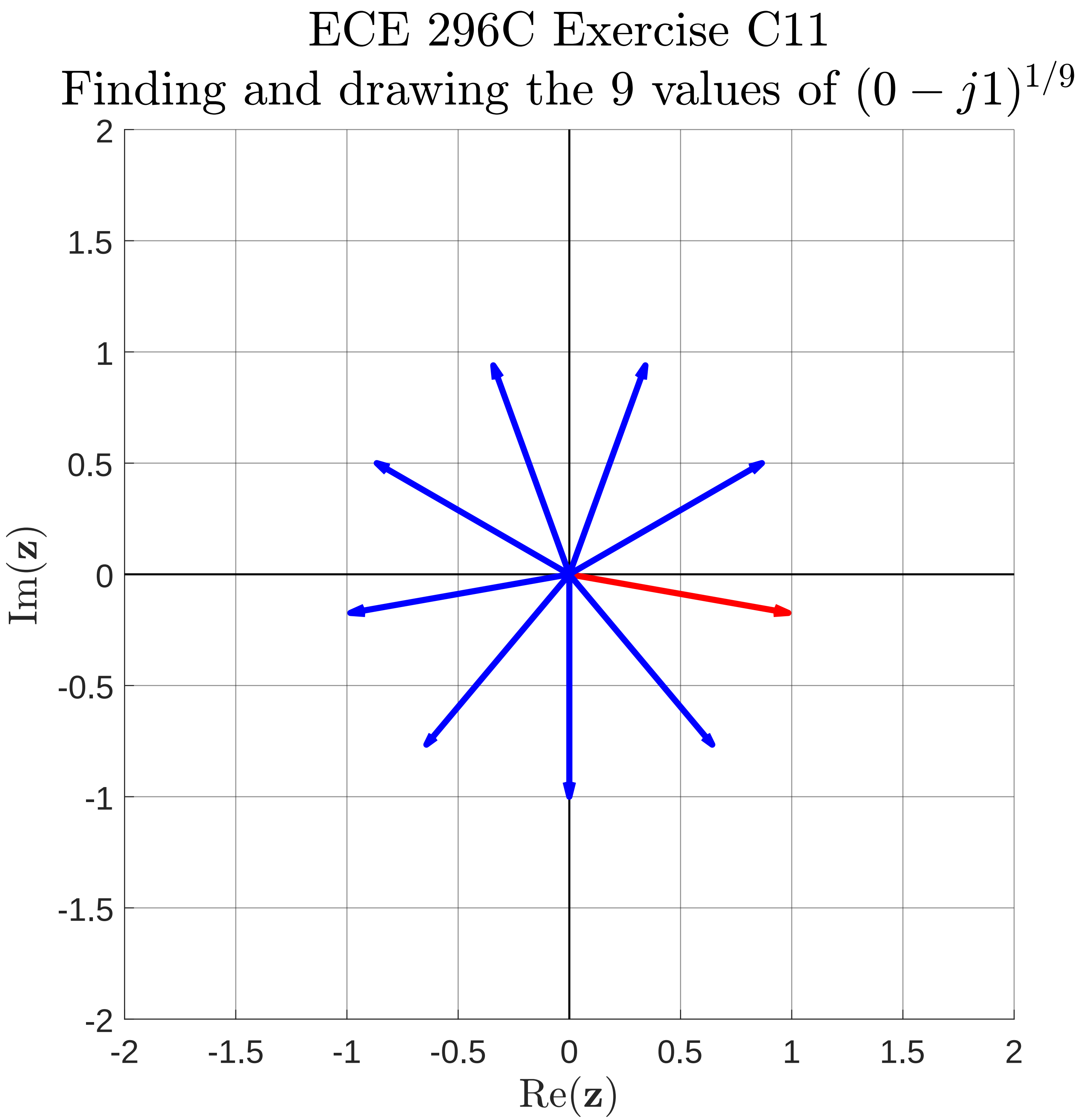
ECE 296C Exercise C11

Finding and drawing the 3 values of $(-1 + j0)^{1/3}$





ECE 296C Exercise C11
Finding and drawing the 3 values of $(-1 - j0.0001)^{1/3}$
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Im($\mathbf{z}$)
Re($\mathbf{z}$)


ECE 296C Exercise C11
Finding and drawing the 9 values of $(0 - j1)^{1/9}$
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
$\mathrm{Im}(\mathbf{z})$
$\mathrm{Re}(\mathbf{z})$

ECE 296C Exercise C11

Finding and drawing the 7 values of $(5 + j15)^{1/7}$