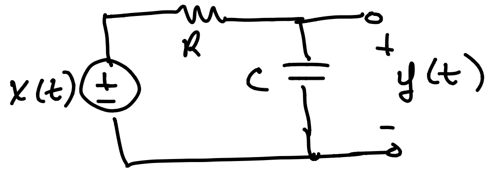
Example 2.4: Find the impulse response $h(t)$ of the RC circuit shown below with $y(0^{-}) = 0$.



Solution: Kirchhoff's voltage law gives

$$RC\frac{dy(t)}{dt} + y(t) = x(t), \tag{E1}$$

or

$$\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t). \tag{E2}$$

The impulse response satisfies

$$\frac{dh(t)}{dt} + \frac{1}{\tau_c}h(t) = \frac{1}{\tau_c}\delta(t), \tag{E3}$$

where $\tau_c = RC$ is the time constant.

A zero initial condition means that

$$h(t) = 0,\ t < 0. \tag{E4}$$

At any time $t > 0$, (E3) becomes

$$\frac{dh(t)}{dt} + \frac{1}{\tau_c}h(t) = 0. \tag{E5}$$

The solution to the homogeneous differential equation (E5) is

$$h(t) = Ce^{-t/\tau_c},\ t > 0 \tag{E6}$$

for any constant C.

At any time $t \neq 0$, $h(t)$ is given by (E4) and (E6). Here, only a single value of the constant C satisfies (E3) at all t, including $t = 0$. Integrate (E3) from $t = 0^-$ to $t = 0^+$:

$$\int_{0^-}^{0^+} \frac{dh(t)}{dt}dt + \int_{0^-}^{0^+} \frac{1}{\tau_c}h(t)dt = \int_{0^-}^{0^+} \frac{1}{\tau_c}\delta(t)dt, \tag{E7}$$

or

$$h(0^+) - h(0^-) = C - 0 = \frac{1}{\tau_c}. \tag{E8}$$

Finally, we find

$$h(t) = \frac{1}{\tau_c}e^{-t/\tau_c}u(t),\ \tau_c = RC. \tag{E9}$$