Announcements, Goals, and Reading

Announcements:

- HW03 due Tuesday October 4th, 11:59 pm on Mastering Physics
- Midterm 1: Thursday 10/20, 7-9PM

Goals for Today:

- Projectile Motion
- Relative Motion
- Circular Motion

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 4: Kinematics in 2D

Midterm 1 October 20th, 7-9PM

- **Covers Chapters 1-5* from Knight textbook, Homework 1-5***
- **Key topics: Motion, Acceleration, Position, Velocity, Kinematics, Projectile Motion, Circular Motion, and Forces*.** *No questions about sig. figs or relative motion.*
- Location depends on 1st letter of your last name:
  - **HAS20 – Last Name A-F**
  - **HAS124 - Last Name G-H**
  - **ISB135 - Last Name I-M**
  - **ILCN151 - Last Name N-T**
  - **HAS126 - Last Name U-Z**
  - **HAS138 - Reduced distraction / Extra time accommodation**
  - **Online-only students (UWW/FLEX sections): You will be contacted about details this week.**
  - ***If you have extra time accommodations, please take the exam in HAS 20. I will come at the end to proctor the extra time. You can also take the exam with Disability Services. If you need other disability accommodations, please contact me.***
- **Allowed a Calculator and an 8.5”×11” reference sheet with handwritten notes on both sides.** Be sure to bring a calculator as there are never enough spares.
- ~25 Multiple Choice Questions; **Bring a #2 pencil**
- Practice problems will be posted on Moodle and Mastering Physics
- SI/TA exam review sessions will be held on exam week.
- Makeup Exams: If you have a conflict with another exam, please let us know as soon as possible. Friday 10/14 will be the last day to request a makeup exam without penalty. E-mail our TA, Joanna Wuko (jwuko@physics.umass.edu) and CC me.

**Questions about Force will be limited in number, scope and complexity.*

Projectile Motion

Special case of 2D motion with constant acceleration

Acceleration due to gravity in vertical direction, no acceleration in horizontal

x = direction of horizontal motion

y = height above ground

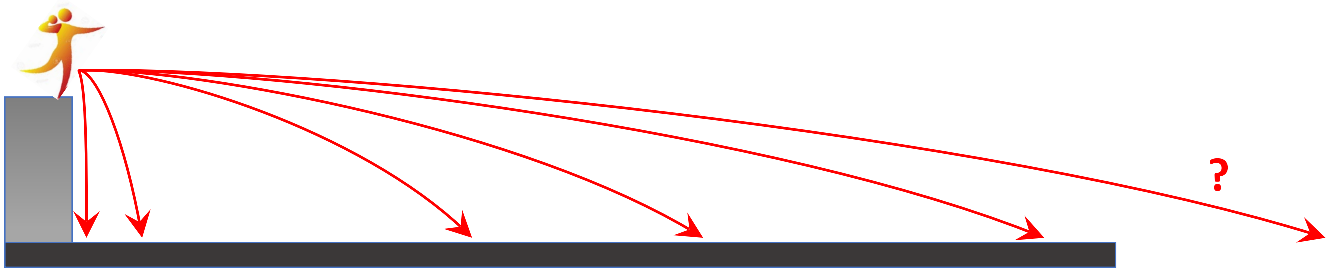
$$a_x = 0$$
$$a_y = -g$$

Equations for projectile motion

$$x(t) = x_0 + v_{0x}t$$
$$y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2$$
$$v_x(t) = v_{0x}$$
$$v_y(t) = v_{0y} - gt$$

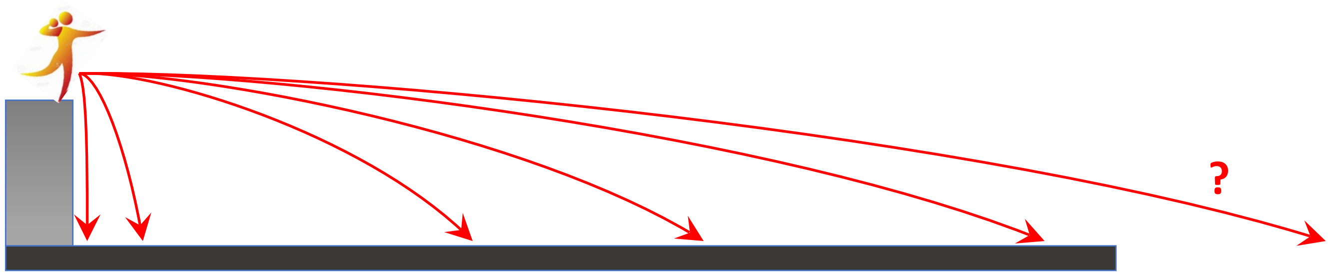
Horizontal velocity stays constant

Fast Projectile Motion: Satellites



- After 1 second, ball falls 5 m, ***independent* of initial horizontal velocity**
- Suppose we throw ball with incredible velocities – few km/second – what changes?

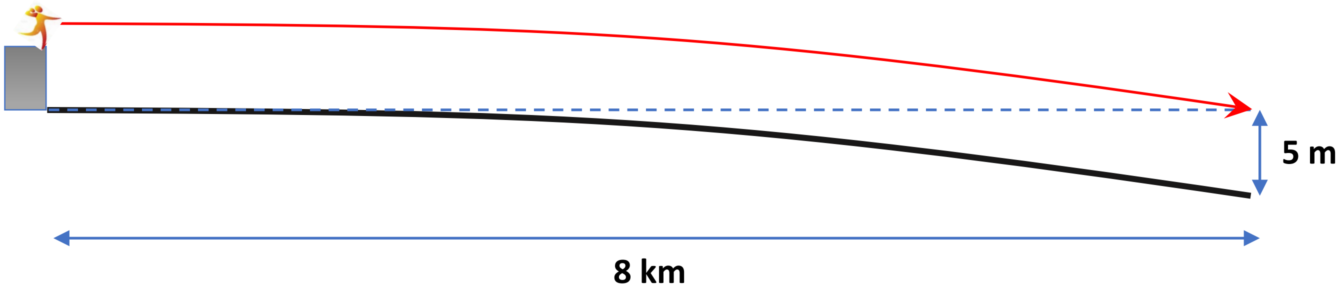
Fast Projectile Motion: Satellites



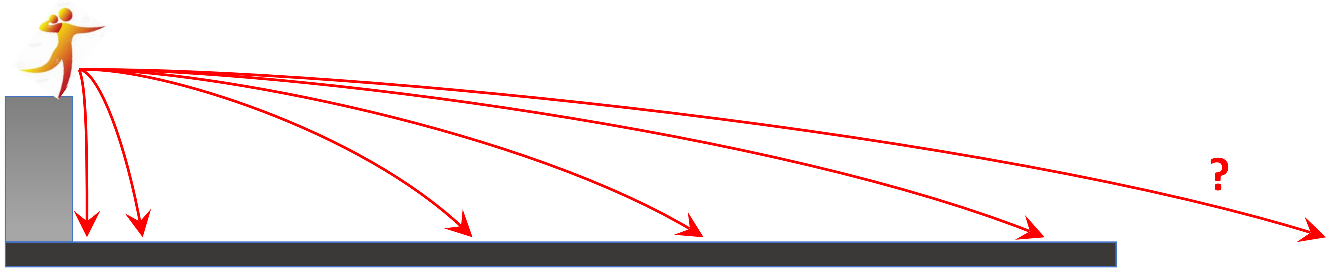
- After 1 second, ball falls 5 m, ***independent*** **of initial horizontal velocity**

- Suppose we throw ball with incredible velocities – few km/second – what changes?

- **Earths curvature**: surface falls 5 m for every 8000 m tangent to the surface



Fast Projectile Motion: Satellites



- After 1 second, ball falls 5 m, ***independent* of initial horizontal velocity**
- Suppose we throw ball with incredible velocities – few km/second – what changes?

- **Earths curvature**: surface falls 5 m for every 8000 m tangent to the surface
- If you launch an object so that it goes 8000 m in 1 second, it would fall 5 m in the same time and match the earth's surface
- **A satellite constantly *falls* while orbiting around the earth without ever crashing into it – falling distance must match earth's curvature**
- 8 km/s = 18 000 mph! At sea level air resistance would burn object – need to put satellites many miles up
- Gravity is still nearly as strong 200 km up – we usually don't put satellites up high to get away from Earth gravity

Relative Motion

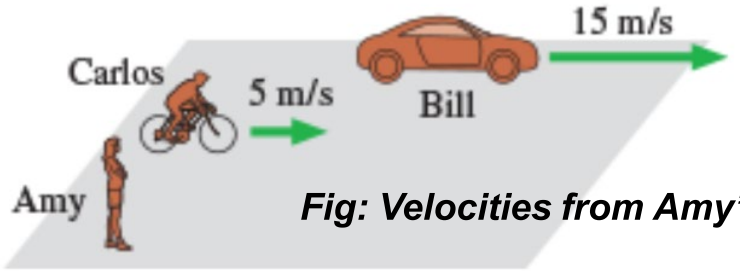
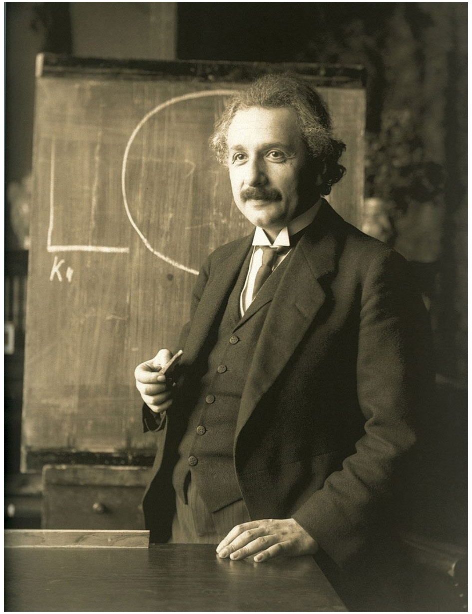


Fig: Velocities from Amy's frame of reference



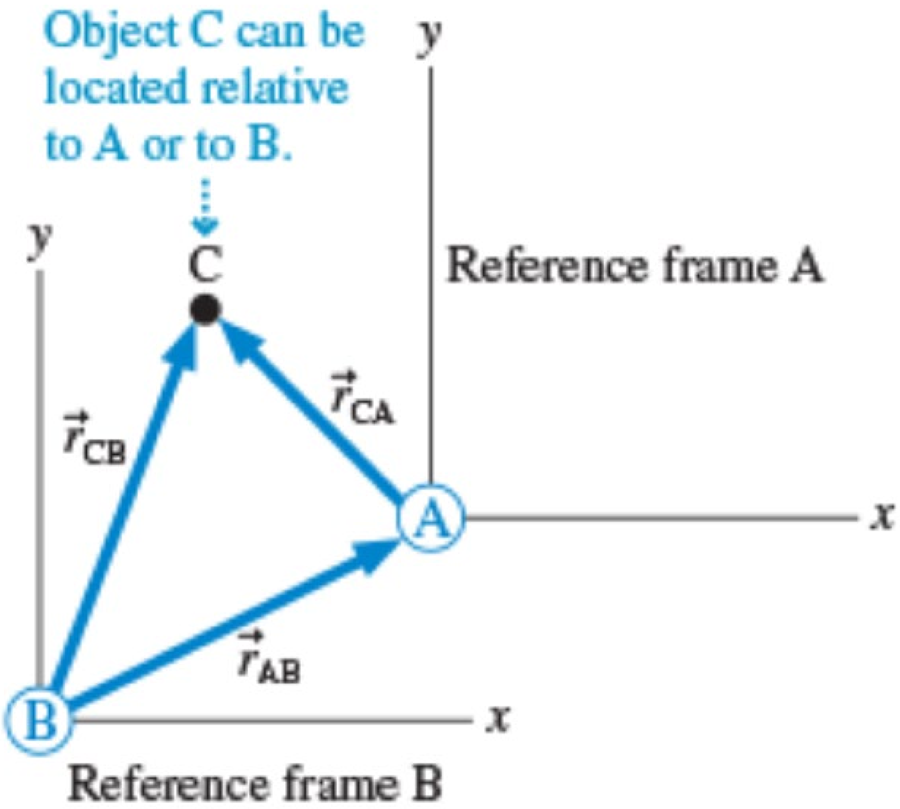
-How fast does Bill appear to be moving compared to Carlos?
-Carlos compared to Bill?
-*Amy* compared to Bill?

From Bill/Carlos' perspective, Amy is the one who is moving. Who's right?

Einstein says: They **all** are right.

***All** inertial (non accelerating) frames are totally equivalent for the performance of **all** physical experiments.*

Relative Motion



$$\boldsymbol{r}_{CB} = \boldsymbol{r}_{CA} + \boldsymbol{r}_{CB}$$

$$\vec{v}_{\mathrm{CB}} = \vec{v}_{\mathrm{CA}} + \vec{v}_{\mathrm{AB}}$$

The velocity of C relative to B is equivalent to the vector sum of the velocity of C relative to A + the velocity of A relative to B.

Relative Velocity: Why is it important?

CNN

-"Angle of attack" sensors on two Boeing 737 Max flights failed Oct 29,2018 and Mar 10,2019
-flight systems didn't know the angle of the plane relative to the air (GPS doesn't help)
-planes crashed

-Air France Flight 447: Rio de Janeiro to Paris, June 1, 2009, Airbus A330
-inconsistent airspeed measurements (velocity of plane relative to air, GPS doesn't help)
- crashed in Atlantic

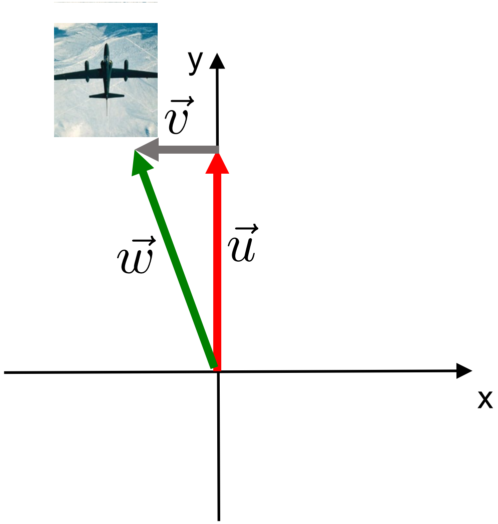
Relative Velocity

Common type of vector problem

Plane in a crosswind (or boat in a current)

A plane sets a course straight north at 600mph **relative to the air**.
Wind is blowing from the east at 100mph.

What is the resulting velocity of the plane relative to the ground?



Plane is carried along by wind, off its original course

Resulting velocity is the sum of the plane's velocity relative to the air and the wind velocity

$\vec{u} = 600\hat{j}$ Plane's velocity **relative to air**

$\vec{v} = -100\hat{i}$ Wind velocity

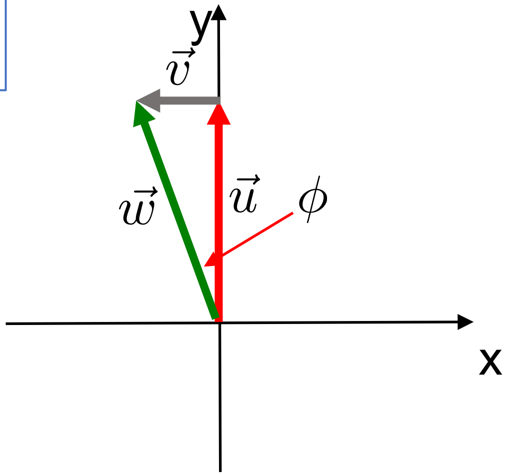
$\vec{w} = \vec{u} + \vec{v} = -100\hat{i} + 600\hat{j}$ Plane's velocity relative to ground

$$\vec{w} = \vec{u} + \vec{v} = -100\hat{i} + 600\hat{j}$$ Plane's velocity relative to ground

What are magnitude and direction of plane's resulting velocity?

$$|\vec{w}| = \sqrt{w_x^2 + w_y^2}$$

$$= \sqrt{(-100)^2 + (600)^2} = 608$$



opposite

$$\tan\phi = \frac{|\vec{v}|}{|\vec{u}|} = \frac{100}{600} = 0.167 \Rightarrow \phi = 9.5^0$$ west of north

adjacent

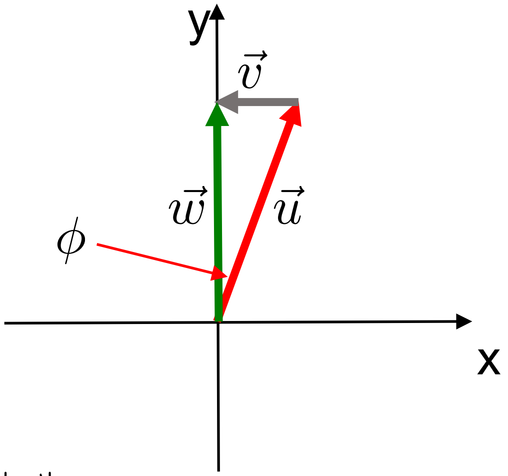
Relative Velocity

Common variant

Plane in a crosswind

A plane can fly at 600 mph (relative to air).
Wind is blowing from the east at 100 mph.

What heading should plane fly at, so that resulting velocity relative to the ground is straight north?



$\vec{u} = u_x\hat{i} + u_y\hat{j}$ Plane velocity relative to air $|\vec{u}| = 600$

$\vec{v} = -100\hat{i}$ Wind velocity

$\vec{w} = \vec{u} + \vec{v}$ Plane velocity relative to ground is straight north $\Rightarrow$ $w_x = 0$

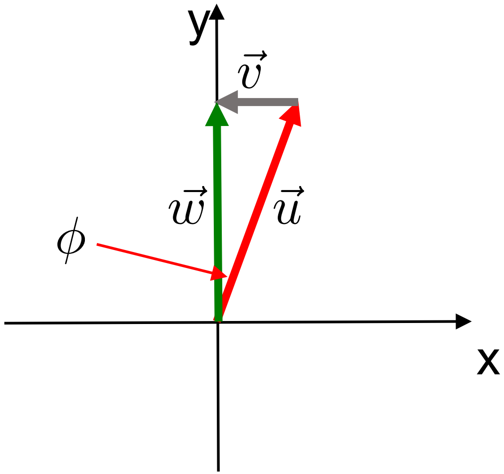
$\vec{w} = (u_x - 100)\hat{i} + u_y\hat{j}$ $\Rightarrow$ $u_x = 100$

Relative Velocity

Common variant

Plane in a crosswind

A plane can fly at 600mph (relative to air).
Wind is blowing from the east at 100mph.



What heading should plane fly at, so that resulting velocity is straight north?

$$u_x = 100 \qquad |u| = 600$$

$$\sin\phi = \frac{u_x}{|u|} = 100/600 \quad \Rightarrow \quad \phi = 9.6^0 \text{ east of north}$$

You may also do it in a more tedious way:

$$u_y = \sqrt{|u|^2 - u_x^2} = \sqrt{600^2 - 100^2} = 592$$

$$\tan\phi = \frac{u_x}{u_y} = 0.169 \quad \Rightarrow \quad \phi = 9.6^0 \text{ east of north}$$

Relative Velocity

Getty Images

Feb 9, 2020, BBC News

Experts are hailing a British Airways flight as the fastest subsonic New York to London journey.

The Boeing 747-436 reached speeds of 825 mph (1,327 km/h) as it rode a jet stream accelerated by Storm Ciara.

The four hours and 56 minutes flight arrived at Heathrow Airport 80 minutes ahead of schedule on Sunday morning.

The jet stream reached speeds of 260 mph (418 km/h) on Sunday morning, according to BBC Weather.

Despite travelling faster than the speed of sound the plane would not have broken the sonic barrier as it was helped along by fast-moving air.

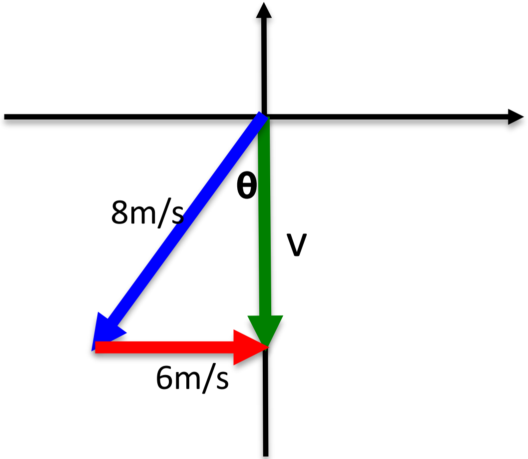
Relative to the air, the plane was travelling slower than 801mph.

Relative Motion

A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s.

A wise elder duck finally realizes that the solution is to fly at an angle to the wind.

If the ducks fly at 8.0 m/s **relative to the air**, what direction should they head in order to move directly south?
How fast will they be flying relative to the ground?



$$\sin\theta = \frac{6m/s}{8m/s} = 0.75$$

$$\theta = 49^o$$

$$v = \sqrt{(8m/s)^2 - (6m/s)^2} = 5.3m/s$$

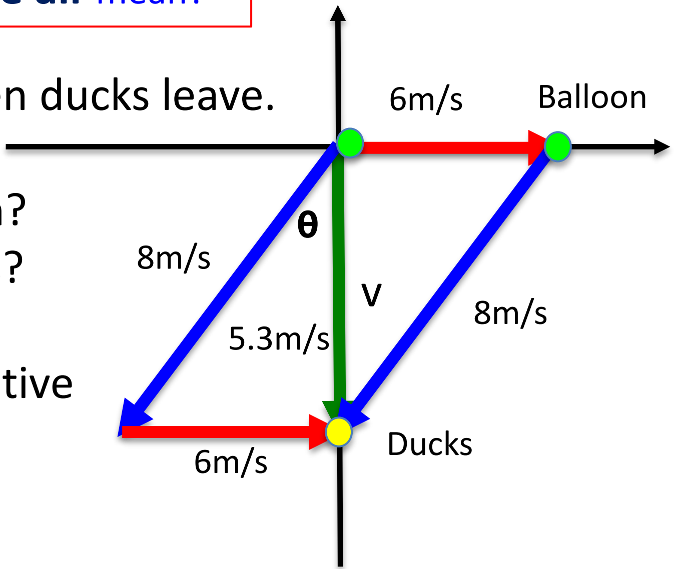
Relative Motion

A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s. They can fly at 8.0 m/s relative to air.

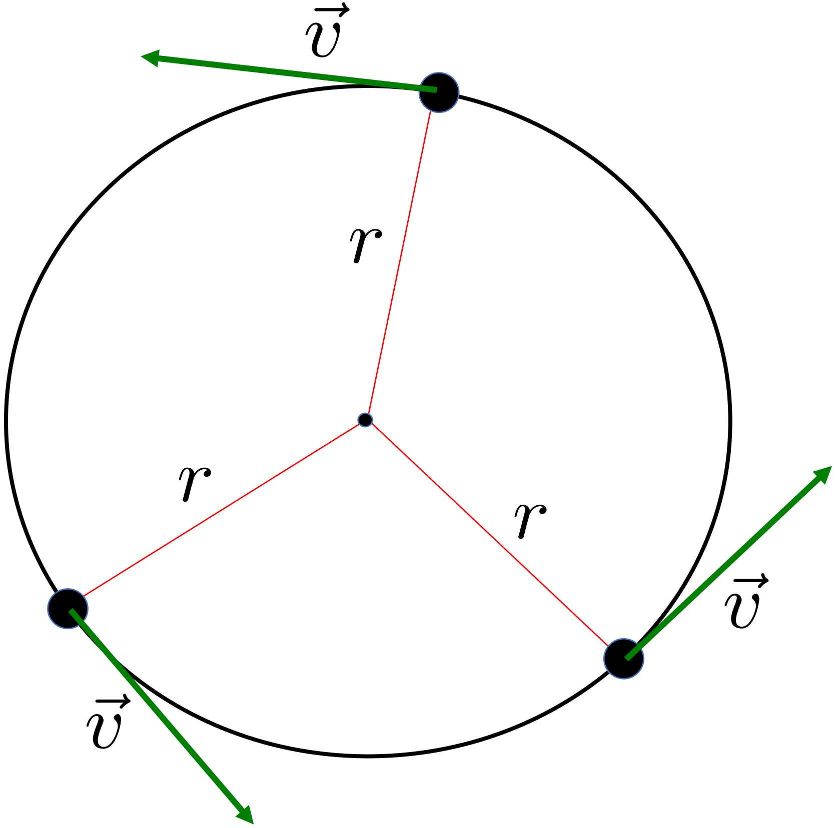
What does flying at **8.0 m/s relative to the air** mean?

Imagine let balloon go from origin when ducks leave.

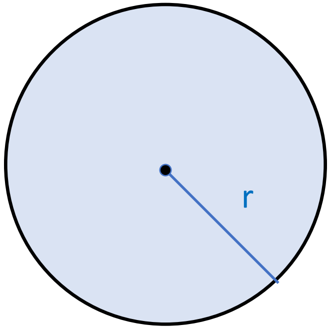
(A) 1 second later, where is the balloon?
(B) 1 second later, where are the ducks?
(C) How far apart are they?
(D) What is the speed of the ducks relative to the balloon?



Next topic: Uniform Circular Motion

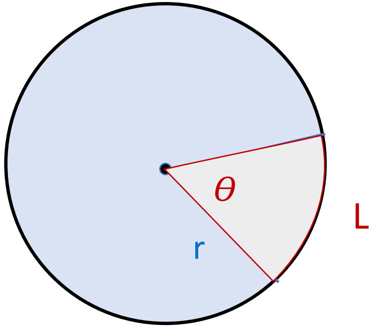


Circles – Essential Info



Circumference $C = 2\pi r$ (circle perimeter length)

Arc length $L = \theta r \Longleftrightarrow \theta = \dfrac{L}{r}$



length of portion of circle subtended by angle θ

Potentially confusing point!

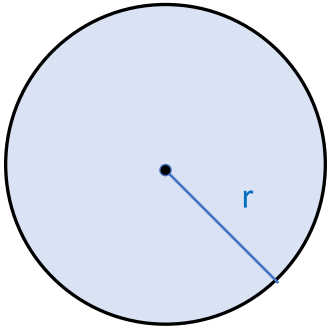
What are the dimensions of angles?

Infer from arc length that angles are dimensionless

Nonetheless, apart from **radians** (just a label attached to a number to signal that it is an angle) we also use **degrees** and **revolutions** to measure angles

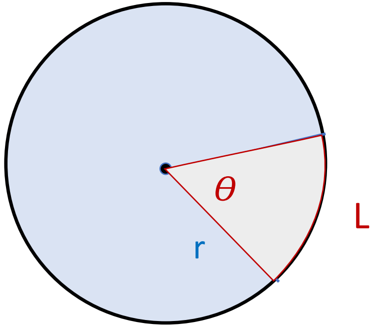
Circles – Essential Info

Circumference $C = 2\pi r$ (circle perimeter length)



Arc length $L = \theta r \Longleftrightarrow \theta = \frac{L}{r}$

length of portion of circle subtended by angle θ



2π radians gives a full circle.

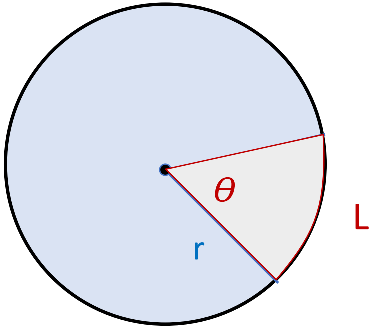
Hence circumference is arc length of full circle

Taking θ=2π in arc length reproduces circumference

For fixed angle θ ⇒ Increasing radius increases arc length, and vice-versa

Important to keep this in mind

Circles – Essential Info



3 Units for angles

Radians, Degrees, and Revolutions

360 degrees = 2π radians = 1 revolution

Example: A turntable spins at 33.3 revolutions per minute (rpm).

How many radians per second is this?

1 revolution is going one full circle around the center

$$\left(33.3\frac{rev}{min}\right)\left(\frac{2\pi rad}{1rev}\right)\left(\frac{1min}{60s}\right)$$

$$= 3.49rad/s$$

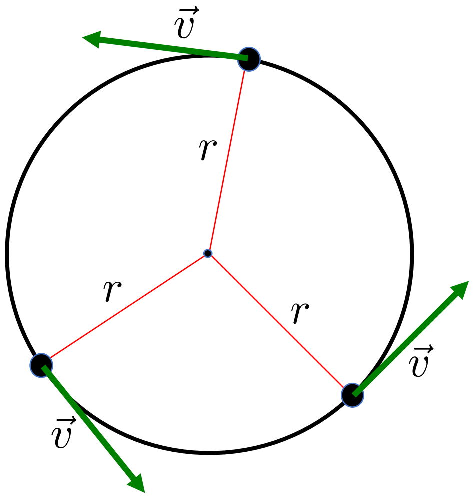
Uniform Circular Motion

Particle moving in a circular path with constant speed

Magnitude of velocity stays constant, but direction changes

Particle is accelerating!

Come back to this



Characterizing uniform circular motion…

Radius ➡ r

Period ➡ T time required for one full revolution $= \dfrac{\text{Total Time}}{\text{Total Number of Revolutions}}$

Speed ➡ $v = \dfrac{\text{circumference}}{\text{period}} = \dfrac{2\pi r}{T}$

$|\vec{v}| = v$ Wherever particle is on circle

Example

Rotating cylinder

Revolutions per Minute

Diameter 4.0 cm

Turns at 2400 rpm = 2400 turns in 60 seconds

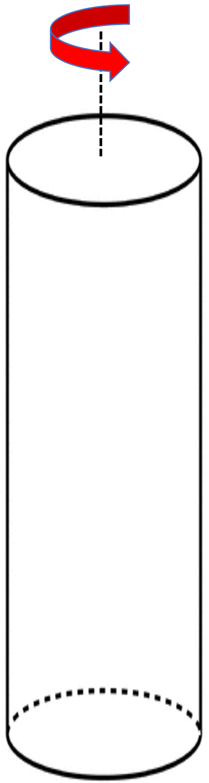
Find the speed of a point on its surface?

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{T}$$

Radius r = .02m

Period $T = \frac{60s}{2400} = 0.025s$

Speed $v = \frac{2\pi(.02m)}{(.025s)} = 5.0m/s$



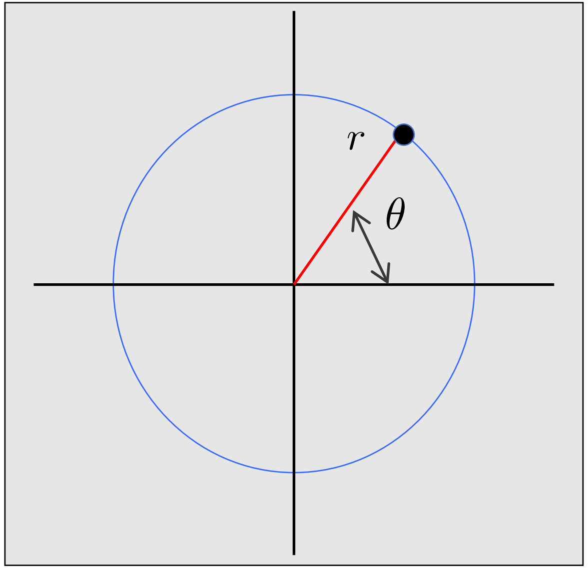
Particle on a circular path

Also talk about more general motion for particle constrained to move on a circle

Not necessarily at a constant rate

Define angle θ with respect to positive x-axis

Particle path can be described by angle as a function of time: $\theta(t)$



Position of particle described by a single function of time

Very similar to 1D motion described by x(t)

Introduce angular analogues of velocity and acceleration

How fast is particle going around circle at any point in time?
Is it speeding up or slowing down?

Pose similar problems as in 1D case

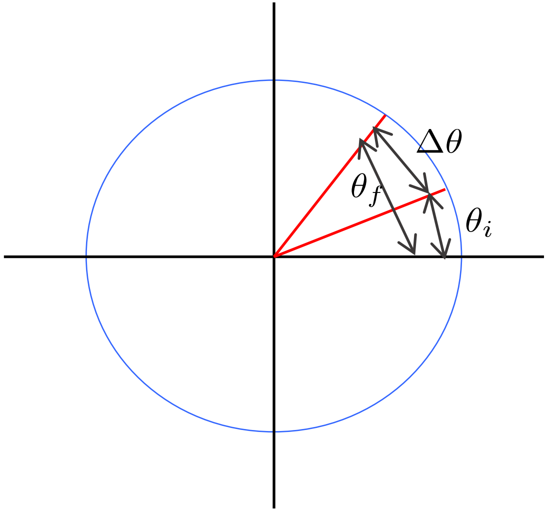
Call these "angular velocity" and "angular acceleration"

Particle on a circular path

Particle path can be described by angle as a function of time $\theta(t)$

Develop "angular velocity" as rotational analogue of linear velocity

Between times t_i and t_f
angle of particle changes from θ_i to θ_f



Angular displacement $\Delta\theta = \theta_f - \theta_i$

Change in time $\Delta t = t_f - t_i$

Analogous to definition of average linear velocity as displacement over change in time

Average angular velocity:

$$\omega_{avg} = \frac{\text{angular displacement}}{\text{change in time}} = \frac{\Delta\theta}{\Delta t}$$

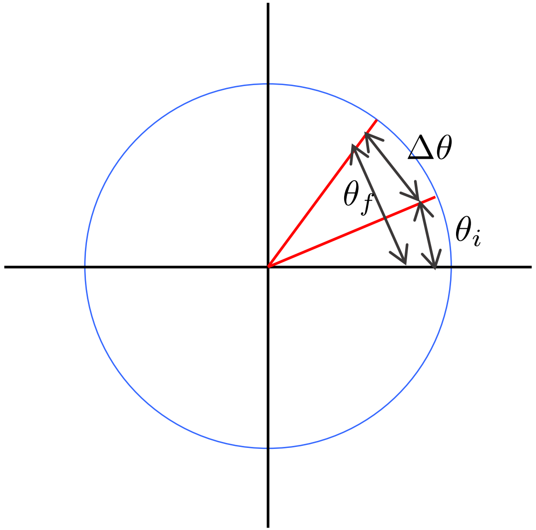
Particle on a circular path

$$\theta(t)$$

Between times t_i and t_f
Angle of particle changes from θ_i to θ_f

Angular displacement $\Delta\theta = \theta_f - \theta_i$

Change in time $\Delta t = t_f - t_i$



Average angular velocity

$$\omega_{avg} = \frac{\text{angular displacement}}{\text{change in time}} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity

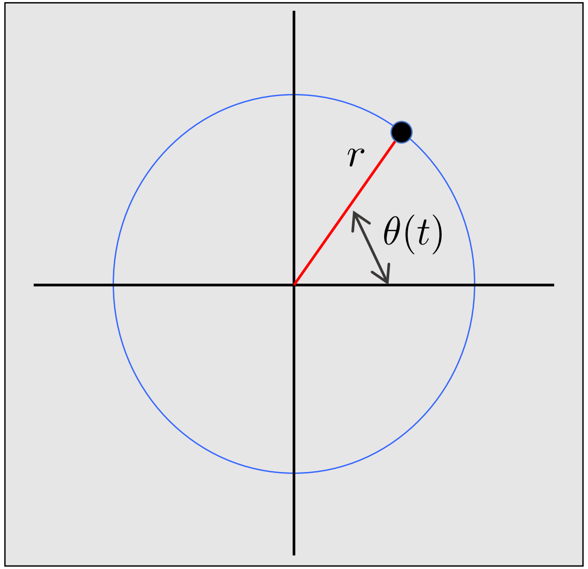
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Analogous to limiting process that defined instantaneous linear velocity

Think of circular motion as like 1D motion, but with position on a circle instead of a line

Particle on a circular path

$\theta(t)$



Instantaneous angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

What are units of angular velocity?

Angles are actually dimensionless

Not length, mass or time!

Change in arc length for a certain change in angle is proportional to radius of circle

Take "units" of angle = radians

Units of angular velocity = radians/second

1 revolution of full circle = 2π radians
1 radian ~ 57.3 degrees

Example…

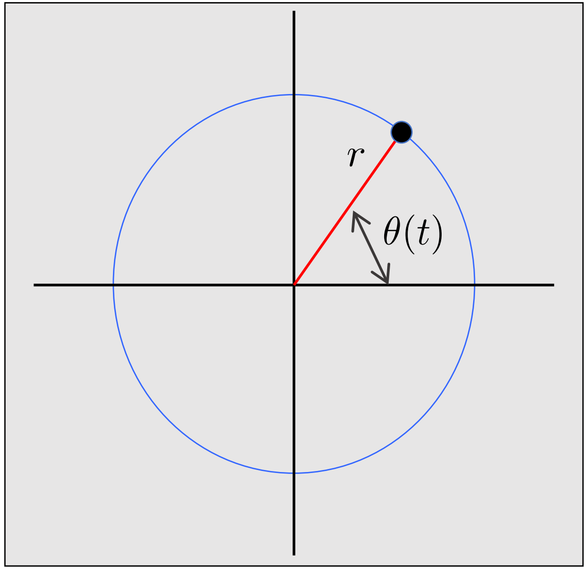
Particle goes around circle 1 time per each second

$$\omega = 2\pi \,\mathrm{rad}/s$$

Particle on a circular path $\theta(t)$

Often angular velocities are given in revolutions/second or revolutions/minute (rpm)

Convert these to radians/second

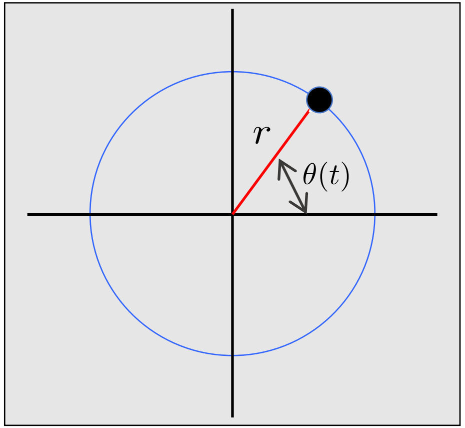


$$17\,rev/s = (17\,rev/s)\left(\frac{2\pi\,rad}{1\,rev}\right)$$

$$= 34\pi\,rad/s = 110\,rad/s$$

$$1.00\,RPM = \left(1\frac{rev}{min}\right)\left(\frac{2\pi\,rad}{1\,rev}\right)\left(\frac{1\,min}{60\,s}\right) = 0.105\,rad/s$$

Particle on a circular path $\theta(t)$



Define **angular acceleration**

Rate of change of angular velocity

Analogous to ordinary acceleration in 1D

ω_i Initial angular velocity at time t_i

ω_f Final angular velocity at time t_f

$$\Delta\omega = \omega_f - \omega_i$$ Change in angular velocity

$$\Delta t = t_f - t_i$$ Change in time

Average angular *acceleration*

$$\alpha_{avg} = \frac{\text{change in angular velocity}}{\text{change in time}} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular *acceleration*

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

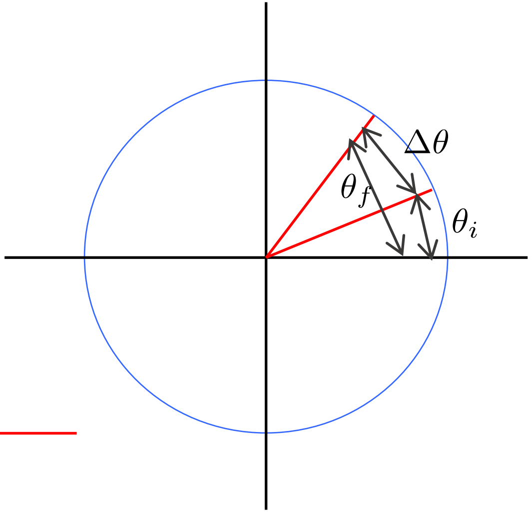
Particle on a circular path

$$\theta(t)$$

Between times t_i and t_f
angle of particle changes from θ_i to θ_f

Angular displacement $$\Delta\theta = \theta_f - \theta_i$$

Change in time $$\Delta t = t_f - t_i$$



Average angular velocity:

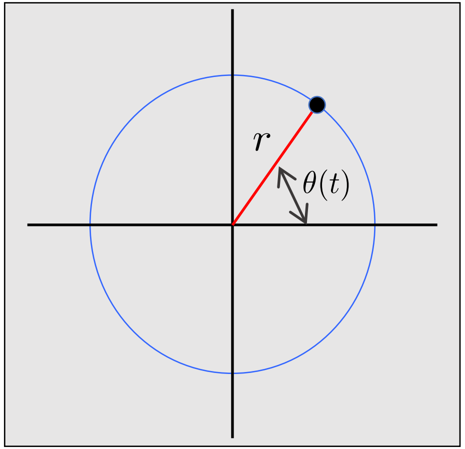
$$\omega_{avg} = \frac{\text{angular displacement}}{\text{change in time}} = \frac{\Delta\theta}{\Delta t}$$

Particle goes around circle once per second:
ω = 2π rad/s
ω = 360 degrees/s
ω = 1 rev/s

Instantaneous angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Particle on a circular path $\theta(t)$



Define **angular acceleration**

Rate of change of angular velocity

Analogous to ordinary acceleration in 1D

$t_i \longrightarrow \omega_i$ Initial angular velocity

$t_f \longrightarrow \omega_f$ Final angular velocity

$$\Delta\omega = \omega_f - \omega_i$$ Change in angular velocity

$$\Delta t = t_f - t_i$$ Change in time

Average angular *acceleration*

$$\alpha_{avg} = \frac{\text{change in angular velocity}}{\text{change in time}} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular *acceleration*

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

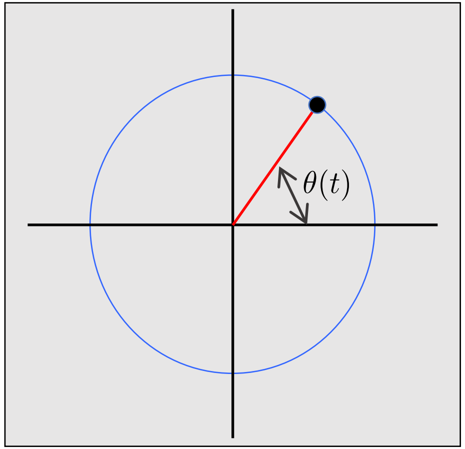
Very important analogy between linear motion and circular motion

Circular motion with constant angular acceleration

Analogous to linear motion with constant acceleration

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = v_0 + at$$



$$\theta(t) = \theta_0 + \omega_0 \, t + \frac{1}{2} \alpha \, t^2$$

$$\omega(t) = \frac{d\theta}{dt} = \omega_0 + \alpha \, t$$

$$\alpha = \frac{d\omega}{dt} = \alpha \quad \checkmark$$

θ_0	Angular position at t=0
ω_0	Angular velocity at t=0
α	Constant angular acceleration

(if time permits..)

Example Problem

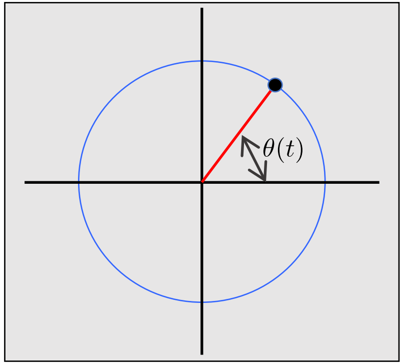
counterclockwise

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

What is the drill's angular acceleration?

Assume constant angular acceleration

$$\omega(t) = \frac{d\theta}{dt} = \omega_0 + \alpha\, t$$



Counterclockwise rotation → Θ is increasing with time

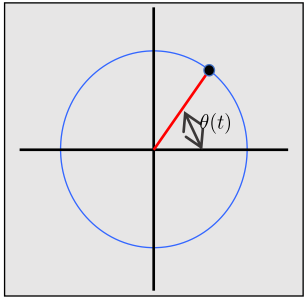
ω_0 is positive

Slows to rest → α is negative

Clockwise rotation has angular velocity $\omega<0$

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

What is the drill's angular acceleration?



$$\omega(t) = \omega_0 + \alpha\, t$$

Find ω_0 in rad/sec

$$\omega_0 = (2400 \frac{rev}{min})(\frac{2\pi\ rad}{1\ rev})(\frac{1\ min}{60\ s}) = 251.3\, rad/s$$

Find angular acceleration

$$\omega(3s) = \omega_0 + \alpha(3\, s) = 0$$

Solve for α in rad/s²

Correct

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

$$\omega(t) = \omega_0 + \alpha\, t$$

Easiest to redo part A $\longrightarrow$ Express angular acceleration in rev/s^2 rather than rad/s^2

$$\omega_0 = (2400\frac{rev}{min})(\frac{1\,min}{60\,s}) = 40.0\,rev/s$$

$$\omega(T) = 0 \Rightarrow \alpha = -\frac{\omega_0}{T} = -\frac{40.0\,rev/s}{3.00\,s} = -13.3\,rev/s^2$$

$$T = 3.00\,s$$

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

Now look at angular position at T=3.00s

$$\theta(t) = \theta_0 + \omega_0\, t + \frac{1}{2}\alpha\, t^2$$

$$\omega_0 = 40.0\, rev/s$$

$$\alpha = -13.3\, rev/s^2$$

$$\theta_0 = 0$$

Don't really care about angle at t=0

Plug in to get…

$$\theta(3s) = (40.0\, rev/s)(3\, s) + \frac{1}{2}(-13.3\, rev/s^2)(3\, s)^2 = 60\, rev$$

If we had found θ(3s) in radians…

$\longrightarrow$ Divide by 2π to convert to revolutions

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

Plot results…

Probably should have started by drawing a graph!

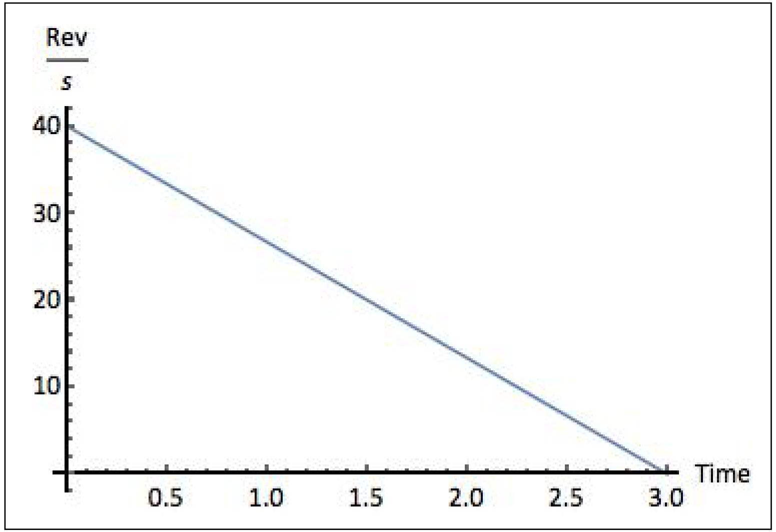
Plot angular velocity as a function of time first

Straight line corresponding to constant angular acceleration

Can also find number of revolutions before stopping from area under angular velocity curve

(40 rev/s)(3 s)/2 = 60 rev ✓

$$\omega(t) = 40.0\,rev/s + (-13.3\,rev/s^2)t$$



A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

Plot results…

$$\theta(t) = (40.0\, rev/s)t + \frac{1}{2}(-13.3\, rev/s^2)t^2$$

Now plot angular position as function of time

Angular velocity is slope of angular position graph

Slope gradually decreases to zero as drill slows down