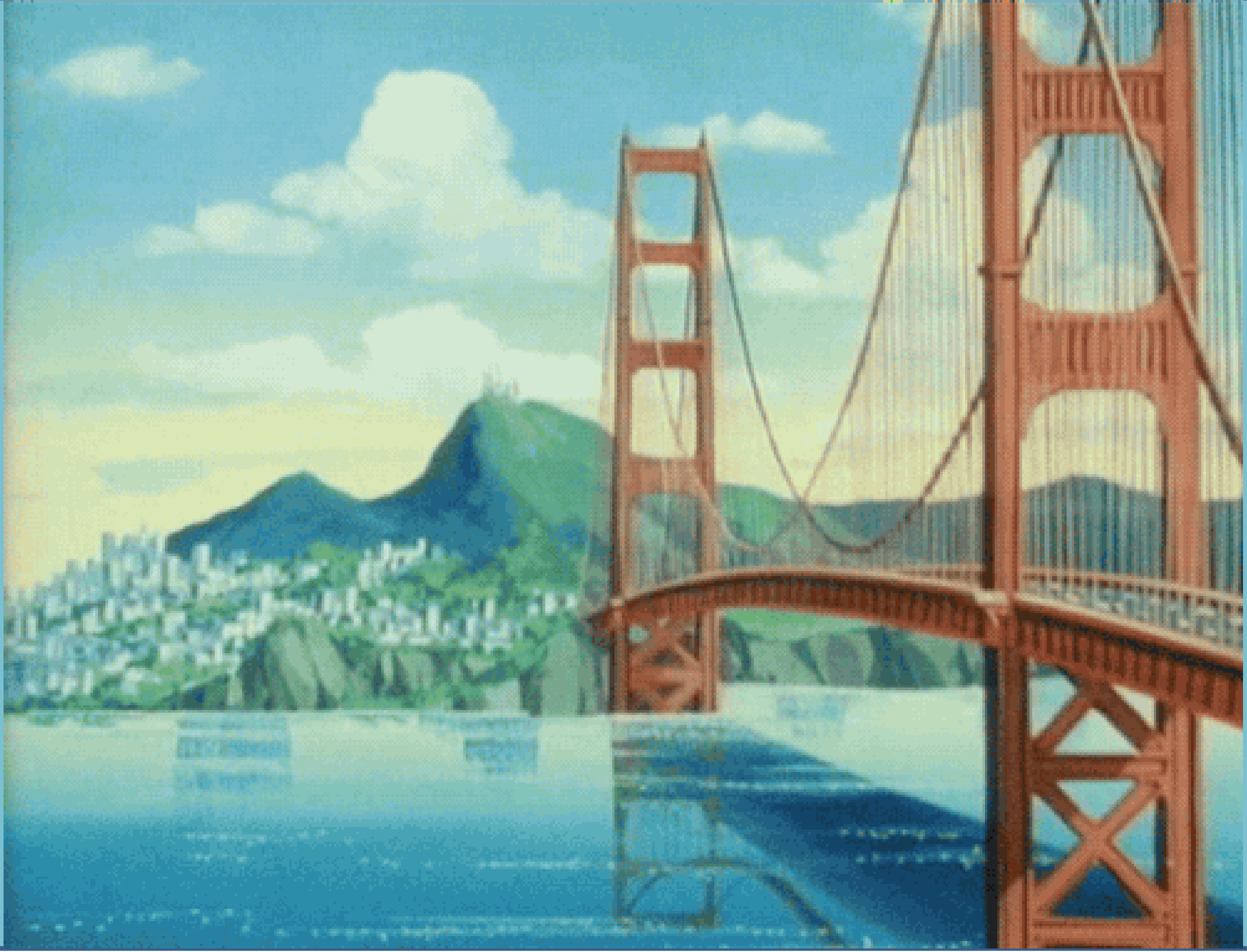


Announcements, Goals, and Reading

Announcements:

- HW05 due Tuesday 10/18. *(These topics will be on the midterm…)*
- Midterm 1: Thursday 10/20, 7-9PM
- SI Sam's review session: Wednesday, October 19th, 7-9pm, Thompson Hall 106
- TA Joanna's review session: Monday, October 17th, 5-7pm, Hasbrouck 113
- Prof. Hamilton review lecture: Wednesday, October 19th during the normal class time

Goals for Today:

- Forces
- Newton’s Second Law

Reading (Physics for Scientists and Engineers 4/e by Knight)

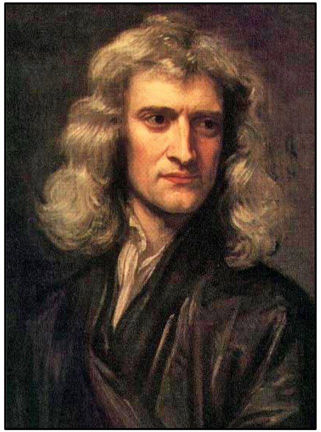
- Chapter 6: Dynamics I: Motion along a Line

Midterm 1 October 20th, 7-9PM

- **Covers Chapters 1-5* from Knight textbook, Homework 1-5***
- **Key topics: Motion, Acceleration, Position, Velocity, Kinematics, Projectile Motion, Circular Motion, and Forces*.** *No questions about sig. figs or relative motion.*
- Location depends on 1st letter of your last name:
  - **HAS20 – Last Name A-F**
  - **HAS124 - Last Name G-H**
  - **ISB135 - Last Name I-M**
  - **ILCN151 - Last Name N-T**
  - **HAS126 - Last Name U-Z**
  - **HAS138 - Reduced distraction / Extra time accommodation**
  - **Online-only students (UWW/FLEX sections): You will be contacted about details this week.**
  - *If you have extra time accommodations, please take the exam in HAS 138. I will come at the end to proctor the extra time. You can also take the exam with Disability Services. If you need other disability accommodations, please contact me.*
- **Allowed a Calculator and an 8.5”×11” reference sheet with handwritten notes on both sides.** Be sure to bring a calculator as there are never enough spares.
- ~25 Multiple Choice Questions; **Bring a #2 pencil**
- <u>A practice exam is now available on Moodle.</u>
- SI/TA exam review sessions will be held on exam week. Next Wed will be a review lecture.
- Makeup Exams: Our TA, Joanna Wuko (jwuko@physics.umass.edu) has contacted you with the time/date/location.

**Questions about Force will be limited in number, scope and complexity.*

Newton’s Laws of Motion

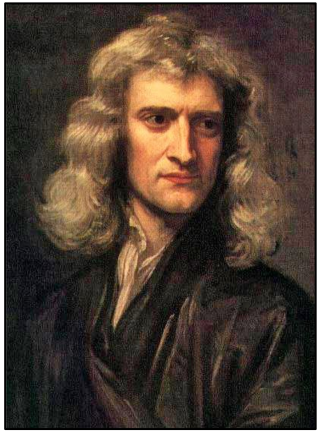


1st law – In the absence of a net external force, an object at rest will remain at rest, and an object in motion will remain in motion with a constant velocity.

2nd law – The acceleration of an object is proportional to the net external force acting on it, and is inversely proportional to its mass.

3rd law – When two objects interact, they exert equal and opposite forces on each other

2nd law – The acceleration of an object is proportional to the net external force acting on it, and is inversely proportional to its mass.



The acceleration of an object with mass m will be

$$\vec{a} = \frac{1}{m}\vec{F}_{net}$$

More commonly written as

$$\vec{F}_{net} = m\vec{a}$$

And still more commonly as

$$\boxed{\vec{F} = m\vec{a}}$$

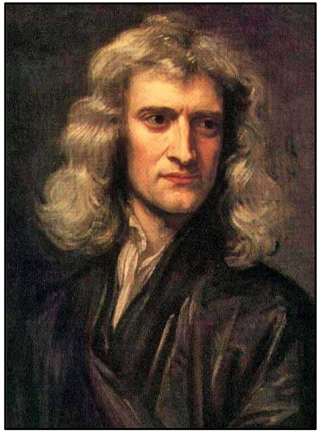
Where we tacitly understand that the force on the left hand side is the net force.

Think of the forces as being the "causes" and acceleration is the mass-dependent "effect"

Free Body Diagrams

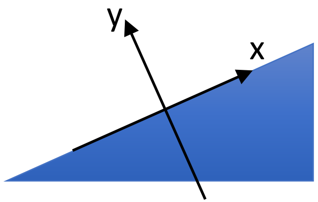
$$\vec{F}_{net} = m\vec{a}$$

Systematic way to attack force & acceleration problems



1. Isolate the object being analyzed and draw all ***forces that act ON the object***.

2. Draw conveniently oriented coordinate axes and find components of forces along these axes.



Example – with inclined plane, orient axes parallel and perpendicular to the inclined plane

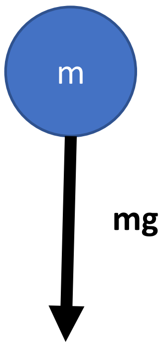
3. Apply 2nd law. Determine acceleration along each of the axes.

Alternatively - may be told acceleration, and need to determine one of the forces.

Some problems will also require free body diagrams for more than one object

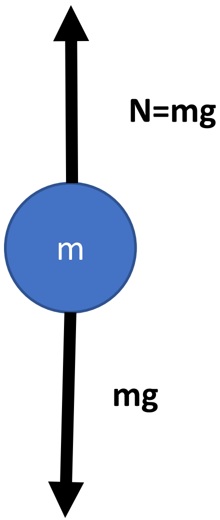
Simple examples..

Free Body Diagram: Object in Free Fall



$$\boldsymbol{a} = -g\hat{\boldsymbol{y}}$$

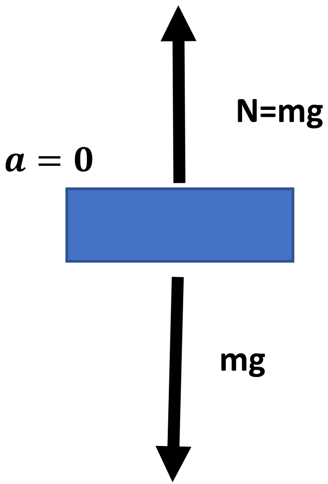
Free Body Diagram: Student Sitting At Desk



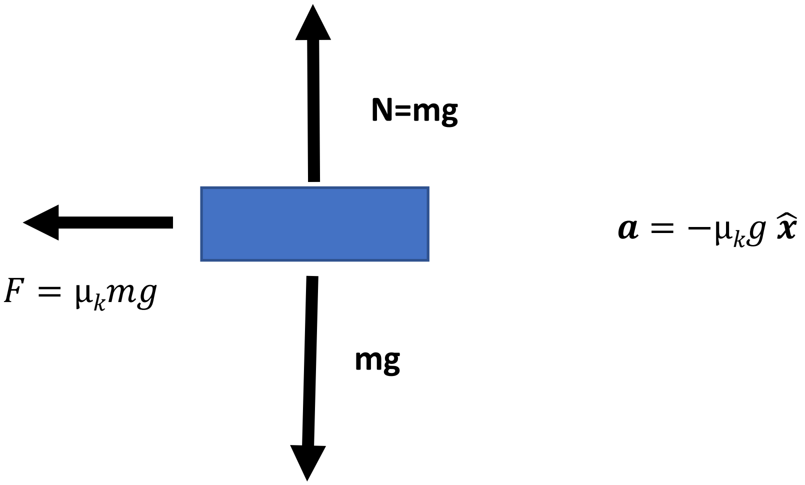
$$\boldsymbol{a} = \mathbf{0}$$

Simple examples..

Free Body Diagram: Puck Sliding on Ice

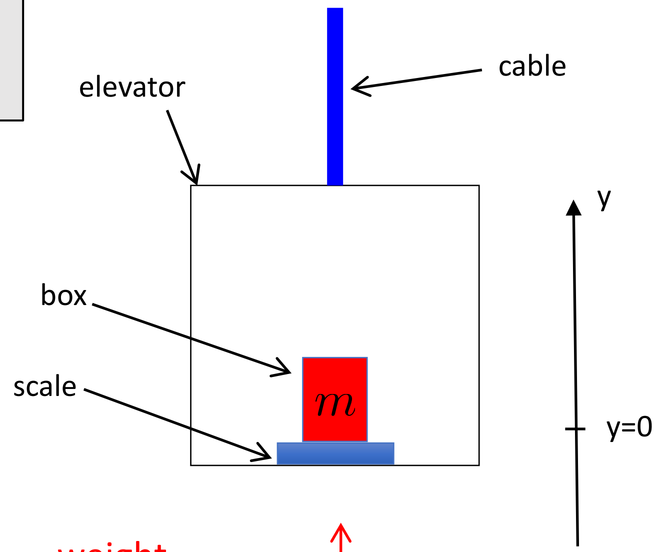


Free Body Diagram: Puck Sliding on Dirt



Note: Free body diagrams don't include velocities, but you can imagine the puck is moving this way---------->

Normal force problem:
but now object accelerating



How much does box of mass m weigh in an elevator when it is…

Accelerating up? a>0
Accelerating down? a<0
Moving at constant speed? a=0

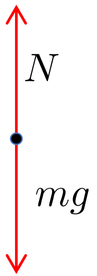
Key points

Scale exerts normal force on box that holds it up

weight

Reading on scale equals this normal force

Draw free body diagram



2nd law ➡ $N - mg = ma$

➡ $N = mg + ma$ ➡

Heavier for a>0
Lighter for a<0
Weight unchanged for a=0

Example of Free Body Diagram

$$\vec{F}_{net} = m\vec{a}$$

A particle of mass 3 kg, lying on the frictionless surface, is acted on by two forces parallel to the ground.

The first has magnitude 10 N and points straight north

The second has magnitude 15 N and points 35° east of south

Find the acceleration of the particle.

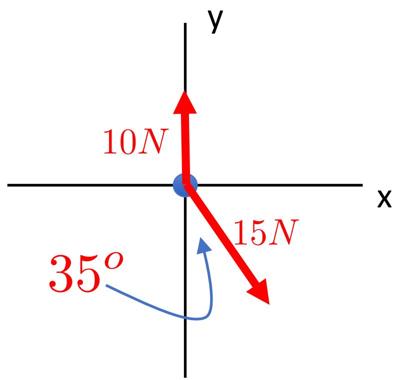
$$\vec{F}_1 = (10N)\hat{j}$$

$$\vec{F}_2 = (15N)\sin 35^o\,\hat{i} - (15N)\cos 35^o\,\hat{j}$$

$$= 8.6N\,\hat{i} - 12.3N\,\hat{j}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$= 8.6N\,\hat{i} - 2.3N\,\hat{j}$$



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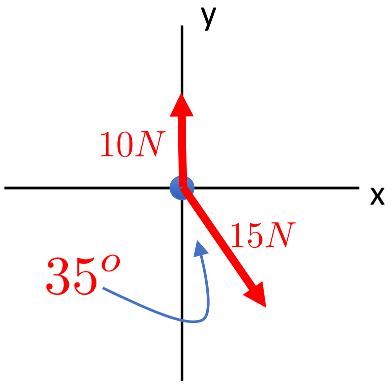


$$\vec{F}_{net} = 8.6N\,\hat{i} - 2.3N\,\hat{j}$$

$$\vec{a} = \frac{1}{m}\vec{F}_{net}$$

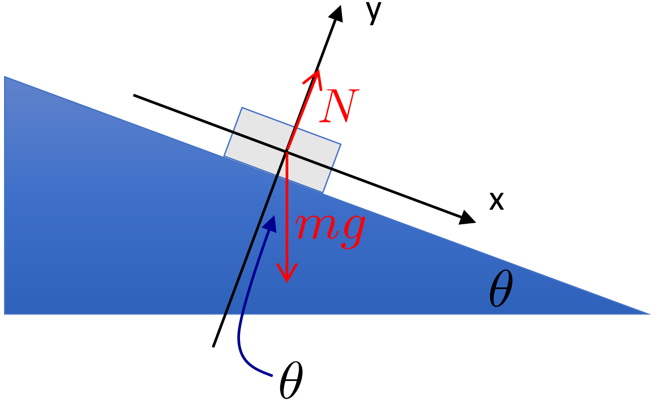
$$= \frac{1}{3kg}(8.6N\,\hat{i} - 2.3N\,\hat{j})$$

$$= (\underset{\substack{\uparrow \\ a_x}}{2.9m/s^2})\,\hat{i} + (\underset{\substack{\uparrow \\ a_y}}{-0.77m/s^2})\,\hat{j}$$

$$\boxed{1\frac{N}{kg} = 1\frac{kg\,m/s^2}{kg} = 1m/s^2}$$

Another example...

A block of mass m sits on a frictionless inclined plane at angle θ

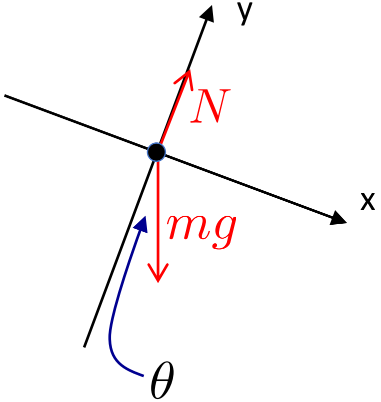


Find acceleration of block down the plane.

Find magnitude of normal force of plane on block.

Draw conveniently oriented coordinate axes

Makes things much easier



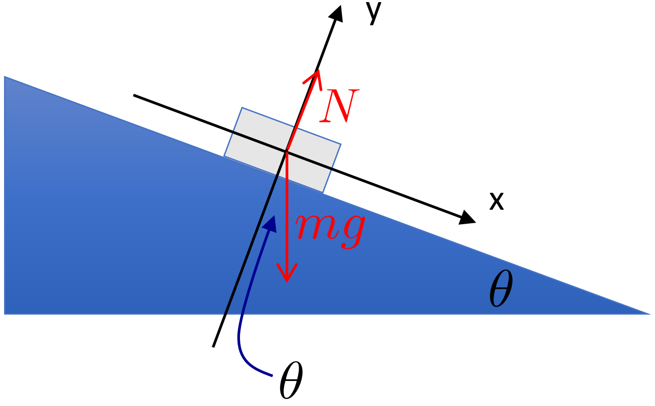
Draw in Forces

Let N be unknown magnitude of Normal force

A block of mass m sits on an inclined plane at angle θ

Find acceleration of block down the plane.

Find magnitude of normal force of plane on block.



Add forces together to find net force

$$\vec{F}_1 = N\,\hat{j}$$

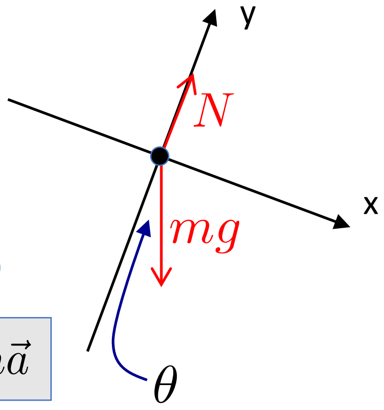
$$\vec{F}_2 = mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$= mg\sin\theta\,\hat{i} + (N - mg\cos\theta)\,\hat{j}$$ (cause)

$$= ma_x\,\hat{i} + ma_y\,\hat{j}$$ (effect)

$$\boxed{\vec{F}_{net} = m\vec{a}}$$



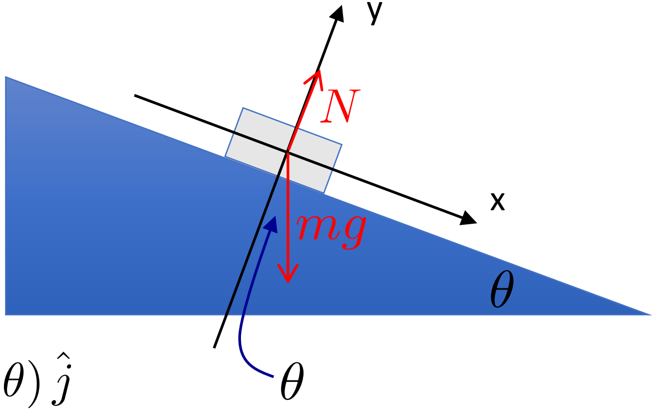
$$a_x = g\sin\theta$$ ✓

As we had assumed in kinematics discussion.

A block of mass m sits on an inclined plane at angle θ

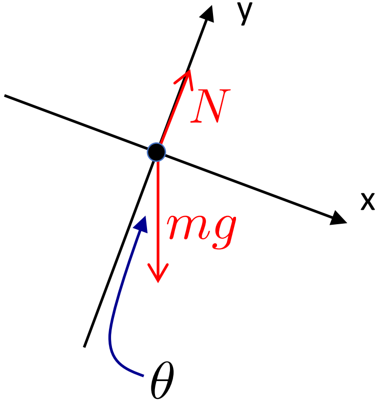
Find acceleration of block down the plane.

Find magnitude of normal force of plane on block.



$$\vec{F}_{net} = mg\sin\theta\,\hat{i} + (N - mg\cos\theta)\,\hat{j}$$
$$= ma_x\,\hat{i} + ma_y\,\hat{j}$$

➡ $a_x = g\sin\theta$ Acceleration down plane



Normal force?

Block is not lifting off or falling through plane ➡ $a_y = 0$

➡ $N = mg\cos\theta$

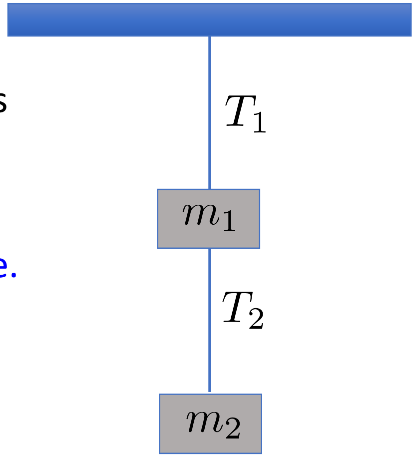
$\theta = 0$ ➡ $N = mg$ ✓ Correct for horizontal surface

Keep going with examples

Two boxes with masses m_1 and m_2 are suspended by ropes

Find the tensions T_1 and T_2 in the two ropes

Magnitude of tension is the same everywhere in a rope.
Direction? Tension acts to pull contact points together.



What should we expect?

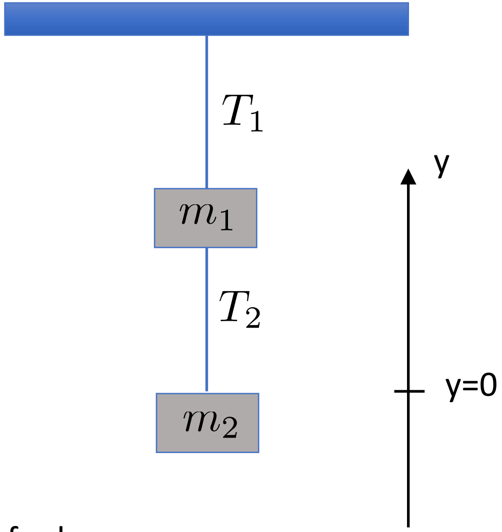
Upper rope supports both masses ➡ $T_1 = m_1 g + m_2 g$

Lower rope supports only lower mass ➡ $T_2 = m_2 g$

See how this comes out of free body diagram analysis

Always good to understand simple cases first

Two boxes with masses m_1 and m_2 are suspended by ropes

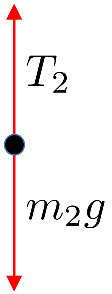


Find the tensions T_1 and T_2 in the two ropes

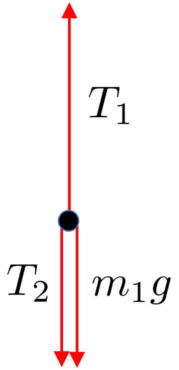
Need to draw a free body diagram for each mass

1D problem ➡ Only necessary to choose which direction is positive

Lower mass



Upper mass



2nd law for lower mass

$$T_2 - m_2g = m_2a = 0$$

$$\Rightarrow T_2 = m_2g \quad \checkmark$$

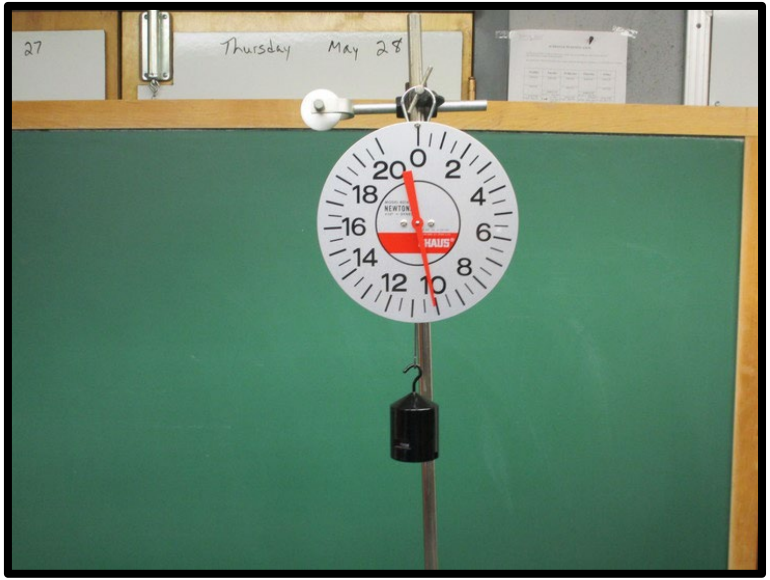
Not moving

2nd law for upper mass

$$T_1 - T_2 - m_1g = m_1a = 0$$

$$\Rightarrow T_1 = m_1g + T_2 = m_1g + m_2g \quad \checkmark$$

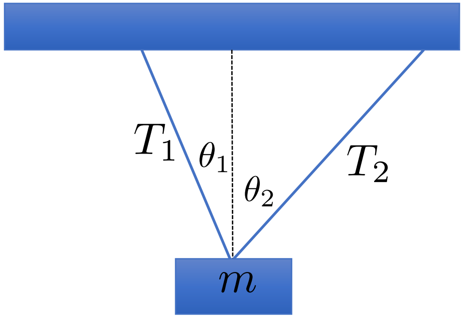
Demo: Tension in a string



A box of mass m=30 kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



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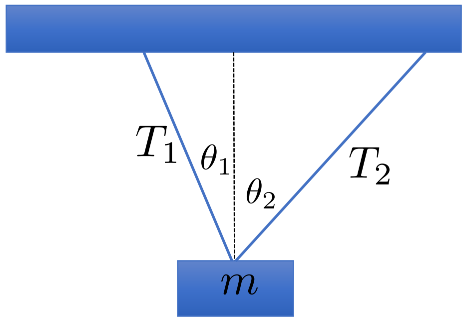
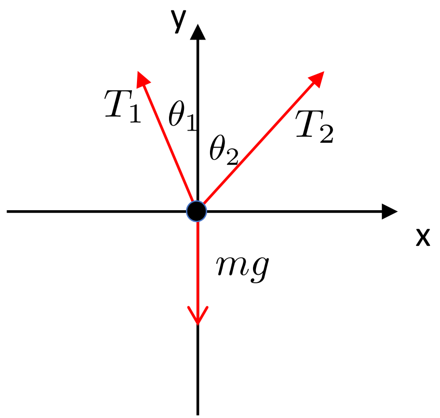


Write out components of individual forces

$$\vec{F}_1 = -mg\,\hat{j}$$
$$\vec{F}_2 = -T_1 \sin\theta_1\,\hat{i} + T_1 \cos\theta_1\,\hat{j}$$
$$\vec{F}_3 = +T_2 \sin\theta_2\,\hat{i} + T_2 \cos\theta_2\,\hat{j}$$



Compute net force

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$
$$= (-T_1 \sin\theta_1 + T_2 \sin\theta_2)\hat{i} + (T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg)\hat{j}$$

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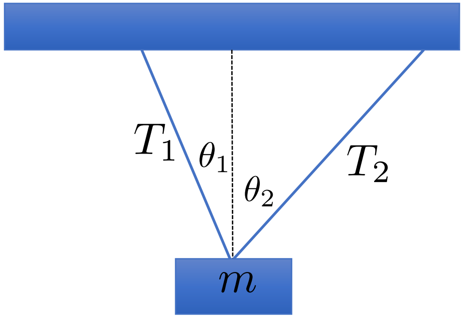
Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



$$\vec{F}_{net} = (-T_1 \sin\theta_1 + T_2 \sin\theta_2)\hat{i} + (T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg)\hat{j}$$
$$= m\vec{a} = 0$$

Not moving!

Both x and y components of net force must vanish ➡ 2 equations with 2 unknowns

$$-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0$$
$$T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg = 0$$

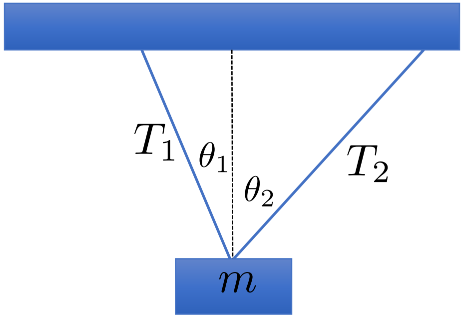
Solve 1st equation for T_2 and plug into 2nd equation

➡ $$T_2 = \frac{\sin\theta_1}{\sin\theta_2} T_1$$

A box of mass m=30kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



$$-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0 \quad \Rightarrow \quad T_2 = \frac{\sin\theta_1}{\sin\theta_2} T_1$$

$$T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg = 0$$

Plug in T_1 to obtain T_2

Plugging T_2 into 2nd equation gives…

$$T_1 (\cos\theta_1 + \frac{\cos\theta_2 \sin\theta_1}{\sin\theta_2}) - mg = 0$$

Plug in numbers for m, θ_1, θ_2 to get final answers

Solve for T_1

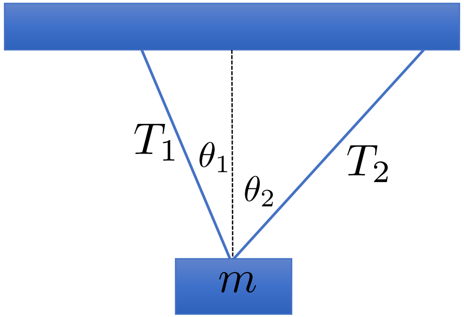
$$T_1 = \frac{mg\sin\theta_2}{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}$$

$$T_2 = \frac{mg\sin\theta_1}{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}$$

A box of mass m=30kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where... $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



$$T_1 = \frac{mg\sin\theta_2}{\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1}$$

$$T_2 = \frac{mg\sin\theta_1}{\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1}$$

Interesting limiting cases...

Equal angles $\theta_1 = \theta_2 \Rightarrow T_1 = T_2 = \dfrac{mg}{2\cos\theta}$

So that y-components are equal to mg/2

One vertical rope $\theta_1 = 0 \Rightarrow \sin\theta_1 = 0,\ \cos\theta_1 = 1 \Rightarrow T_1 = mg,\ T_2 = 0$

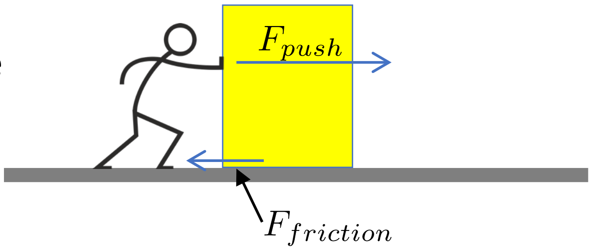
Vertical rope supports all the weight

Friction

Opposes motion of object across a surface

Depends surface properties (e.g. rough or smooth)

Coefficient of friction: μ



Two types of friction:

Static friction – when the object isn't moving

Coefficient of static friction - μ_s

Tells how hard we have to push to start the object moving

Kinetic friction – when the object is moving

Coefficient of kinetic friction - μ_k

Tells how friction force impacts object in motion

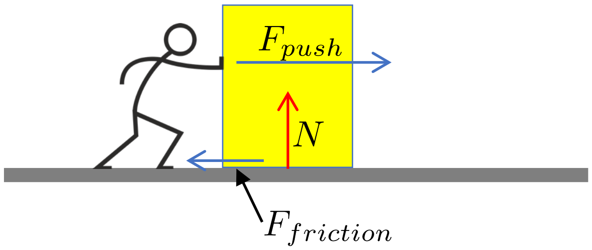
Easier to keep something moving than to start it moving ➡ $\mu_k < \mu_s$

Friction

Frictional force also proportional to normal force of surface on object

Larger for things that weigh more

But normal force can increase for other reasons too!



Magnitude of friction force

Start with **Kinetic Friction**

Slightly more complicated for static friction

$$\boxed{F_f = \mu_k N}$$

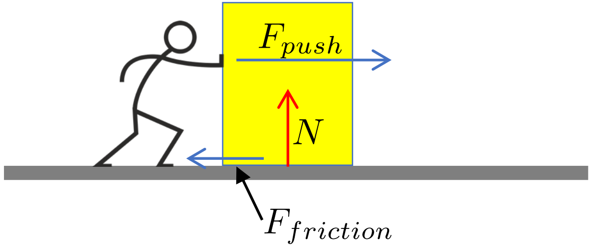
Coefficient of kinetic friction

Normal force

Kinetic friction examples…

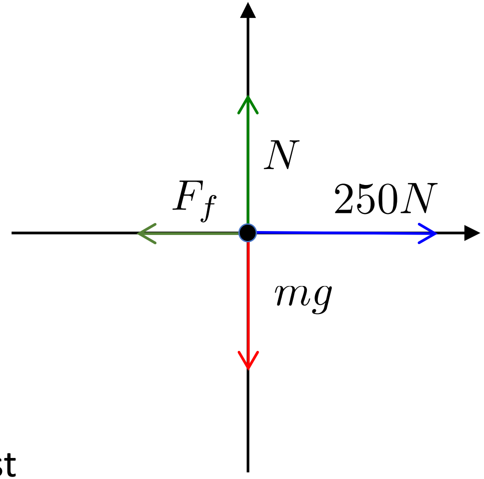
$$F_f = \mu_k N$$

A 30 kg box is pushed along a surface with coefficient of kinetic friction μ_k= 0.4 by a horizontal force of 250 N.



What is its acceleration?
How hard should it be pushed to move with constant speed?

Find net force



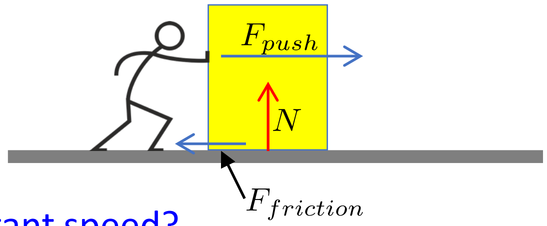
$$F_{net} = (250N - F_f)\hat{i} + (N - mg)\hat{j}$$
$$= ma_x\,\hat{i} + ma_y\,\hat{j}$$

Need normal force ➡ Look at y-direction first

$$a_y = 0 \quad \Longrightarrow \quad N = mg$$

Obvious in this case, but there are more complicated problems to come

A 30 kg box is pushed along a surface with coefficient of kinetic friction μ_k= 0.4 by a horizontal force of 250N.

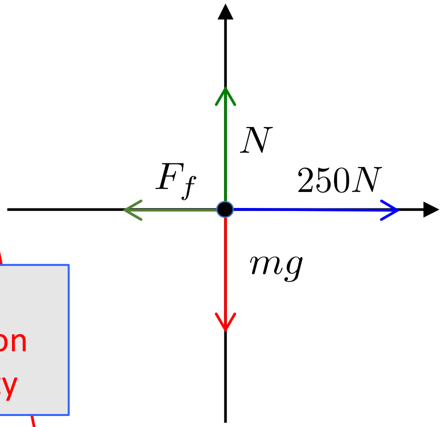


What is its acceleration?

How hard should it be pushed to move with constant speed?

$$F_{net} = (250N - F_f)\hat{i} + (N - mg)\hat{j}$$
$$= ma_x\,\hat{i} + ma_y\,\hat{j}$$

$$a_y = 0 \quad \Rightarrow \quad N = mg$$



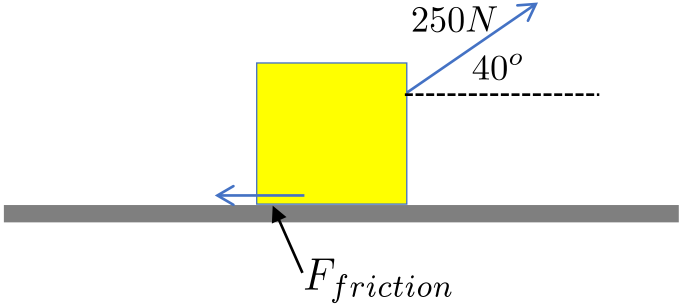
Now look at x-direction

Pushing with a force equal to kinetic friction gives constant velocity

$$a_x = \frac{1}{m}(250N - F_f)$$
$$= \frac{1}{30kg}(250N - 120N)$$
$$= 4.3m/s^2$$

$$F_f = \mu_k N = \mu_k mg$$
$$= (0.4)(30kg)(9.8m/s^2)$$
$$= 120N$$

Kinetic friction examples…



A 30 kg box is pulled across a floor with friction coefficient μ_k=0.4 by a force of 250N applied at angle 40^0

Find the magnitude of the normal force.
Find the acceleration of the box.

Forces…

What's going on - vertical component of applied force decreases normal force, which decreases friction force

$$\vec{F}_1 = -mg\,\hat{j}$$

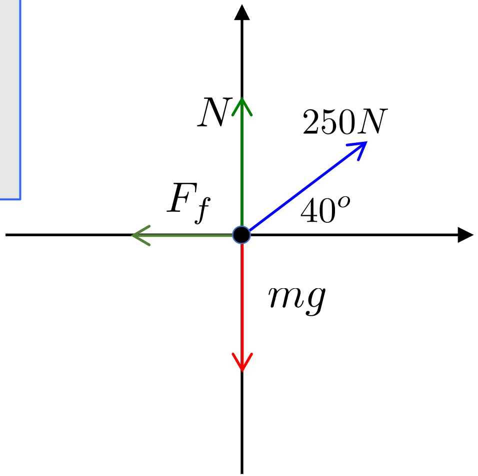
$$\vec{F}_2 = N\,\hat{j}$$

$$\vec{F}_3 = -F_f\,\hat{i}$$

$$\boxed{F_f = \mu_k N}$$

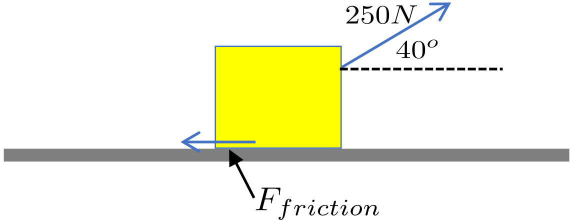
$$\vec{F}_4 = (250N\cos 40^o)\,\hat{i} + (250N\sin 40^o)\,\hat{j}$$

$$= 192\ N\,\hat{i} + 161N\,\hat{j}$$



$$F_{net} = (192N - F_f)\,\hat{i} + (N + 161N - mg)\,\hat{j}$$

Kinetic friction examples…



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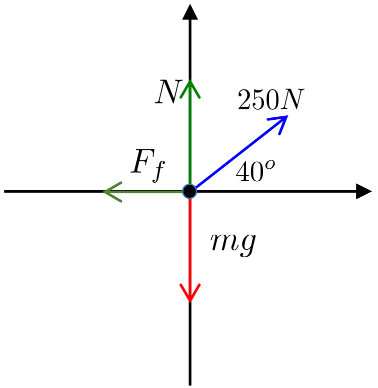


$$F_{net} = (192N - F_f)\,\hat{i} + (N + 161N - mg)\,\hat{j}$$
$$= ma_x\,\hat{i} + ma_y\,\hat{j}$$

Use y-direction to find normal force

$$a_y = 0 \Rightarrow N = mg - 161N = 294N - 161N = 133N$$

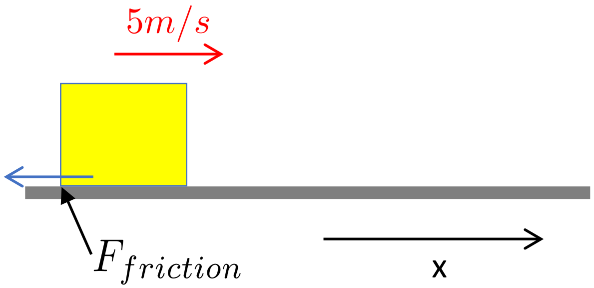
Use normal force to find friction force $F_f = \mu_k N = (0.4)(133N) = 53N$

Find acceleration

$$a_x = \frac{1}{m}(192N - F_f) = \frac{1}{30kg}(192N - 53N) = 4.6m/s^2$$

Another standard problem

A 3kg block is initially sliding at 5 m/s across a surface with coefficient of friction μ_k=0.2

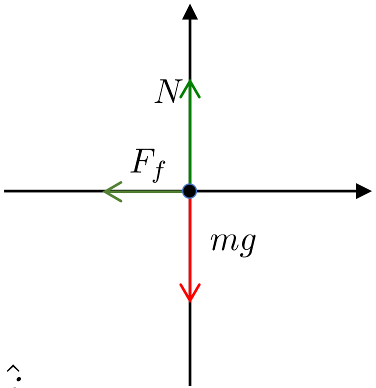


How far does it slide before coming to rest?

Problem doesn't mention time…

Use $D = \dfrac{(v_1^2 - v_0^2)}{2a}$

$$v_0 = 5m/s$$
$$v_1 = 0$$
$$a = ?$$



Draw free body diagram…

$$F_{net} = -F_f\,\hat{i} + (N - mg)\,\hat{j} = ma_x\,\hat{i} + ma_y\,\hat{j}$$

y-direction ➡ $N = mg$

x-direction ➡ $a_x = -\dfrac{1}{m}F_f = -\dfrac{1}{m}(\mu_k N) = -\dfrac{1}{m}(\mu_k mg) = -\mu_k g$

$$= -(0.2)(9.8m/s^2) = -2.0m/s$$

Another standard problem

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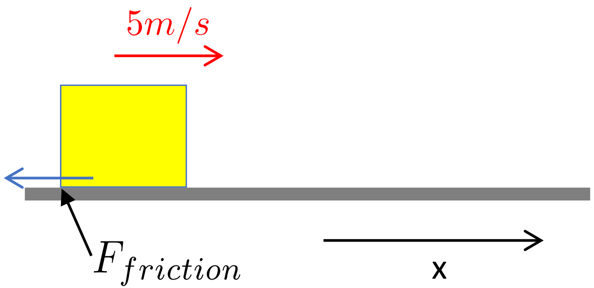
How far does it slide before coming to rest?



x-direction

$$a_x = -\frac{1}{m}F_f = -\frac{1}{m}(\mu_k N) = -\frac{1}{m}(\mu_k mg) = -\mu_k g$$

$$= -(0.2)(9.8m/s^2) = -2.0m/s$$

Note the acceleration is independent of the mass of the object

Problem doesn't mention time…

Use $D = \frac{(v_1^2 - v_0^2)}{2a}$

$$v_0 = 5m/s$$
$$v_1 = 0$$
$$a = -2.0m/s^2$$

$$D = \frac{(0\,\frac{m}{s})^2 - (5\,\frac{m}{s})^2}{2 \times (-\frac{2m}{s^2})} = 6.25\text{ m}$$

What about *static friction*?

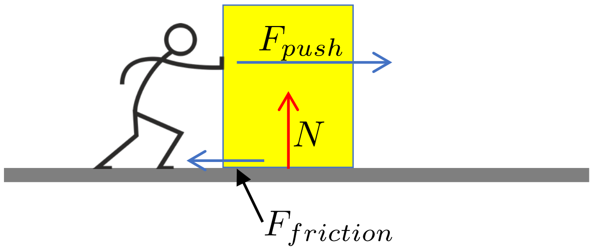
Need to push hard enough to overcome static friction

Static friction force can vary in response to an applied force, up to a maximum value.

$$F_f^{(max)} = \mu_s N$$

For static friction

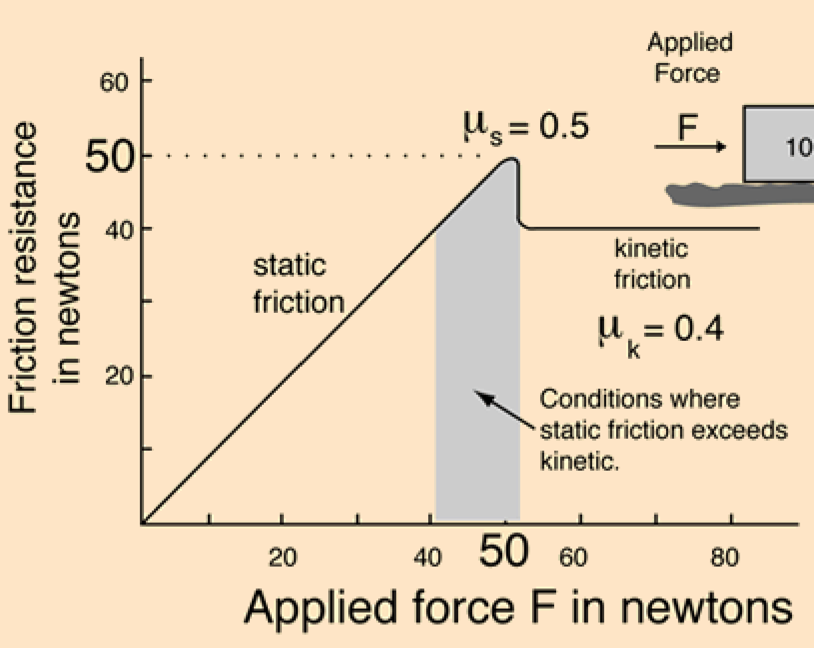
$$F_f \leq F_f^{(max)}$$



You push harder, static friction pushes back harder

Until you reach the maximum value and the object will start to move

Static vs Kinetic Friction (from hyperphysics.phy-astr.gsu/edu)



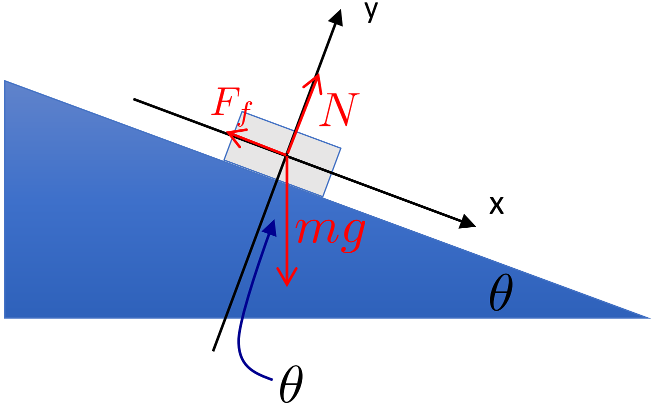
F	f
20	20
40	40
50	50
60	40
80	40

- Value of maximum static friction depends on conditions
- For a tire, could go from 0.7 on dry roads to 0.4 on wet
- Race car tires can be 0.9 on dry, 0.1 on wet

Static friction problem

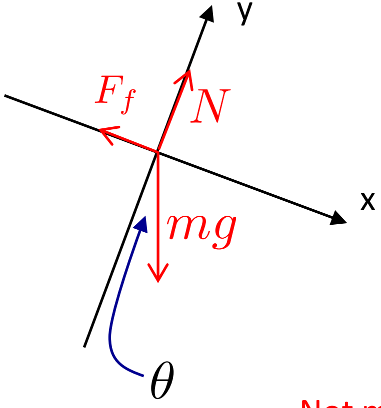
A block of mass m sits at rest on an inclined plane with coefficient of static friction μ_s

What is the maximum angle θ_{max} for which this is possible?



$$\theta = \theta_{max} \Rightarrow F_f = F_f^{(max)} = \mu_s N$$

Redo earlier analysis adding in friction



$$\vec{F}_1 = N\,\hat{j}$$

$$\vec{F}_2 = mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j}$$

$$\vec{F}_3 = -F_f\,\hat{i}$$

Not moving!

$$\vec{F}_{net} = (mg\sin\theta - F_f)\,\hat{i} + (N - mg\cos\theta)\,\hat{j} = m\vec{a} = 0$$

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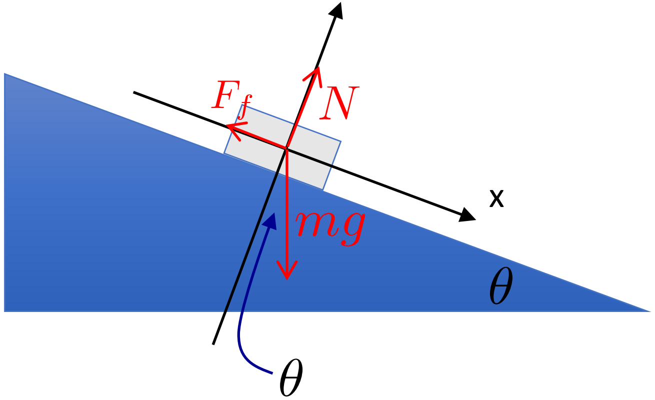
What is the maximum angle θ_{max} for which this is possible?



$$\theta = \theta_{max} \Rightarrow F_f = F_f^{(max)} = \mu_s N$$

$$\vec{F}_{net} = (mg\sin\theta - F_f)\,\hat{i} + (N - mg\cos\theta)\,\hat{j} = m\vec{a} = 0$$

y-direction $\Rightarrow$ $N = mg\cos\theta$ As before...

$\Rightarrow$ $F_f = \mu_s mg\cos\theta$ Friction force depends on angle through normal force

x-direction $\Rightarrow$ $\cancel{mg}\sin\theta - \mu_s\cancel{mg}\cos\theta = 0$

$\Rightarrow$ $\boxed{\tan\theta_{max} = \mu_s}$ Same for any mass

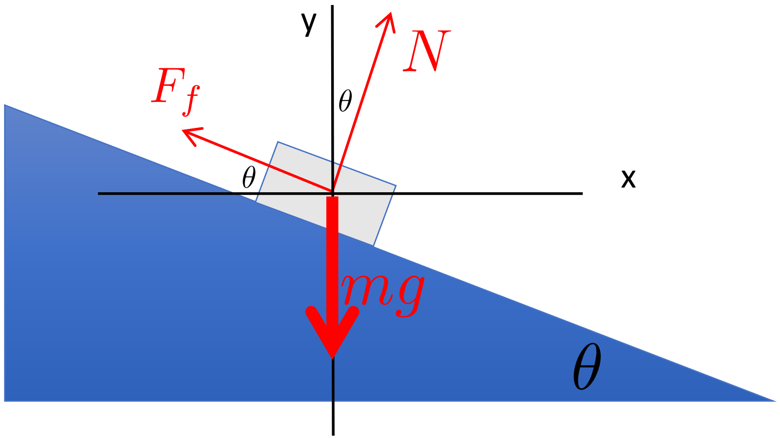
Also same on any planet!

Static friction problem

Mass m sits at rest on inclined plane with coefficient of static friction μ_s

What is the maximum angle θ_{max} for which this is possible?

Use horizontal & vertical coordinates



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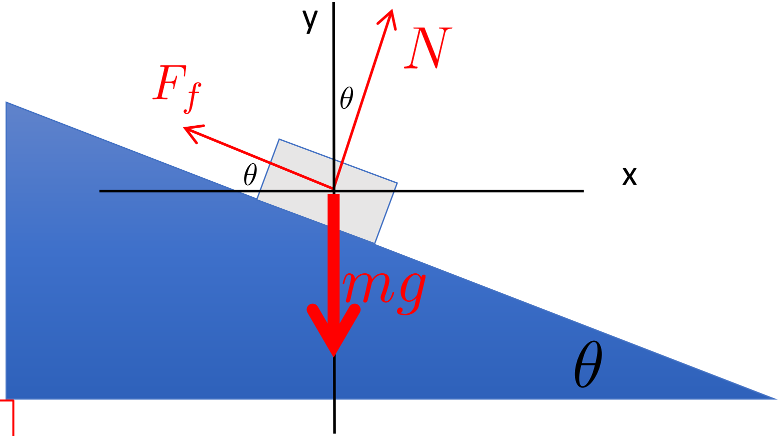

$$
\begin{aligned}
\vec{F}_g &= -mg\hat{y} \\
\vec{F}_N &= N\sin\theta\hat{x} + N\cos\theta\hat{y} \\
\vec{F}_f &= -F_f\cos\theta\hat{x} + F_f\sin\theta\hat{y}
\end{aligned}
$$

$$\hat{x}: \quad N\sin\theta - F_f\cos\theta = 0 \quad \Rightarrow \quad N = F_f\frac{\cos\theta}{\sin\theta}$$

$$\hat{y}: \quad -mg + N\cos\theta + F_f\sin\theta = 0 \quad \Rightarrow \quad -mg + F_f\frac{\cos^2\theta}{\sin\theta} + F_f\sin\theta = 0$$

$$F_f = mg\sin\theta = \mu_s N = \mu_s mg\sin\theta\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \quad \mu_s = \tan\theta$$

Angle of Repose: $\tan(\theta)=\mu_s$

Angle of Repose: $\tan(\theta)=\mu_s$