
Lillehammer
POPSUGAR

Announcements, Goals, and Reading

Announcements:

- HW12 due Tuesday 12/13
- *Last day to turn in late homework for partial credit: Tuesday 12/20, 11:59pm.*
- Forward FOCUS survey is open—please provide feedback.

Goals for Today:

- Angular Momentum
- Review & Examples

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 12: Rotation of a Rigid Body

E-mail me with questions for Monday's review!

Final Exam

Monday 12/19, 1-3PM (Section 2) | Tuesday 12/20, 10:30a-12:30pm (Section 1)

- **Covers Chapters 9-12, Lectures through the end of this week, Homework 9-12**
- **Key topics: Momentum, Energy, Work, Springs, Rotational Dynamics, Angular Momentum**

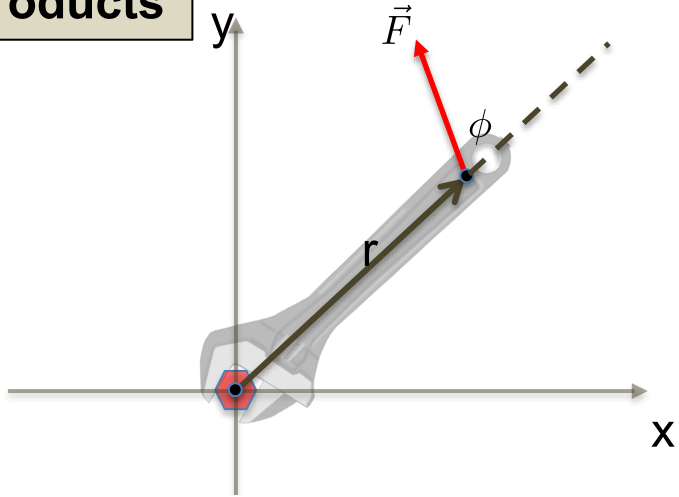
- Location: HAS 20 (Request pending for a 2nd room..)
- *If you have extra time/reduced distraction accommodations, come to the Reduced Distraction room (HAS109 for Dec 20 final, HAS130 for Dec 19 final)*

- **Allowed a Calculator and an 8.5”×11” reference sheet with handwritten notes on both sides.** Be sure to bring a calculator as there are never enough spares.
- ~25 Multiple Choice Questions; **Bring a #2 pencil**
- Practice problems are up on Moodle and Mastering Physics
- **TA Review Session: TBA**
- **SI Review Sessions:** 12/18 4-6PM & 7-9PM (HAS 126)
- The last lecture will be review focused. E-mail me any practice questions or topics you’d like me to go over.
- Makeup Exams: If you have another final exam scheduled at the same time slot, please notify me via email and we can discuss alternative arrangements.

Review: Torque and Cross Products

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = rF\sin\phi$$



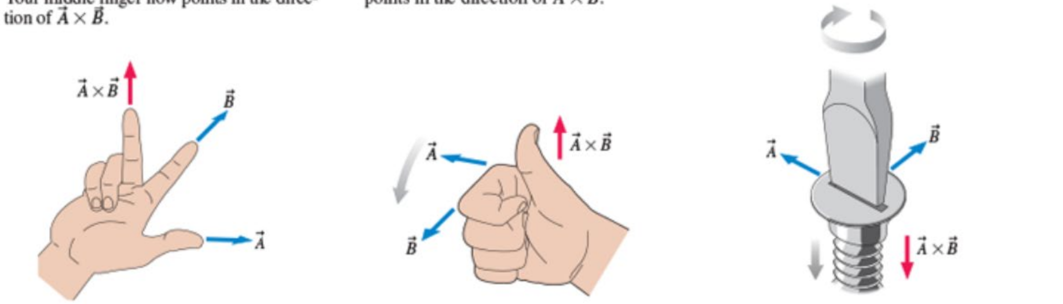
right hand rule gives torque pointing in + z direction

Using the right-hand rule

Spread your *right* thumb and index finger apart by angle α. Bend your middle finger so that it is *perpendicular* to your thumb and index finger. Orient your hand so that your thumb points in the direction of $\vec{A}$ and your index finger in the direction of $\vec{B}$. Your middle finger now points in the direction of $\vec{A} \times \vec{B}$.

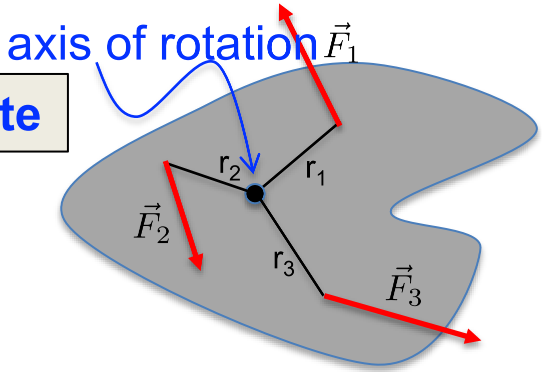
Make a loose fist with your *right* hand with your thumb extended outward. Orient your hand so that your thumb is perpendicular to the plane of $\vec{A}$ and $\vec{B}$ and your fingers are curling *from* the line of vector $\vec{A}$ *toward* the line of vector $\vec{B}$. Your thumb now points in the direction of $\vec{A} \times \vec{B}$.

Imagine using a screwdriver to turn the slot in the head of a screw from the direction of $\vec{A}$ to the direction of $\vec{B}$. The screw will move either "in" or "out." The direction in which the screw moves is the direction of $\vec{A} \times \vec{B}$.



Review: Rotational Dynamics

Torque causes things to rotate



What is relation between torque and rotational motion?

Can generally be some number of torques acting on object

$$\vec{\tau}_i = \vec{r}_i \times \vec{F}_i \qquad i = 1, 2, \ldots, N$$

Net torque on an object around a given axis is the sum of the individual torques

$$\vec{\tau}_{net} = \vec{\tau}_1 + \cdots + \vec{\tau}_N \qquad \tau_{net} = |\vec{\tau}_{net}|$$

angular analogue of

$$F_{net} = ma$$

Can show that Newton's second law

$$\boxed{\tau_{net} = I\alpha}$$

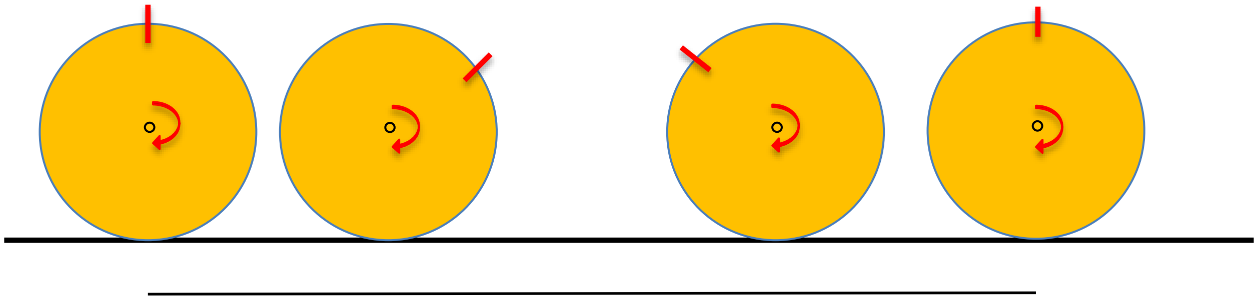
net torque

moment of inertia

angular acceleration

Important special case: "Rolling without Slipping"

Here $v=R\omega$ (the velocity of a point on the circumference) = velocity of center of mass



Distance traveled in one revolution = 1 circumference

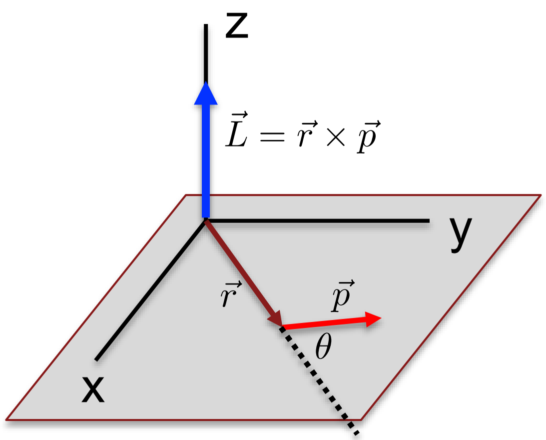
Center of mass will travel 1 full circumference in 1 period: v_{cm}=circumference/period

$$v_{cm} = \frac{2\pi R}{T} = 2\pi R \frac{1}{T} = 2\pi R\nu = 2\pi\nu R = \omega R = R\omega$$

Angular momentum

Rotational analogue of momentum

Recall – if no net external forces act on a system, then momentum is conserved



Angular momentum is conserved if no net external torques act on system

For a particle at position $\vec{r}$ with momentum $\vec{p} = m\vec{v}$

Angular momentum is defined to be

$$\boxed{\vec{L} = \vec{r} \times \vec{p}} \Rightarrow |\vec{L}| = rmv\sin\theta$$

By “right hand rule” see that angular momentum is perpendicular to plane of motion

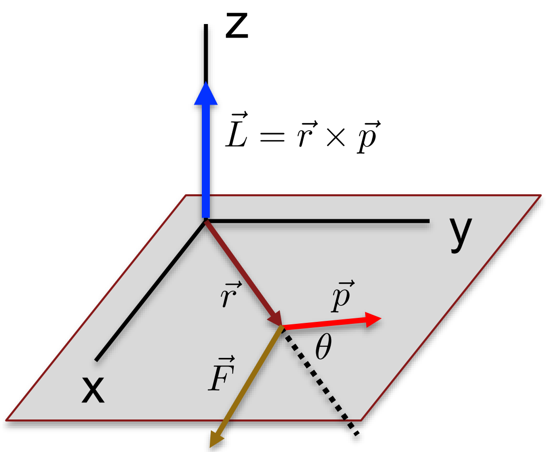
Counterclockwise rotation $\Rightarrow$ Angular momentum in + z-direction

Angular momentum

For a particle at position $\vec{r}$
with momentum $\vec{p} = m\vec{v}$

Angular momentum is defined to be

$$\boxed{\vec{L} = \vec{r} \times \vec{p}}$$



If forces acts on the particle, can show from Newton's laws that

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

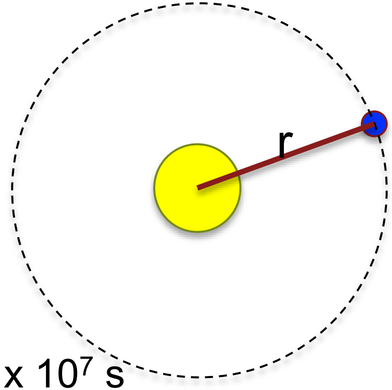
Rotational analogue of $\dfrac{d\vec{p}}{dt} = \vec{F}_{net}$

Net torque gives rate of change of angular momentum

Vanishin g net torque $\quad \vec{\tau}_{net} = 0 \Rightarrow \dfrac{d\vec{L}}{dt} = 0$ Angular momentum of particle is conserved

Angular momentum example

What is the angular momentum associated with the Earth's orbit around the sun?



Mass of earth $m = 6 \times 10^{24}$ kg

Radius of Earth's orbit $r = 1.5 \times 10^{11}$ m

Period of Earth's orbit $T = 1$ year $= 3.2 \times 10^{7}$ s

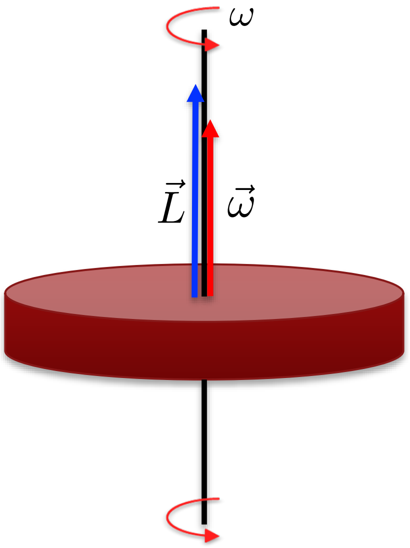
$$L = rp = rmv = rm(\frac{2\pi r}{T}) = (\frac{2\pi m r^2}{T}) = 2.7 \times 10^{40} Js$$

Angular Momentum of Rigid Body

Need to add up the angular momenta of all parts of the body to get the total angular momentum

Result $\boxed{\vec{L} = I\vec{\omega}}$

$$\vec{\omega} = \frac{\vec{r} \times \vec{v}}{r^2}$$



mom
ent of
inertia

angular velocity vector points along rotational axis in direction given by right hand rule

Rotational analogue of $\vec{p} = m\vec{v}$

Relation between torque and angular momentum still holds for rigid bodies

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

For vanishing net torque angular momentum is conserved $\frac{d\vec{L}}{dt} = 0$

Example

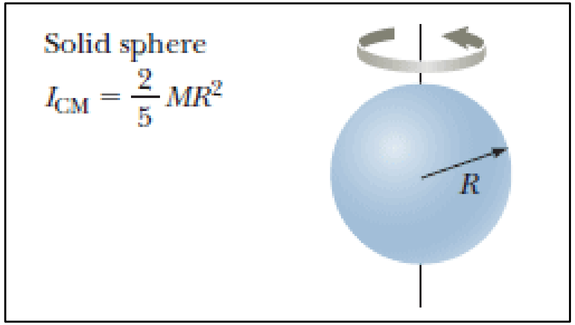
What is the angular momentum associated with the Earth's spin about its axis?

Mass of earth $m = 6 \times 10^{24}$ kg

Radius of earth $r = 6.4 \times 10^{6}$ m

Period of rotation T= 1 day = 86,000 s

$$L = I\omega$$
$$= (\frac{2}{5}mr^2)(\frac{2\pi}{T})$$
$$= 7.2 \times 10^{33}\ Js$$



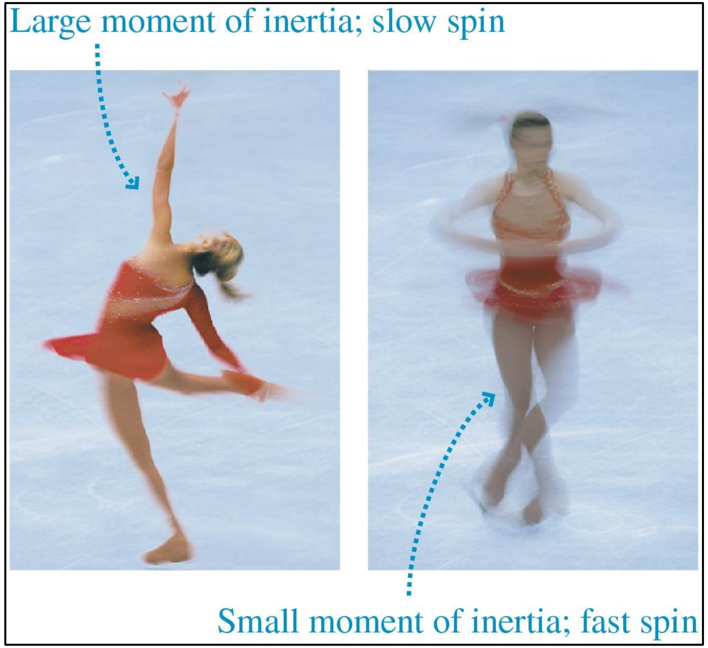
Demo: Conservation of Angular Momentum

Why does the angular velocity change as the weights are moved in/out?

Classic example: Conservation of Angular Momentum

As an ice skater spins, external torque is small, so her angular momentum is almost constant.

By drawing in her arms and legs to reduce her moment of inertia, she increases her angular velocity



$$L = I\omega$$

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow I_i\omega_i = I_f\omega_f$$

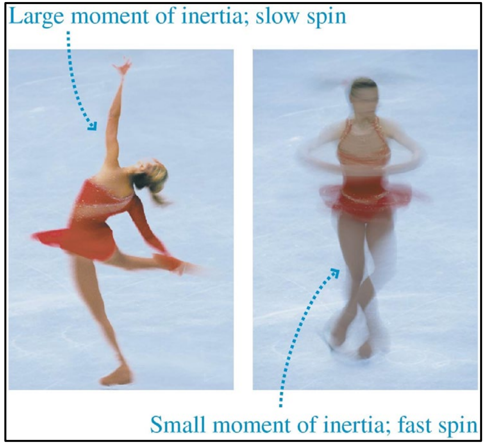
$$\omega_f = \omega_i\left(\frac{I_i}{I_f}\right)$$

$$I_f < I_i \Rightarrow \omega_f > \omega_i$$

Angular momentum example

A figure skater has moment of inertia I_i = 2kgm^2 when her arms are extended and I_f = 1kgm^2 when her arms are fully pulled in.

She is initially spinning at 20rpm with her hands out



What is her angular velocity when she pulls them in?

$$I_i\omega_i = I_f\omega_f \quad \Rightarrow \quad \omega_f = \omega_i\left(\frac{I_i}{I_f}\right)$$

$$20rpm = 2.1\, rad/s$$

$$\omega_f = (20rpm)\left(\frac{2\, kgm^2}{1\, kgm^2}\right) = 40rpm$$

Does her kinetic energy change in this process?

Skater must do work to pull her arms in!

$$K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(2\, kgm^2)(2.1\, rad/s)^2 = 4.4\, J$$

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(1\, kgm^2)(4.2\, rad/s)^2 = 8.8\, J$$

$\Rightarrow$ Yes

Demo: Conservation of Angular Momentum: Your turn!

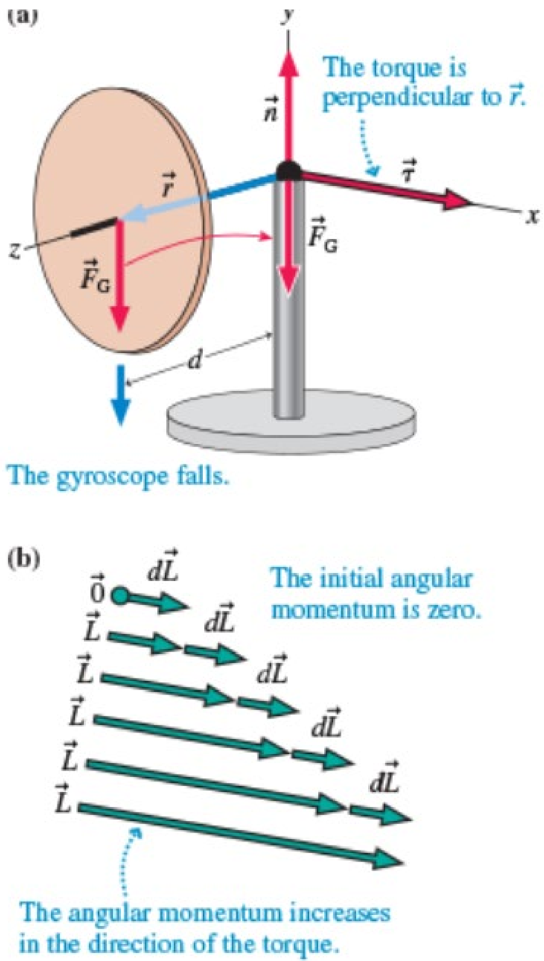
What should happen when the spinning wheel is slowly flipped over?

Bicycle Wheel Gyroscope

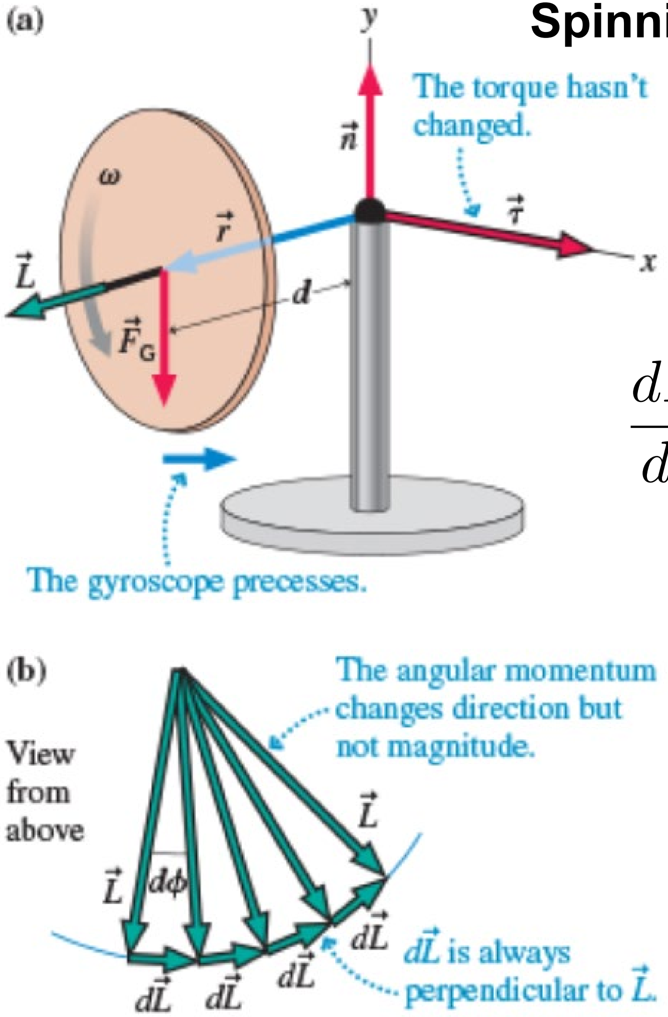
What is going on here? Is there a torque on this system?

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

Non spinning tire



Spinning tire

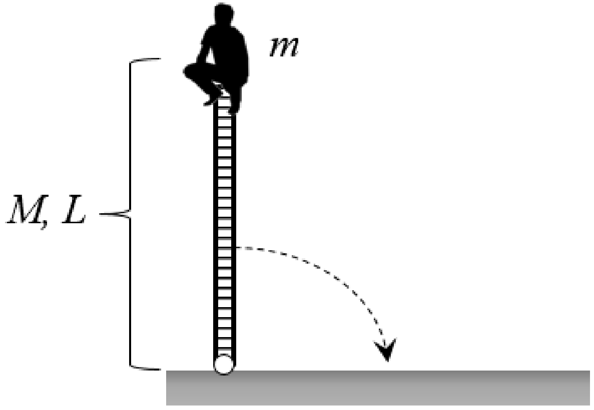


$$\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$$

Rotational Kinetic Energy

Bob is sitting (attached) on a ladder when it falls over (with the pivot point at the bottom of the ladder staying fixed)

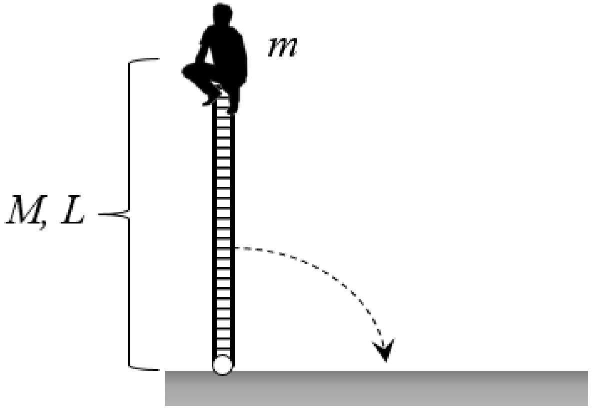
How fast will Bob be moving when he hits the ground?



Rotational Kinetic Energy

Bob is sitting (attached) on a ladder when it falls over (with the pivot point at the bottom of the ladder staying fixed)

How fast will Bob be moving when he hits the ground?



Use Conservation of Energy

Initial energy is all potential energy

$$E_i = Mg(L/2) + mgL$$

Final energy is all (rotational) kinetic energy

$$E_f = \frac{1}{2} I \omega^2$$

Want to find ω

Moment of inertia

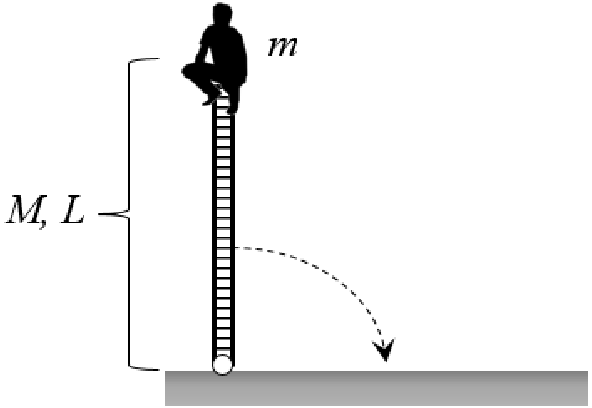
$$I = I_{ladder} + I_{bob}$$

$$I_{ladder} = \frac{1}{3} ML^2$$

$$I_{bob} = mL^2$$

$$I = \frac{1}{3} ML^2 + mL^2$$

Rotational Kinetic Energy



Bob is sitting (attached) atop a ladder when it falls over (with the pivot point at the bottom of the ladder staying fixed)

How fast will Bob be moving when he hits the ground?

$$E_i = Mg(L/2) + mgL$$

$$I = \frac{1}{3}ML^2 + mL^2$$

$$E_f = \frac{1}{2}I\omega^2$$

$$E_i = E_f \Rightarrow \omega^2 = \frac{2(MgL/2 + mgL)}{I}$$

Plug in expression for I and solve for ω

Bob's speed ⇒ $v = \omega L$

Consider a uniform solid sphere of radius R and mass M rolling without slipping. Which form of its kinetic energy is larger, translational or rotational?

A) Its translational kinetic energy is larger than its rotational kinetic energy.
B) Its rotational kinetic energy is larger than its translational kinetic energy.
C) Both forms of energy are equal.
D) You need to know the speed of the sphere to tell.

Consider a uniform solid sphere of radius R and mass M rolling without slipping. Which form of its kinetic energy is larger, translational or rotational?

A) Its translational kinetic energy is larger than its rotational kinetic energy.
B) Its rotational kinetic energy is larger than its translational kinetic energy.
C) Both forms of energy are equal.
D) You need to know the speed of the sphere to tell.

$$\begin{aligned}
\mathrm{KE(translational)} &= \frac{1}{2}Mv^2 \\
\mathrm{KE(rotational)} &= \frac{1}{2}I\omega^2, \quad v = R\omega \\
&= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 \\
&= \frac{1}{5}Mv^2
\end{aligned}$$

A long thin rod of length L has a linear density $\lambda(x) = Ax$ where x is the distance from the left end of the rod.

(a) How far is the center of mass of the rod from the left end of the rod?
(b) What is the mass of the rod?

A long thin rod of length L has a linear density $\lambda(x) = Ax$ where x is the distance from the left end of the rod.

(a) How far is the center of mass of the rod from the left end of the rod?

(b) What is the mass of the rod?

$$\text{Mass} = \int_0^L \Lambda(x)dx = \int_0^L Axdx = \frac{1}{2}AL^2$$

$$\text{Center of Mass} = \frac{\int_0^L \Lambda(x)xdx}{\text{Mass}} = \frac{\int_0^L Ax^2dx}{\text{Mass}} = \frac{\frac{1}{3}AL^3}{\text{Mass}}$$

$$= \frac{\frac{1}{3}AL^3}{\frac{1}{2}AL^2}$$

$$\text{Center of Mass} = \frac{2}{3}L$$

Rotational dynamics example

A bicycle wheel has radius R=0.35m and mass M=0.44kg is initially spinning at 100rpm on a truing stand

Torque comes from 0.8N force of ball bearings rubbing on edge of axle at r=0.0026m

Make approximation that all mass is at the rim

How long does wheel take to come to rest?

Moment of inertia

$$I = MR^2 = (0.44kg)(0.35m)^2 = 0.054\,kgm^2$$

opposing rotational motion

Torque

$$\tau = Fr = -(0.8N)(0.0026m) = -0.0021\,Nm$$

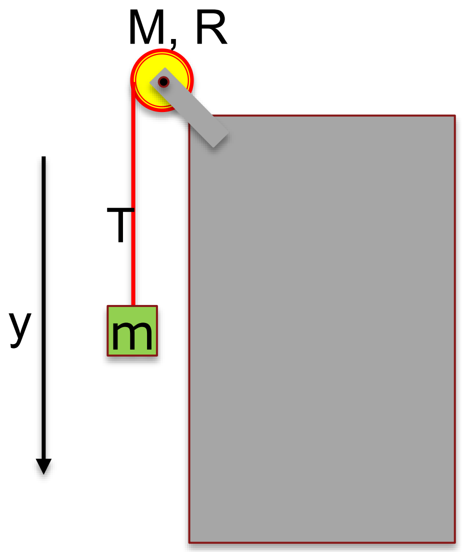
Find angular acceleration

$$\tau = I\alpha \Rightarrow \alpha = \frac{\tau}{I} = \frac{-0.0021\,Nm}{0.054\,kgm^2} = -0.038 rad/s^2$$

A solid disk with mass M=2.5kg and radius R=0.2m has massless rope wrapped around it

Block of mass m=1.2kg descends with rope as it unravels from disk

Find the acceleration of the block and the tension in the rope



Block → $\boxed{mg - T = ma}$

Disk → Feels torque from rope $\tau = TR$

Moment of inertia $I = \frac{1}{2}MR^2$

Angular acceleratio n $\alpha R = a$ → $\alpha = \frac{a}{R}$ as rope unspools, disk spins

key point!

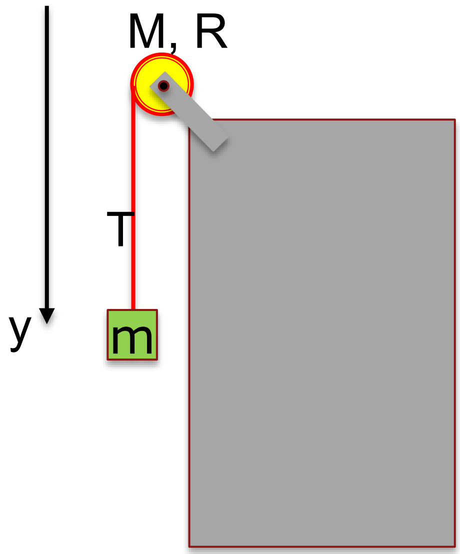
Subtlety Alert → Positive rotation is counterclockwise

Made the y-axis point down so that a>0 coincides with α>0

A solid disk with mass M=2.5kg and radius R=0.2m has massless rope wrapped around it

Block of mass m=1.2kg hangs from rope as it unravels from disk

Find the acceleration of the block and the tension in the rope



Block → $\boxed{mg - T = ma}$

Disk →

Feels torque from rope $\tau = TR$

Moment of inertia $I = \frac{1}{2}MR^2$

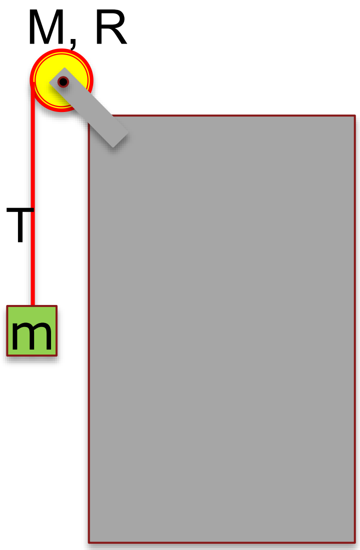
Angular acceleratio n $\alpha R = a$ → $\alpha = \frac{a}{R}$

$\tau = I\alpha$ → $TR = (\frac{1}{2}MR^2)(\frac{a}{R})$ → $\boxed{T = \frac{1}{2}Ma}$

A solid disk with mass M=2.5kg and radius R=0.2m has massless rope wrapped around it

Block of mass m=1.2kg hangs from rope as it unravels from disk

Find the acceleration of the block and the tension in the rope



$$\boxed{mg - T = ma}$$

$$\boxed{T = \frac{1}{2}Ma}$$

2 equations with 2 unknowns

Solve to find…

$$a = \frac{m}{m + \frac{1}{2}M}g$$

$$T = \frac{2mM}{2m + M}g$$

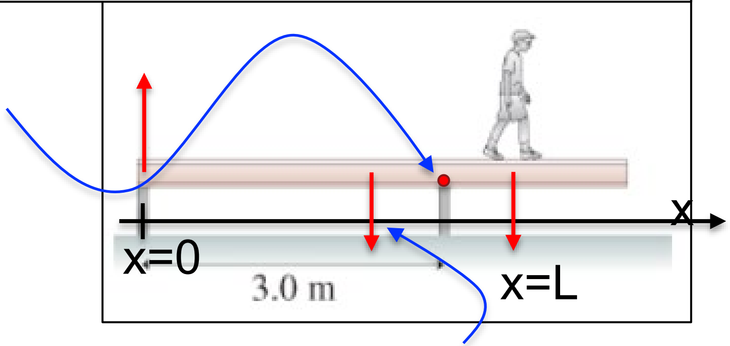
M=0 → Nothing restrains block from falling

→ $a = g$

$T = 0$

A 40 kg , 5.1-m-long beam is supported, but not attached to, the two posts in the figure (Figure 1) . A 20 kg boy starts walking along the beam.

How close can he get to the right end of the beam without it falling over?



x=2.55m

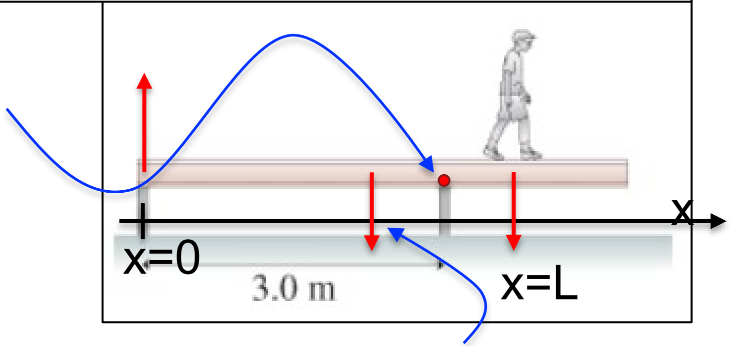
A 40 kg , 5.1-m-long beam is supported, but not attached to, the two posts in the figure (Figure 1) . A 20 kg boy starts walking along the beam.

How close can he get to the right end of the beam without it falling over?

Compute torques from mass of beam and boy about axis above right support post

If net torque is positive (counterclockwise), it can be countered by negative (clockwise) torque from left support post

If net torque is negative (clockwise), beam will fall over



Gravity acts on beam at center of mass @ x=2.55m

Boy's center of mass @ x=L

$$\tau_{net} = +(3m - 2.55m)(40kg)g - (L - 3m)(20kg)g > 0$$

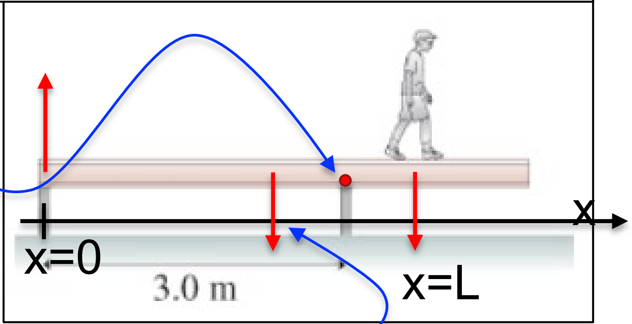
distance of beam cm from pivot point

distance of boy from pivot point

A 40 kg , 5.1-m-long beam is supported, but not attached to, the two posts in the figure (Figure 1) . A 20 kg boy starts walking along the beam.

How close can he get to the right end of the beam without it falling over?

Compute torques from mass of beam and boy about axis above right support post



x=2.55m

$$\tau_{net} = +(3m - 2.55m)(40kg)\cancel{g} - (L - 3m)(20kg)\cancel{g} > 0$$

$$18\,kgm - (20kg)L + 60\,kgm > 0$$

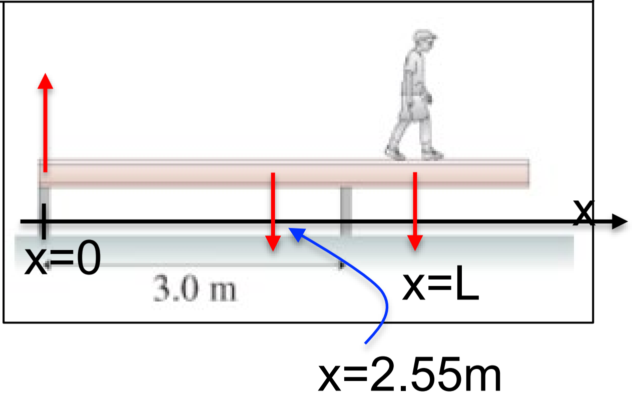
$$3.9\,m > L$$

Minimum safe distance from end

$$d = 5.1\,m - 3.9\,m = 1.2\,m$$

A 40 kg , 5.1-m-long beam is supported, but not attached to, the two posts in the figure (Figure 1) . A 20 kg boy starts walking along the beam.

How close can he get to the right end of the beam without it falling over?



Slightly alternative reasoning

Find center of mass x_{cm} of combined beam and boy system

If x_{cm} < 3m then the torque around the right post will be positive and can be countered by torque from left post