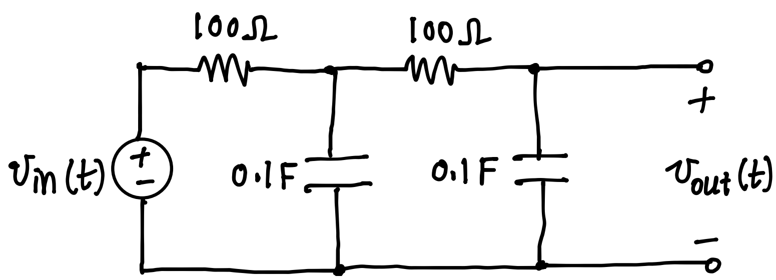
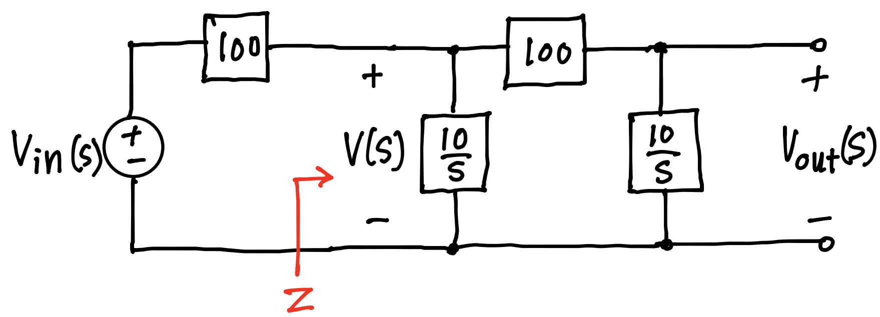
Example 4.1b: Consider the circuit shown below.



(a) Find the transfer function $H(s)$.

(b) Find $v_{\text{out}}(t)$ if $v_{\text{in}}(t) = 2e^{-t}u(t)$.

<u>Solution</u>:
(a) First convert to the s-domain circuit.



From voltage division,

$$V_{\text{out}}(s) = V(s)\frac{\frac{10}{s}}{100 + \frac{10}{s}} = V(s)\frac{1}{10s+1}. \tag{E1}$$

The impedance Z is

$$Z = \frac{1}{\frac{s}{10} + \frac{1}{100+\frac{10}{s}}} = \frac{1}{\frac{s}{10} + \frac{s}{100s+10}} = \frac{10(100s+10)}{s(100s+10)+10s}$$
$$= \frac{100s+10}{10s^2+2s} = \frac{50s+5}{5s^2+s}. \tag{E2}$$

Hence, the voltage division between $V_{\text{in}}(s)$ and $V(s)$ is

$$\begin{aligned} V(s) = V_{\text{in}}(s)\frac{Z}{100+Z} &= V_{\text{in}}(s)\frac{\frac{50s+5}{5s^2+s}}{100+\frac{50s+5}{5s^2+s}} \\ &= V_{\text{in}}(s)\frac{50s+5}{100(5s^2+s)+50s+5} \\ &= V_{\text{in}}(s)\frac{10s+1}{100s^2+30s+1}. \end{aligned} \tag{E3}$$

From (E1) and (E3),

$$V_{\text{out}}(s) = V_{\text{in}}(s)\frac{1}{100s^2+30s+1} \tag{E4}$$

or

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{100s^2+30s+1}. \tag{E5}$$

(b) Using $V_{\text{in}}(s) = \frac{2}{s+1}$, we have

$$V_{\text{out}}(s) = H(s)V_{\text{in}}(s) = \frac{0.02}{(s+1)(s^2+0.3s+0.01)}. \tag{E6}$$

Since the roots of $s^2 + 0.3s + 0.01$ are $p_1 = -0.2618$ and $p_2 = -0.0382$, we can write

$$\begin{aligned} V_{\text{out}}(s) &= \frac{0.02}{(s+1)(s+0.2618)(s+0.0382)} \\ &= \frac{A_1}{s+1} + \frac{A_2}{s+0.2618} + \frac{A_3}{s+0.0382}. \end{aligned} \tag{E7}$$

The coefficients are

$$A_1 = \left.\frac{0.02}{s^2+0.3s+0.01}\right|_{s=-1} = 0.0282, \tag{E8}$$

$$A_2 = \left.\frac{0.02}{(s+1)(s+0.0382)}\right|_{s=-0.2618} = -0.1212, \tag{E9}$$

$$A_3 = \left.\frac{0.02}{(s+1)(s+0.2618)}\right|_{s=-0.0382} = 0.093. \tag{E10}$$

Hence,

$$V_{\text{out}}(s) = \frac{0.0282}{s+1} - \frac{0.1212}{s+0.2618} + \frac{0.093}{s+0.0382}. \tag{E11}$$

So, we find

$$v_{\text{out}}(t) = (0.0282e^{-t} - 0.1212e^{-0.2618t} + 0.093e^{-0.0382t})u(t). \tag{E12}$$