Simulation - Simple Harmonic Motion

Overview

Computer simulation has become a standard practice in physics. We live in a time when computing power, the number of and speed of computations, has increased exponentially. This has given us the capability of doing many complex computations in a very short amount of time. It also means we can make physical predictions of things not yet observed or measured.

In this lab you will learn some of the basic ideas of computer simulation and use it to predict the properties of a simple harmonic oscillator: the period of a mass oscillating on a spring.

Experimental Design - Hooke's Law

The first part of a simulation is to develop a model of the system you are simulating. In this lab the system is a mass hanging on a spring. The hanging mass feels the force of weight F_W pulling down and the tension in the spring F_S pulling up.

1. Draw a free body diagram (FBD) of a mass hanging on a spring. Label the forces acting on the mass. You can draw the FBD on the whiteboard in the lab, then take a picture and paste it in your report.

2. Use Newton's 2nd Law to write a mathematical formula for the net force on the hanging mass. If the mass hanging on the spring is in equilibrium (not accelerating).

Fspring=mg

Netforce = mg-Fspring

The force of the spring F_S is proportional to how much the spring is stretched. The more you pull down on the spring, the stronger the spring pulls back. The relationship between the tension of the spring and the stretch of the spring is given by Hooke's Law:

$$F_S = -ky$$

Here y is the amount the hanging spring is stretched and k is what is known as the spring constant. It is the constant of proportionality between the tension in the spring and the amount of stretch. There is a minus sign (-) in the equation above. The meaning of the minus sign is the tension in the always pull back to the equilibrium length of the spring. Therefore the spring force is described as a *restoring* force. The tension in the spring is trying to restore the spring back to equilibrium.

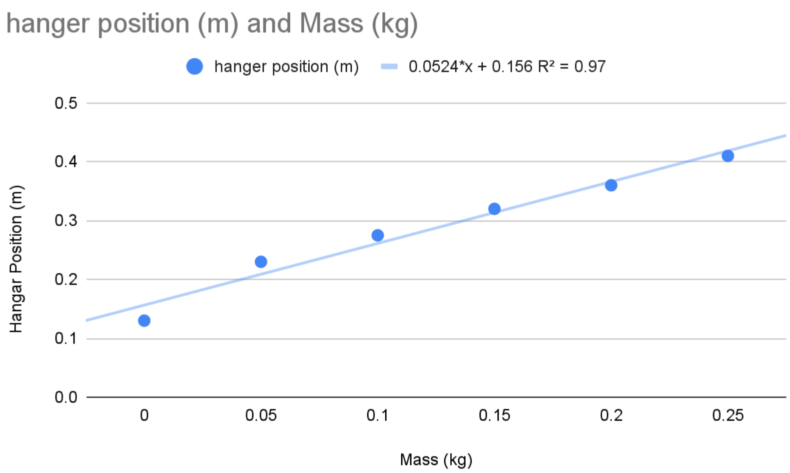
Simulations often use actual data in the model. In your model of simple harmonic motion you will need to know the spring constant of the mass-spring system.

- Hang a 50 gram hanger from the end of the spring. Record the position of the bottom of the hanger. Note: you can see the position of the hanger in the mirror next to the centimeter scale. This helps make a more accurate measurement. Record your values in the spreadsheet "Simulation - Simple Harmonic Motion - Data" on the sheet tabbed at the bottom "Hooke's Law."

3. Record the position of the bottom of the hanger for additional masses added to the hanger in 50 gram increments. Make about 6 measurements up to 250 grams of additional mass.

4. For each value of mass, calculate the weight hanging.

5. Make a scatter plot of the hanging weight vs. the position of the hanger. Copy and paste the plot here. Describe the relationship between the weight and the position of the hanger. Is this relationship consistent with Hooke's Law? Explain why.



The weight is linearly proportional to the hangar position, this is consistent with hooke's law because hooke's law is a simple linear relationship between x position and the spring constant.

6. Based on the relationship between the weight and position of the bottom of the hanger, use the function LINEST() to determine the best fit parameters. What quantity in Hooke's Law is related to the slope?

The slope is what is related to the spring constant.

Now you should have a value of the spring constant of the spring you are going to model in your simulation of the simple harmonic motion of a mass on a spring.

Simulation of Mass and Spring Oscillating

Now let's start building the simulation. In the spreadsheet "Simulation - SHM - Student Copy" on the sheet tabbed at the bottom "Mass-Spring."

- Enter the value of the spring constant in N/m you found in the previous part on Hooke's Law.

What is the value of mass you use in your simulation? It turns out that that question is a little complicated. It is not just the mass of the hanger and additional mass added that oscillates. When the mass is oscillating, the spring is oscillating too! So we need to include the mass of the spring in the simulation. It turns out that about 1⁄3 of the mass is in effect oscillating with the hanging mass. So the *total* mass in the simulation is:

$$M_{total} = m_{hanger} + 150grams + \frac{1}{3}m_{spring}$$

- Weigh the mass of the spring. Record the total mass of hanger, 150 grams and third of spring mass in the spreadsheet.

The simulation of a mass on spring represents the motion of the mass-spring system as it oscillates. The simulation calculates the position, velocity and acceleration of the system in specific increments of time. Here is how it works:

	A	B	C	D	E
1	Spring Constant (N/m) =			Total Mass (kg) =	
2					
3	time (sec)	position y (m)	velocity y (ms)	acceleration (m/s^2)	Spring Force (N)
4	0.000	-0.05	0		
5					

- Enter the initial values of time, position, and velocity into the spreadsheet from the table above (row 4).
- Calculate the spring force at initial time 0.0 in cell E4. When you make a calculation you always start with an equal sign (=). The spring force is the force according to Hooke's Law, which we are using to model a spring. Enter into cell E4 the formula =-$B$1*B4. The dollar symbols ($) holds the address of the column B and row 1 fix. It will not be incremented.
- Calculate the acceleration at initial time 0.0 in cell D4. Enter the formula =E4/$E$1.

Now we want to calculate the position, velocity, and acceleration for the next increment of time. We will increment in steps of 0.001 sec.

- Calculate in cell A5 the next increment of time. Enter the formula =A4+0.001
- Calculate in cell B5 the next increment of position. Enter the formula =B4+C4*0.001
- Calculate in cell C5 the next increment of velocity. Enter the formula =C4+D4*0.001
- Calculate in cell D5 the next increment of acceleration. Enter the formula =E5/$E$1
- Calculate in cell E5 the next increment of spring force. Enter the formula =-$B$1*B5

You can probably guess that this becomes very tedious if you have entered these formulas a thousand times, which is what you will have to do! Fortunately, there is a way to make the spreadsheet do all the work for you. Most spreadsheets are programmed to do calculation *iteratively*. That means the spreadsheet will make copies of the formulas you entered into the cell and iterate the variables. Here is how to do it. Click on cell A5. The cell will have a border around it with a little square in the lower right corner. Click and hold the square with the mouse and drag it down a few rows. When you release, the spreadsheet will copy/paste the formula you entered in cell A5 to the rows below and increment the value of time by 0.001 sec.

7. Copy/paste the formula for time (Column A) for 1200 rows.

8. Copy/paste the formula for position (Column B) for 1200 rows.

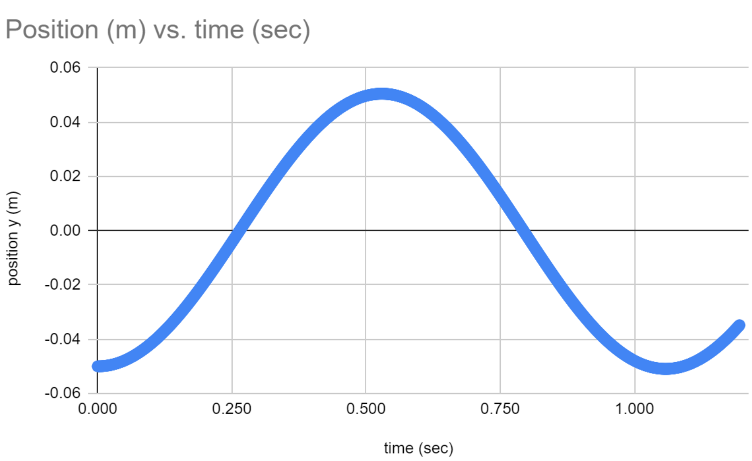
9. Copy/paste the formula for velocity (Column C) for 1200 rows.

10. Copy/paste the formula for acceleration (Column D) for 1200 rows.

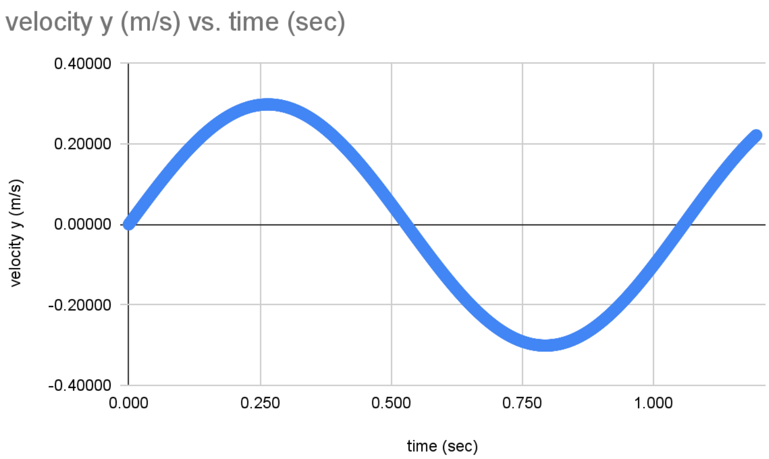
11. Copy/paste the formula for force (Column E) for 1200 rows.

Now you have simulated the motion of a mass on a spring for a duration of about 1.2 seconds.

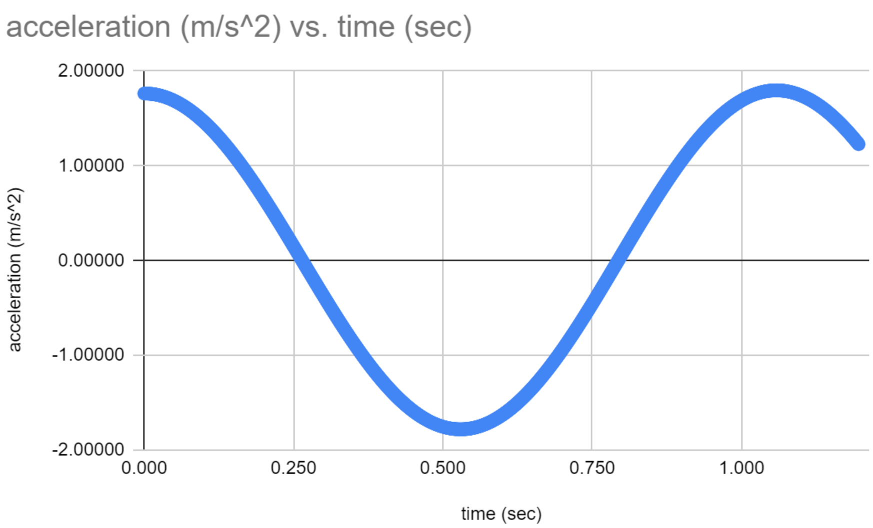
12. Make a plot of position vs. time of the simulation. How would you describe the motion of the mass on a spring based upon the simulation? Is it what you expected? Explain why
    The motion will be moving upwards, which is expected because the weight is not enough to keep the spring stretched to that length, and the spring will begin moving back and forth in a negative cosine motion.



13. Make a plot of velocity vs. time of the simulation. How would you describe the motion of the mass on a spring based upon the simulation? Is it what you expected? Explain why.
    This is exactly what is expected because the derivative of position is velocity, and the derivative of -cosine is sine, this looks like a textbook sine wave



14. Make a plot of acceleration vs. time of the simulation. How would you describe the motion of the mass on a spring based upon the simulation? Is it what you expected? Explain why.
    This graph is also exactly within expectation because it seems like a cosine wave, and the derivative of sine is cosine, as is the derivative of velocity is acceleration.



Simulations can be used to make predictions of the nature of the system represented by the simulation. Let's see what value the simulation predicts for the period of oscillation. The period is the duration of time for an oscillating system to complete one oscillation. The system returns to the same state (same position, same velocity) as it was in when it started oscillating. The simulation started with an initial position of -0.05 meters and initial velocity of zero.

15. Scroll down the spreadsheet and find the time when the simulation has returned to its initial position AND initial velocity. What is the value of that time?

row number 1063 seems very close to initial position and velocity. 1.059 seconds

Measuring the Period of Oscillation

Now let's measure the period of oscillation. Click the tab labeled “Mass-Spring Period” at the bottom of the spreadsheet. Record your measurements of period on this page of the spreadsheet.

- Put 150 grams on the hanger and hang the hanger at the end of the spring.
- Pull the mass down 0.05 meters below the equilibrium position and release it.
- Use the lab timer or even your own phone to measure the time it takes the mass oscillating on the spring to complete ten oscillations.
- Repeat the measurement a total of 5 times.

16. What is the average and standard deviation of the period of oscillation? Use the function AVERAGE() and STDEV() in your spreadsheet to calculate these values.

17. How does the measured period compare to the period predicted in the simulation?
    The simulation predicted that the 1.059 seconds while the measured period is .8838, the measured period is about 83% of the predicted time.