Announcements, Goals, and Reading

Announcements:

- HW05 is due Tuesday 10/18.
- HW04 was due Tuesday. Grace period ends tonight.
- Midterm 1: Thursday 10/20, 7-9PM
- SI Aditya's review session: Sunday, October 16th, 7-9pm, Thompson Hall 102
- SI Sam's review session: Wednesday, October 19th, 7-9pm, Thompson Hall 106
- TA review session: TBA (Monday)

Goals for Today:

- Forces
- Newton's Second Law

Reading (Physics for Scientists and Engineers 4/e by Knight)

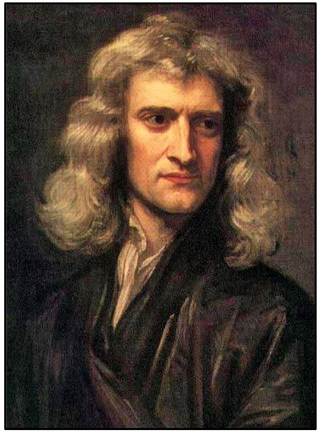
- Chapter 5: Force and Motion
- Chapter 6: Dynamics I: Motion along a Line

Midterm 1 — October 20th, 7-9PM

- **Covers Chapters 1-5* from Knight textbook, Homework 1-5***
- **Key topics: Motion, Acceleration, Position, Velocity, Kinematics, Projectile Motion, Circular Motion, and Forces*.** *No questions about sig. figs or relative motion.*
- Location depends on 1st letter of your last name:
  - **HAS20 – Last Name A-F**
  - **HAS124 - Last Name G-H**
  - **ISB135 - Last Name I-M**
  - **ILCN151 - Last Name N-T**
  - **HAS126 - Last Name U-Z**
  - **HAS138 - Reduced distraction / Extra time accommodation**
  - **Online-only students (UWW/FLEX sections): You will be contacted about details this week.**
  - *If you have extra time accommodations, please take the exam in HAS 138. I will come at the end to proctor the extra time. You can also take the exam with Disability Services. If you need other disability accommodations, please contact me.*
- **Allowed a Calculator and an 8.5”×11” reference sheet with handwritten notes on both sides.** Be sure to bring a calculator as there are never enough spares.
- ~25 Multiple Choice Questions; **Bring a #2 pencil**
- <u>A practice exam is now available on Moodle.</u>
- SI/TA exam review sessions will be held on exam week. Next Wed will be a review lecture.
- Makeup Exams: If you have a conflict with another exam, please let us know as soon as possible. Friday 10/14 will be the last day to request a makeup exam without penalty. E-mail our TA, Joanna Wuko (jwuko@physics.umass.edu) and CC me.

**Questions about Force will be limited in number, scope and complexity.*

Newton’s Laws of Motion

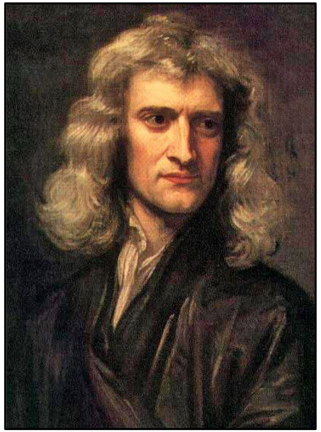


1st law – In the absence of a net external force, an object at rest will remain at rest, and an object in motion will remain in motion with a constant velocity.

2nd law – The acceleration of an object is proportional to the net external force acting on it, and is inversely proportional to its mass.

3rd law – When two objects interact, they exert equal and opposite forces on each other

1st law – In the absence of a ***net external force***, an object at rest will remain at rest, and an object in motion will remain in motion with a constant velocity.



Net external force

"**External**" – Don't need to consider internal forces which hold an object together

Don't affect the motion of an object. Ignore them from now on.

"**Net**" – Need to add all external forces together to get the net force.

Constant velocity $\Rightarrow$ No acceleration

1st Law $\Rightarrow$ No net force on an object means no acceleration

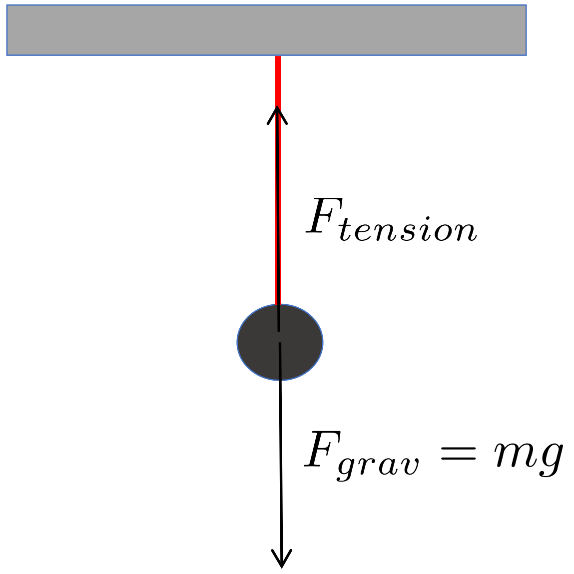
Conversely, if an object is not accelerating, there must be no net force on it

Can learn things about forces from the 1st law.

Object of mass m near the surface of the earth has a downward force on it due to Earth's gravity.

The object is suspended from the ceiling by a red rope.

The rope exerts upwards force on the object whose magnitude is the tension in the rope.



Object isn't moving: net force is zero

$$\vec{F}_{\text{tension}} + \vec{F}_{\text{gravity}} = 0$$

$$F_{\text{tension}}\,\hat{y} - mg\,\hat{y} = 0$$

$$F_{\text{tension}} - mg = 0$$

$$\Rightarrow \quad F_{tension} = mg$$

We can hang objects with different masses from rope

Note → Tension in rope can vary, up to a maximum possible value that makes it break

Similar application of 1st law

A block of mass m sits on the floor

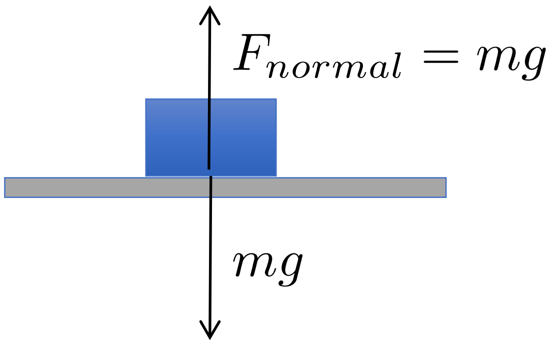
Gravitational force acts downwards

No acceleration ➡ $\vec{F}_{net} = 0$

Must be a force of same magnitude exerted upwards by floor on block

Called normal force

"Normal" here means perpendicular



The floor can exert any normal force necessary to keep an object from falling through it.

At least up to the breaking point of the floor

Conceptual Questions:

Q1) Bob was transporting an open box of cupcakes to a school party. The car in front of him stopped suddenly; he applied the brakes immediately. He was wearing his seat belt and suffered no physical harm, but the cupcakes flew into the dashboard and became “smushcakes.” Explain what happened.

Q2) A rock is thrown straight up. At the top of the trajectory, the velocity is momentarily zero. Does this imply that the force acting on the object is zero? Explain your answer.

A crate is located in the center of a flatbed truck. The truck accelerates to the east, and the crate moves with it, not sliding at all. What is the direction of the friction force exerted by the truck on the crate?

1) To the west

2) To the east

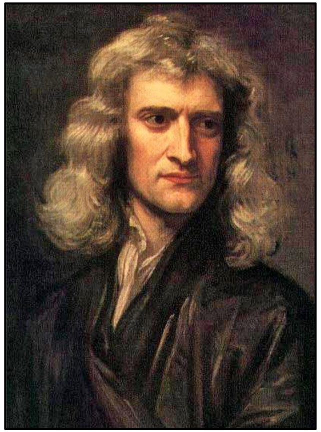
3) Up

4) Down

5) No friction force exists because the crate is not sliding.

6) None of the above

2nd law – The acceleration of an object is proportional to the net external force acting on it and is inversely proportional to its mass.



Acceleration and Force are 3D vectors

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

There may be some number of external forces acting on an object

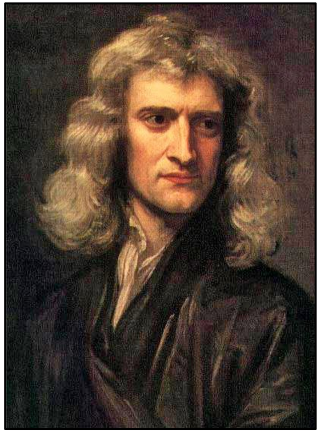
$$\vec{F}_i \qquad i = 1, 2, \ldots, N$$

Total number of external forces

Net external force

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N = \sum_{i=1}^{N} \vec{F}_i$$

2nd law – The acceleration of an object is proportional to the net external force acting on it, and is inversely proportional to its mass.



The acceleration of an object with mass m will be

$$\vec{a} = \frac{1}{m}\vec{F}_{net}$$

More commonly written as

$$\vec{F}_{net} = m\vec{a}$$

And still more commonly as

$$\boxed{\vec{F} = m\vec{a}}$$

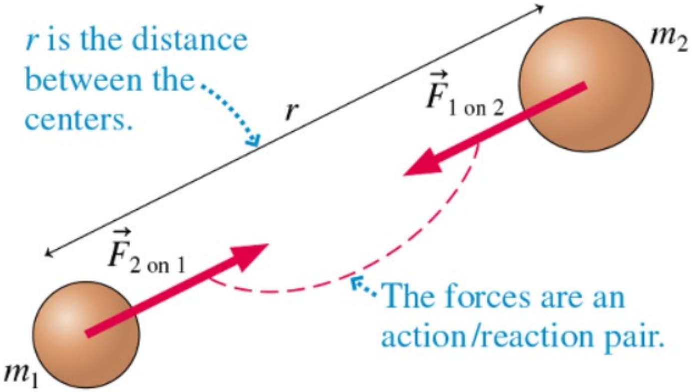
Where we tacitly understand that the force on the left hand side is the net force.

Think of the forces as being the "causes" and acceleration is the mass-dependent "effect"

Gravitational Force

For any 2 objects distance r apart…

$$F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = \frac{G m_1 m_2}{r^2}$$



…where G is the "gravitational constant."

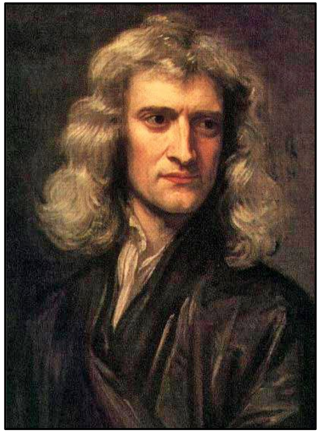
In this course, we assume we are approximately near the Earth's surface, so to calculate Earth's force of gravity we set $R=R_E$ and one of the masses to m_E.

So an object with mass m_1 would experience $|F| = m_1 a = \frac{G m_1 m_E}{R_E^2} = \mathrm{m}_1 \left(\frac{\mathrm{Gm_E}}{\mathrm{R_E^2}}\right) = \mathrm{m}_1 \mathrm{g}$

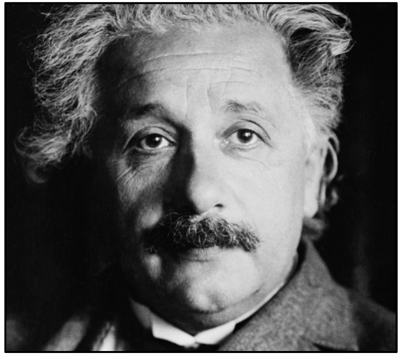
So a=g pointing toward earth for ANY mass as long as $r \sim R_E$

Mass

Second Law: **mass** is a measure of inertia, a reluctance to accelerate.



Also, **mass** is a measure of the amount of "stuff" in an object because this "stuff" interacts via gravitational force

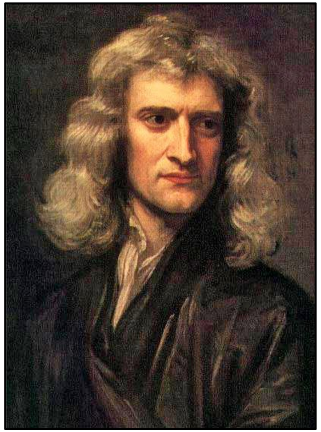


Although these are two distinct concepts, inertial and gravitational mass are the same!

$$-G\frac{mM_{Earth}}{R_{Earth}^2} = ma \Rightarrow a = -g; \quad g = G\frac{M_{Earth}}{R_{Earth}^2} = 9.8\frac{m}{s^2}$$

Dimension & Units of Force

$$\vec{F} = m\vec{a}$$



Dimension of left hand side = Dimension of right hand side

RHS ➡ $$\frac{(\mathrm{mass})(\mathrm{length})}{(\mathrm{time})^2}$$

So this is also the dimension of force

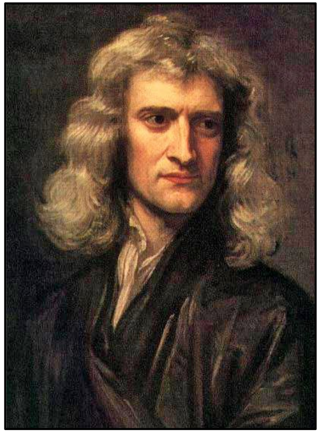
SI unit of force

$$1\,\mathrm{Newton} = 1\,\frac{kg\,m}{s^2}$$

Abbreviated as $1\,N = 1\,\frac{kg\,m}{s^2}$

Free Body Diagrams

$$\vec{F}_{net} = m\vec{a}$$

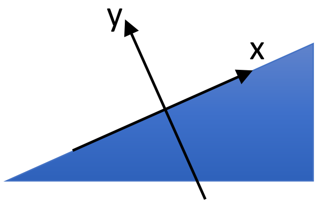


Systematic way to attack force & acceleration problems

1. Isolate the object being analyzed and draw all ***forces that act ON the object***.

2. Draw conveniently oriented coordinate axes and find components of forces along these axes.

Example – with inclined plane, orient axes parallel and perpendicular to the inclined plane



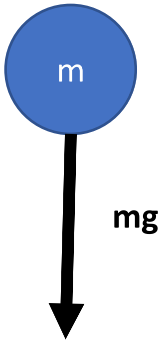
3. Apply 2nd law. Determine acceleration along each of the axes.

Alternatively - may be told acceleration, and need to determine one of the forces.

Some problems will also require free body diagrams for more than one object

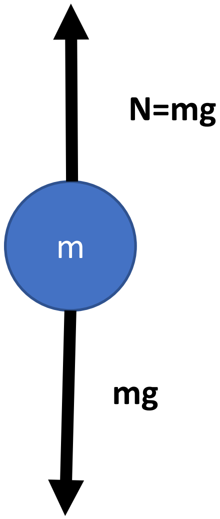
Simple examples..

Free Body Diagram: Object in Free Fall



$$\boldsymbol{a} = -\boldsymbol{g}\hat{\boldsymbol{y}}$$

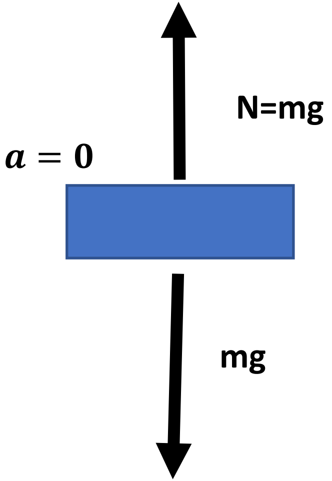
Free Body Diagram: Student Sitting At Desk



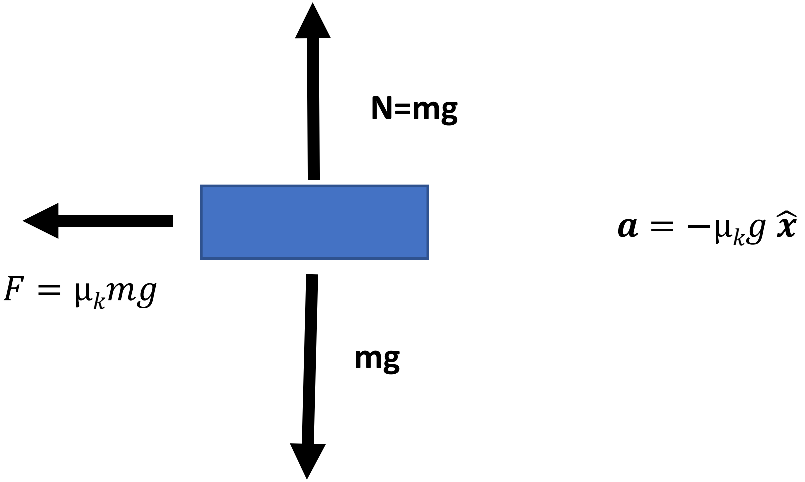
$$\boldsymbol{a} = \mathbf{0}$$

Simple examples..

Free Body Diagram: Puck Sliding on Ice



Free Body Diagram: Puck Sliding on Dirt



Note: Free body diagrams don't include velocities, but you can imagine the puck is moving this way---------->

Example of Free Body Diagram

$$\vec{F}_{net} = m\vec{a}$$

A particle of mass 3 kg, lying on the frictionless surface, is acted on by two forces parallel to the ground.

The first has magnitude 10 N and points straight north

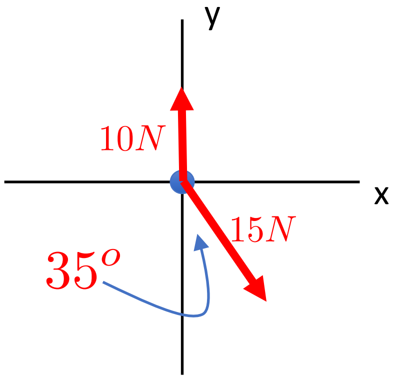
The second has magnitude 15 N and points 35^o east of south

Find the acceleration of the particle.

$$\vec{F}_1 = (10N)\hat{j}$$

$$\vec{F}_2 = (15N)\sin 35^o\,\hat{i} - (15N)\cos 35^o\,\hat{j}$$

$$= 8.6N\,\hat{i} - 12.3N\,\hat{j}$$



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$= 8.6N\,\hat{i} - 2.3N\,\hat{j}$$

Example of Free Body Diagram

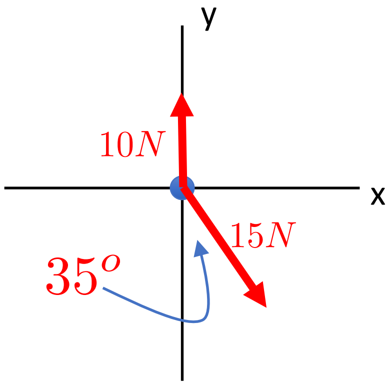
$$\vec{F}_{net} = m\vec{a}$$

A particle of mass 3 kg, lying on the frictionless surface, is acted on by two forces parallel to the ground.

The first has magnitude 10N and points straight north

The second has magnitude 15N and points 35° east of south

Find the acceleration of the particle.



$$\vec{F}_{net} = 8.6N\,\hat{i} - 2.3N\,\hat{j}$$

$$\vec{a} = \frac{1}{m}\vec{F}_{net}$$

$$= \frac{1}{3kg}(8.6N\,\hat{i} - 2.3N\,\hat{j})$$

$$= \underbrace{(2.9m/s^2)}_{a_x}\,\hat{i} + \underbrace{(-0.77m/s^2)}_{a_y}\,\hat{j}$$

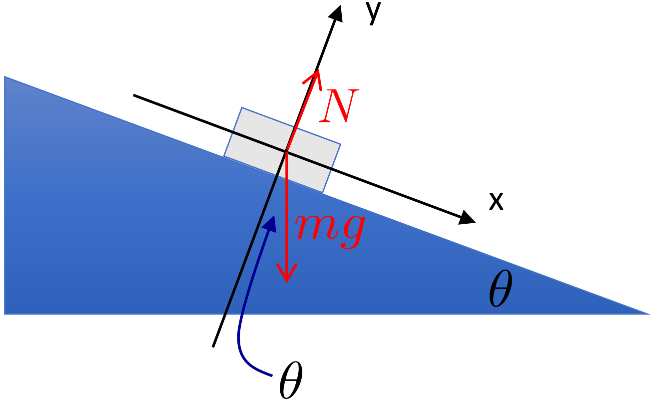
$$1\frac{N}{kg} = 1\frac{kg\,m/s^2}{kg} = 1m/s^2$$

Another example…

A block of mass m sits on a frictionless inclined plane at angle θ

Find acceleration of block down the plane.

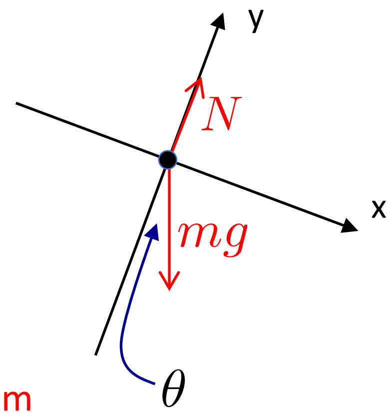
Find magnitude of normal force of plane on block.



Draw conveniently oriented coordinate axes

Makes things much easier

Draw in Forces



Let N be unknown magnitude of Normal force

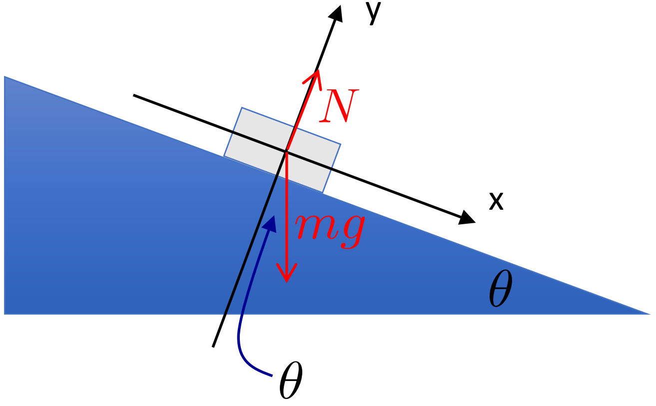
Free body "purists" would draw a separate diagram

But it is also fine to include forces as part of larger diagram

A block of mass m sits on an inclined plane at angle θ

Find acceleration of block down the plane.

Find magnitude of normal force of plane on block.



Add forces together to find net force

$$\vec{F}_1 = N\,\hat{j}$$

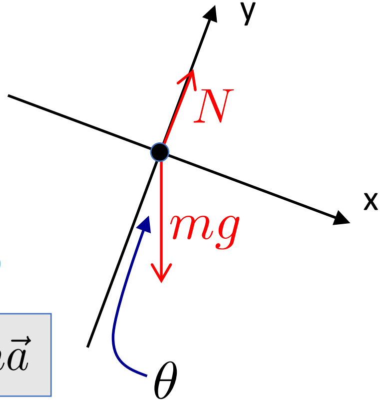
$$\vec{F}_2 = mg\sin\theta\,\hat{i} - mg\cos\theta\,\hat{j}$$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

$$= mg\sin\theta\,\hat{i} + (N - mg\cos\theta)\,\hat{j}$$ (cause)

$$= ma_x\,\hat{i} + ma_y\,\hat{j}$$ (effect)

$$\boxed{\vec{F}_{net} = m\vec{a}}$$



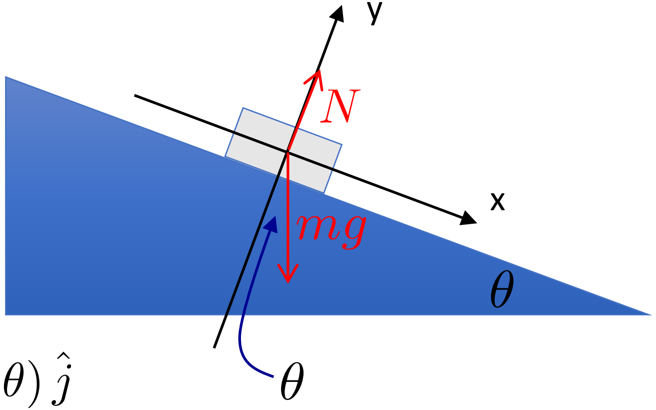
$\Rightarrow$ $a_x = g\sin\theta$ ✓

As we had assumed in kinematics discussion.

A block of mass m sits on an inclined plane at angle θ

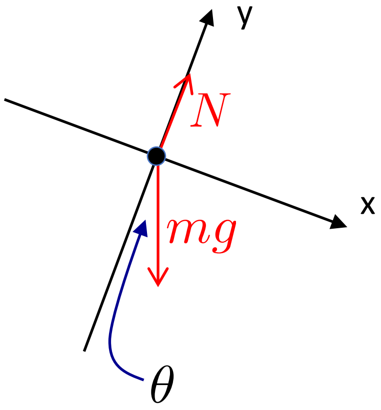
Find acceleration of block down the plane.

Find magnitude of normal force of plane on block.



$$\vec{F}_{net} = mg\sin\theta\,\hat{i} + (N - mg\cos\theta)\,\hat{j}$$
$$= ma_x\,\hat{i} + ma_y\,\hat{j}$$

➡ $a_x = g\sin\theta$ Acceleration down plane



Normal force?

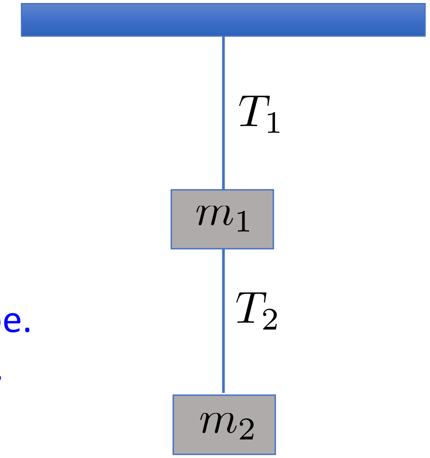
Block is not lifting off or falling through plane ➡ $a_y = 0$

➡ $N = mg\cos\theta$

$\theta = 0$ ➡ $N = mg$ ✓ Correct for horizontal surface

Blank

Keep going with examples



Two boxes with masses m_1 and m_2 are suspended by ropes

Find the tensions T_1 and T_2 in the two ropes

Magnitude of tension is the same everywhere in a rope.
Direction? Tension acts to pull contact points together.

What should we expect?

Upper rope supports both masses

$$T_1 = m_1 g + m_2 g$$

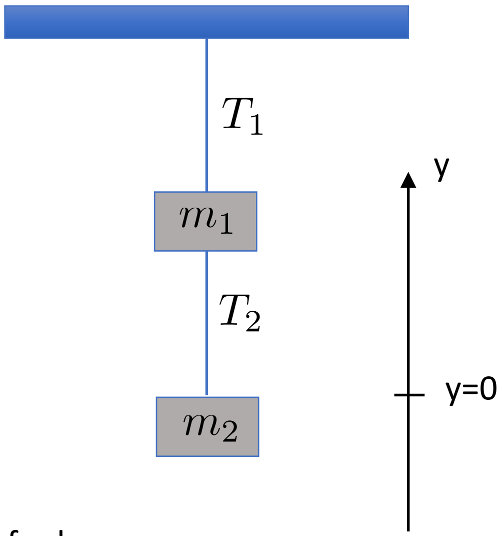
Lower rope supports only lower mass

$$T_2 = m_2 g$$

See how this comes out of free body diagram analysis

Always good to understand simple cases first

Two boxes with masses m_1 and m_2 are suspended by ropes

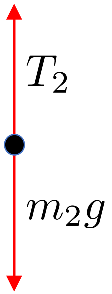


Find the tensions T_1 and T_2 in the two ropes

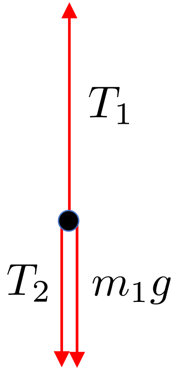
Need to draw a free body diagram for each mass

1D problem ➡ Only necessary to choose which direction is positive

Lower mass



Upper mass



2nd law for lower mass

$$T_2 - m_2 g = m_2 a = 0$$

$$\Rightarrow T_2 = m_2 g \quad \checkmark$$

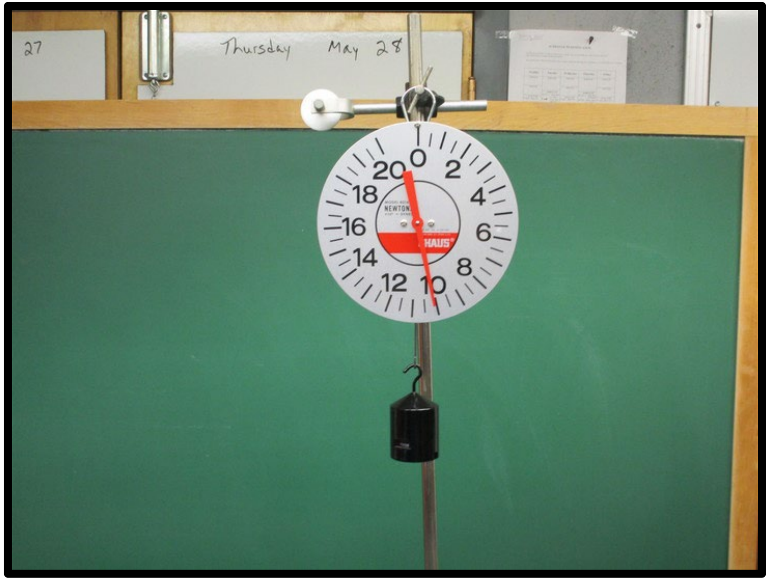
Not moving

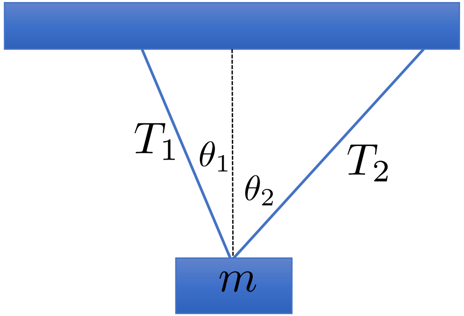
2nd law for upper mass

$$T_1 - T_2 - m_1 g = m_1 a = 0$$

$$\Rightarrow T_1 = m_1 g + T_2 = m_1 g + m_2 g \quad \checkmark$$

Demo: Tension in a string

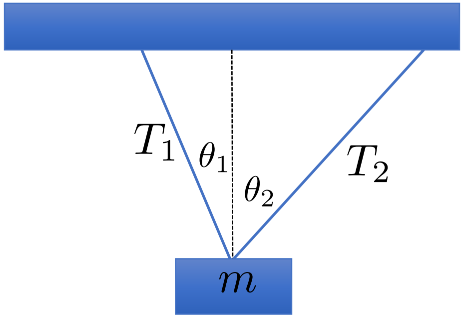




A box of mass m=30 kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

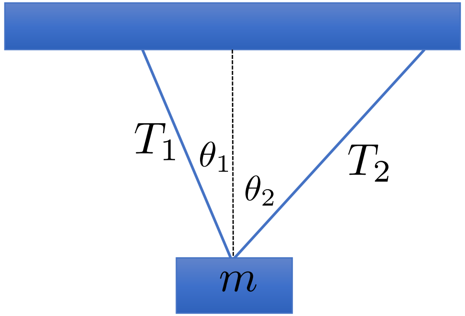
Find T_1 and T_2



A box of mass m=30 kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

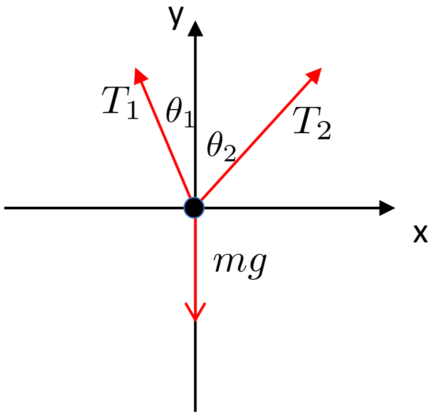
Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



Write out components of individual forces

$$\vec{F}_1 = -mg\,\hat{j}$$
$$\vec{F}_2 = -T_1 \sin\theta_1\,\hat{i} + T_1\cos\theta_1\,\hat{j}$$
$$\vec{F}_3 = +T_2 \sin\theta_2\,\hat{i} + T_2\cos\theta_2\,\hat{j}$$



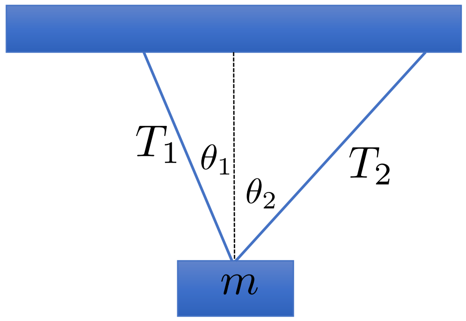
Compute net force

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$
$$= (-T_1\sin\theta_1 + T_2\sin\theta_2)\hat{i} + (T_1\cos\theta_1 + T_2\cos\theta_2 - mg)\hat{j}$$

A box of mass m=30 kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where... $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2

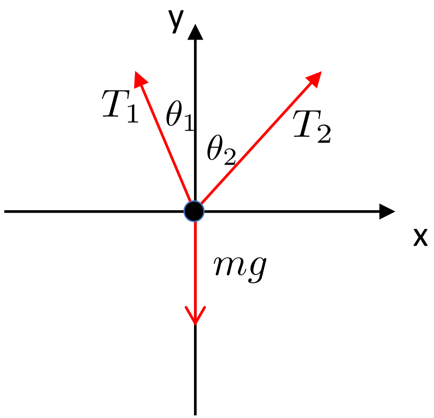


Write out components of individual forces

$$\vec{F}_1 = -mg\,\hat{j}$$

$$\vec{F}_2 = -T_1 \sin\theta_1\,\hat{i} + T_1\cos\theta_1\,\hat{j}$$

$$\vec{F}_3 = +T_2 \sin\theta_2\,\hat{i} + T_2\cos\theta_2\,\hat{j}$$



Compute net force

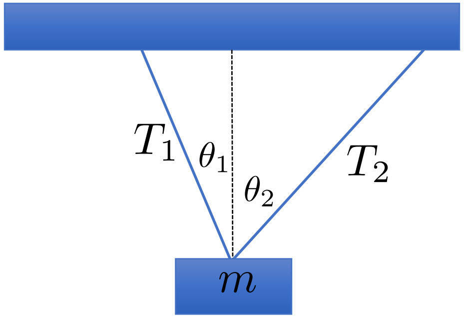
$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= (-T_1\sin\theta_1 + T_2\sin\theta_2)\hat{i} + (T_1\cos\theta_1 + T_2\cos\theta_2 - mg)\hat{j}$$

A box of mass m=30kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



$$\vec{F}_{net} = (-T_1 \sin\theta_1 + T_2 \sin\theta_2)\hat{i} + (T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg)\hat{j}$$
$$= m\vec{a} = 0$$

Not moving!

Both x and y components of net force must vanish → 2 equations with 2 unknowns

$$-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0$$
$$T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg = 0$$

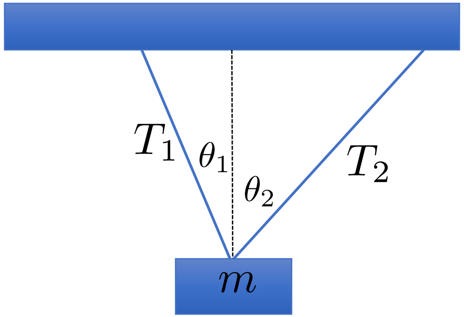
Solve 1st equation for T_2 and plug into 2nd equation

→ $$T_2 = \frac{\sin\theta_1}{\sin\theta_2} T_1$$

A box of mass m=30kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where... $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



$$\vec{F}_{net} = (-T_1 \sin\theta_1 + T_2 \sin\theta_2)\hat{i} + (T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg)\hat{j}$$
$$= m\vec{a} = 0$$

Not moving!

Both x and y components of net force must vanish → 2 equations with 2 unknowns

$$-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0$$
$$T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg = 0$$

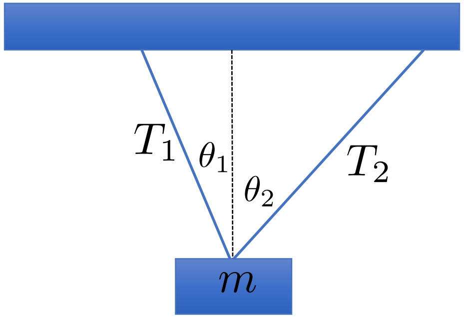
Solve 1st equation for T_2 and plug into 2nd equation

$$\Rightarrow T_2 = \frac{\sin\theta_1}{\sin\theta_2} T_1$$

A box of mass m=30kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



$$-T_1 \sin\theta_1 + T_2 \sin\theta_2 = 0 \quad \Rightarrow \quad T_2 = \frac{\sin\theta_1}{\sin\theta_2} T_1$$

$$T_1 \cos\theta_1 + T_2 \cos\theta_2 - mg = 0$$

Plug in T_1 to obtain T_2

Plugging T_2 into 2nd equation gives…

$$T_1 \left(\cos\theta_1 + \frac{\cos\theta_2 \sin\theta_1}{\sin\theta_2}\right) - mg = 0$$

Plug in numbers for m, θ_1, θ_2 to get final answers

Solve for T_1

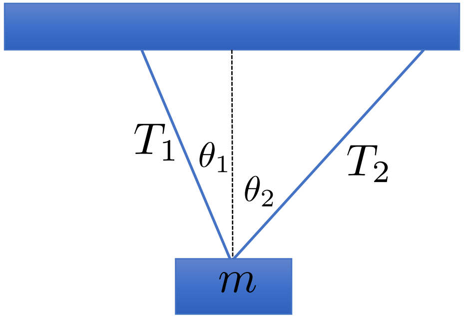
$$T_1 = \frac{mg \sin\theta_2}{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}$$

$$T_2 = \frac{mg \sin\theta_1}{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}$$

A box of mass m=30kg is suspended from 2 ropes with tensions T_1 and T_2 at angles θ_1 and θ_2

Where… $\theta_1 = 30^o \quad \theta_2 = 45^o$

Find T_1 and T_2



$$T_1 = \frac{mg\sin\theta_2}{\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1}$$

$$T_2 = \frac{mg\sin\theta_1}{\cos\theta_1\sin\theta_2 + \cos\theta_2\sin\theta_1}$$

Interesting limiting cases…

Equal angles $\theta_1 = \theta_2$ ➡ $T_1 = T_2 = \dfrac{mg}{2\cos\theta}$

So that y-components are equal to mg/2

One vertical rope $\theta_1 = 0$ ➡ $\sin\theta_1 = 0$, $\cos\theta_1 = 1$ ➡ $T_1 = mg$, $T_2 = 0$

Vertical rope supports all the weight