Introduction to Electrical and Computer Engineering

Feedback Control

(hidden principle/technology)

Gyro Boy

Segue

Booster Vehicle during Ascent

Balancing a Stick

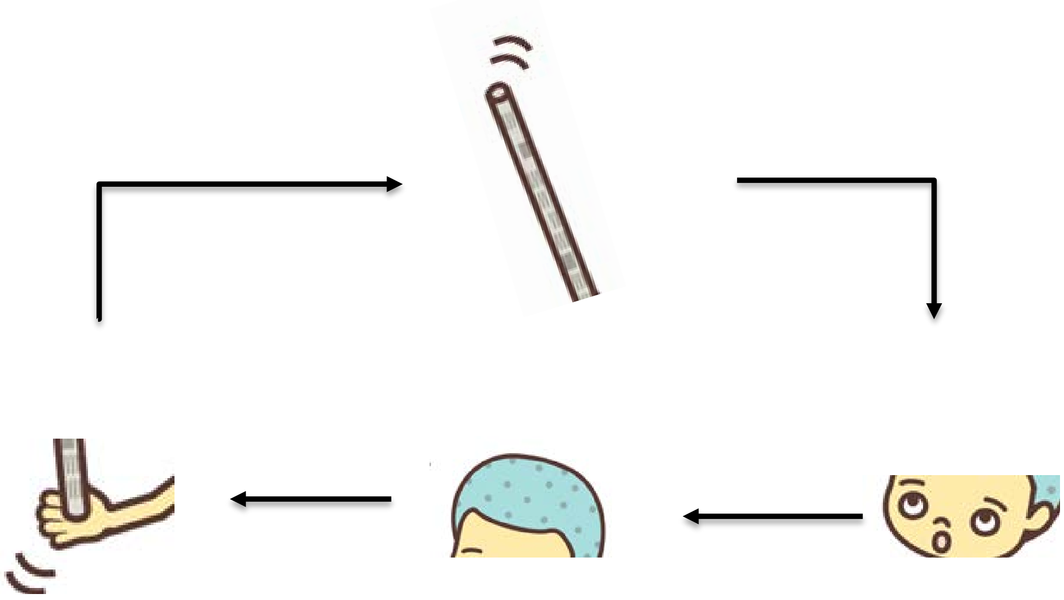
Questions

- Which sticks harder to balance?
  - Smaller lengths
- What did the “Balancer” watch?
  - Stick angle
- Did the “Balancer’s” position change?
  - Yes

“Balancer’s Principal Parts

- Visual system – measure stick angle
- Hand – provides restoring force
- Brain – computes the necessary hand commands

Balancer in Feedback with Stick

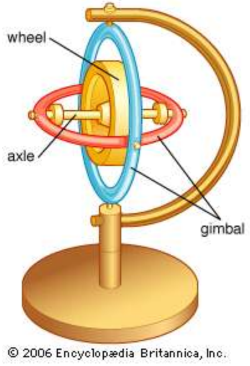


Gyro Boy’s Principal Parts

- Gyros – measure imbalance
- Motors – provide restoring force (wheels)
- Computer – computes the necessary motor commands

Gyro Boy’s Gyros

Gyros



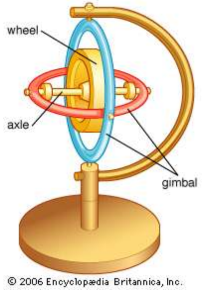
Gyro Boy’s Servo Motors

Motors (servo motors)

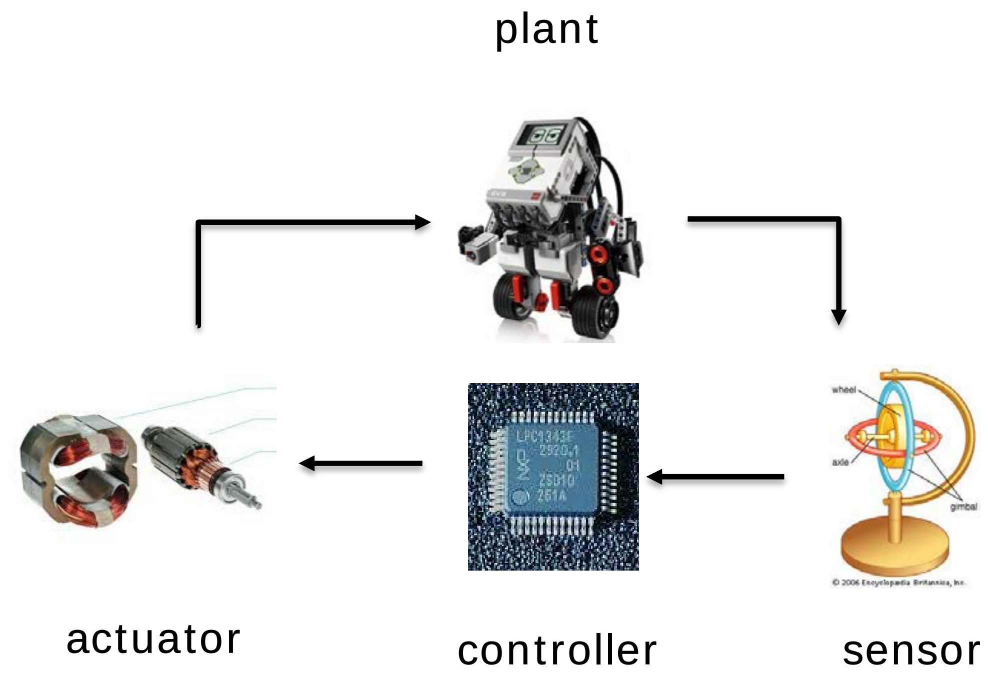
Gyro Boy’s Computer

Microcontroller (ARM family)

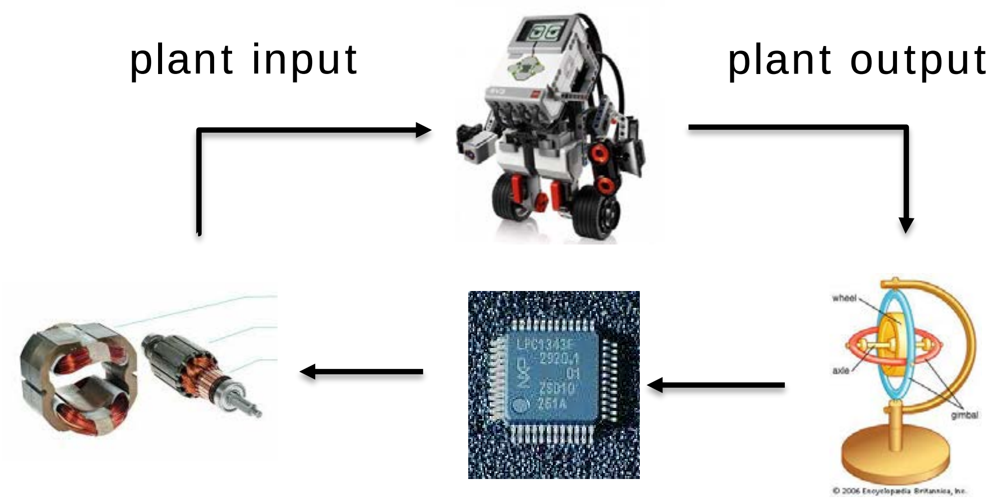
Gyro Boy’s Feedback Loop



Feedback Loop Jargon



Feedback Loop Jargon (more)

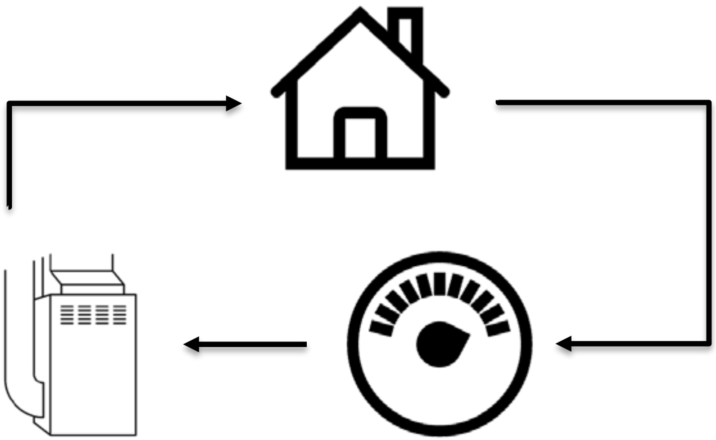


- plant input = ?
  "force" produced by motors

- plant output = ?
  gyro boy's "tilt angle"

Example: Home Heating System

plant = house



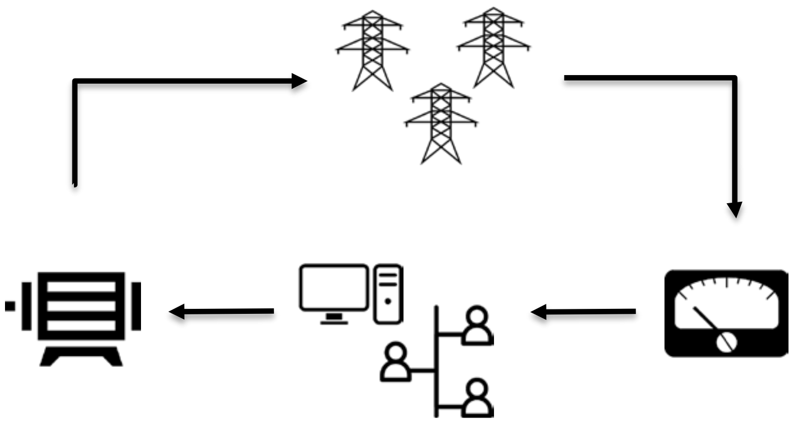
actuator
= furnace

controller + sensor
= thermostat

- plant input = ?
  "heat" produced and distributed by furnace
- plant output = ?
  house's temperature

Example: Electric Grid

plant = electric grid



actuators
= generators

controllers
= computer & humans

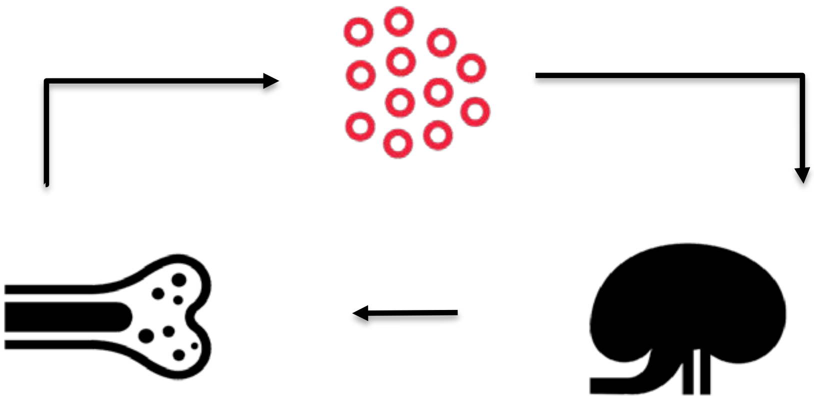
sensors = meters

- plant input = ?
  "electric power"
  produced by
  generators

- plant output = ?
  "electric power"
  carried by grid &
  delivered to users

Example: Blood Oxygen

plant = red blood cell pool



actuators
= bone marrow

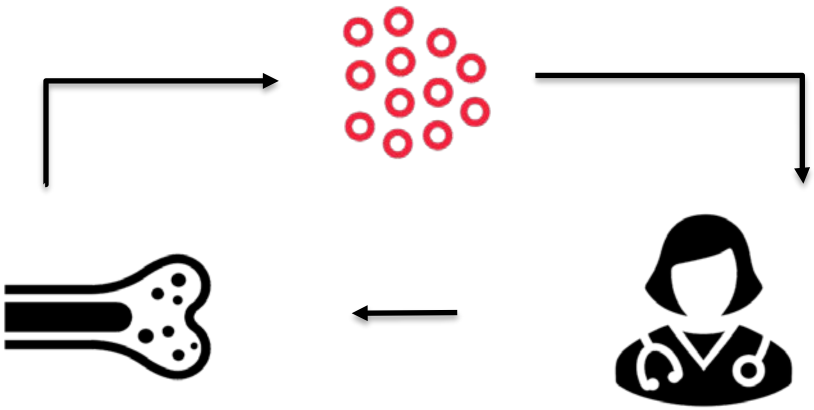
sensor + controller
= kidneys

- plant input =
  new red blood cells

- plant output =
  total blood hemoglobin

Example: Kidney Failure

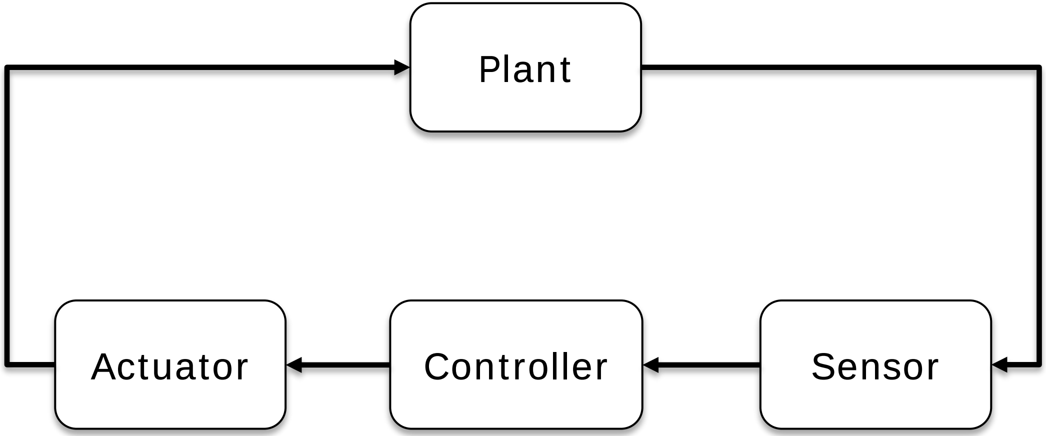
plant = red blood cell pool



actuators
= bone marrow

sensor + controller
= doctor prescribes erythropoietin (EPO)

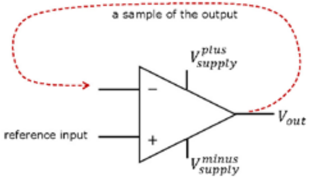
Standard Feedback Loop Block Diagram



Challenge: Express "Feedback Amplifier" as a Standard Feedback Loop

Adding Feedback to the OpAmp

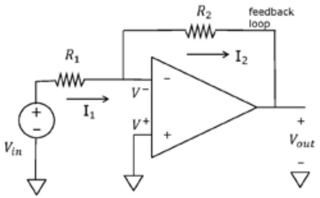
- Providing a sample of output voltage of the OpAmp, back to its input, is a way that it can be controlled to keep it from reaching its extremes
- This is what is called feedback control, and is the topic of next week's module.



- feedback occurs when you connect part of the output back to the input
- positive feedback is when the return path reinforces the input signal
- negative feedback is when the return path partially cancels the input signal

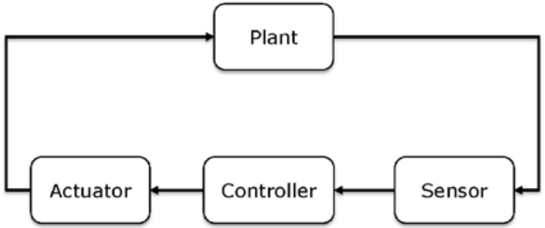
Basic Op Amp Rules

- There are two basic rules for working with Op Amps when feedback is used
- 1.) Voltage $V^+ = V^-$
- 2.) Current $I_1 = I_2$

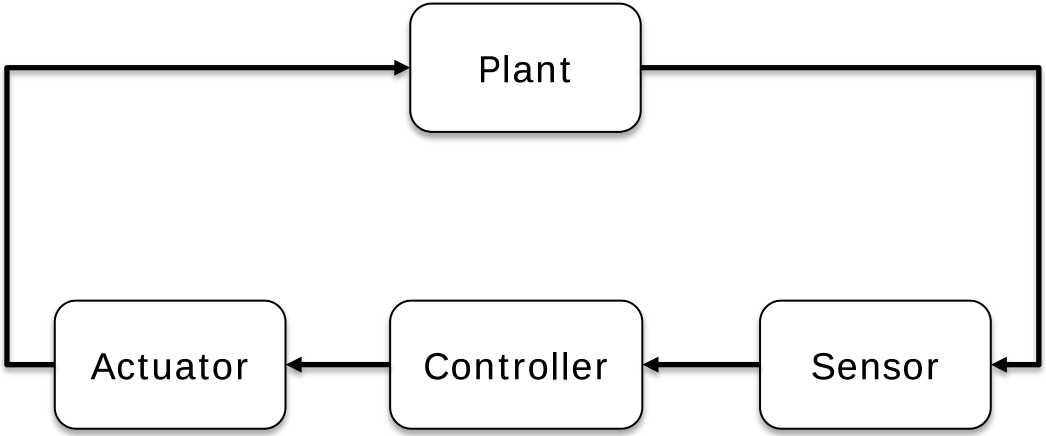


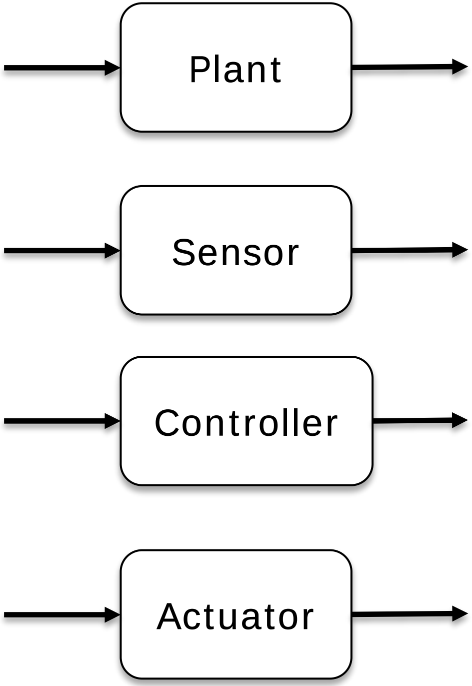
The reasons why these two rules exist, requires some in-depth analysis. This analysis is not part of this course. For now, just assume that these two rules govern the behavior of the Op Amp

Standard Feedback Loop Block Diagram

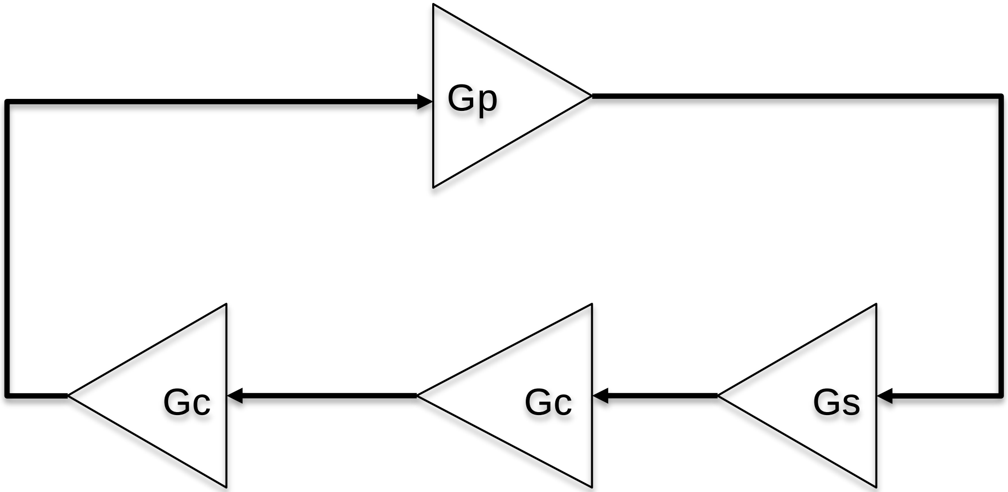


Systems Blocks

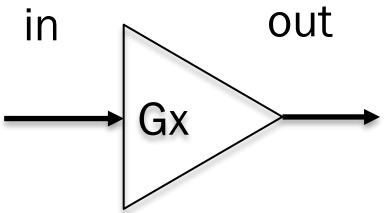




Example: Blocks are Amplifiers

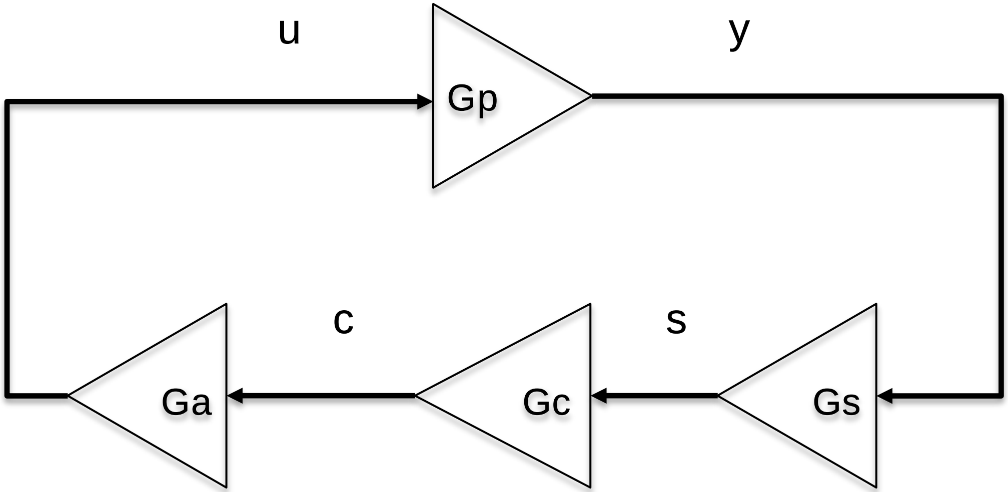


Rule



out = Gx * in

Basic Feedback Loop Equation



Closed Loop Equations

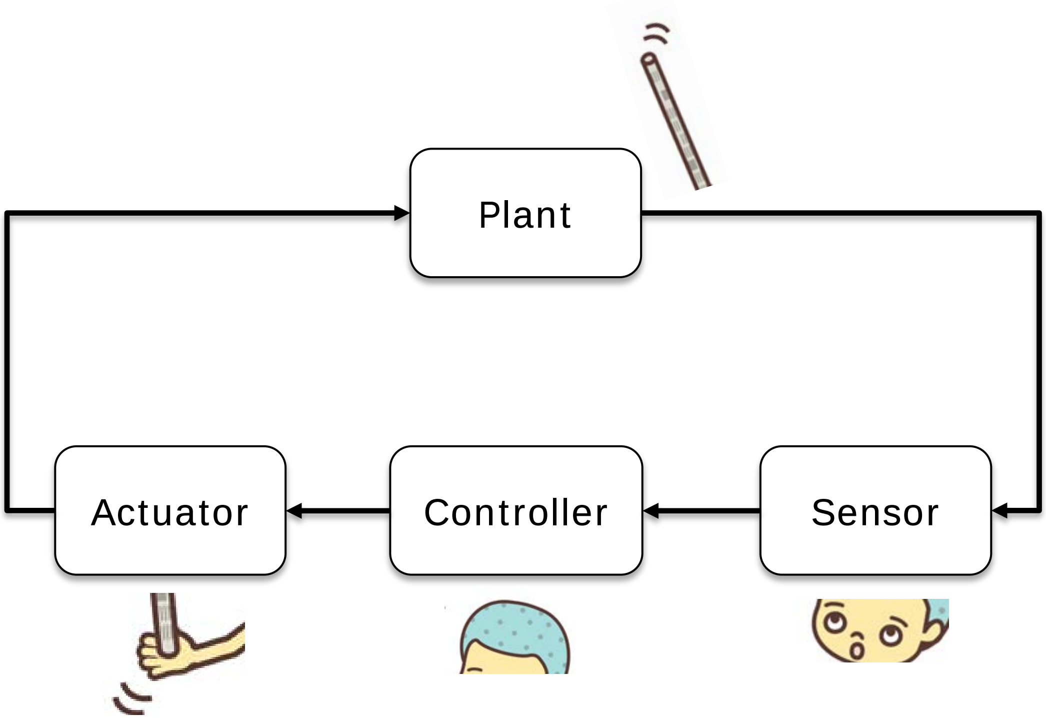
y = Gp*u

s = Gs*y

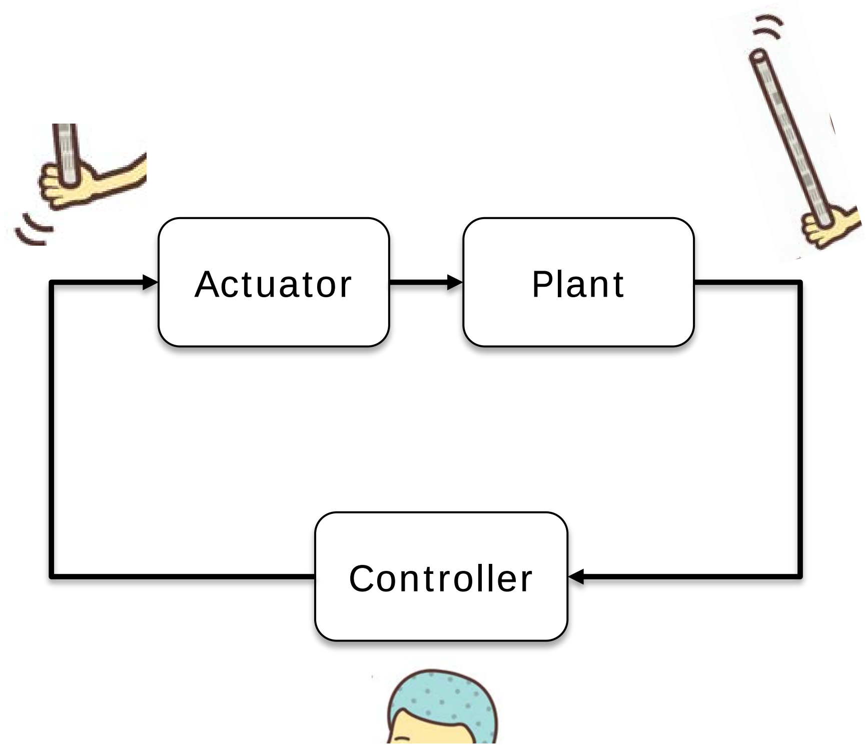
c = Gc*s

u = Ga*c

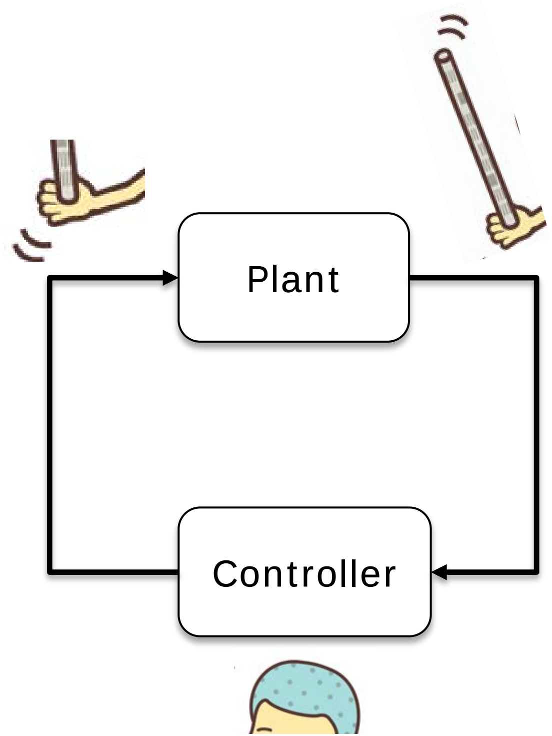
Feedback Loop – Balancing a Stick



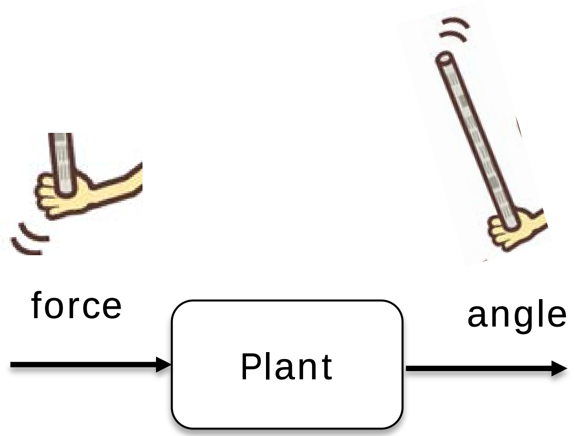
Feedback Loop – Balancing a Stick



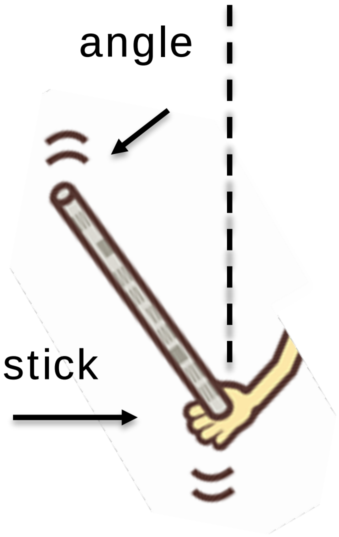
Feedback Loop – Balancing a Stick - Simplify



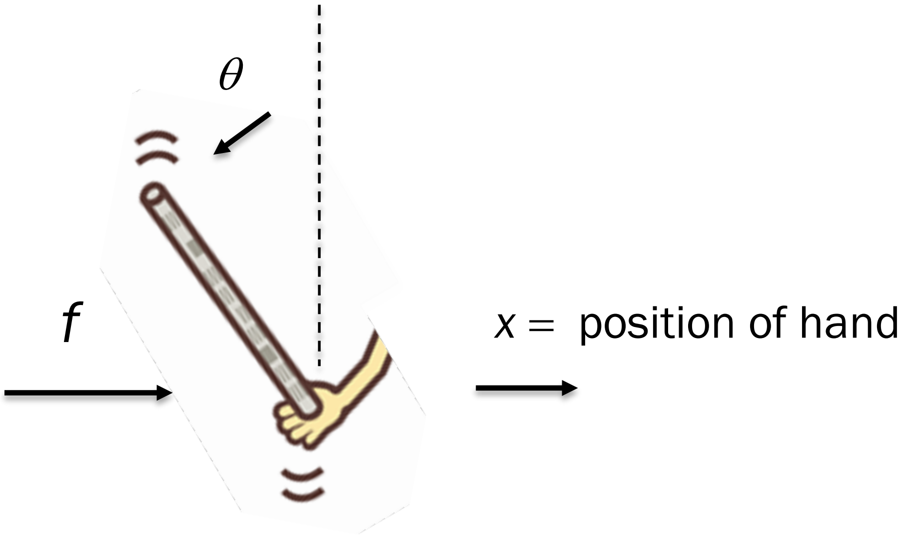
Modelling the plant



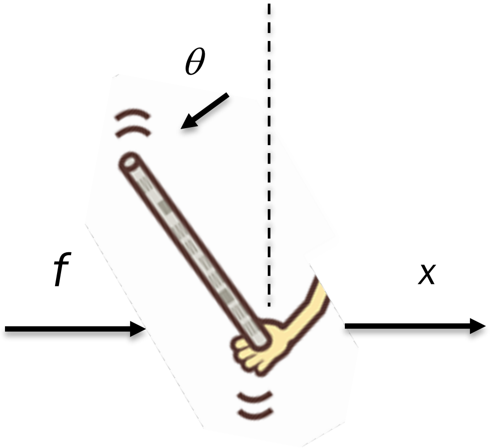
Modelling the plant



Motions (rotational & translational)



Newton's Laws of Motion



$$Ma = \sum forces; \quad \text{translational}$$

$$I\alpha = \sum torques; \quad \text{rotational}$$

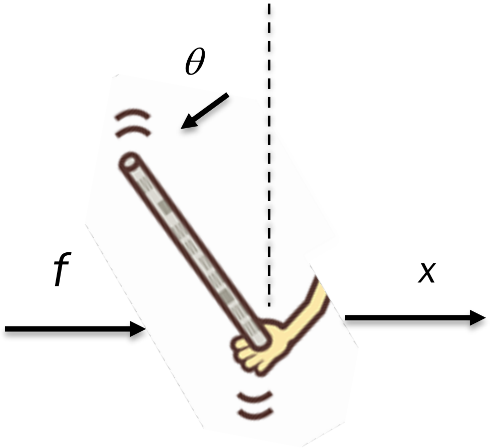
Law of Motion as Differential Equation (D.E.)

$$Ma = \sum forces$$

$$
\begin{aligned}
a &= \frac{\Delta \text{ velocity}}{\Delta t} \\
&= \frac{d}{dt}\text{velocity} \\
&= \frac{d}{dt}\ \frac{\Delta \text{ position}}{\Delta t} \\
&= \frac{d}{dt}\ \frac{d}{dt}\ \text{position} \\
&= \frac{d^2}{dt^2}\ x \\
&\equiv \ddot{x}
\end{aligned}
$$

$$M\ddot{x} = \sum forces$$

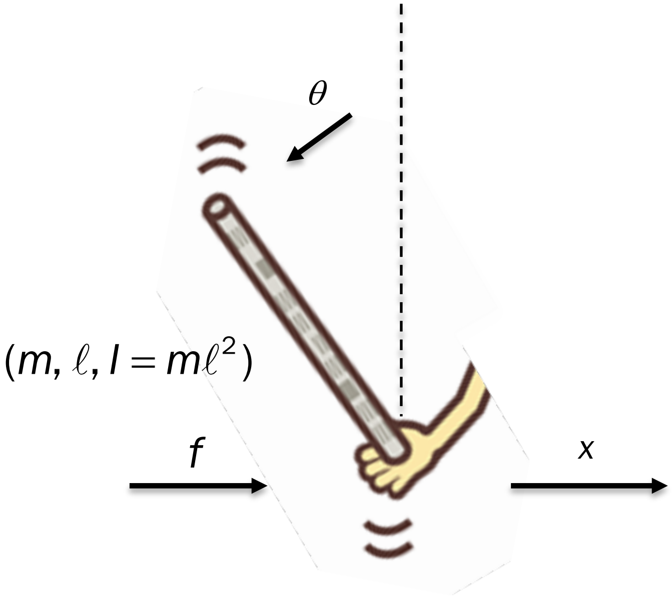
Newtons Law of Motion



$$Ma = \sum forces; \quad \text{translational}$$

$$I\alpha = \sum torques; \quad \text{rotational}$$

D.E. for the Hand-Stick

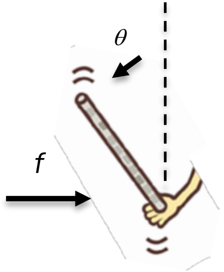


$$\ddot{\theta} - \frac{g}{\ell}\theta = \frac{1}{m\ell} f$$

$$\ddddot{x} - \frac{g}{\ell}\ddot{x} = \ddot{f} - \frac{2g}{\ell} f$$

g = gravitational acceleration

D.E. Model of the plant



$$f \longrightarrow \boxed{\ddot{\theta} - \frac{g}{\ell}\theta = \frac{1}{m\ell} f} \longrightarrow \theta$$

D.E. Solution

$$\ddot{\theta} - \frac{g}{\ell}\theta = \frac{1}{m\ell} f$$

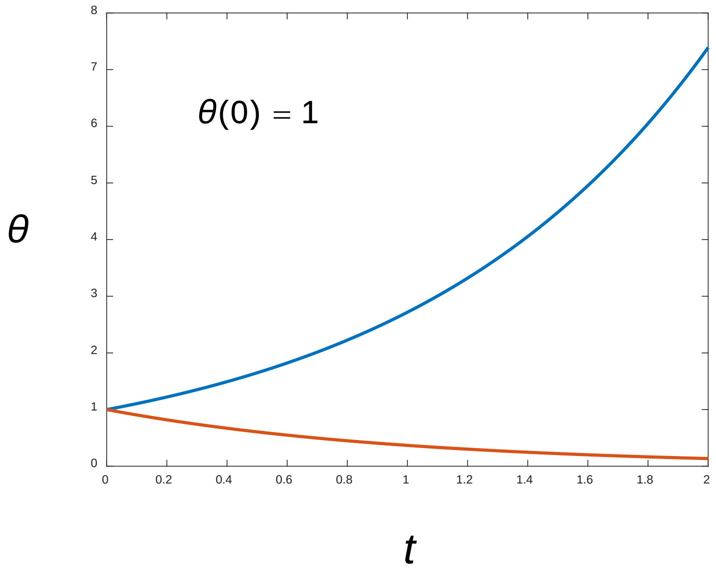
take $f = 0$

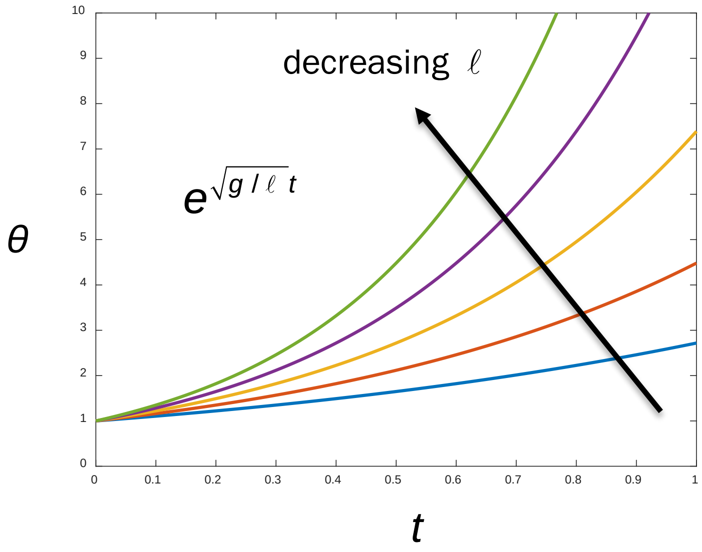
$$\ddot{\theta}(t) - \frac{g}{\ell}\theta(t) = 0; \quad \theta(0) \neq 0$$

$$\theta(t) = \frac{\theta(0)}{2}\left(e^{-\sqrt{g/\ell}\,t} + e^{\sqrt{g/\ell}\,t}\right)$$

D.E. Solution

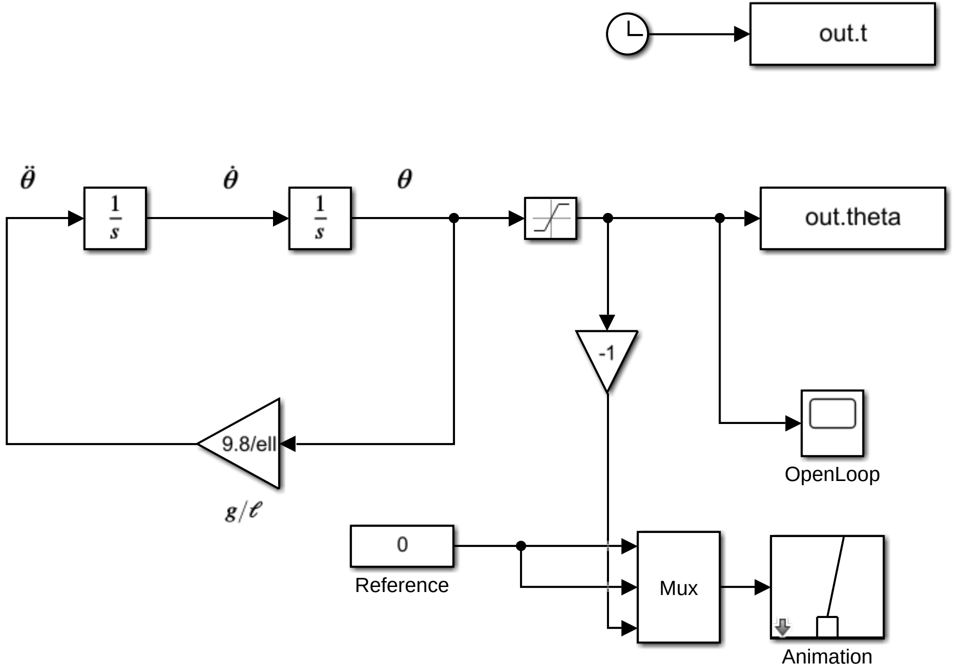
$$\theta(t) = \frac{\theta(0)}{2}\left(e^{-\sqrt{g/\ell}\,t} + e^{\sqrt{g/\ell}\,t}\right)$$



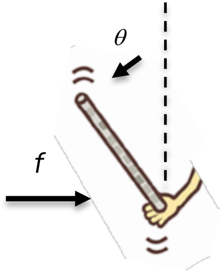


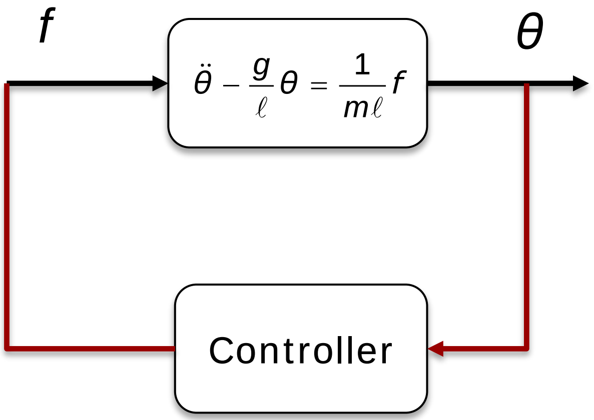
Simulation

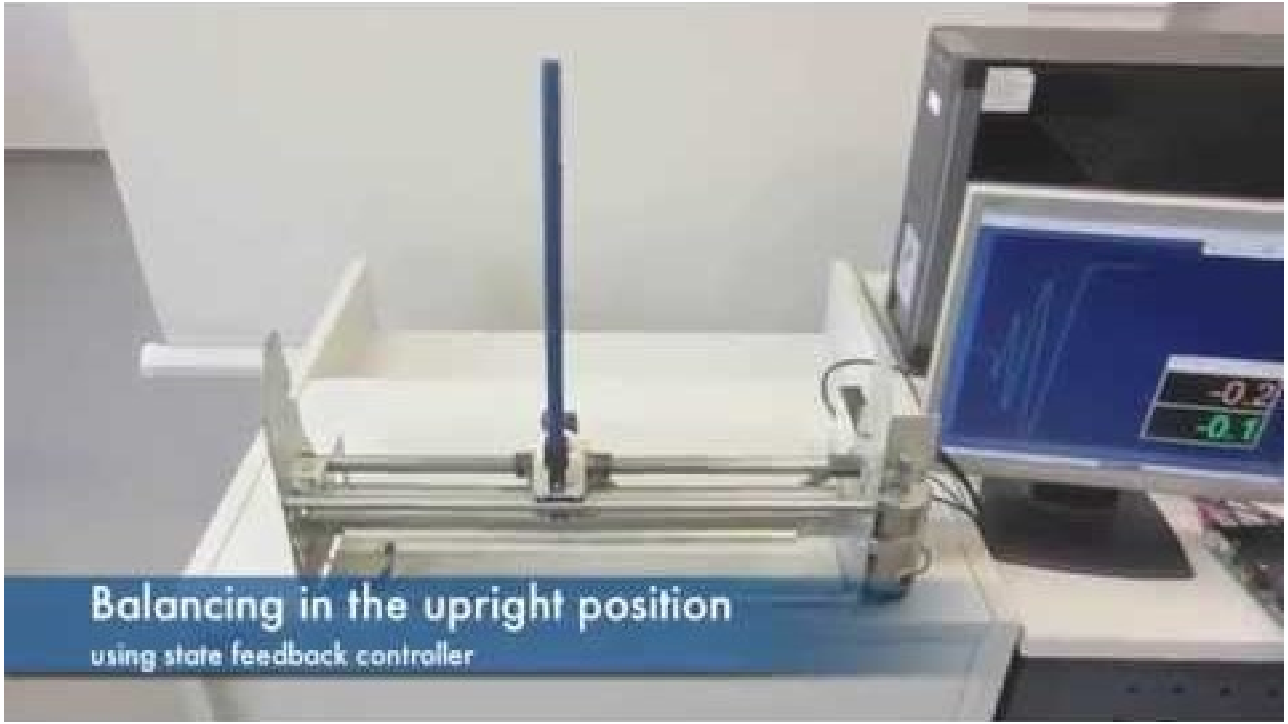
$$\ddot{\theta}(t) - \frac{g}{\ell}\theta(t) = 0; \quad \theta(0) \neq 0$$



Stabilizing the stick

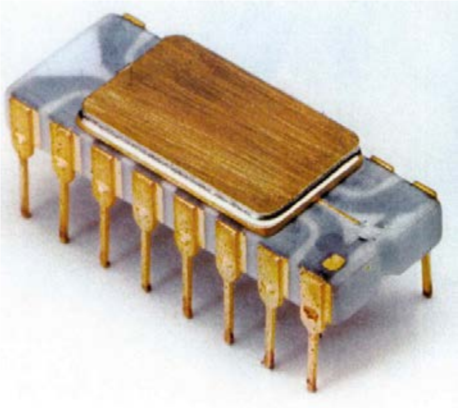





-0.2
-0.1
Balancing in the upright position
using state feedback controller


TU

The Chip That Changed the World

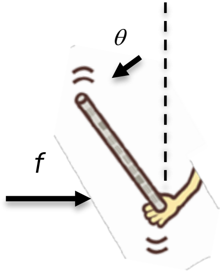


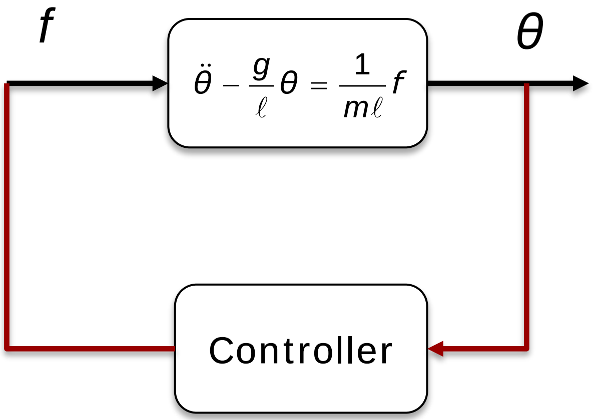
The Intel 4004 microprocessor, 1971.
Photo: Getty Images

Intel 4044
2.3K transistors
92K operations/sec

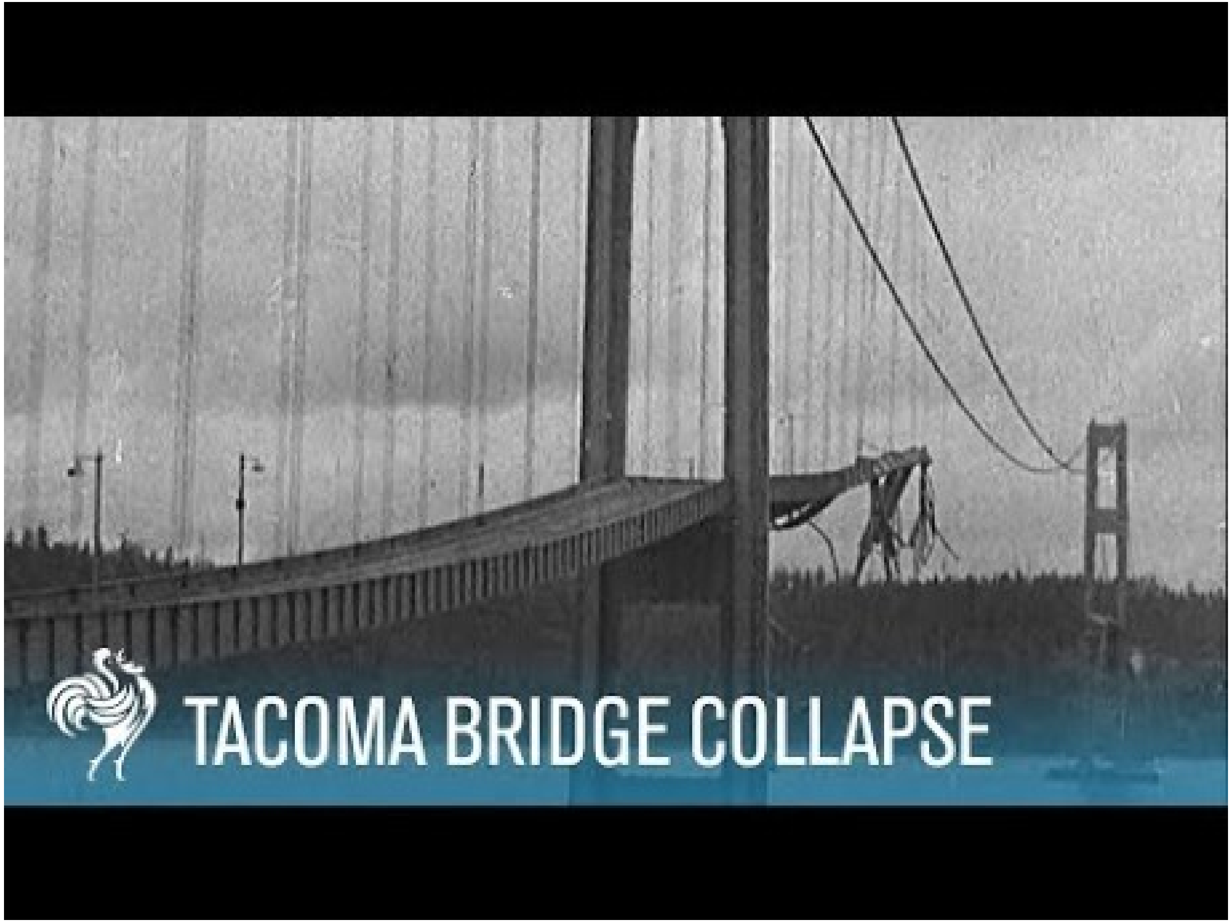
Apple M1 Max
57B transistors
10.4T flops

Stabilizing the stick

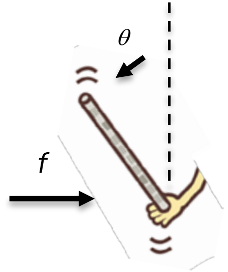


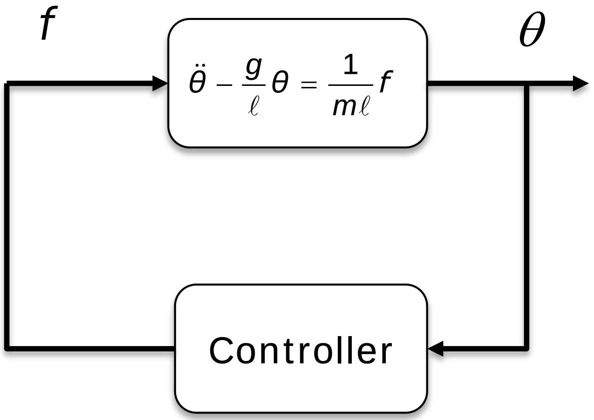



600


TACOMA BRIDGE COLLAPSE

Stabilizing the stick

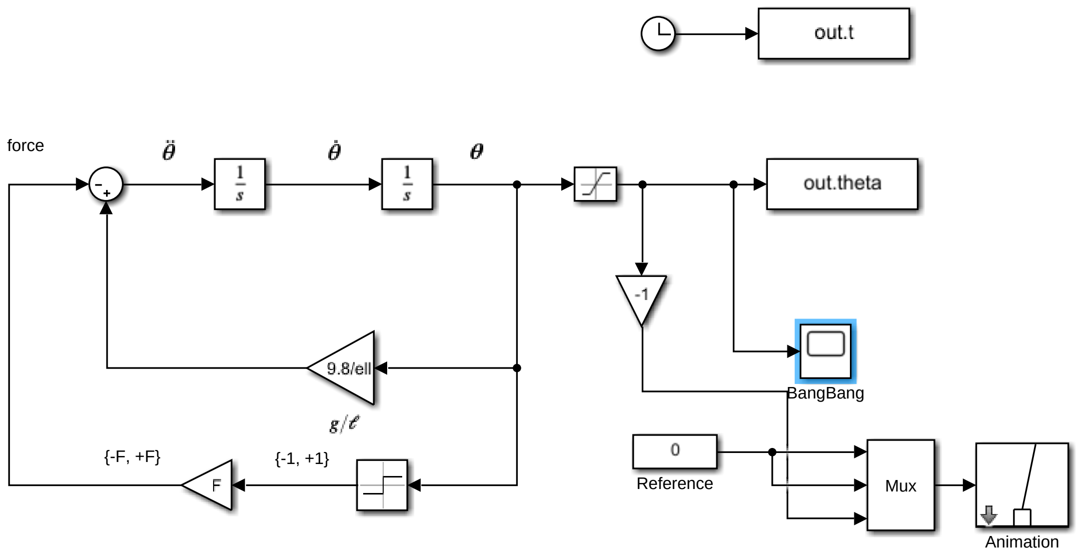




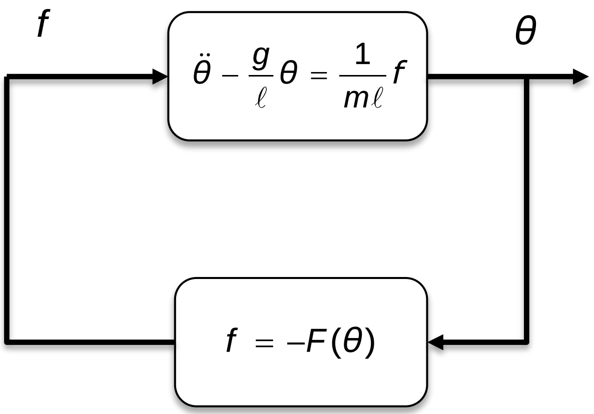
Bang-Bang Controller

$$\text{force} = \begin{cases} -F, & \text{if } \theta > 0 \\ +F, & \text{if } \theta < 0 \end{cases}$$

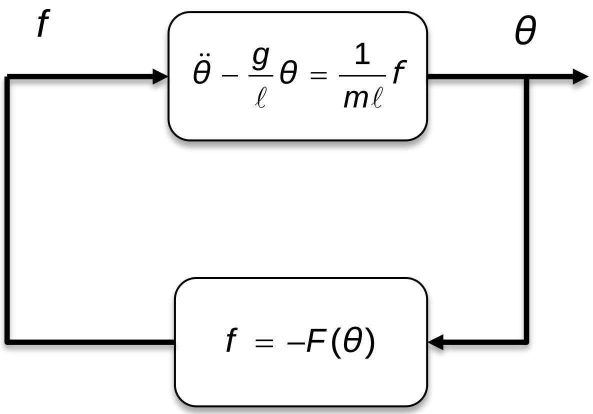
$$F = \text{given value}$$



Secret Sauce #1



Secret Sauce #1 (the closed-loop DE)

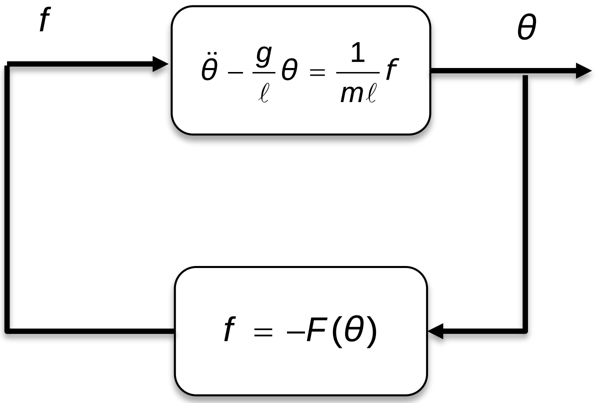


$$\ddot{\theta} - \frac{g}{\ell}\theta = \frac{1}{m\ell} f$$

$$= -\frac{1}{m\ell} F(\theta)$$

$$\ddot{\theta} - \frac{g}{\ell}\theta + \frac{1}{m\ell} F(\theta) = 0; \qquad \theta(0) \neq 0$$

Secret Sauce #2 (rule of positive coefficients)



$$\boxed{\ddot{\theta} - \frac{g}{\ell}\theta + \frac{1}{m\ell}F(\theta) = 0; \quad \theta(0) \neq 0}$$

Feedback system is stable, if the coefficients of

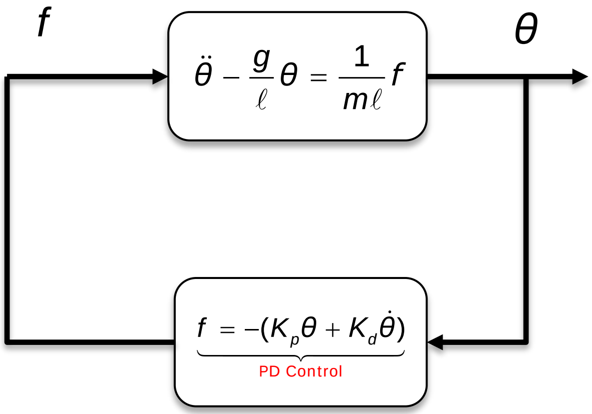
$$\theta, \dot{\theta} \text{ and } \ddot{\theta}$$

are strictly positive.

$$\theta \to 0 \quad \text{as} \quad t \to \infty$$

stick returns to 12'oclock

Stabilization of Hand-Stick (PD Control) -1



$$\ddot{\theta} - \frac{g}{\ell}\theta + \frac{1}{m\ell}F(\theta) = 0; \quad \theta(0) \neq 0$$

PD control: $F(\theta) = \underbrace{K_p\theta}_{\text{P term}} + \underbrace{K_d\dot{\theta}}_{\text{D term}}$

$$\ddot{\theta} + \frac{K_d}{m\ell}\dot{\theta} + \left(\frac{K_p}{m\ell} - \frac{g}{\ell}\right)\theta = 0; \quad \theta(0) \neq 0$$

The hand-stick system is stabilized, if both:

$$\frac{K_p}{m\ell} - \frac{g}{\ell} > 0;$$

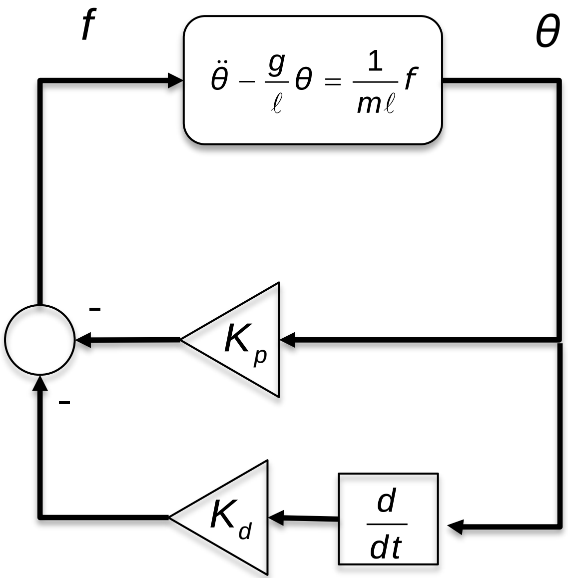
$$\frac{K_d}{m\ell} > 0$$

Examples

Suppose K_p *and* K_d are chosen so that the closed-loop D.E. are:

Closed-loop D.E.	Hand-stick system stable?
$\ddot{\theta} - \dot{\theta} + \theta = 0; \quad \theta(0) \neq 0$	No, unstable
$\ddot{\theta} + \dot{\theta} + \theta = 0; \quad \theta(0) \neq 0$	Yes, stable
$\ddot{\theta} + \dot{\theta} = 0; \quad \theta(0) \neq 0$	No, unstable
$\ddot{\theta} + \dot{\theta} - \theta = 0; \quad \theta(0) \neq 0$	No, unstable

Stabilization using PD control



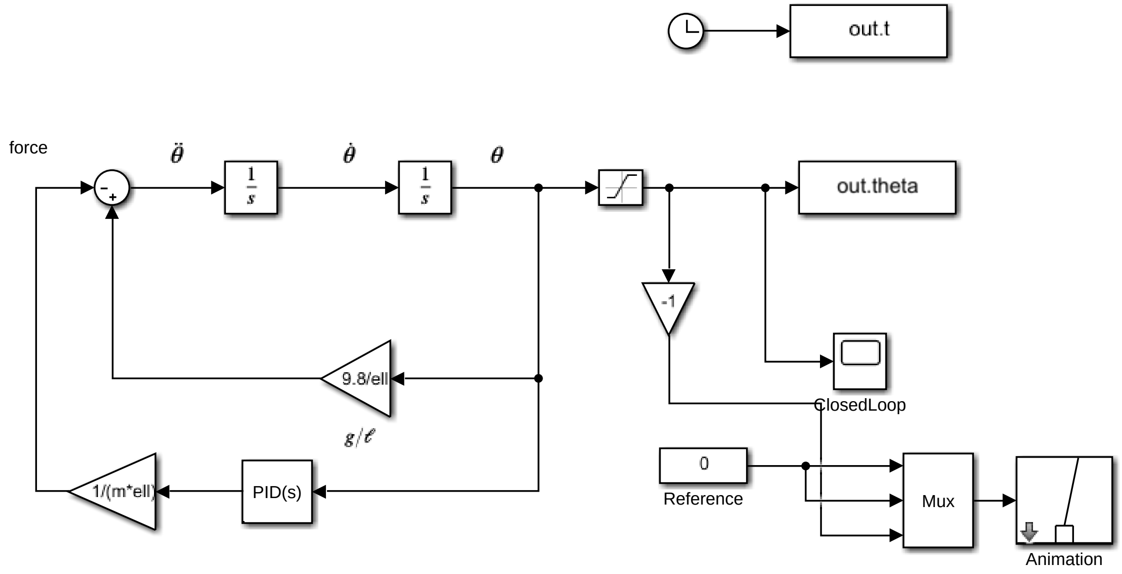
$$\ddot{\theta} + \frac{K_d}{m\ell}\dot{\theta} + \left(\frac{K_p}{m\ell} - \frac{g}{\ell}\right)\theta = 0; \quad \theta(0) \neq 0$$

The hand-stick system is stabilized when:

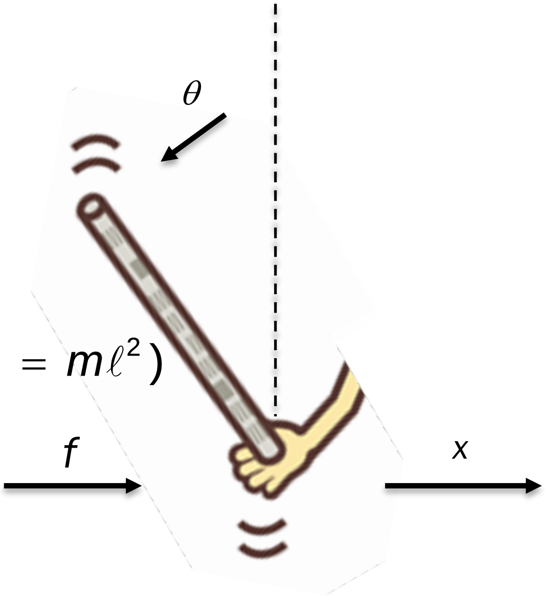
K_p is positive enough

K_d is positive

Stabilization of Hand-Stick (PD Control) - 2



D.E. for the Hand-Stick



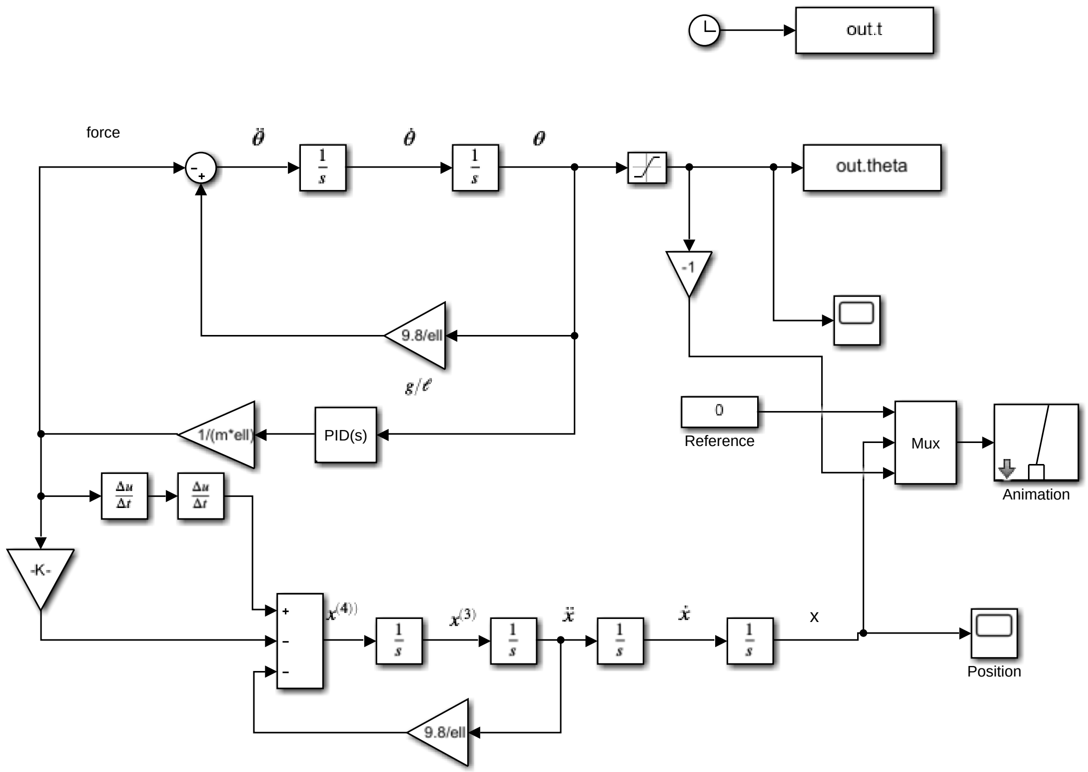
$(m,\ \ell,\ I = m\ell^2)$

$$\ddot{\theta} - \frac{g}{\ell}\theta = \frac{1}{m\ell} f$$

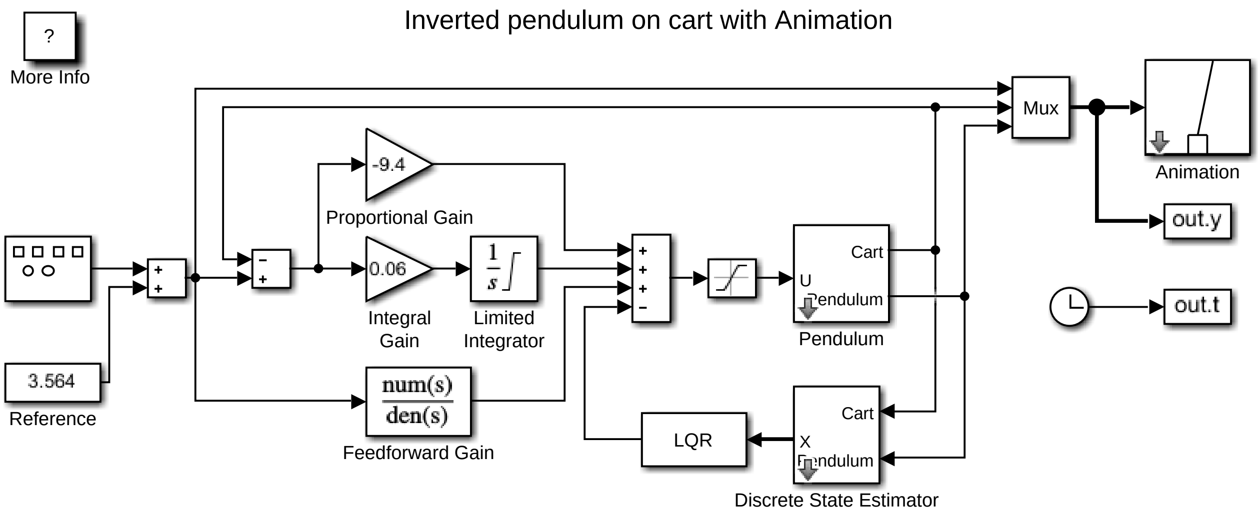
$$\ddddot{x} - \frac{g}{\ell}\ddot{x} = \ddot{f} - \frac{2g}{\ell} f$$

g = gravitational acceleration

Stabilization of Hand-Stick (PD Control) - 3



Gyro Boy


Inverted pendulum on cart with Animation
?
More Info
Reference
3.564
Proportional Gain
-9.4
0.06
Integral
Gain
Limited
Integrator
num(s)
den(s)
Feedforward Gain
Mux
Animation
out.y
out.t
Cart
U
Pendulum
Pendulum
Cart
X
Pendulum
LQR
Discrete State Estimator

Self-Balancing Robot (Principles of Feedback Control)

1. Provided examples of feedback control systems and identified their principal parts (plant, sensor, controller and actuator)

2. Modeled the hand-stick system with a D.E.

3. Stabilized the hand-stick system with a PD controller