ECE124: Discussion

Discussion #4

Yeonsik Noh, PhD

- Simplifying Boolean expression:

(a) xy + (xy)'

$xy = A$

$F(x,y) = xy + (xy)' = A + A' = 1$

$= xy + (x' + y')$

cf $(x \cdot y)' = x' + y'$

$= xy + x' + y'$

$= \underbrace{(x + x')}_{1} \cdot (y + x') + y'$

$= y + x' + y'$

$= 1 + x' = 1$

(b) xy + x'y'

$* \ (xy)' \neq x'y'$

$(xy)' = x' + y'$

x	y	minterm	F (XNOR)
0	0	$x'y'$	1
0	1	$x'y$	0
1	0	xy'	0
1	1	xy	1

$F(x,y)$

$F(x,y) = xy + x'y'$

$\Rightarrow F(x,y) = xy + x'y'$

$= (xy + x') \cdot (xy + y')$

$= \{\underbrace{(x + x')}_{1} \cdot (y + x')\} \cdot \{(x + y') \cdot \underbrace{(y + y')}_{1}\}$

$= (1 \cdot (y + x')) \cdot ((x + y') \cdot 1)$

$= (y + x') \cdot (x + y')$

$= (y + x')x + (y + x')y'$

$= (yx + \underbrace{x'x}_{0}) + (\underbrace{y \cdot y'}_{0} + x'y')$

$= xy + x'y'$

2.8 Find the complement of F = wx + yz; then show that FF' = 0 and F + F' = 1.

$F(w,x,y,z)$

$$F\cdot F' = (wx+yz)\cdot(wx+yz)'$$

$$\Rightarrow = (wx+yz)\cdot\{(wx)'\cdot(yz)'\}$$

$$= (wx+yz)\cdot\{(w'+x')\cdot(y'+z')\}$$

$$= \{wx\cdot\underbrace{(w'+x')}_{(wx)'}(y'+z')\} + \{yz\cdot(w'+x')\underbrace{(y'+z')}_{(yz)'}\}$$

$$= \{\underbrace{(wx)\cdot(wx)'}_{0}\cdot(y'+z')\} + \{\underbrace{(yz)(yz)'}_{0}\cdot(w'+x')\}$$

$$= 0\cdot\_ + 0\cdot\_$$

$$= \text{"}0\text{"}$$

$$F+F' = (wx+yz)+(wx+yz)'$$

$$= wx+yz+\{(wx)'\cdot(yz)'\}$$

$$= wx+\{(yz+(wx)')\cdot(yz+(yz)')\}$$

$$= wx+yz+(wx)'$$

$$= 1+yz = \text{"}1\text{"}$$

2.9 Find the complement of the following expression.

(a) $F(x,y) = xy' + x'y$ "XOR"

$F'(x,y)$

$F'(x,y) = xy + x'y'$ "XNOR"

$F = x'y + xy'$

$F = xy' + x'y$

$F' = (xy' + x'y)'$

$= (xy')' \cdot (x'y)'$

$= (x'+y) \cdot (x+y')$

$= (x'+y)x + (x'+y)y'$

$= (x'x) + (yx) + (x'y') \cdot (y \cdot y')$

$= xy + x'y'$

x	y	minterm	F' (XNOR)	F (XOR)
0	0	$x'y'$	1	0
0	1	$x'y$	0	1
1	0	xy'	0	1
1	1	xy	1	0

$F' = x'y' + xy$

XOR

$F(x,y) = \underset{01}{x'y} + \underset{10}{xy'} = m_1 + m_2 = \Sigma(1,2) = M_0 + M_3 = \Pi(0,3)$

2.9 Find the complement of the following expression.

(b) $F(a,b,c) = (a + c)(a + b')(a' + b + c')$ POS

$F'(a,b,c) = \{(a+c)\cdot(a+b')\cdot(a'+b+c')\}'$

$= (a+c)' + (a+b')' + (a'+b+c')'$

$= (a'\cdot c') + (a'\cdot b) + (ab'c)$

SOP

$F'(a,b,c) = a'c' + a'b + ab'c$

$= a'c'(b+b') + a'b(c+c') + ab'c$

$= \underset{010}{a'bc'} + \underset{000}{a'b'c'} + \underset{011}{a'bc} + \cancel{\underset{010}{a'bc'}} + \underset{101}{ab'c}$

$F' = a'bc' + a'b'c' + a'bc + ab'c$

$= m_2 + m_0 + m_3 + m_5 \Rightarrow F' = 1$

a	b	c	m	F	F'
0	0	0	m_0	0	1
0	0	1	m_1	1	0
0	1	0	m_2	0	1
0	1	1	m_3	0	1
1	0	0	m_4	1	0
1	0	1	m_5	0	1
1	1	0	m_6	1	0
1	1	1	m_7	1	0

2.9 Find the complement of the following expression.

(c) $F(x,y,z,v,w) = z + z'(v'w + xy)$ = $z + z'v'w + z'xy$ sop

$$F' = \{z + z'(v'w + xy)\}'$$

$$= z' \cdot \{z'(v'w + xy)\}'$$

$\vdots$

$$F' = z' \cdot (z + v + w') \cdot (z + x' + y')$$

$(z' + 0)$

pos

2.10 Given the Boolean functions F_1 and F_2, show that

(a) The Boolean function $E_1 = F_1 + F_2$ contains the sum of the minterms of F_1 and F_2.

$F_1 = OR \Rightarrow F_1(x,y) = x + y$

$F_2 = AND \Rightarrow F_2(x,y) = xy$

x	y	F_1 (OR)	F_2 (AND)	E_1 (F_1+F_2)	E_2 $(F_1 \cdot F_2)$
0	0	0	0	0	0
0	1	1	0	1	0
1	0	1	0	1	0
1	1	1	1	1	1

$$E_1(x,y) = F_1 + F_2 = x + y + xy$$
$$= x(y+y') + y(x+x') + xy$$
$$= xy + xy' + \cancel{xy} + x'y + \cancel{xy}$$
$$= \underset{11}{xy} + \underset{10}{xy'} + \underset{01}{x'y}$$
$$= m_3 + m_2 + m_1$$
$$= \Sigma(1,2,3)$$

$$E_2 = F_1 \cdot F_2$$
$$= (x+y) \cdot xy$$
$$= x(xy) + y(xy)$$
$$= xxy + xyy$$
$$= xy + xy = xy = m_3$$

2.10 Given the Boolean functions F_1 and F_2, show that

(b) The Boolean function $E = F_1F_2$ contains only the minterms that are common to F_1 and F_2.