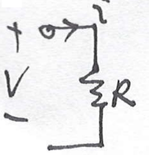
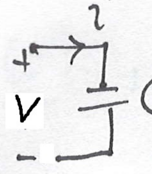
RC circuits.



$i = \frac{1}{R} V.$



$i = C \cdot \frac{dV}{dt}.$ $\boxed{V_c(0^-) = V_c(0^+)}$

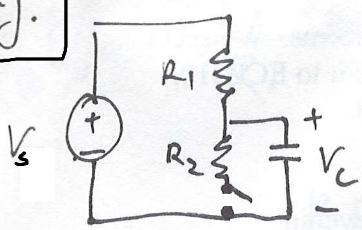
General procedure:

① write down diff equation

② find General solution

③ Use initial condition.

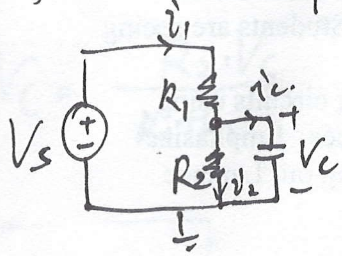
$\boxed{\text{e.g.}}$



ⓐ $t=0$, the switch closes.

find $V_c(t)$; $t>0$.

① find the diff. eq for $t>0$.



$i_1 - i_2 - i_c = 0.$

$$\frac{V_s - V_c}{R_1} - \frac{V_c}{R_2} - C \cdot \frac{dV_c}{dt} = 0.$$

$$C \frac{dV_c}{dt} + \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}_{\frac{1}{R_{11}}} \cdot V_c = \frac{V_s}{R_1}.$$

$$C \frac{dV_c}{dt} + \frac{1}{R_{11}} \cdot V_c = \frac{V_s}{R_1}.$$

$$\frac{dV_c}{dt} + \underbrace{\frac{1}{R_{11} \cdot C}}_{A} V_c = \underbrace{\frac{V_s}{R_1 C}}_{B}$$

② General solution.

$$V_c(t) = \frac{V_s \cdot R_{11}}{R_1} + k \cdot e^{-\frac{t}{R_{11} \cdot C}}$$

③ find k:

$$V_C(0^+) = V_C(0^-).$$

$$\underline{V_C(0^-) = V_s.}$$

So: $$V_C(0^+) = V_s = \frac{V_s \cdot R_{11}}{R_1} + k \Rightarrow$$

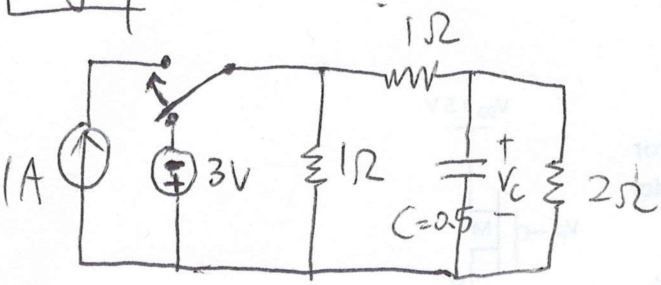
$$k = \left(1 - \frac{R_{11}}{R_1}\right) \cdot V_s$$

$$V_C = \frac{R_{11} \cdot V_s}{R_1} + \left(1 - \frac{R_{11}}{R_1}\right) \cdot V_s \cdot e^{-\frac{t}{R_{11} C}}$$

$$R_{11} = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_C = \frac{R_2 \cdot V_s}{R_1 + R_2} + \frac{R_1 \cdot V_s}{R_1 + R_2} \cdot e^{-\frac{(R_1 + R_2) \cdot t}{R_1 R_2 C}}$$

e.g ②

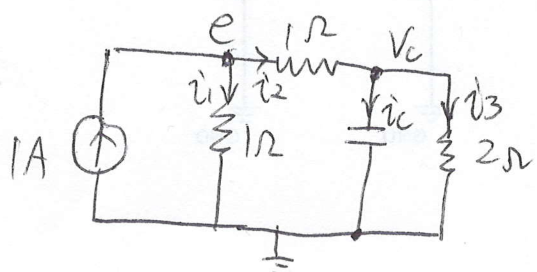


When $t<0$; switch connected to 3V

@ $t=0$; switch turns to 1A

find $V_c(t)$ $(t>0)$

① re-draw circuit @ $t>0$:



@ V_c : KCL:

$i_2 - i_c - i_3 = 0 \Rightarrow \frac{e-V_c}{1} - C\frac{dV_c}{dt} - \frac{V_c}{2} = 0$

$\Rightarrow e - 0.5\frac{dV_c}{dt} - \frac{3}{2}V_c = 0$ --- ①

@ e : KCL:

$1A - i_1 - i_2 = 0 \Rightarrow 1 - \frac{e}{1} - \frac{e-V_c}{1} = 0 \Rightarrow e = \frac{1+V_c}{2}$

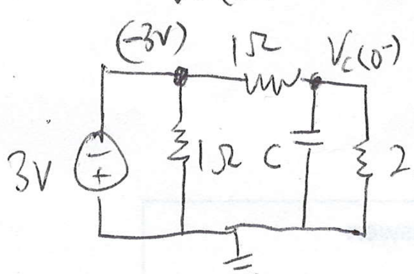
from ①, we have: (subsituting e with $\frac{1+V_c}{2}$)

$\frac{1+V_c}{2} - 0.5\frac{dV_c}{dt} - \frac{3}{2}V_c = 0 \Rightarrow -0.5\frac{dV_c}{dt} - V_c + \frac{1}{2} = 0$

② $\Rightarrow \frac{dV_c}{dt} + 2V_c = 1 \Rightarrow V_c = \frac{1}{2} + k \cdot e^{-2t}$

③ Initial conditions:

$V_c(0^+) = V_c(0^-)$ (re-drawn circuit @ $t<0$)



Using voltage divider:

$V_c(0^-) = \frac{2}{1+2} \cdot (-3) = -2V$

So: $V_c(0^+) = V_c(0^-) = -2 = \frac{1}{2} + k \Rightarrow k = -2.5$

So $V_c(t) = \frac{1}{2} - 2.5 \cdot e^{-2t}$.