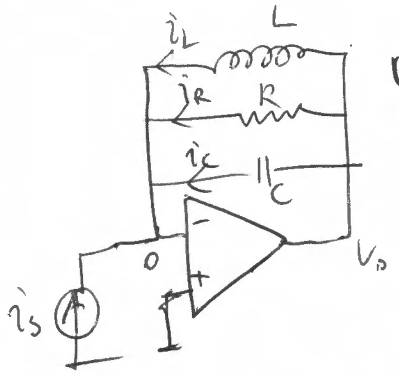
Problem 1



(a)

KCL:

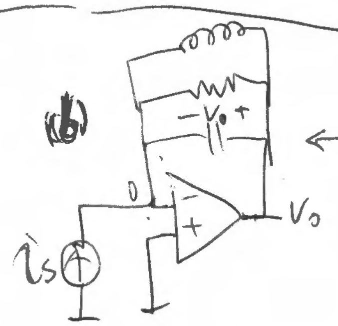
$$i_s + i_C + i_R + i_L = 0$$

$$i_s + C\frac{dV_o}{dt} + \frac{V_o}{R} + i_L(0^+) + \frac{1}{L}\int_0^t V_o\,dt = 0$$

$$\frac{di_s}{dt} + C\frac{d^2V_o}{dt^2} + \frac{1}{R}\frac{dV_o}{dt} + \frac{V_o}{L} = 0$$

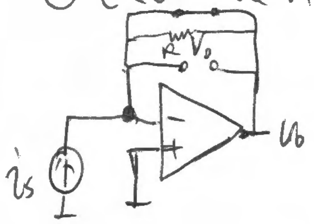
$$\frac{d^2V_o}{dt^2} + \frac{1}{RC}\frac{dV_o}{dt} + \frac{1}{LC}\cdot V_o = -\frac{di_s}{dt}\cdot\frac{1}{C}$$

(b)



(1) $V_o(0^+) = V_o(0^-)$

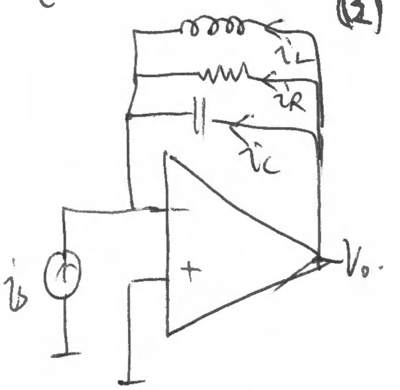
(a) $t<0$ the steady-state circuit.



$\Rightarrow V_o(0^-) = 0$

So: $V_o(0^+) = V_o(0^-) = 0$.

$t = 0^+$



(2)

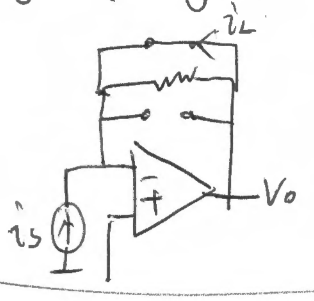
$$\frac{dV(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-i_s(0^+) - i_R(0^+) - i_L(0^+)}{C}$$

KCL: since: $i_s(0^+) + i_C(0^+) + i_R(0^+) + i_L(0^+) = 0$

$$= -\frac{1A}{C} - \underbrace{\frac{V_o(0^+)}{R}}_{i_R(0^+)}\cdot\frac{1}{C} - \frac{1}{C}\cdot i_L(0^-)$$

$$= -\frac{1}{C} - 0 - \frac{1}{C}\cdot i_L(0^-)$$

$t = 0^-$ (steady state)



$i_s(0^-) + i_L(0^-) = 0$

$\Rightarrow i_L(0^-) = -i_s(0^-) = -\frac{1}{2}A$

So: $\frac{dV(0^+)}{dt} = -\frac{1}{C}(1 + i_L(0^-))$

$$= -\frac{1}{C}\left(1 - \frac{1}{2}\right)$$

$$= -\frac{1}{2}\cdot\frac{1}{C}$$

$$= -2\ (V\cdot s^{-1})$$

(c): $2\alpha = \frac{1}{RC} \Rightarrow \alpha = \frac{1}{2RC} = \frac{1}{2\times 2.5} = 0.2\ Hz$

$\omega_0^2 = \frac{1}{LC} = \frac{1}{1\times\frac{1}{4}} = 4\ Hz^2$

$\alpha^2 < \omega_0^2 \Rightarrow$ Underdamped

$\omega_0 = \sqrt{\frac{1}{LC}} = 2\ Hz$

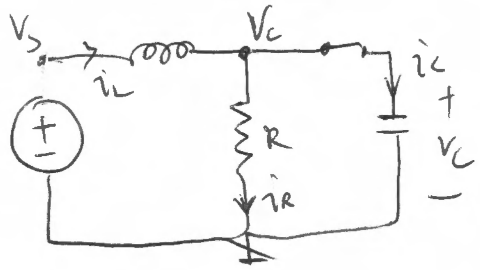
$Q = \frac{\omega_0}{2\alpha} = \frac{2Hz}{2\times 0.2} = 5$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{4 - 0.2^2}$

$= 1.99\ Hz$

$t = Q\cdot T = 5\times\frac{1}{f} = 5\times\frac{2\pi}{\omega_d} = 5\times\frac{2\pi}{1.99} = 15.8\ (s)$

Problem 2



(a) ~~$t=0^+$~~, for $t>0$

$$i_L - i_R - i_C = 0$$

$$i_L(0^+) + \frac{1}{L}\int_0^t (V_s - V_c)\,dt - \frac{V_c}{R} - C\cdot\frac{dV_c}{dt} = 0$$

Diff.:
$$\frac{V_s - V_c}{L} - \frac{1}{R}\frac{dV_c}{dt} - C\frac{d^2V_c}{dt^2} = 0$$

$$\frac{d^2V_c}{dt^2} + \frac{1}{CR}\cdot\frac{dV_c}{dt} + \frac{1}{LC}V_c = \frac{V_s}{LC}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 10\times\frac{1}{4}} = 0.2\text{ Hz}. \quad W_0^2 = \frac{1}{LC} = \frac{1}{1\times\frac{1}{4}} = 4\text{ Hz}^2$$

$\alpha^2 - W_0^2 < 0$. $\Rightarrow$ Underdamped.

$$W_d = \sqrt{W_0^2 - \alpha^2} = 1.99\text{ Hz} \simeq 2\text{ Hz}$$

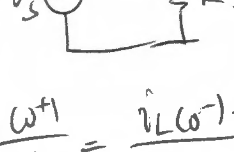
$$V_C(t) = V_s + \cancel{A_1}\cdot e^{-0.2t}\cdot(A_1\cos 2\cdot t + A_2\cdot\sin 2\cdot t) = 2 + e^{-0.2t}\cdot(A_1\cos 2t + A_2\cdot\sin 2\cdot t)$$

$$V_c(0^+) = V_c(0^-) = 1 \Rightarrow 2 + A_1 = 1 \Rightarrow A_1 = -1$$

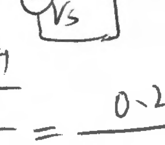
So: $V_c(t) = 2 + e^{-0.2t}\cdot(-\cos 2\cdot t + A_2\cdot\sin 2\cdot t)$

$$\frac{dV_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{i_L(0^+) - i_R(0^+)}{C} = \frac{i_L(0^-) - \frac{V_c(0^+)}{R}}{C}$$

For $i_L(0^-)$:



$\Rightarrow$ (steady state:)



$$i_L(0^-) = \frac{V_s}{R} = \frac{2}{10} = 0.2\,A$$

$$\text{So: } \frac{dV_c(0^+)}{dt} = \frac{i_L(0^-) - \frac{V_c(0^+)}{R}}{C} = \frac{0.2A - \frac{1}{10}}{C} = \frac{0.1A}{C} = \frac{0.1}{1/4} = 0.4\ V.s^{-1}$$

$\Rightarrow A_2 = 0.1$

$$i_R(t) = \frac{V_c(t)}{R} = 0.2 + 0.1\cdot e^{-0.2t}\cdot(-\cos 2t + 0.1\sin 2t)$$