

Example 2.7: Determine if each of the following LTI systems with the given impulse response is stable or not.

- (a) An ideal integrator with $h(t) = u(t)$.
- (b) A finite-length integrator with $h(t) = u(t) - u(t - T)$.¹
- (c) An RC circuit with $h(t) = \frac{1}{RC}e^{-t/RC}u(t)$.

Solution:

(a)

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_0^{\infty} 1 dt = \infty. \quad (\text{E1})$$

Hence, it is unstable.

(b)

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_0^T 1 dt = T < \infty. \quad (\text{E2})$$

Hence, it is stable.

(c)

$$\begin{aligned} \int_{-\infty}^{\infty} |h(t)| dt &= \int_0^{\infty} \frac{1}{RC} e^{-t/RC} dt \\ &= - \left[e^{-t/RC} \right]_0^{\infty} = -(0 - 1) = 1 < \infty. \end{aligned} \quad (\text{E3})$$

Hence, it is stable.

¹The input-output relation is $y(t) = \int_{t-T}^t x(\tau) d\tau$.