ECE 214 - Probability and Statistics, Spring 2024

Homework #4

Due: 03/04/24, 11:59 pm

1. A car rental service operates every day and experiences varying demand. On days with high demand (H), there is a higher likelihood of customers returning cars late, while on low demand (L) days, cars are generally returned on time. The probability of a car being returned late on a high demand day is 0.3, whereas on a low demand day, it is 0.1. There is the conditional independence of car returns given the day. The probability of having a high demand day is 0.6 and a low demand day is 0.4.

   (a) What is the probability that exactly two out of four rented cars are returned late on a given day?

   (b) Given that exactly two cars are returned late, what is the probability that it was a high demand day?

   (c) What is the probability that at most one car is returned late out of four on a given day?

2. Consider a discrete random variable X that represents the outcome of a simple game. In this game, a six-sided fair die is rolled. Depending on the outcome, points are awarded as follows:

   - If the die shows 1 or 2, then $X = 0$ points.
   - If the die shows 3 or 4, then $X = 1$ point.
   - If the die shows 5, then $X = 2$ points.
   - If the die shows 6, then $X = 3$ points.

   (a) Define the probability mass function (PMF) of X.

   (b) What is the probability $P(X = 1)$?

   (c) Calculate the probability $P(X > 1)$.

   (d) Given that $X > 0$, what is the conditional probability that $X = 2$?