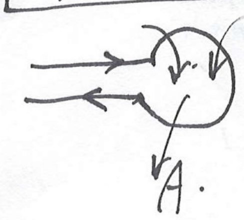
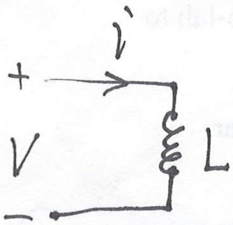
Inductor:



Φ (magnetic flux) $= B \cdot A = \underline{L} \cdot i$

Inductance. $L = \frac{\Phi}{i}$ (Henry).

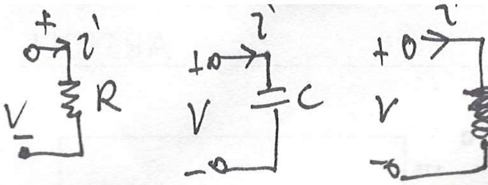
$$V = \frac{d\Phi}{dt} = L \cdot \frac{di}{dt}$$



$$V = L \cdot \frac{di}{dt} \quad \left(\text{Compare} \quad \begin{array}{l} i = \frac{1}{R} V \\ i = C \frac{dv}{dt} \end{array}\right)$$

It integral form.

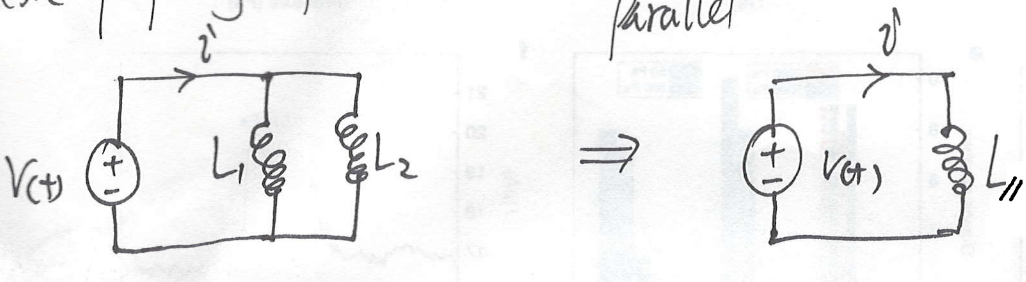
$$\int_0^t v\,dt = L\,(i(t) - i(0)) \Rightarrow i(t) = i(0) + \frac{1}{L}\int_0^t v(t)\,dt.$$



$$i = \frac{V}{R} \qquad i = C \cdot \frac{dV}{dt} \qquad i_{(t)} = i_{(0)} + \frac{1}{L}\int_0^t V(t)\, dt.$$

Basic property of Inductor:

Parallel



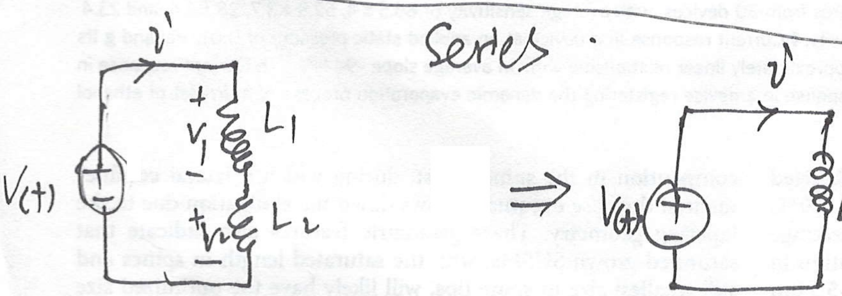
$$i = i_1 + i_2.$$

$$\frac{1}{L_{11}} \cdot \int_0^t V(t)\, dt = \frac{1}{L_1}\int_0^t V(t)\, dt + \frac{1}{L_2}\int_0^t V(t)\, dt$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2}\right) \cdot \int_0^t V(t)\, dt$$

$$\Rightarrow \frac{1}{L_{11}} = \frac{1}{L_1} + \frac{1}{L_2}. \qquad \text{Or: } L_{11} = \frac{L_1 \cdot L_2}{L_1 + L_2}$$

Series.



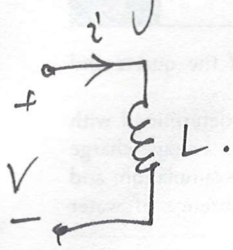
$$V(t) = V_1 + V_2$$

$$= L_1 \frac{di}{dt} + L_1 \frac{di}{dt} = (L_1 + L_2) \cdot \frac{di}{dt}$$

$$= L_{eq} \cdot \frac{di}{dt}.$$

So. $L_{eq} = L_1 + L_2$

Eenergy stored



$$W = \int_0^t p(t)\,dt = \int_0^t i(t)\cdot V(t)\cdot dt$$

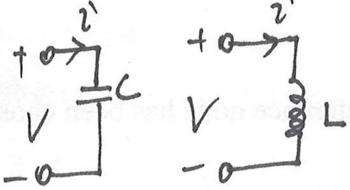
$$= \int_0^t i(t)\cdot L\cdot \frac{di}{dt}\,dt = L\int_0^t i(t)\,di$$

$$= \frac{L}{2}\left(i_{(t)}^2 - i_{(0)}^2\right)$$

Assume: $i(0) \sim 0$

So: $E = W = \frac{1}{2} L\cdot i^2.$

RL Circuits



$i = C\cdot\frac{dv}{dt}$ $\quad V = L\cdot\frac{di}{dt}$ $\Rightarrow i(t) = i(0^+) + \frac{1}{L}\int_0^t v\,dt$.

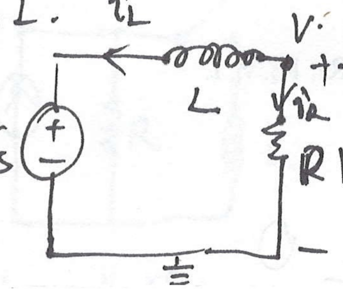
$\Downarrow$

$\boxed{V(0^-) = V(0^+) \qquad i(0^-) = i(0^+)}$ Initial Conditions.

General Solution to 1st order Diff equation:

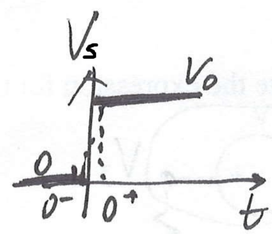
$$\frac{dV(t)}{dt} + A\,V(t) = B \Rightarrow V(t) = \frac{B}{A} + k\cdot e^{-At}.$$

e.g. 1.



$V_s = V_0\cdot u(t)$

find $V(t)$



① for $t>0$. $(V_s = V_0)$

$-i_L - i_R = 0$ or $i_L + i_R = 0.$

$\Downarrow$

$$i_L(0^+) + \frac{1}{L}\int_0^t (V - V_0)\,dt + \frac{V}{R} = 0$$

do differential on both sides.

$$0 + \frac{1}{L}(V - V_0) + \frac{1}{R}\frac{dV}{dt} = 0$$

$$\frac{dV}{dt} + \frac{R}{L}V = \frac{R}{L}V_0$$

$$V(t) = V_0 + k\cdot e^{-\frac{R}{L}\cdot t} \quad \left[\frac{1}{R/L} \rightarrow \text{time constant} \right)$$

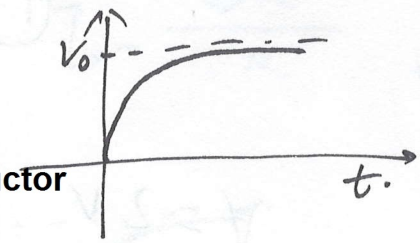
Initial condition: $-i_R - i_L = 0 \Rightarrow i_R(0^+) = -i_L(0^+)$

$$V(0^+) = i_R(0^+)\cdot R = -i_L(0^+)\cdot R = -i_L(0^-)\cdot R$$

$$= 0\cdot R = 0.$$

So: $V(0^+) = 0 = V_0 + k\cdot e^{-\frac{R}{L}\cdot 0} \Rightarrow k = -V_0.$

So: $V(t) = V_0 - V_0\cdot e^{-\frac{R}{L}t}.$



Inference: at steady state, an inductor behaves like a 'short-circuit'