Example 5.4: Find the Fourier transform of the signal

$$x(t) = e^{-t/T}u(t), \; T = \text{constant } (> 0).$$

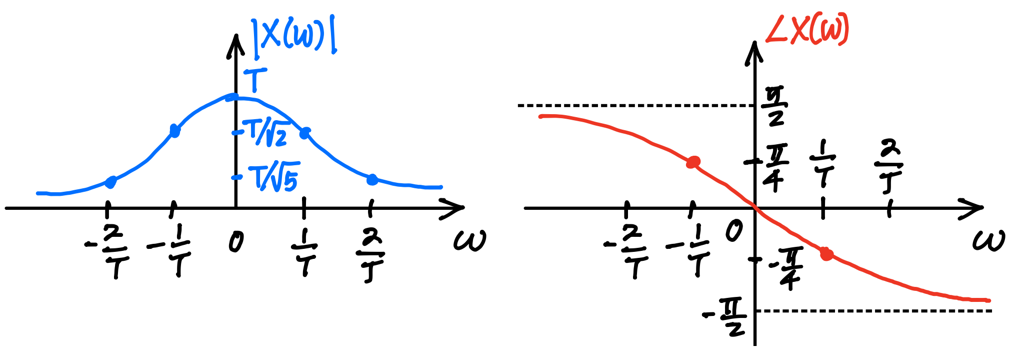
Solution:
By the integral definition,[1]

$$\begin{aligned} X(\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{0}^{\infty} e^{-t/T}e^{-j\omega t}dt \\ &= \int_{0}^{\infty} e^{-(\frac{1}{T}+j\omega)t}dt = -\left[\frac{e^{-(\frac{1}{T}+j\omega)t}}{\frac{1}{T}+j\omega}\right]_0^{\infty} \\ &= \frac{1}{\frac{1}{T}+j\omega} = \frac{T}{1+j\omega T}. \end{aligned} \tag{E1}$$

The magnitude and phase are

$$|X(\omega)| = \frac{T}{\sqrt{1+\omega^2T^2}}, \tag{E2}$$

$$\angle X(\omega) = -\tan^{-1}\omega T. \tag{E3}$$



[1]This is equal to $X(s) = \mathscr{L}\{x(t)\} = 1/(s+1/T)$ evaluated at $s = j\omega$.