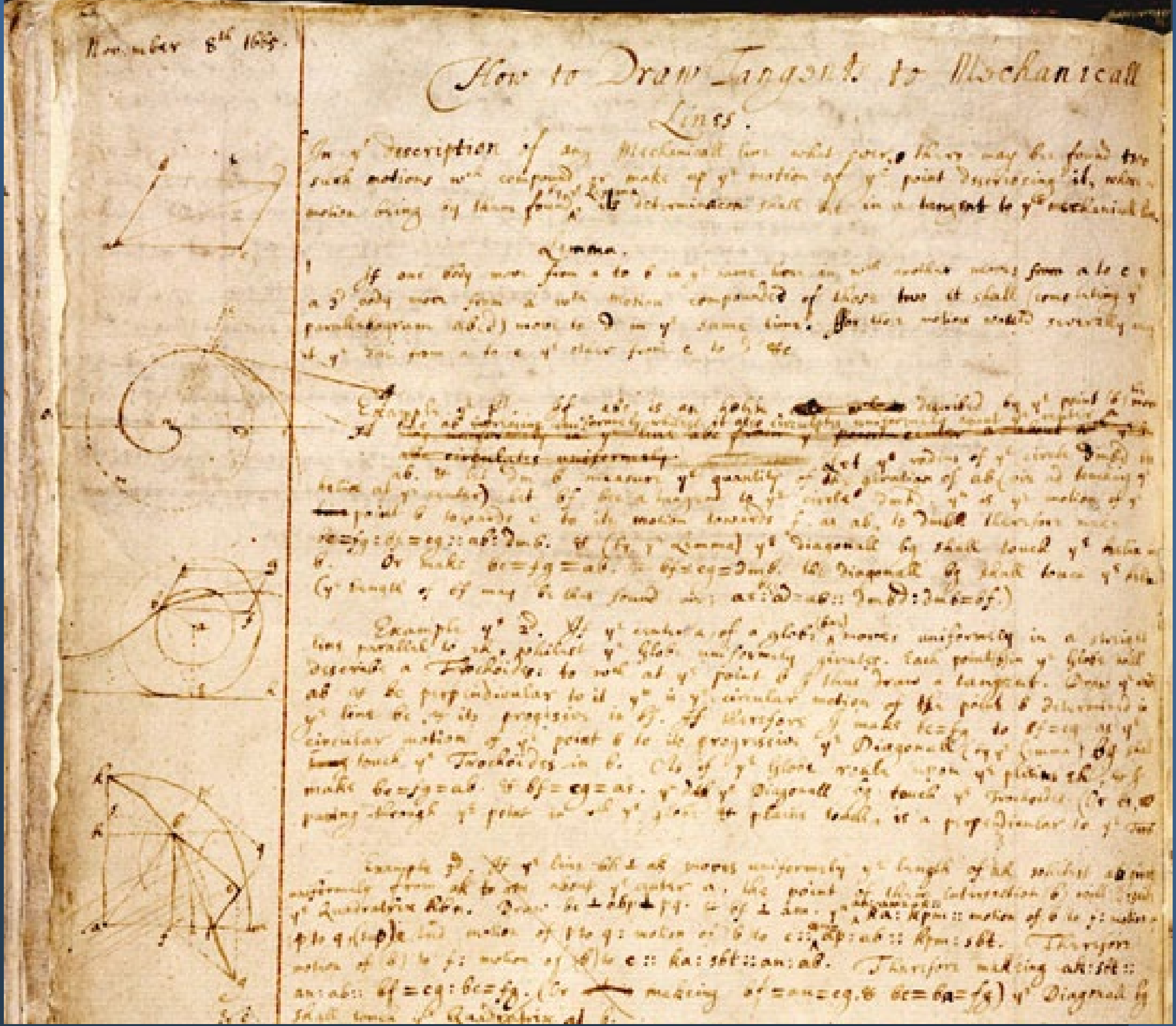


November 8th 1666.

How to Draw Tangents to Mechanicall Lines.

In ye description of any Mechanicall line what ever, there may bee found two such motions wch compound ye motion of ye point describing it, whose motion being by them found, its determination shall bee in a tangent to ye mechanicall line.

Lemma.

If one body move from a to b in ye same time in wch another moves from a to c & a 3d body move from a wth a motion compounded of those two it shall (completing ye parallelogram ab.cd) move to d in ye same time. For these motions assisted severally any of ye 3d from a to c ye other from c to d &c.

Announcements, Goals, and Reading

Announcements:

- HW02 due Tuesday Feb 15th, 11:59 pm on Mastering Physics
- **Help Resources: See next page**

Goals for Today:

- Instantaneous velocity **v(t)**
- Finding position **x(t)** from velocity vs time **v(t)**
- Motion w/ constant acceleration **a**

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 2: Kinematics in One Dimension

Help Resources: Now on Moodle

UMassAmherst Moodle in the Cloud Get Help

PHYSICS151_141186_SP22

- Participants
- Grades
- Moodle home
- Dashboard

Help Resources

Professor's Office Hours: Monday 2:20-3:20PM or by appointment
Location: HAS 123

TA Aditya's Office Hours: Monday 9:00-10:00 AM
Location: Zoom (https://umass-amherst.zoom.us/j/95377658553)

SI Sam's Review Sessions: Tuesday/Thursday 7:00pm-8:15pm

Location: DuBois 1302

Drop in help in the Physics Help Room in Hasbrouck 115

Physics Help Room

Help available for any of the following courses from ANY GRADUATE TA: Physics 100, 115, 118, 131, 132, 151, 152, 281, Astro 105

However, if you are looking for specific help, the course for which each TA is affiliated is listed

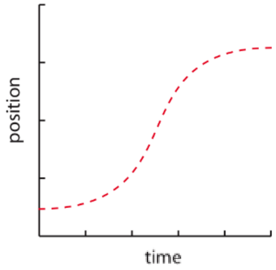
UTA - Undergraduate TA

	Monday	Tuesday	Wednesday	Thursday	Friday
9am - 10am	Eric Lyons 131 UTA	Matthew Maroun 131	Jhih-Ying Su 132 Lab	Matthew Maroun 131	
		Chengzhi Wu Li 131	Eric Lyons 131 UTA	Matthew Harris 131	
10am - 11am	Kripa Anand 132 UTA	Chengzhi Wu Li 131	Nate Hall 131	Chetan Yadav 132 Lab	Kripa Anand 132 UTA
	Dyson Kennedy 131, Astro 105	Jay Sandesara 132, 286	Dyson Kennedy 131, Astro 105	Meghana Vishwanath 152	Y Qiu 131
11am - Noon	Justin Fagoni 132 Lecture	Steven Zhang 131	Justin Fagoni 132 Lecture	Ed van Bruggen 152	Kaifei Ning 132 Lecture
	Kaifei Ning 132 Lecture	Vahini Nareddy 152	Meridth Stone 131	Sanil Raut 131	Y Qiu 131
Noon - 1pm	Justin Fagoni 132 Lecture	Steven Zhang 131	Justin Fagoni 132 Lecture	Nadav Benhamou Goldfajn 131	Kaifei Ning 132 Lecture
	Kaifei Ning 132 Lecture	Chenan Wei 132 Lab	Mingyuan Wang 152	Shrohan Mohapatra 151	Y Qiu 131
1pm - 2pm	Isobel Smith 132 UTA	Baji Jadhav Astro 105	Isobel Smith 132 UTA	Nadav Benhamou Goldfajn 131	Leyna Bajaj 131
	Chetan Yadav 132 Lab	Chenan Wei 132 Lab	Emily Knowlton 132 UTA	Hannah Peltz Smalley 131	Chetan Yadav 132 Lab

	Monday	Tuesday	Wednesday	Thursday
2pm - 3pm	Sili Wu 151	Baji Jadhav Astro 105	Meridth Stone 131	Makayla Vessella 131 Lab, 281
	Zhiyu Yang 151	Bart Szymanowski 131	Ajit Kumar 131	
3pm - 4pm	Sili Wu 151	Sierra Gomez 132 UTA	CLOSED FOR DEPARTMENTAL COLLOQUIUM	Sierra Gomez 132
	Jay Sandesara 132 Lab, 286	Roshan Trivedi 131		Liam Yanulis 131
4pm - 5pm	Sierra Gomez 132 UTA	Emily Knowlton 132 UTA		Ed van Bruggen 152
	Mingyuan Wang 152	Isobel Smith 132 UTA		Shani Perera 131
5pm - 6pm	Sierra Gomez 132 UTA	Isobel Smith 132 UTA		Roshan Trivedi 131
	Ishan Rana 131	Shane Keiser 131		Vivek Chakrabhavi 131
6 pm - 7pm	Sofia Corba 131	Shane Keiser 131	Aditya Kulkarni 131	Vivek Chakrabhavi 131
	Abhishek Kumar 132 Lab	Harith Rathnayaka 151	Abhishek Kumar 132	Harith Rathnayaka 151
7pm - 8pm	Aidan Morehouse 132 UTA	Nicholas Pittman 131, 132 Lab	Nicholas Yazbek 131	Zhiyu Yang 151
	Xiansheng Cai 132 Lab	Mayank Vaghela 132 Lab	Kerry O'Brian 115, 131	Tejas Patwardhan 152
8pm - 9pm	Aidan Morehouse 132 UTA	Nicholas Pittman 131, 132 Lab	Kerry O'Brian 115, 131	Jhih-Ying Su 115
	Likhitha Marlapati 152	Kerry O'Brian 115, 131		Yating Zhang 131

Position vs. time plots (not every plot is legitimate!)

Which of these graphs might represent the motion of a real object (e.g. a bike)?



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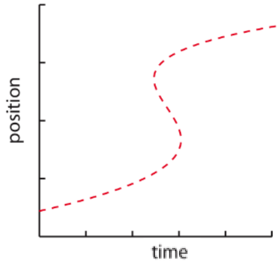
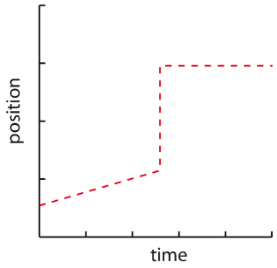


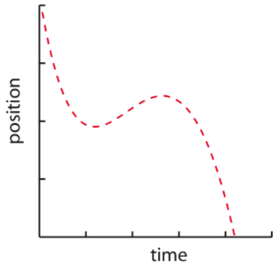
✗

Goes backwards in time!



✗

Changes position instantaneo usly



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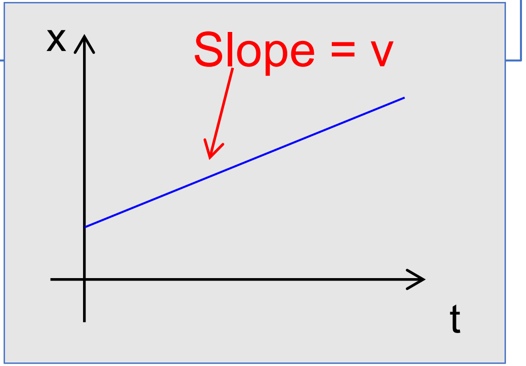
1D motion non-constant velocity ➔ Instantaneous velocity

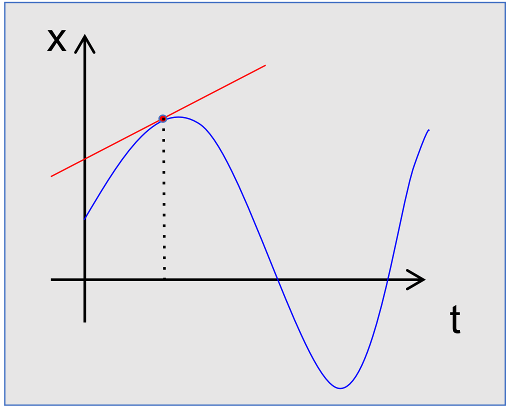
Constant velocity motion

- Straight line motion diagram
- Velocity equals slope of line



More generally

- Velocity changes with time
- **Instantaneous velocity** is slope of line tangent to curve
- Can compute this slope by taking limit of average velocity over shorter and shorter time intervals
- Velocity is the **derivative** of the position curve



1D motion with non-constant velocity

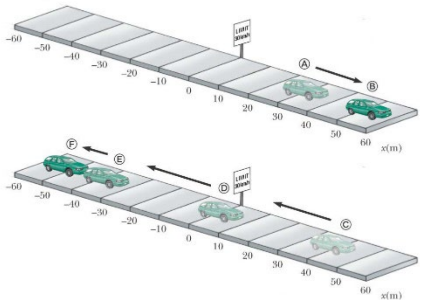
Before "instantaneous velocity" return to…
Average velocity vs. Average speed

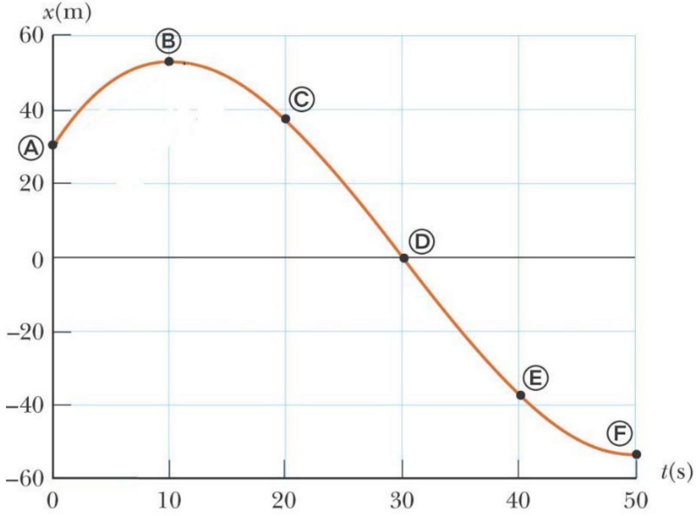
The relationship becomes more important when you can back up…

Cover more distance, without necessarily getting anywhere

Is the velocity zero anywhere? How can you tell?

Where is the velocity the greatest? How can you tell?



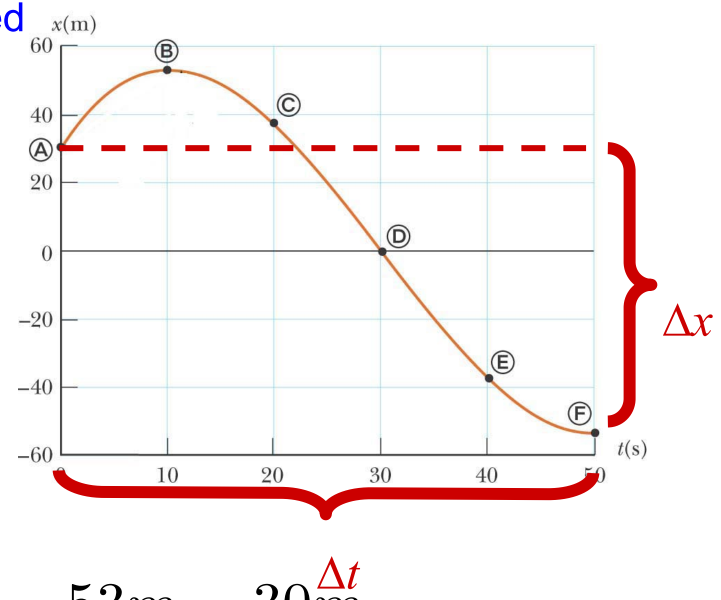


Average Velocity: based on…

Displacement = **Net distance** traveled

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(t_f) - x(t_i)}{t_f - t_i} = \frac{-53m - 30m}{50s - 0s} = -1.7m/s$$

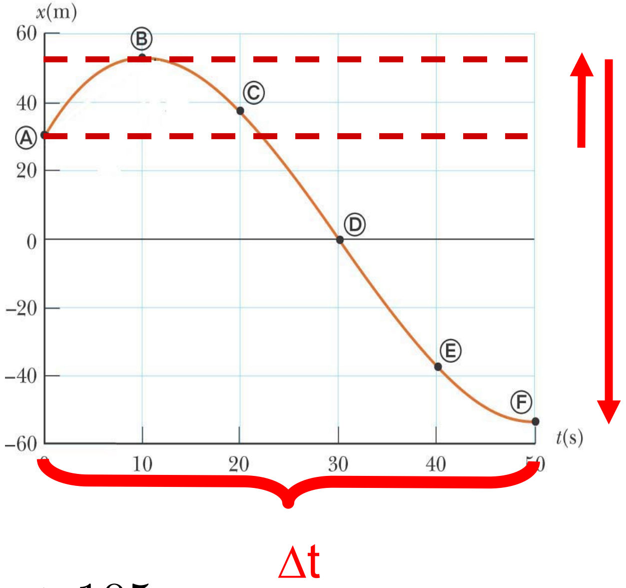
Magnitude is 1.7 m/s; direction is -*x*

Average Speed:

Based on **total distance** travelled

Position of the Car at Various Times

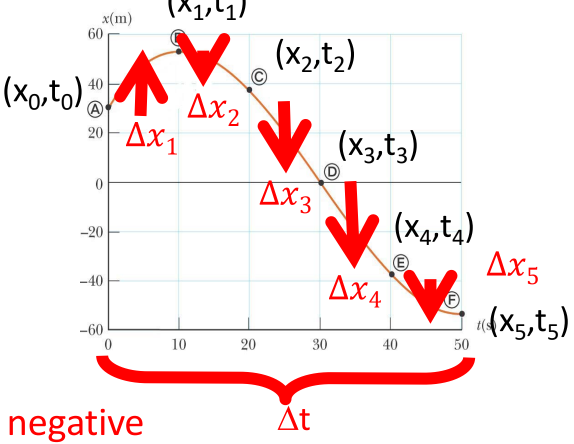
Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



$$s_{avg} = \frac{\text{total distance}}{\text{total time}} = \frac{22m + 105m}{50s} = 2.5m/s$$

No direction and no sign associated with speed.
Never negative, or smaller than |v|.

Dealing with complex 1D motion without calculus



Specify trip with large number N of locations at different times

Object is at positions $x_0, x_1, x_2, \ldots x_N$

at times $t_0, t_1, t_2, \ldots t_N$

Δx_k = **Displacement** on kth part of trip

Can be positive or negative

$\Delta x_k = x_k - x_{k-1}$

$|\Delta x_k|$ = **Distance** traveled on kth part of trip

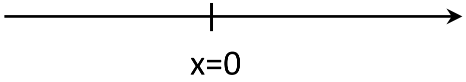
Average speed $$s_{avg} = \frac{\sum_{k=1}^{N} |\Delta x_k|}{t_N - t_0}$$

Total **distance** over total time

Average velocity $$v_{avg} = \frac{\sum_{k=1}^{N} \Delta x_k}{t_N - t_0}$$

Total **displacement** over total time

Example



Jane Austen travels 80 miles east in one hour

70 miles west in ½ hour

50 miles east in ½ hour

$t_0 = 0$ $x_0 = 0$

$t_1 = 1$ $x_1 = +80$

$t_2 = 1.5$ $x_2 = +10$

$t_3 = 2.0$ $x_3 = +60$

Remember:
Distance: always positive
Displacement: can be positive or negative

$$s_{avg} = \frac{80 + 70 + 50}{2} = 100\,mph$$

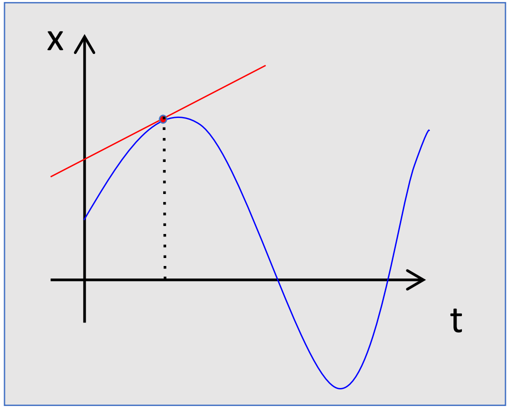
sum of **distances** over total time

$$v_{avg} = \frac{80 - 70 + 50}{2} = 30\,mph$$

sum of **displacements** over total time

Instantaneous Velocity

- What if we want to precisely know the velocity at some point in time, not the average over a finite time interval?
- Compute the *instantaneous velocity* **v(t)** at time t by taking the limit of average velocity over shorter and shorter time intervals
- Instantaneous velocity is slope of tangent line to curve
- Equals **first derivative** of curve- its rate of change vs t.



Compute v_{avg} from t_0 to t_0+T

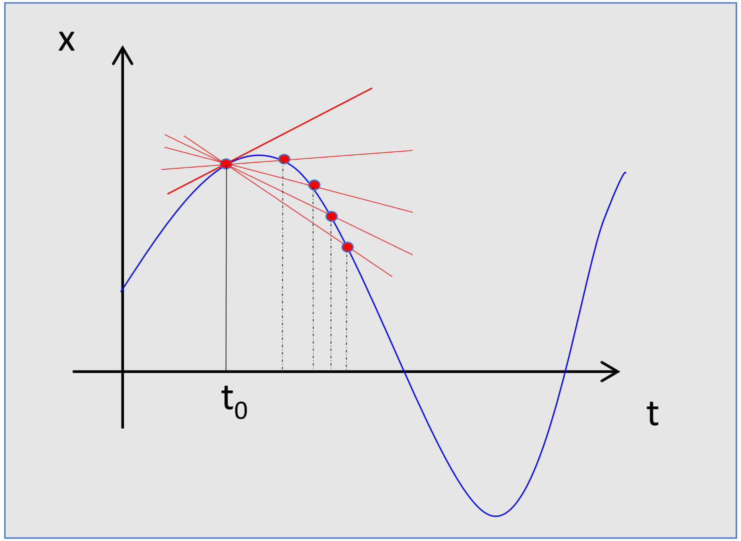
Take limit as T goes to 0

$$v(t_0) = \lim_{T \to 0} \frac{x(t_0 + T) - x(t_0)}{T}$$

Graph ➡ Gives slope of tangent to curve at t_0 ✓

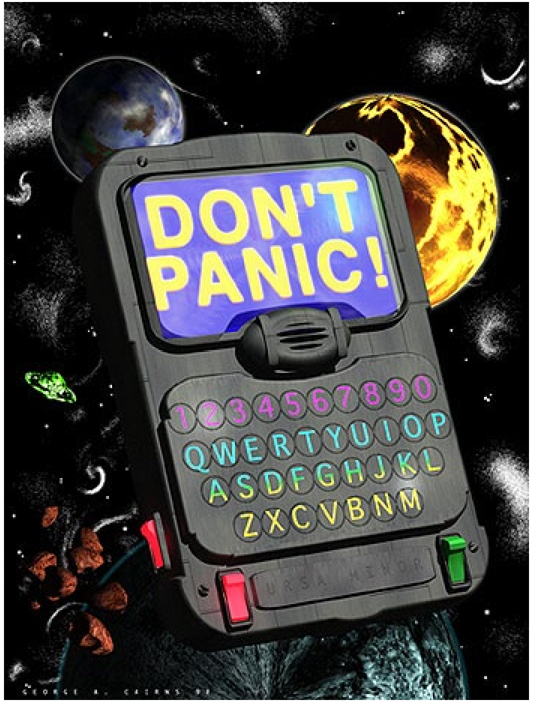
Calculus ➡ This is definition of derivative of curve at t_0 ✓

$$v(t_0) = \left.\frac{dx}{dt}\right|_{t_0}$$



A note on Calculus in this course

- Math 131 (Calculus I) is a co-requisite for this course.
    - However, we will be using some concepts in 151 before they are introduced in 131.
- I will teach a few basics on how to work with derivatives and integrals without going deeply into the details.
    - Think of these as “Tools” or “Special Moves” you can use to solve Physics problems.
    - Some problems will appear on the exam that require you to use these tools. I will keep their number, and their complexity, limited.
- Please make use of office hours and tutoring resources if you need help with this material.



Calculus 'Special Move' #1: The Derivative

This works for any polynomial function.
Why is this the answer? You'll find out in Math 131.
As far as we're concerned…
"It just works!"

The derivative of a polynomial function $f(x) = cx^n$ is given by…

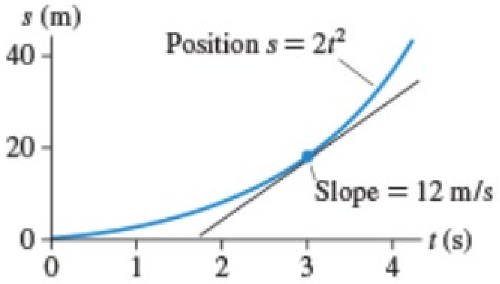
$$\frac{d}{dx}f(x) = \frac{d}{dx}(cx^n) = ncx^{n-1}$$

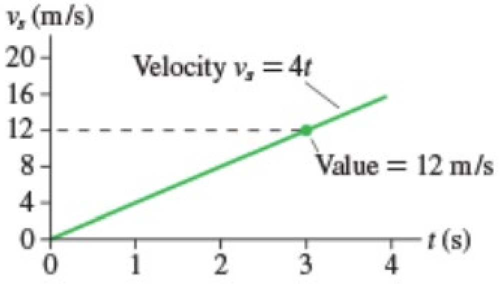
Example: Suppose we have a position s=2t^2.

What is the velocity v_s=ds/dt?

Per above,

$$\frac{d}{dt}(2t^2) = 4t$$





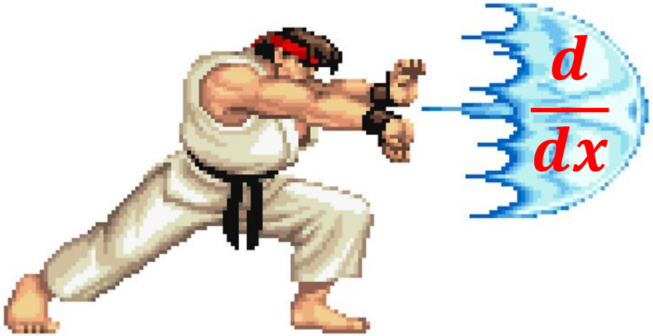
Some Rules for Using Derivatives

Derivative of a constant function c is 0.

$$\frac{d}{dx}c = \frac{d}{dx}(cx^0) = 0$$

Derivative of a sum is the sum of derivatives.

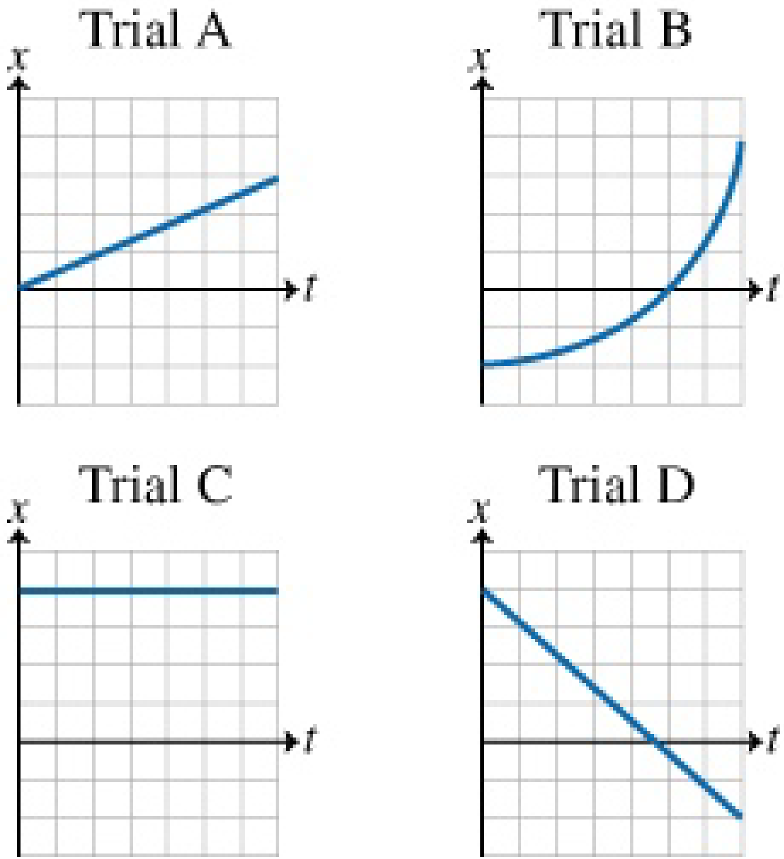
$$\frac{d}{dx}\big(f(x) + g(x)\big) = \frac{df}{dx} + \frac{dg}{dx}$$



Example: **What is the derivative of $x(t)=t^2+4t+16$?**

$$\frac{dx}{dt} = \frac{d}{dt}(t^2) + \frac{d}{dt}(4t) + \frac{d}{dt}(16)$$

$$=2t + 4$$



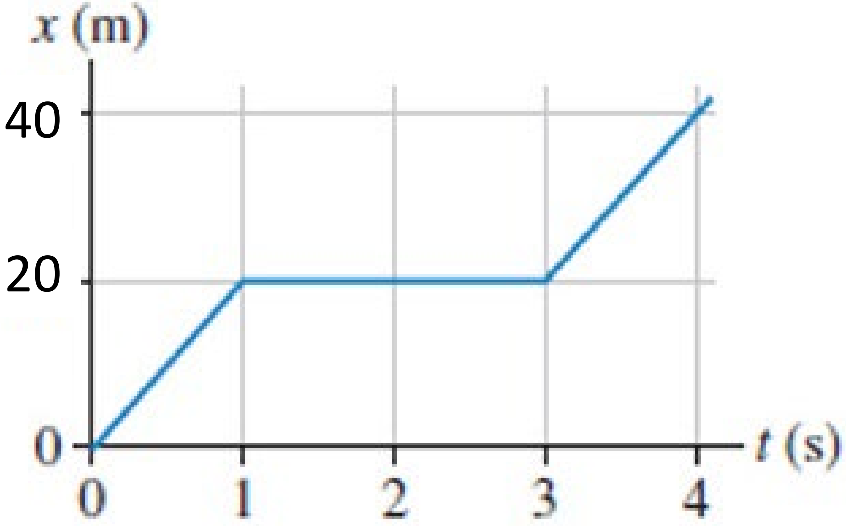
Position versus time

1) Which plots show constant velocity motion?

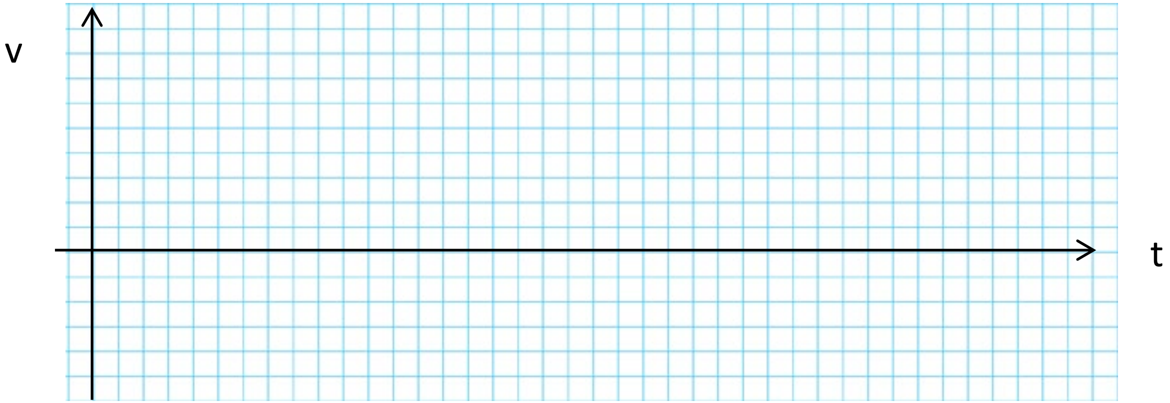
2) How do you tell?

3) How would you compute average velocity?

4) Which plot shows the largest magnitude of average velocity?



Make plot of velocity vs time



Finding Position from Velocity

We can find velocity from x(t) by taking a derivative. Can we infer x(t) from v(t)?
The answer is yes, but we will again need to use calculus.

Velocity is derivative of position ⟷ Position is the integral of velocity

Fundamental theorem of calculus
(Math 131 spoiler warning)

In equations…

$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x_0 + \int_0^t v(t')dt'$$

What does this mean?

Displacement (change in position) is area under velocity curve

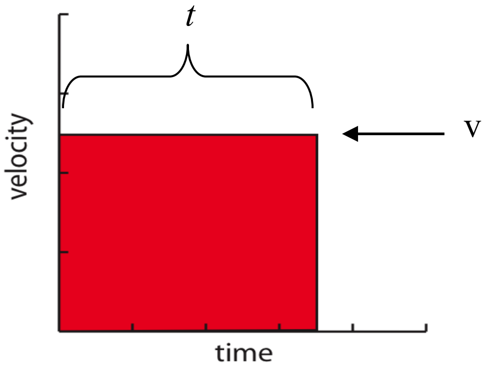
Finding Position from Velocity

$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x_0 + \int_0^t v(t')dt'$$

"Area" under velocity curve between 0 and t

Easy to see for constant velocity ⟶ Velocity curve is horizontal line

Area under curve is simply (height)(length) = v t

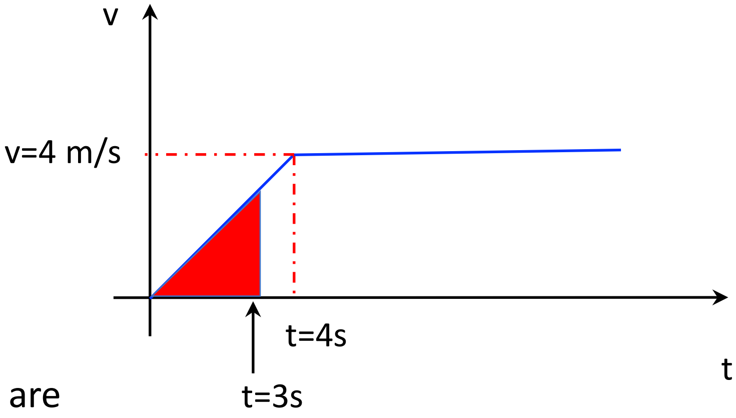


$$x(t) = x_0 + \int_0^t v(t')dt' = x_0 + vt$$

What are the units of "height"? What are the units of "length"? What are the units of the product call here "area"?

Example

Start with graph of velocity vs time



Object accelerates from rest to velocity 4 m/s in 4 s, then moves with constant velocity

Assuming object starts at x=0, what are positions at t =3s and t=7s?

Change in position is area under velocity graph

t = 3s ➡ area of triangle = ½ (base)(height)

slope 1 ➡ height = base = 3

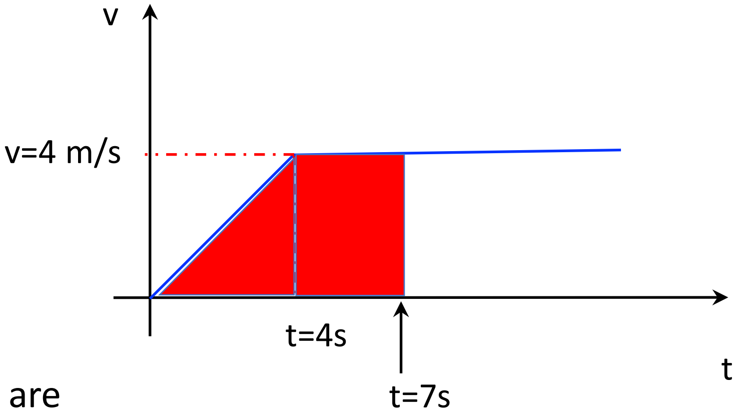
x (3s) = (1/2) x (3 s) x (3 m/s) = 4.5 m

If you are not familiar or comfortable yet with integrals

$$\int_0^t v(t)dt = \int_0^t at\ dt = \frac{at^2}{2}$$

Example

Start with graph of velocity vs time



Object accelerates from rest to velocity 4 m/s, then moves with constant velocity

Assuming object starts at x=0, what are positions at t =3s and t=7s?

Change in position is area under velocity graph

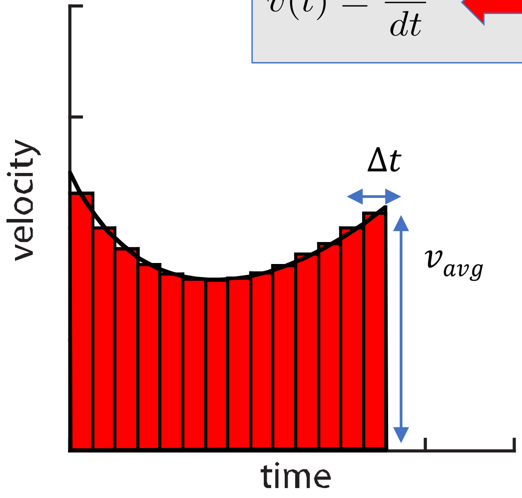
t = 7s ➡ area of triangle + area of rectangle

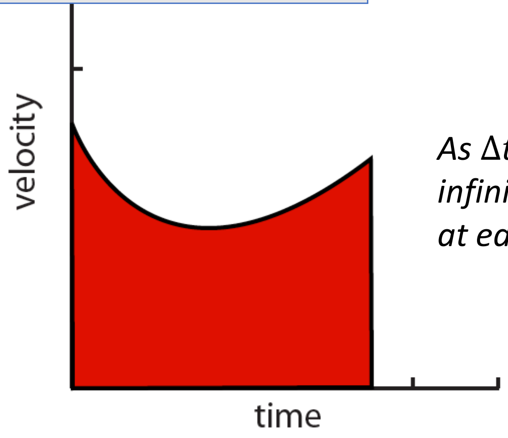
$$x\,(7s) = \underbrace{(1/2) \times (4s) \times (4\ m/s)}_{\text{triangle}} + \underbrace{3(s) \times (4\ m/s)}_{\text{rectangle}} = 8 + 12 = 20\ m$$

What if I want to find the area of a curve that doesn't have simple geometry?

More generally…

$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x_0 + \int_0^t v(t')dt'$$





As $\Delta t \to 0$, Δt becomes the infinitesimally small "dt" at each point in v(t)

- Integral (area under curve) is approximated by by adding up area of rectangles
- Each rectangle acts like constant velocity motion over short time interval
- Exact integral is sum of infinite number of rectangles in limit $\Delta t \to 0$

Height of rectangle

$$\Delta x = v_{avg} \cdot \Delta t$$

Width of rectangle

*That's nice, but how do we **compute** this sum?*

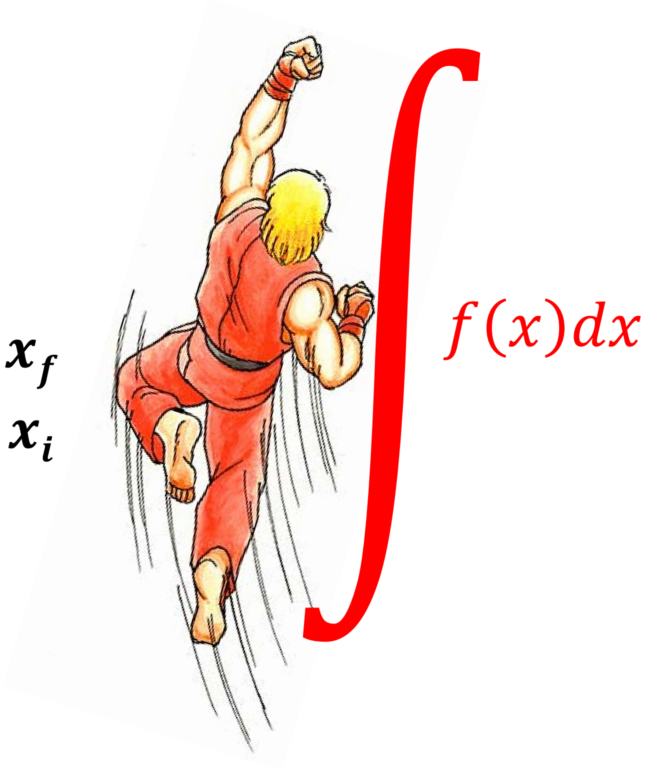
Calculus 'Special Move' #2: The Integral

The integral of a polynomial function
$\boldsymbol{f(x) = cx^n}$ is given by…

$$\int_{x_i}^{x_f} f(x)dx = \int_{x_i}^{x_f} cx^n dx = \left.\frac{cx^{n+1}}{n+1}\right|_{x_i}^{x_f}$$

$$= \frac{c{x_f}^{n+1}}{n+1} - \frac{c{x_i}^{n+1}}{n+1}$$

(For $n \neq -1$)



Example: Suppose we have a velocity v(t)=2t².
Starting at t=0, what is the displacement Δx traveled in 3 seconds?

$$\Delta x = \int_0^3 f(t)dt = \int_0^3 2t^2 dt = \left.\frac{2t^3}{3}\right|_0^{3s} = \frac{2(3)^3}{3}m - 0m = 18m$$

We now know how to relate position and velocity using calculus.

$$v(t) = \frac{dx}{dt} \quad \Leftrightarrow \quad x(t) = x_0 + \int_{t_0}^{t} v(t')\, dt'$$

…how does **acceleration** fit into this?

Instantaneous Acceleration

Recall… $$v(t) = \lim_{T \to 0} \frac{x(t+T) - x(t)}{T}$$

Velocity is rate of change of position with time $\Rightarrow$ $$v = \frac{dx}{dt}$$

Acceleration is defined the same way in terms of velocity

$$a(t) = \lim_{T \to 0} \frac{v(t+T) - v(t)}{T}$$

Acceleration is rate of change of velocity with time $\Rightarrow$ $$a = \frac{dv}{dt} \quad \left(= \frac{d^2x}{dt^2}\right)$$

Acceleration gives **slope of tangent to v(t) curve** at time t
i.e. acceleration is the **derivative** of velocity (which is, in turn the **derivative** of position)
In other words, acceleration is the **second derivative** of position.

More practice with Instantaneous velocity and acceleration:

You won't usually use limits to calculate derivatives outside of math class

Use known results for derivatives of functions:

$$\frac{d}{dt}t^0 = 0$$

$$\frac{d}{dt}t^1 = 1$$

$$\frac{d}{dt}t^2 = 2t$$

"Calculus Special Move #1"

$$\frac{d}{dt}t^n = nt^{n-1}$$

$$x(t) = x_0 + v_0 t \quad \Rightarrow \quad v = \frac{dx}{dt} = v_0$$

Constant velocity motion

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad v = \frac{dx}{dt} = v_0 + at$$

Motion with constant acceleration

$$a(t) = \frac{dv}{dt} = a$$

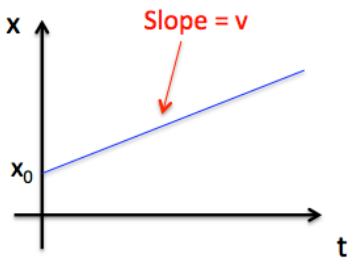
Acceleration

Recall…

Constant velocity

$$x = x_0 + vt$$

Position at t=0

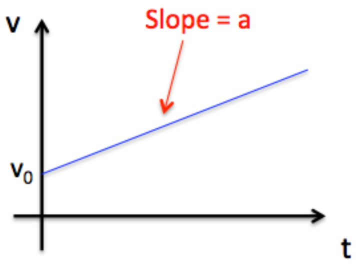


Similarly…

Constant acceleration

$$v = v_0 + at$$

Velocity at t=0



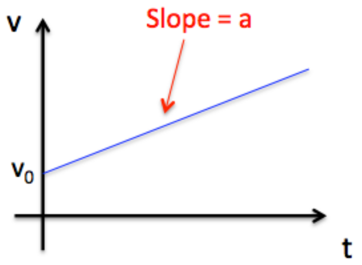
Check…

$$a = \frac{dv}{dt} = \frac{d}{dt}(v_0 + at) = a \quad \checkmark$$

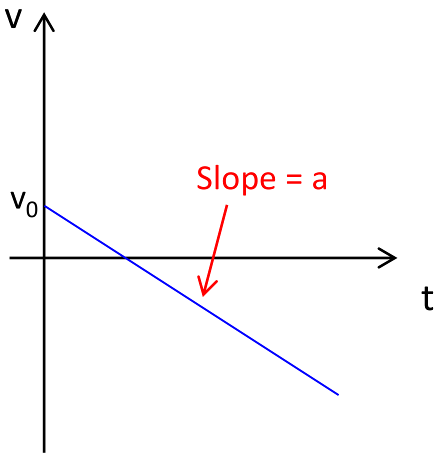
Constant acceleration basics…

$$v = v_0 + at$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(v_0 + at) = a$$



Slope **a** positive ➡ Velocity v increasing with time



Slope **a** negative ➡ Velocity decreasing with time

- When v>0, it is slowing down.
- When v<0, it is actually speeding up, but in the opposite direction

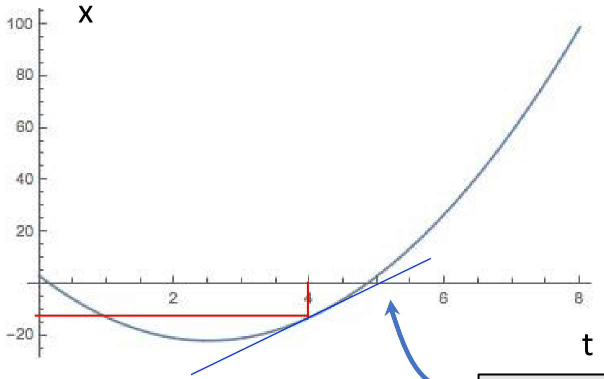
Motion with constant acceleration

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \quad \Rightarrow \quad v = \frac{dx}{dt} = v_0 + at$$

Example

Object starts at position x_0 = 3 m, moving with initial velocity v_0 = -20 m/s and accelerates at a = 8 m/s^2

What are its position and velocity at t = 4s?



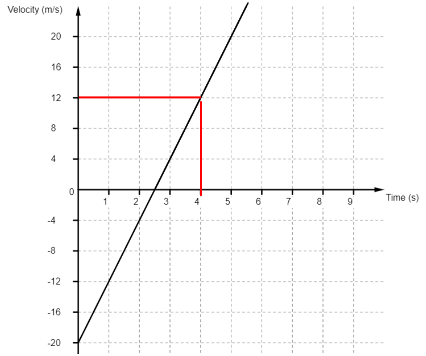
slope of tangent line is instantaneous velocity at t=4s

$$x(t) = 3 - 20t + 4t^2$$

$$x(4) = 3 - 20 \cdot 4 + 4 \cdot 4^2 = -13\,m$$

$$v(t) = -20 + 8t$$

$$v(4) = -20 + 8 \cdot 4 = 12\,m/s$$



Summary

Position x(t)	Velocity v(t)	Acceleration a(t)	Description
$x(t) = x_0$	$v(t) = 0$	$a(t) = 0$	Constant position
$x(t) = x_0+v_0\, t$	$v(t) = v_0$	$a(t) = 0$	Constant velocity
$x(t) = x_0 + v_0\, t + (1/2)\, a_0\, t^2$	$v(t) = v_0 + a_0\, t$	$a(t) = a_0$	Constant acceleration

$$a(t) = \frac{dv}{dt} \Longleftrightarrow v(t) = v_0 + \int_{t_0}^{t} a(t')\, dt'$$

$$v(t) = v_0 + a(t - t_0)$$

$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x_0 + \int_{t_0}^{t} v(t')\, dt'$$

$$\boxed{x(t) = x_0 + v_0(t - t_0) + a(t - t_0)^2 / 2}$$