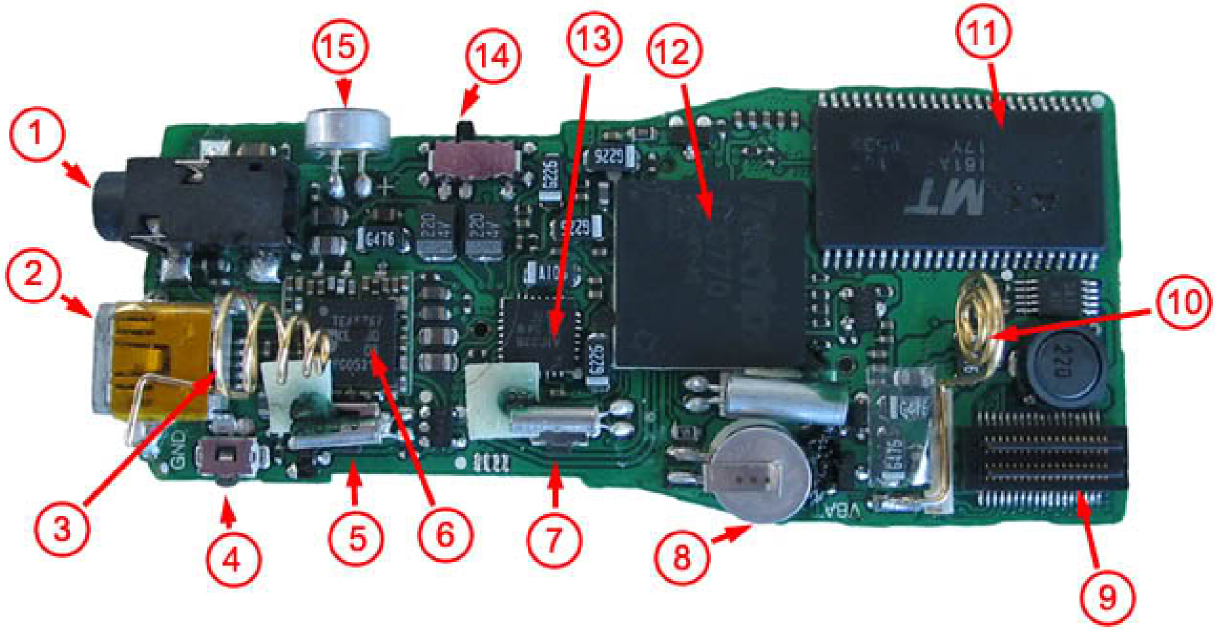
# Introduction to Electrical and Computer Engineering

Sampling and Quantization

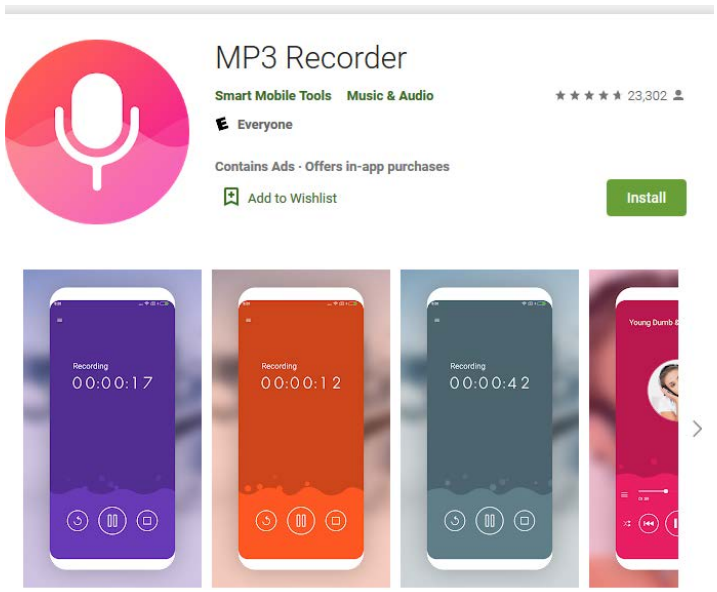
# MP3 Recorder/Player

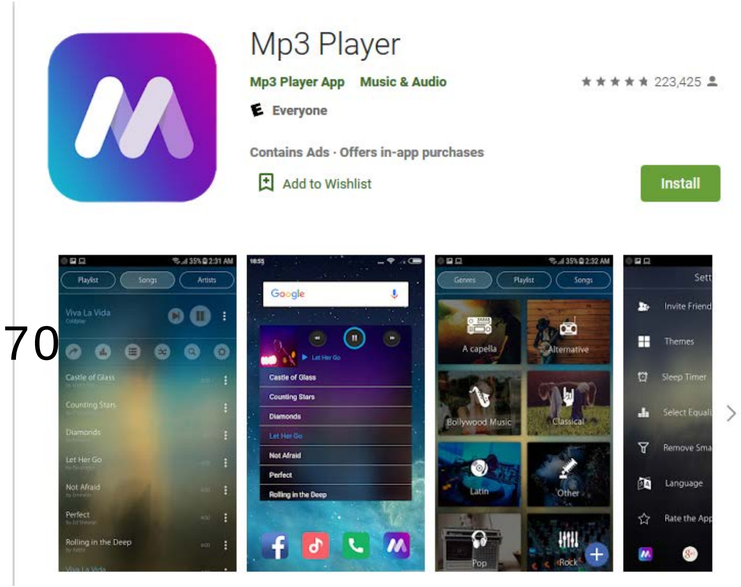
SanDisk Sansa m200 Series

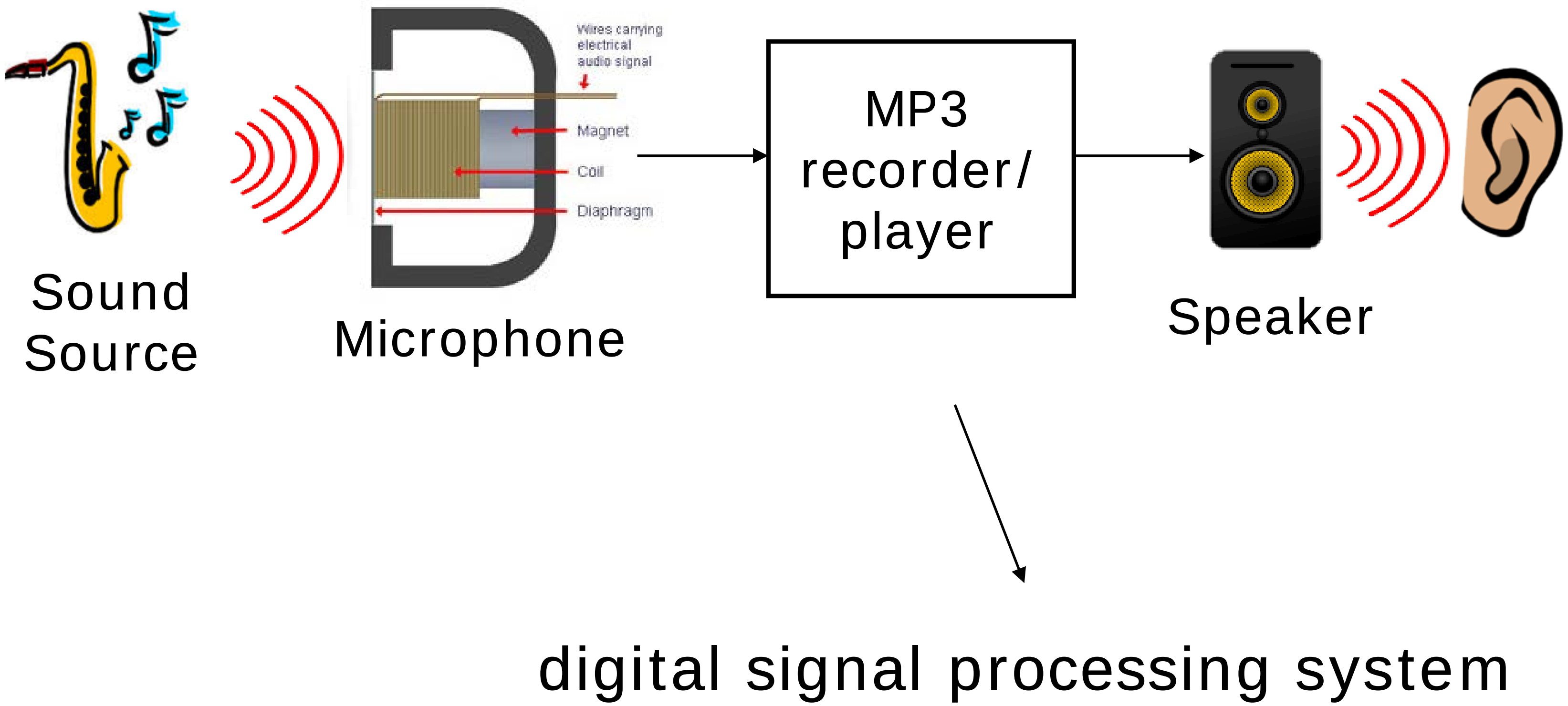



1. Headphone connector
2. USB Connector
6. Philips TEA5767 FM radio
9. Socket for Flash Memory

11. RAM
12. Main Processor – Telechips TCC 770
13. Stereo Amplifier



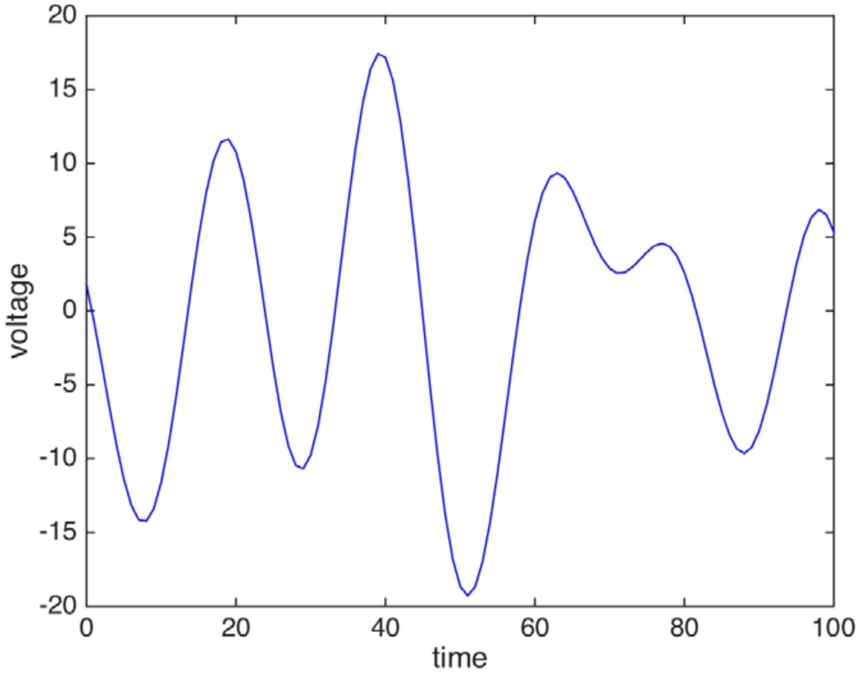

Sound
Source
Wires carrying
electrical
audio signal
Magnet
Coil
Diaphragm
Microphone
MP3
recorder/
player
Speaker
digital signal processing system

# Main Concepts

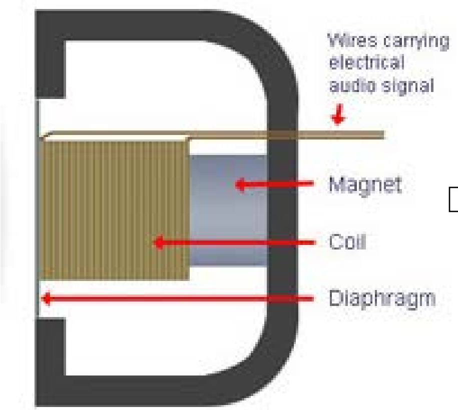
- Signal
- Signal Processing
- Spectrum
- Sampling
- Quantization
- Compression
- Spectrogram

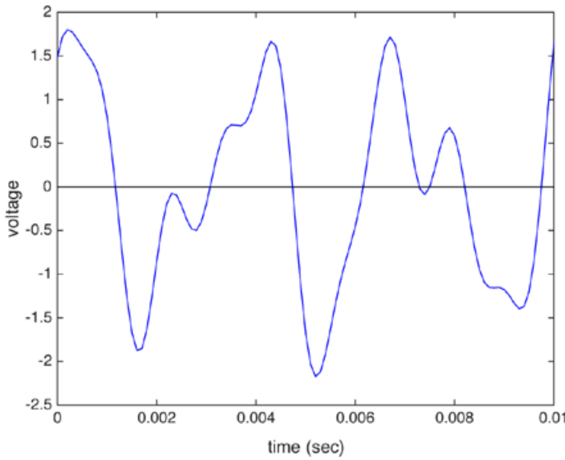
# Signal

A signal is a “time dependent variation” of a characteristic of a physical phenomenon, used to convey information.”



# Audio converted to an electrical signal




sound source

pressure wave

microphone

electrical signal

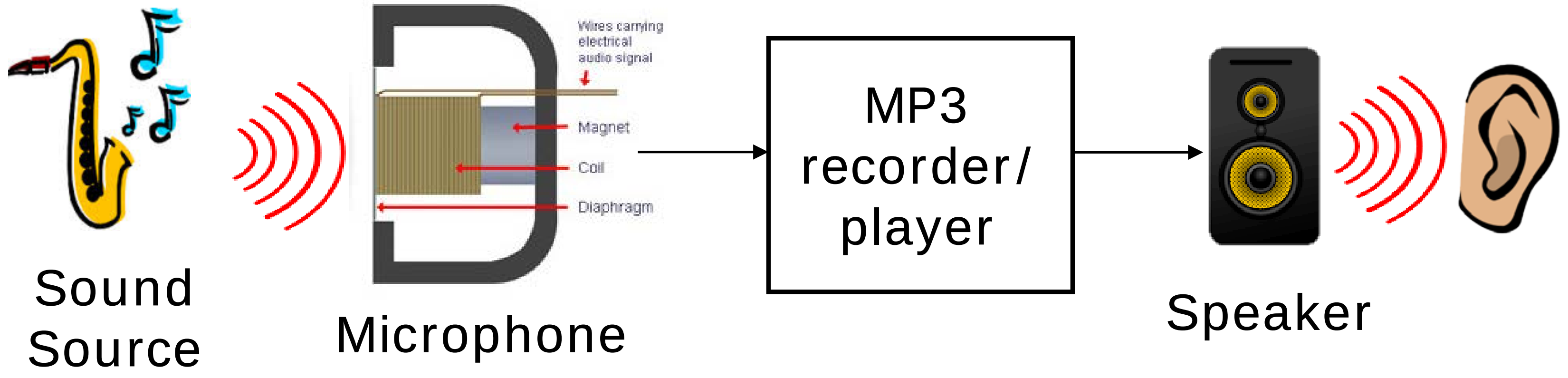
A signal is a “time dependent variation” of a characteristic of a physical phenomenon, used to convey information.”

# Signal Processing

Signal Processors are systems that:

1. extract information from their environments
2. process input signals to put them in more useful forms
3. produce output signals conveying information to a user

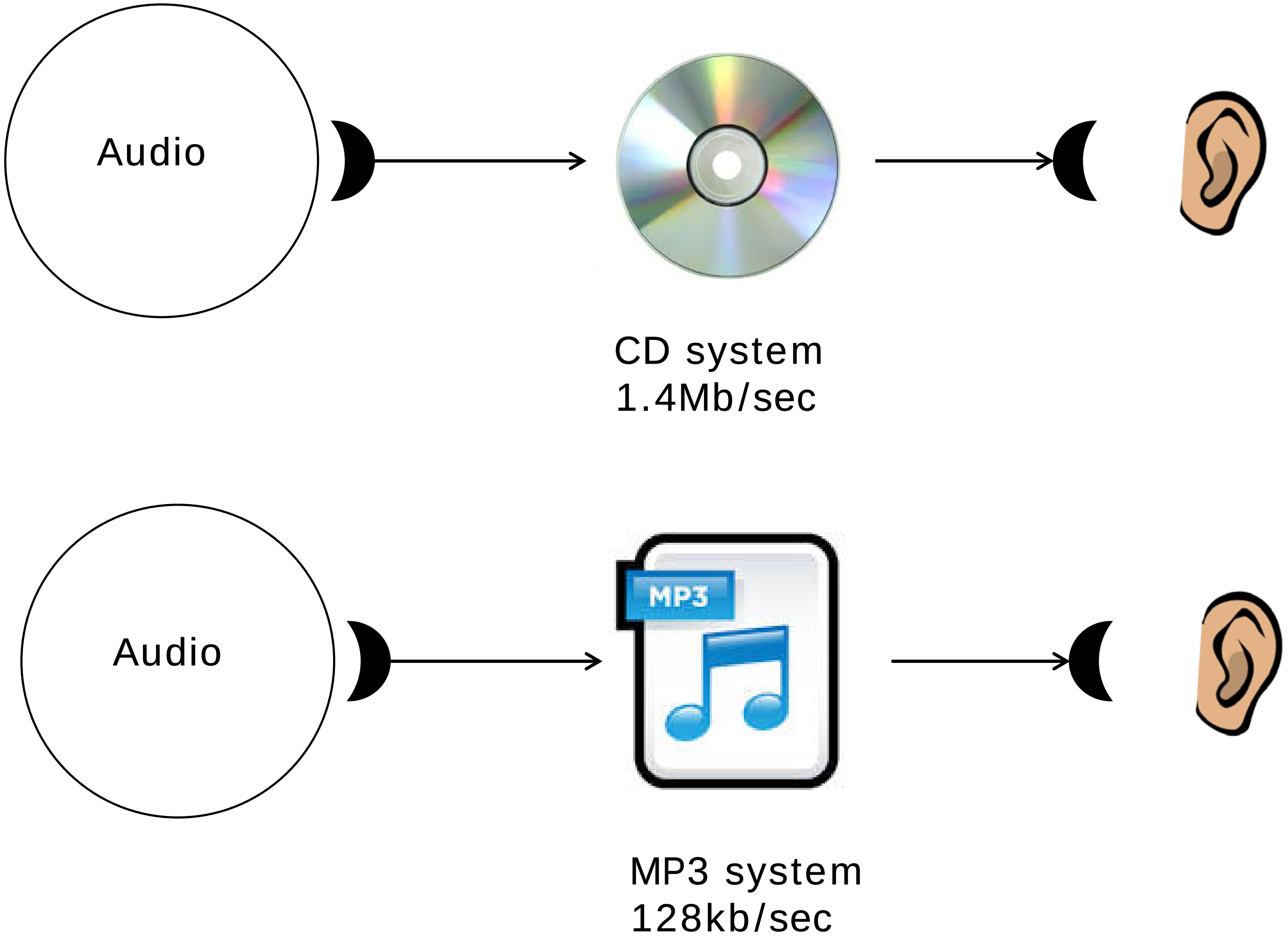
# MP3 recorder/player is a signal processor

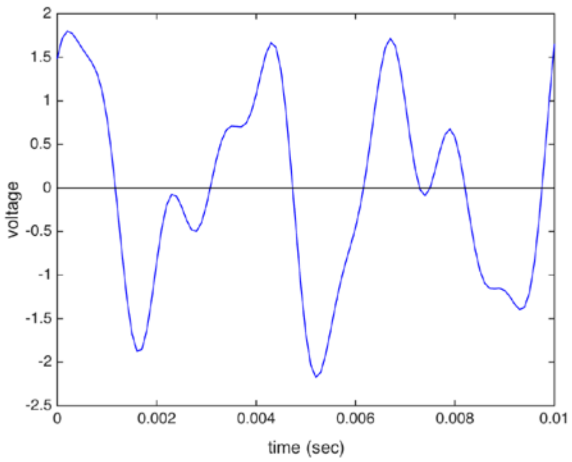



Signal Processor

1. extract information from their environments
2. process input signals to put them in more useful forms
3. produce output signals conveying information to a user

# 2. process input signals to put them in more useful forms

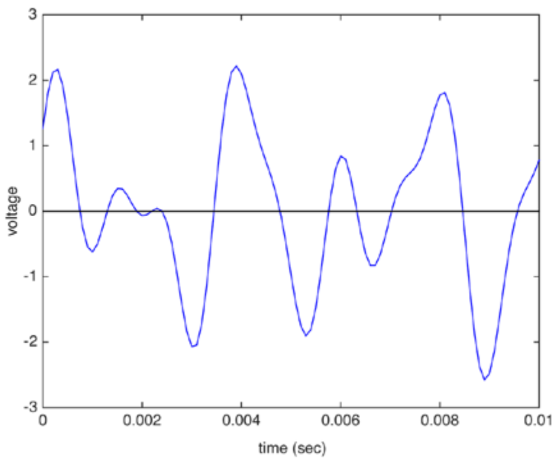





Input
Voltage
Signal

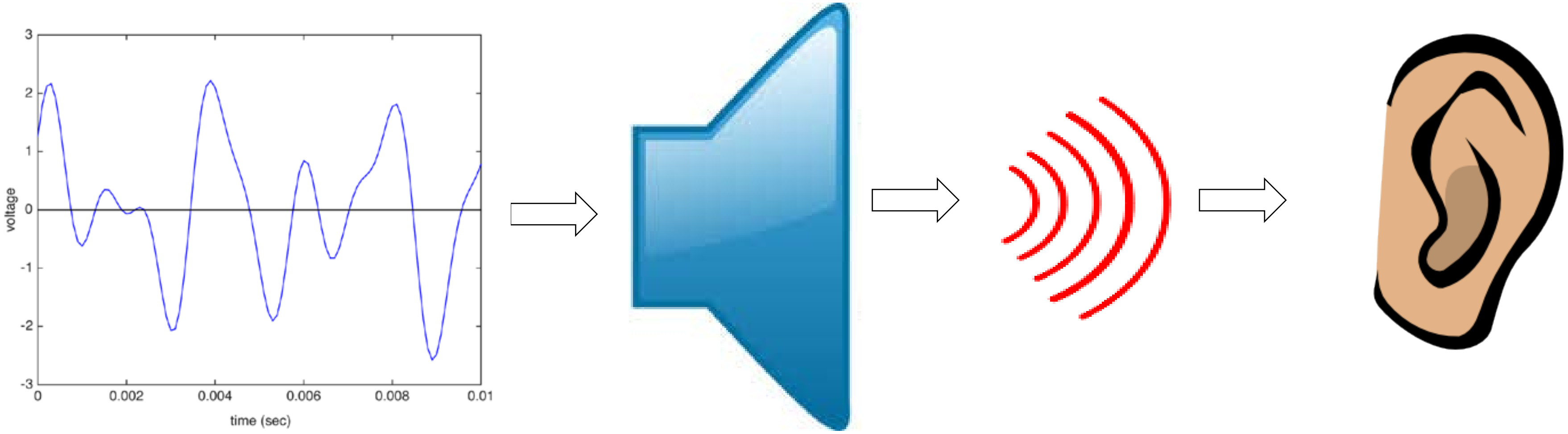
Digital
Signal
Processing
System

Our focus
in this
module




Output
Voltage
Signal

# Electrical signal converted to audio



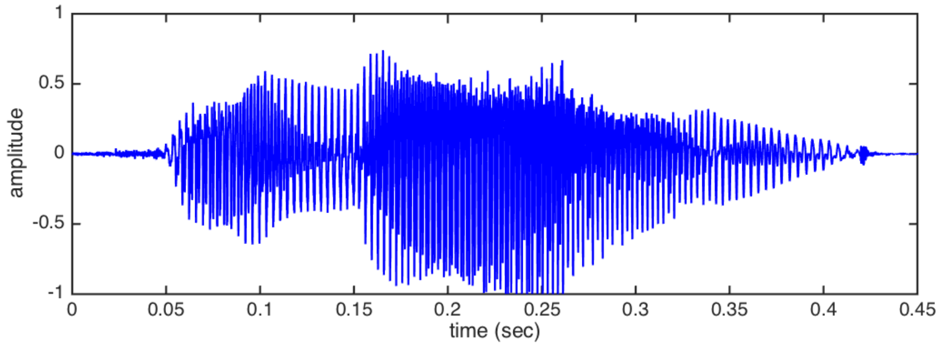

Output Voltage Signal

Speaker

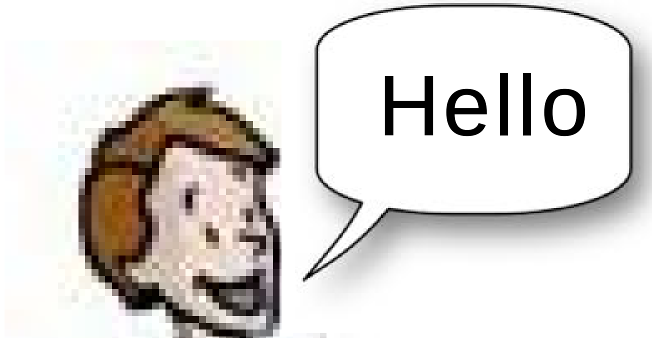
Pressure Wave

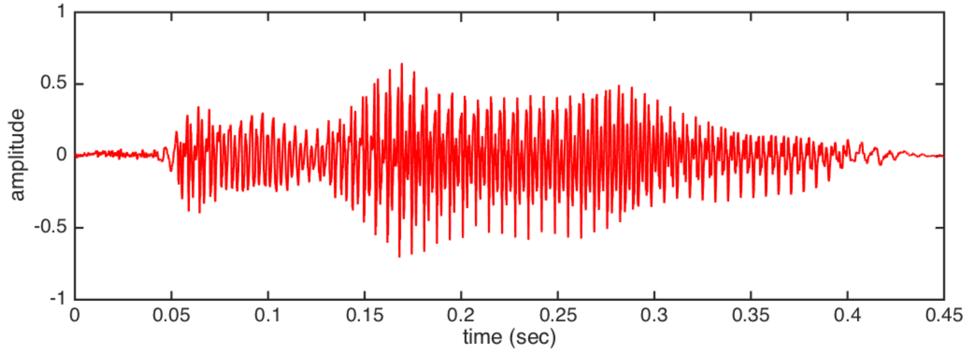
User

# Information in an audio signal

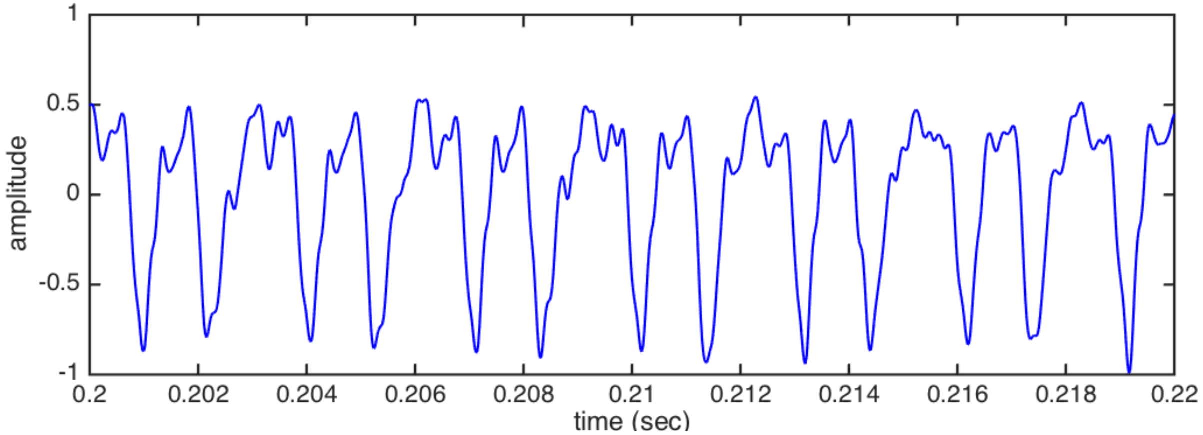



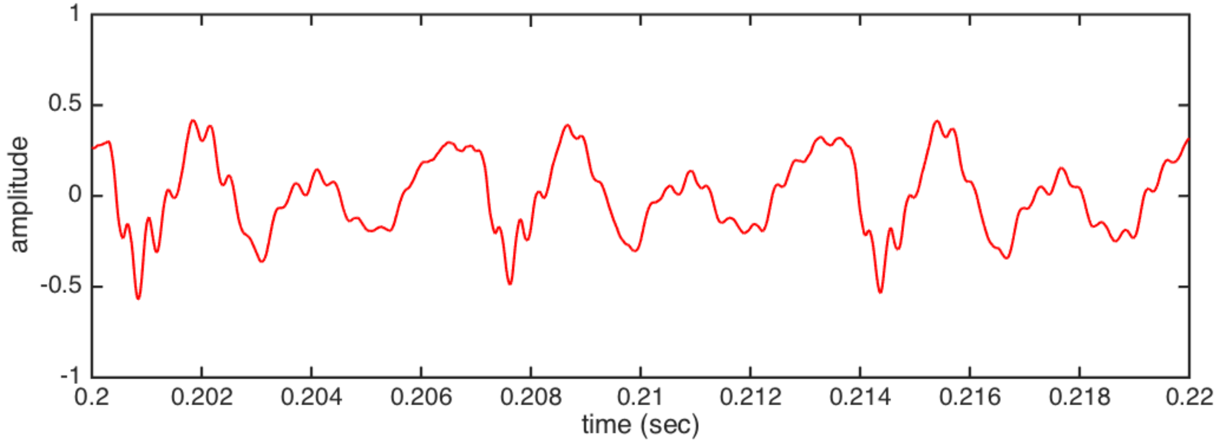

Voltage Signal




Voltage Signal




The two sounds are distinguished by the frequencies of the oscillations in the signals!

# Sinusoids: Basic Building Block of Signals

A sinusoid is a time function of the form

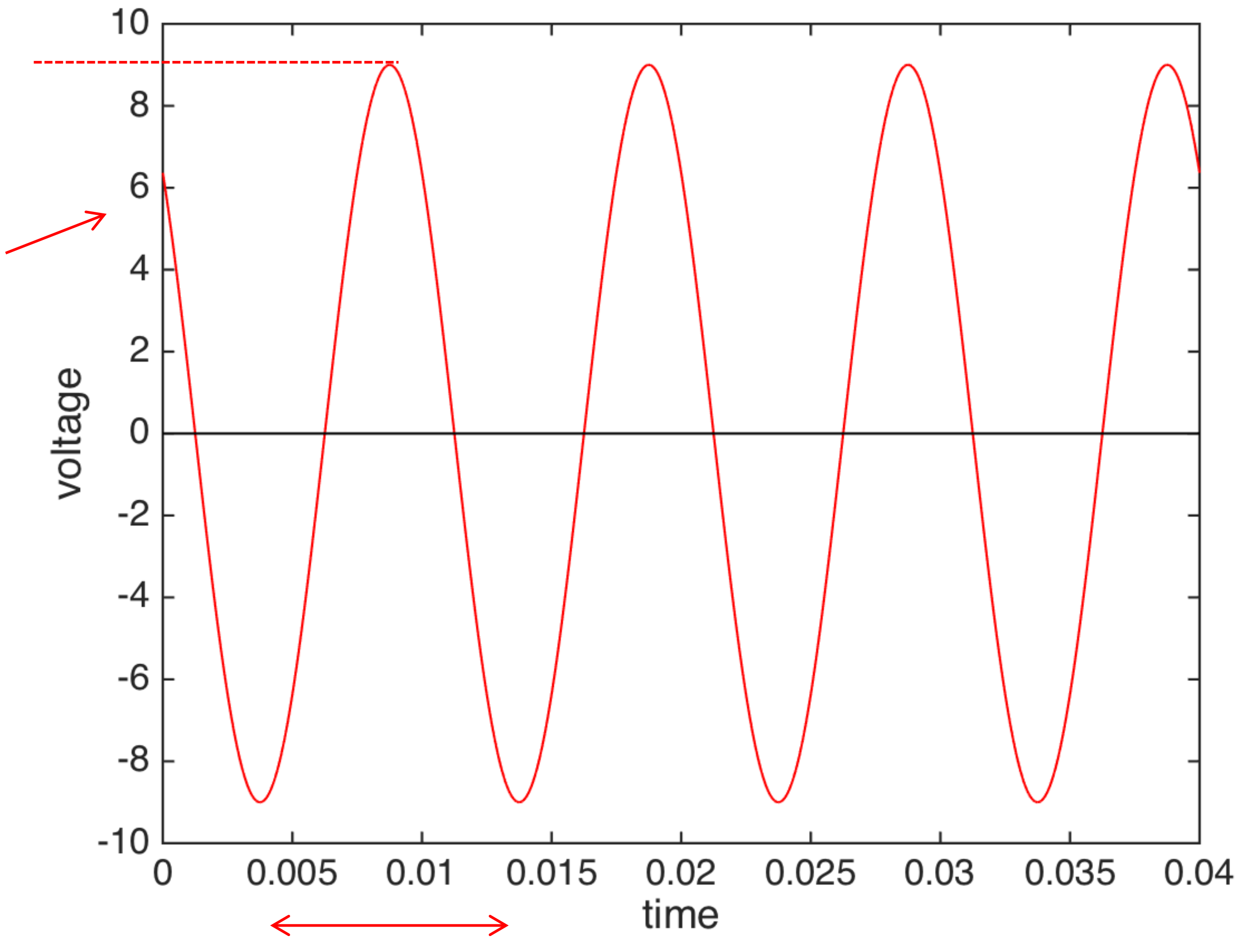
$$c(t) = A\cos(2\pi f t + \theta)$$

- $A$: magnitude (volts)
- $f$: frequency (Hertz – cycles/sec)
- t: time (sec)
- $\theta$: phase (radians)
- $T = 1/f$ sec: period

$$c(t) = 9\cos\left(2\pi(100)t + \frac{\pi}{4}\right)$$

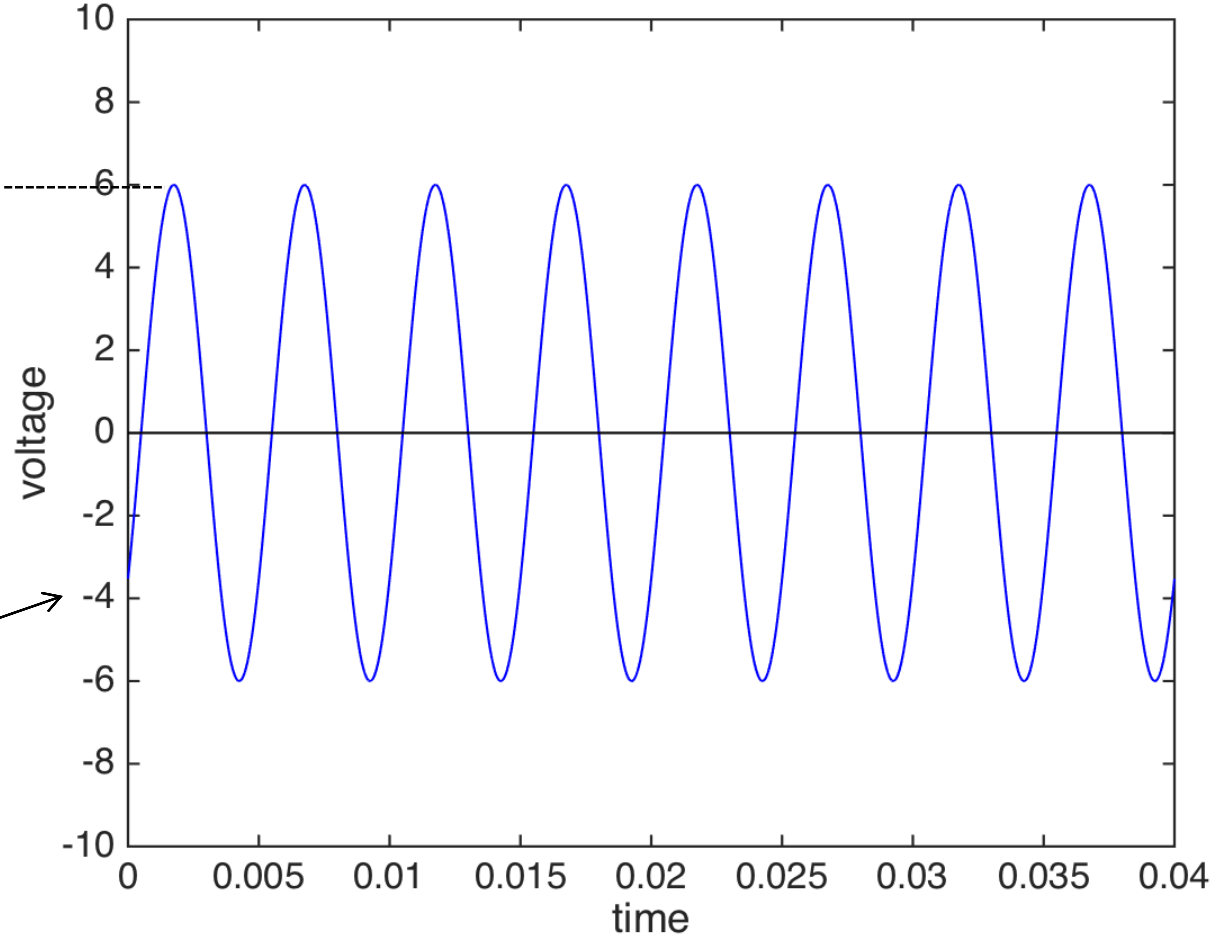
$A$ = 9 Volts

$9\cos(\pi/4) \approx 6.36$ Volts




$T$ = 0.01 sec, ($f$ = 1/$T$ = 100 Hz)

$$c(t) = A\cos(2\pi f t + \theta)$$




6 Volts

$$6\cos(-0.7\pi) \approx -3.53 \text{ Volts}$$

$T$ = 0.005 sec

A = 6
T= 0.005
f = 1/T = 200Hz
Θ = -0.7π

$c(t)=6\cos(2\pi\cdot 200t\ -0.7\pi)$

# Joseph Fourier (1786-1830)

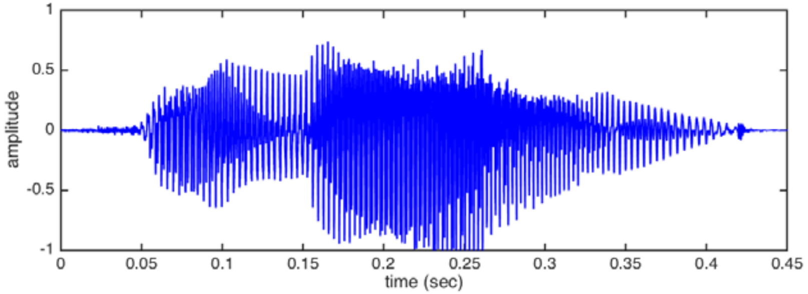
"Every signal can be represented as a sum of sinusoids having different frequencies."

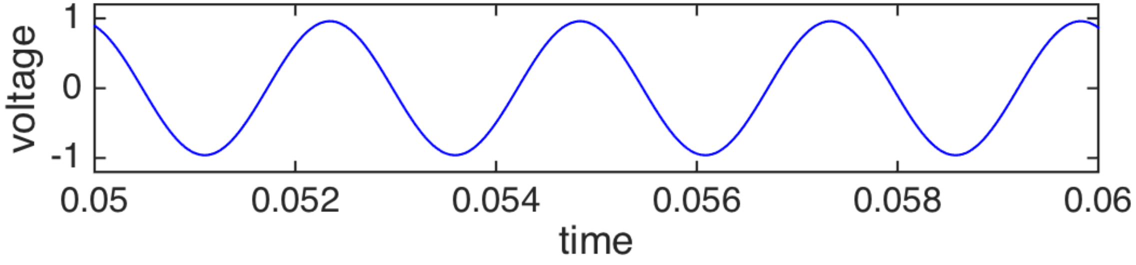
# Fourier Signal Representation

**"Every signal can be represented as a sum of sinusoids having different frequencies"**

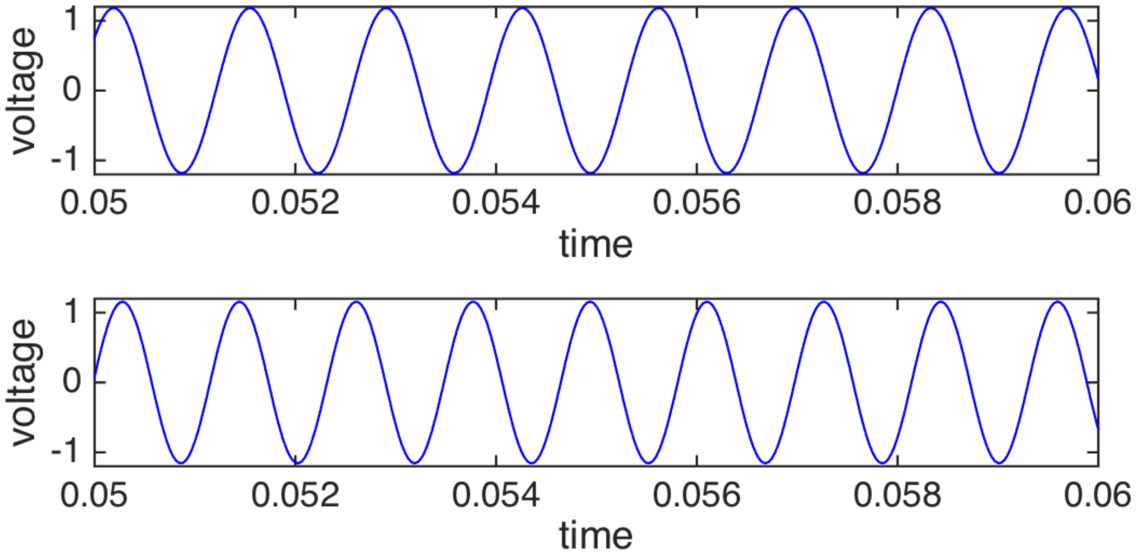
$$s(t) = A_0 \cos\left(2\pi f_0 t + \theta_0\right) + A_1 \cos\left(2\pi f_1 t + \theta_1\right) + A_2 \cos\left(2\pi f_2 t + \theta_2\right) + \ldots$$

$$s(t) \quad \leftrightarrow \quad \left\{ \begin{array}{c} \{A_0, f_0, \theta_0\} \\ \{A_1, f_1, \theta_1\} \\ \{A_2, f_2, \theta_2\} \\ . \\ . \\ . \end{array} \right\}$$


$f$ = 401 Hz;
$A$ = 0.96 V




$f$ = 737 Hz;
$A$ = 1.18 V

$f$ = 859 Hz;
$A$ = 1.25 V

(plus many more)

# Fourier Signal Representation

**"Every signal can be represented as a sum of sinusoids having different frequencies"**

$$s(t) = A_0 \cos\left(2\pi f_0 t + \theta_0\right) + A_1 \cos\left(2\pi f_1 t + \theta_1\right) + A_2 \cos\left(2\pi f_2 t + \theta_2\right) + \ldots$$

$$s(t) = \sum_{k=0}^{\infty} A_k \cos\left(2\pi f_k t + \theta_k\right) \qquad \text{(Fourier Series)}$$

$$s(t) = \int_{-\infty}^{\infty} S(k) e^{j2\pi kt} dk \qquad \text{(Fourier Transform)}$$

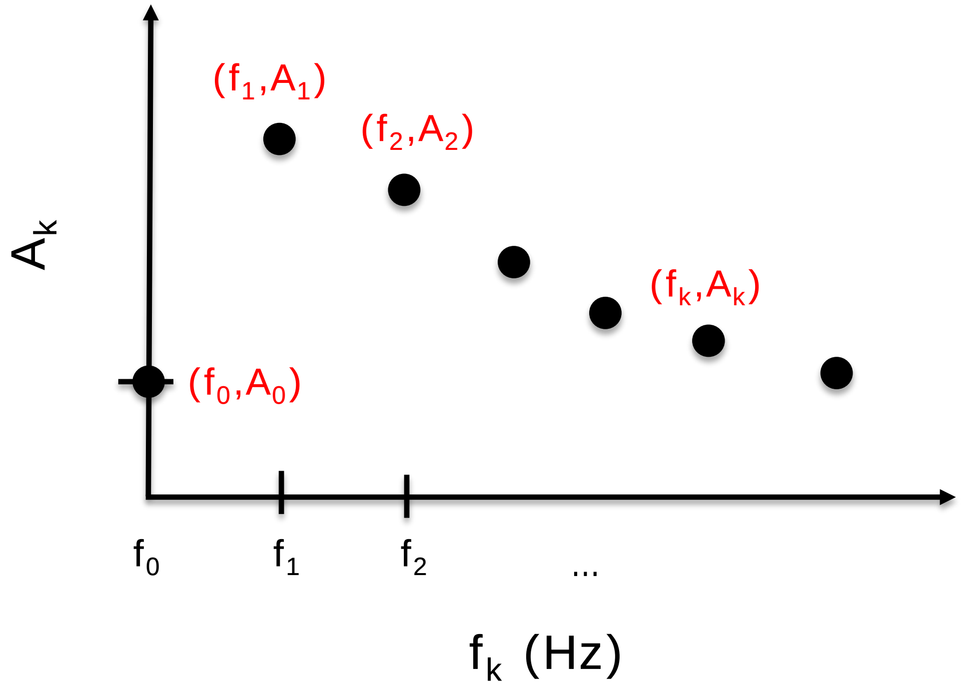
# Main Concepts

- Signal ✓
- Signal Processing ✓
- Spectrum
- Sampling
- Quantization
- Compression
- Spectrogram

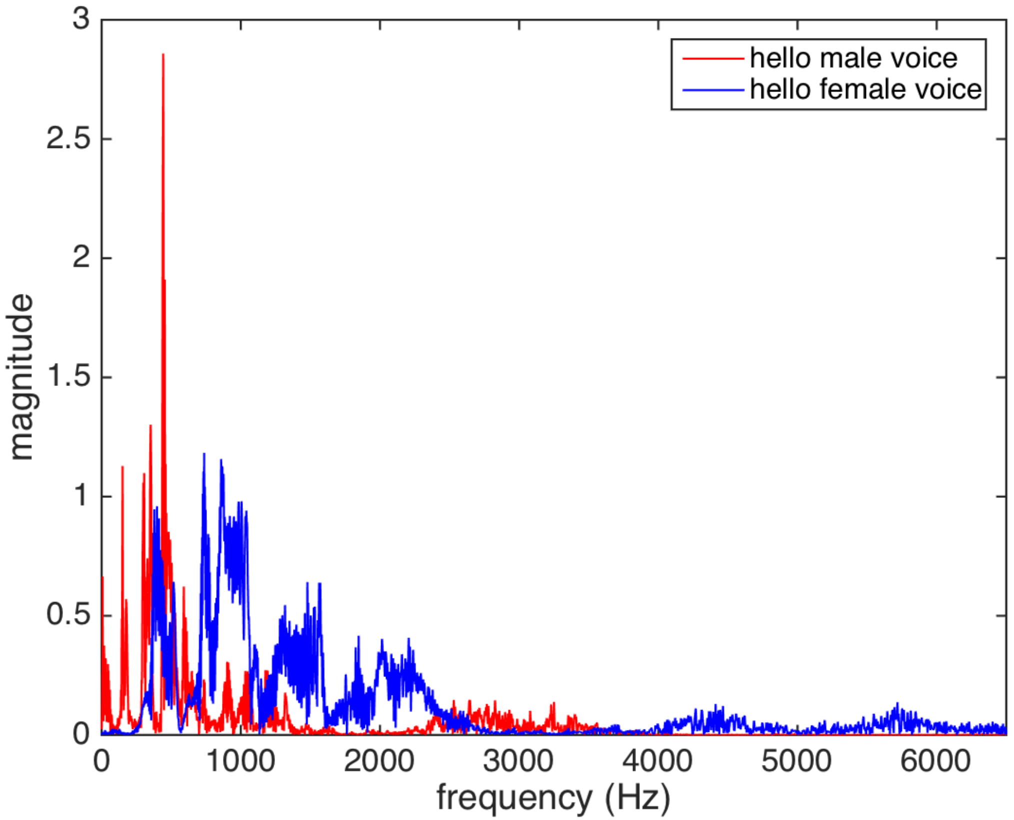
# (Signal) Spectrum

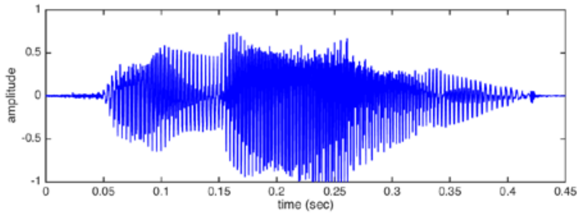
$$s(t) = A_0 \cos\left(2\pi f_0 t + \theta_0\right) + A_1 \cos\left(2\pi f_1 t + \theta_1\right) + A_2 \cos\left(2\pi f_2 t + \theta_2\right) + \ldots$$

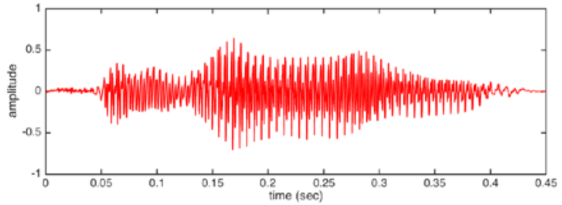
Signal Spectrum is the plot of $A_k$ vs. $f_k$



# Spectra for “Hello”



# Main Concepts

- Signal ✓
- Signal Processing ✓
- Spectrum ✓
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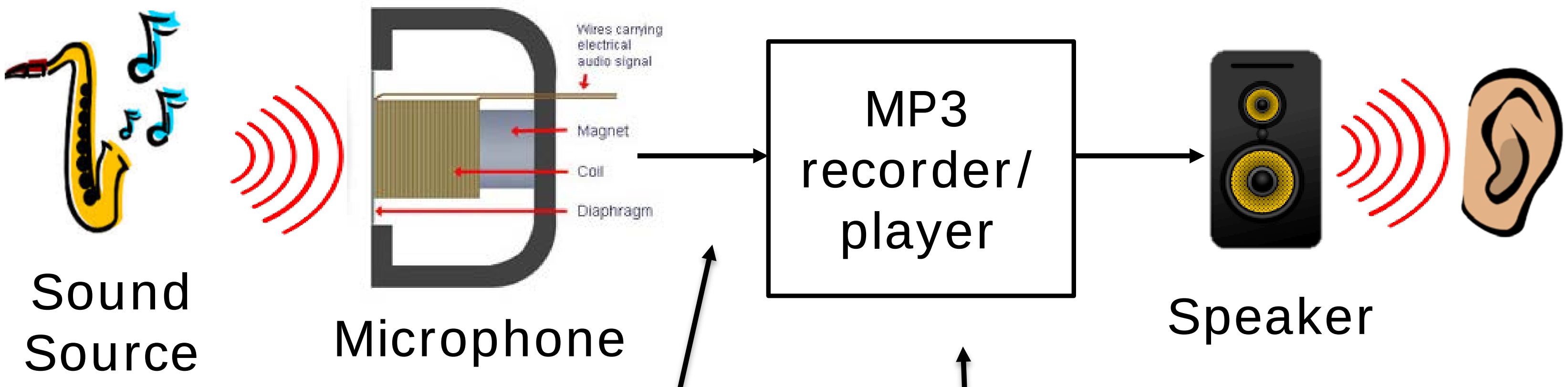
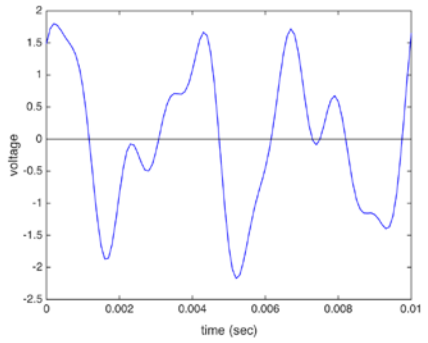
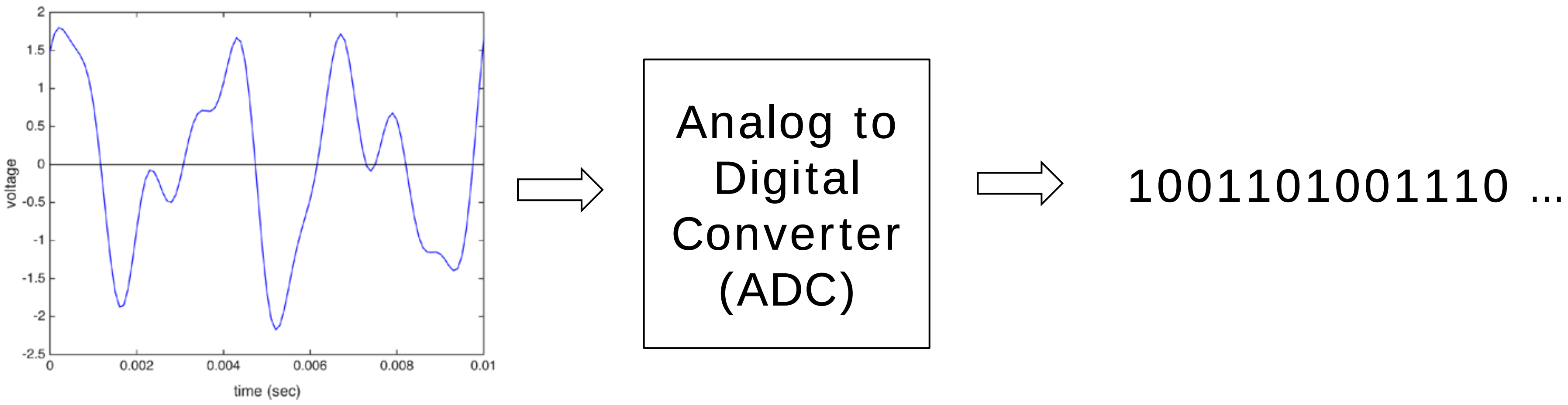
# Analog to Digital Conversion




Analog signal:




Digital System 1001101001110 ...




Analog signal

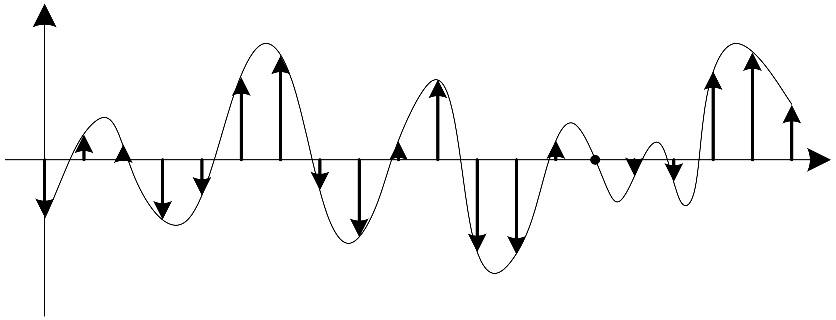
Digital data

# ADC

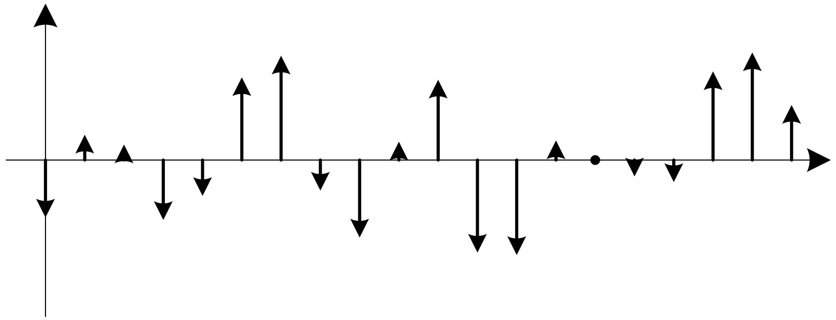
1. Measure the analog signal at discrete times (sampling)
2. Convert voltage level to binary (quantization)

# Sampling

- Measures the signal at equally spaced discrete times

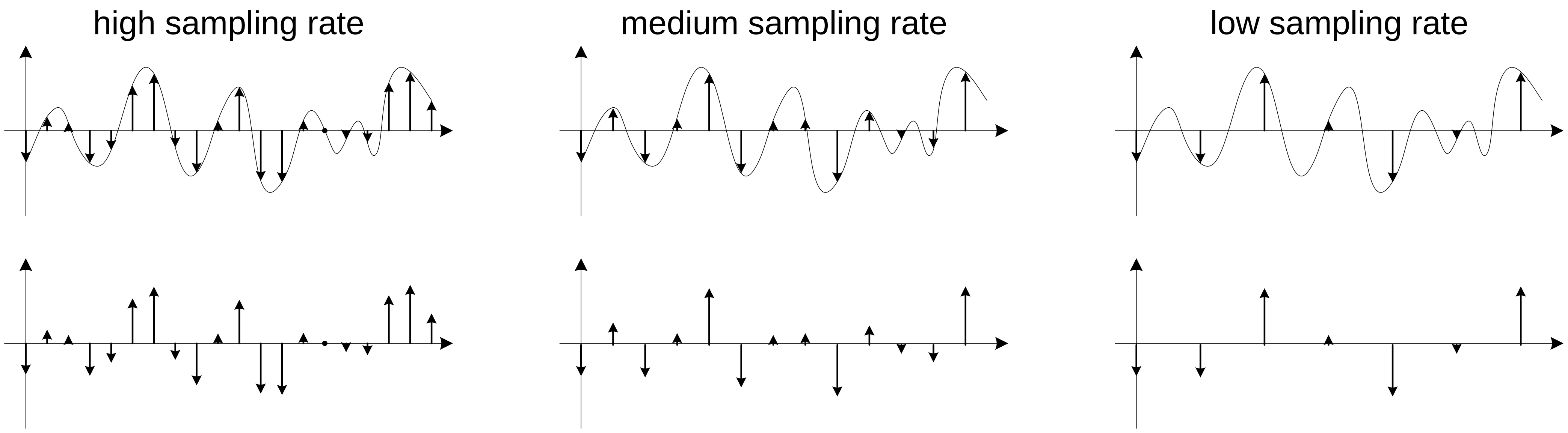


- Samples are representation of the signal



# Sampling Rate (# samples/sec, Hz)

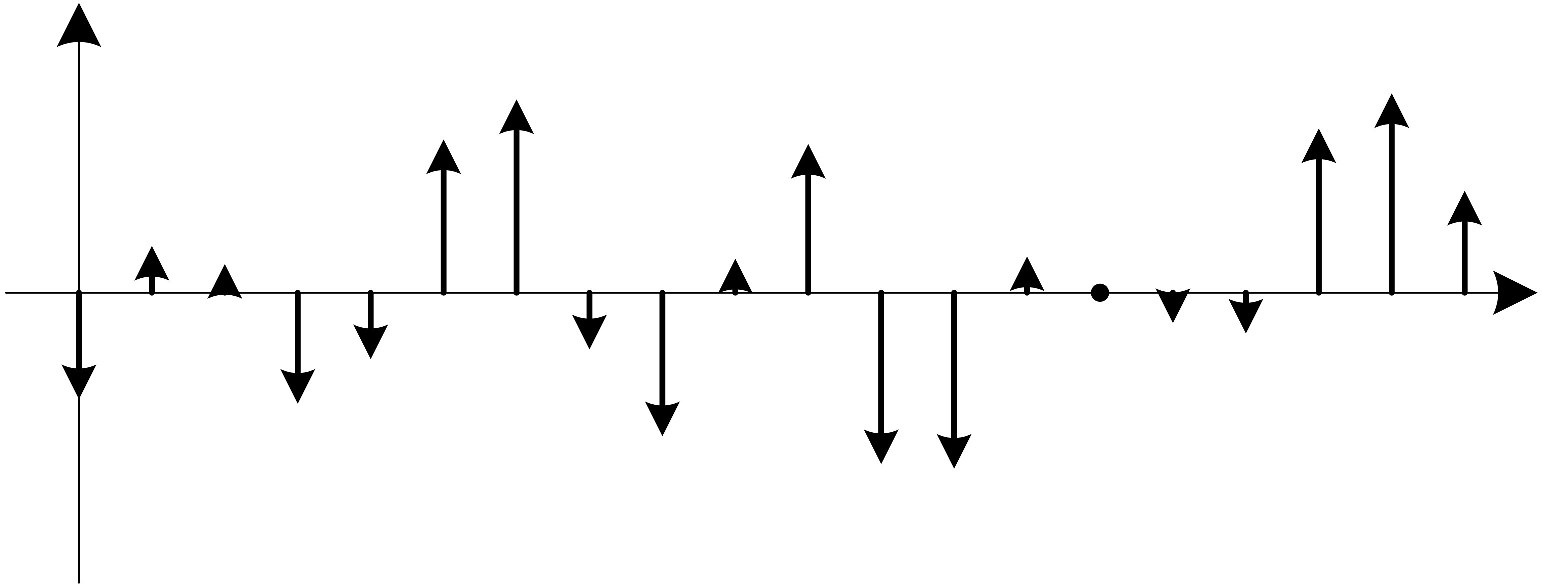
- Sampling rate determines quality of representation




- Low-rate sampling fails to capture high frequencies

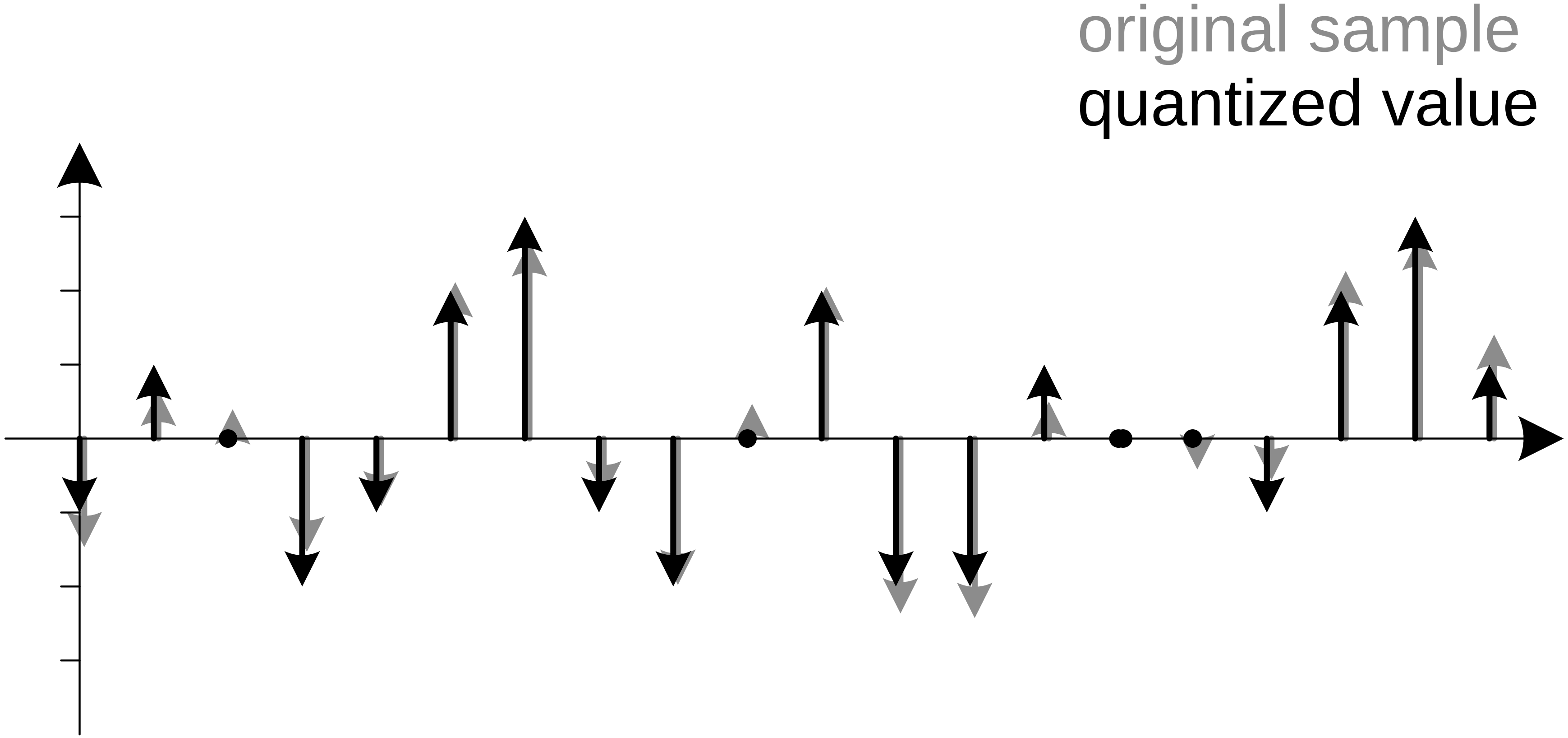
# Quantization

- Samples have continuous values
  - cannot represent digitally with arbitrary precision

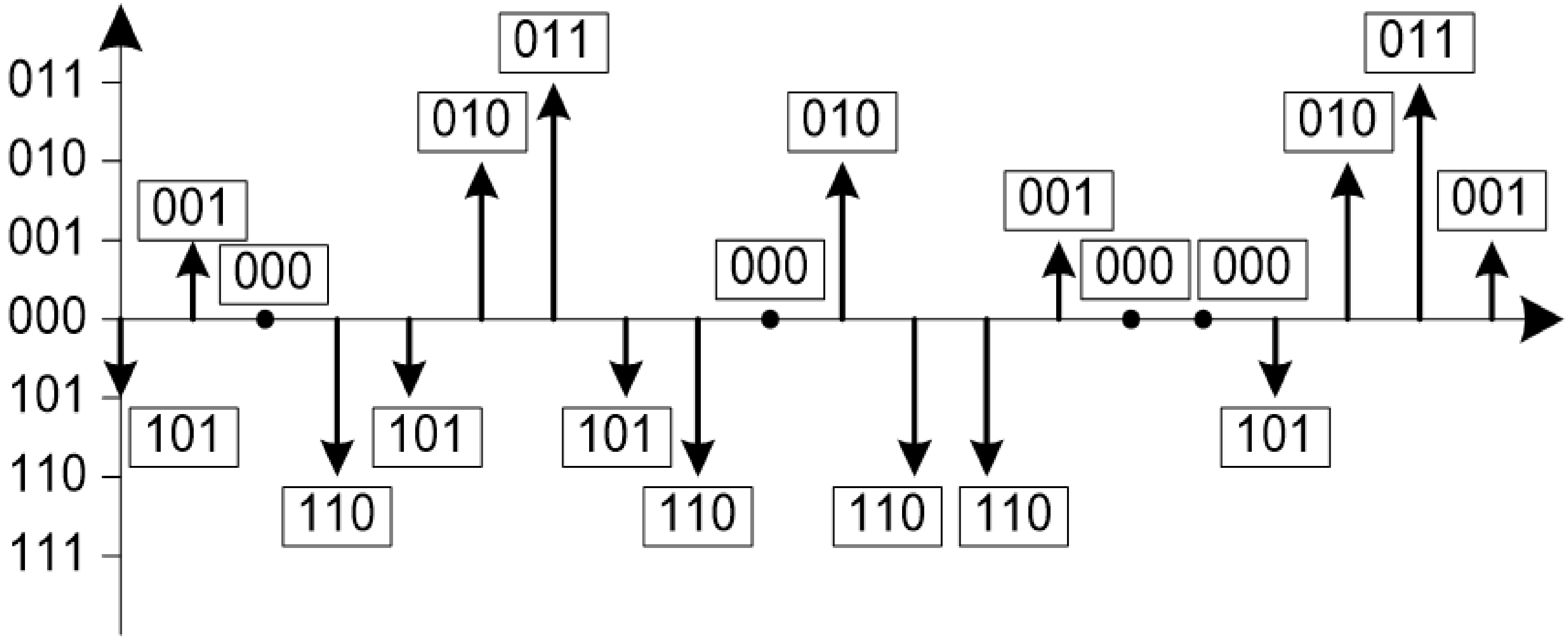


- "Quantization" assigns discrete values to each sample

- ADC (*n*-bit digital output) *n* bits have $2^n$ possible values

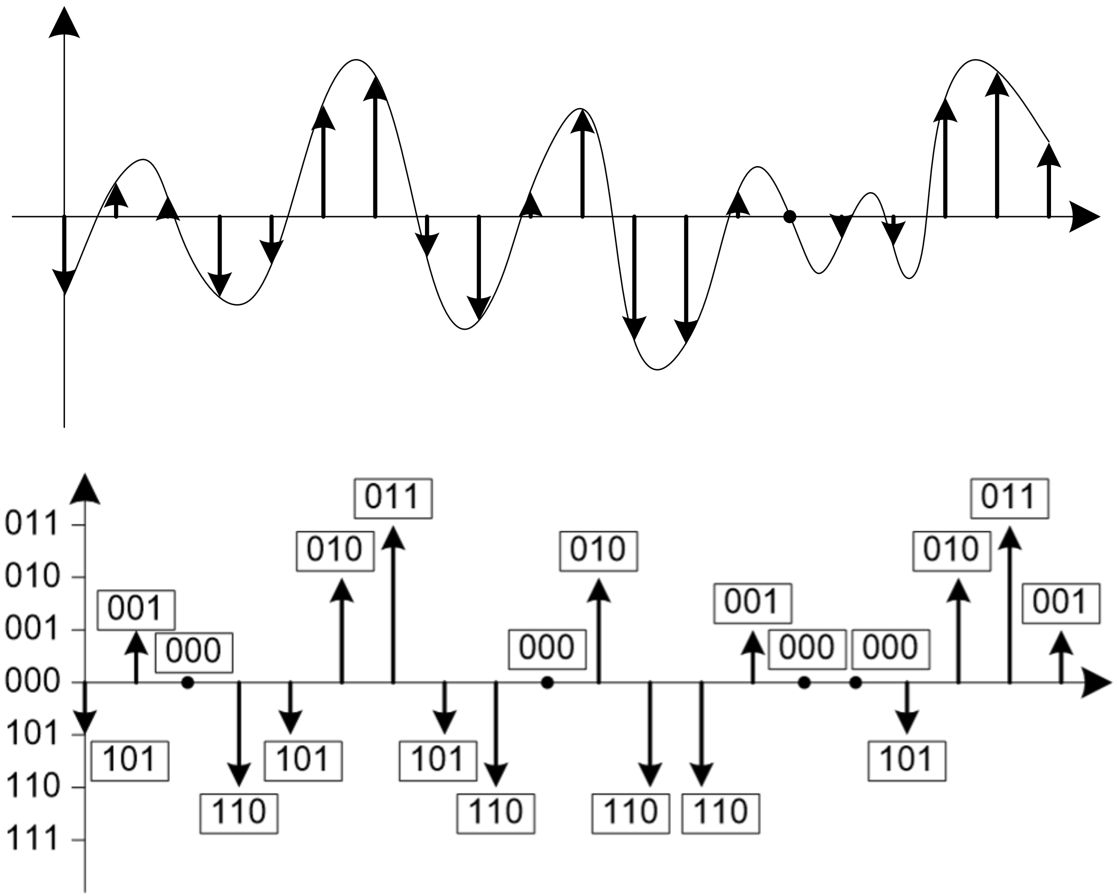


# Digital Representation




| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 101 | 001 | 000 | 110 | 101 | 010 | 011 | 101 | 110 | 000 | 010 | 110 | 110 | 001 | 000 | 000 | 101 | 010 | 011 | 001 |

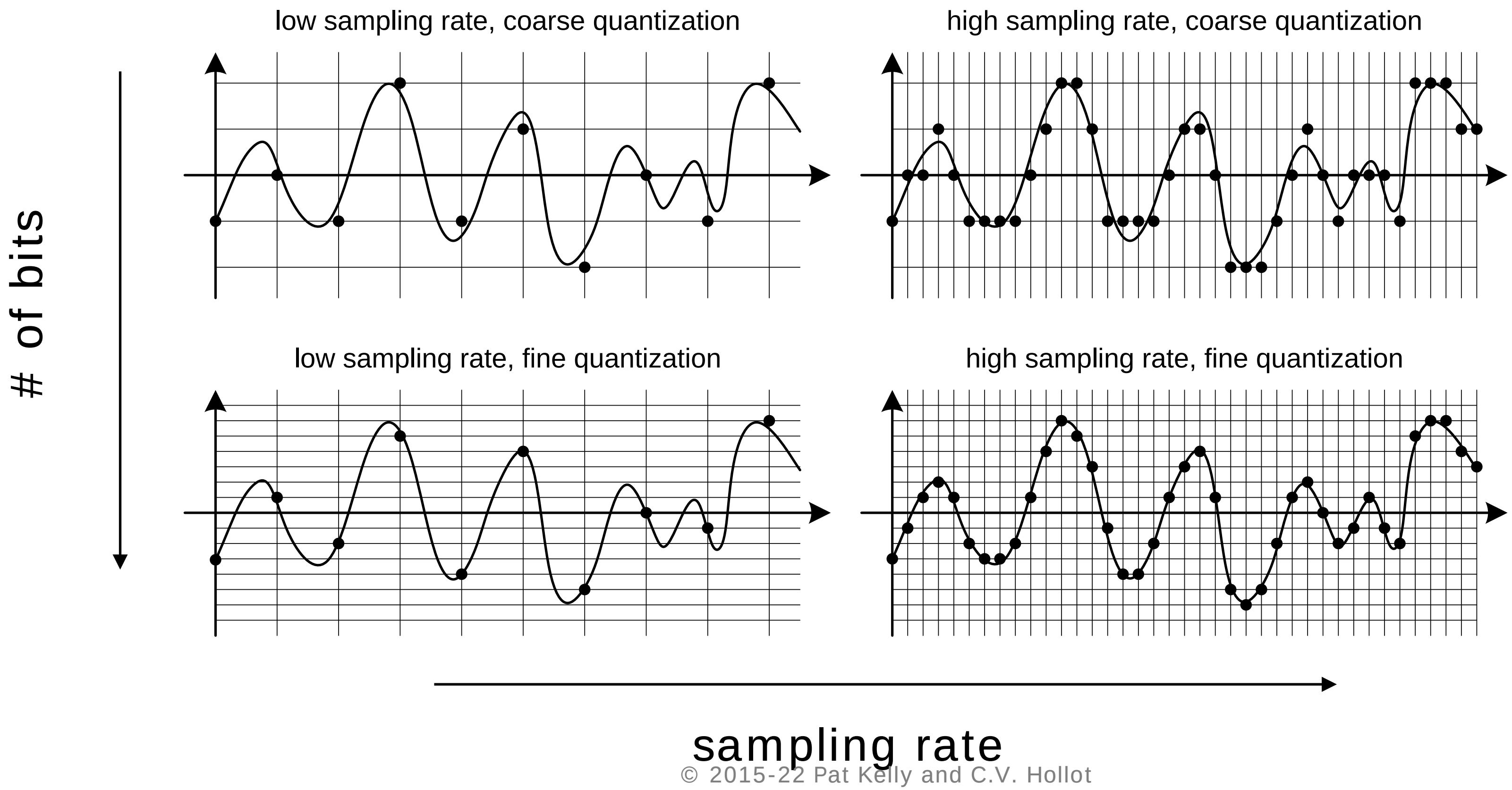
# Analog-Digital Convertor (ADC)




| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 101 | 001 | 000 | 110 | 101 | 010 | 011 | 101 | 110 | 000 | 010 | 110 | 110 | 001 | 000 | 000 | 101 | 010 | 011 | 001 |

# Sampling/Quantization are Lossy

- Lose information in analog signal if:
  - Sampling rate is too low
  - Quantize with too few bits



# Rules of Thumb in Digitization

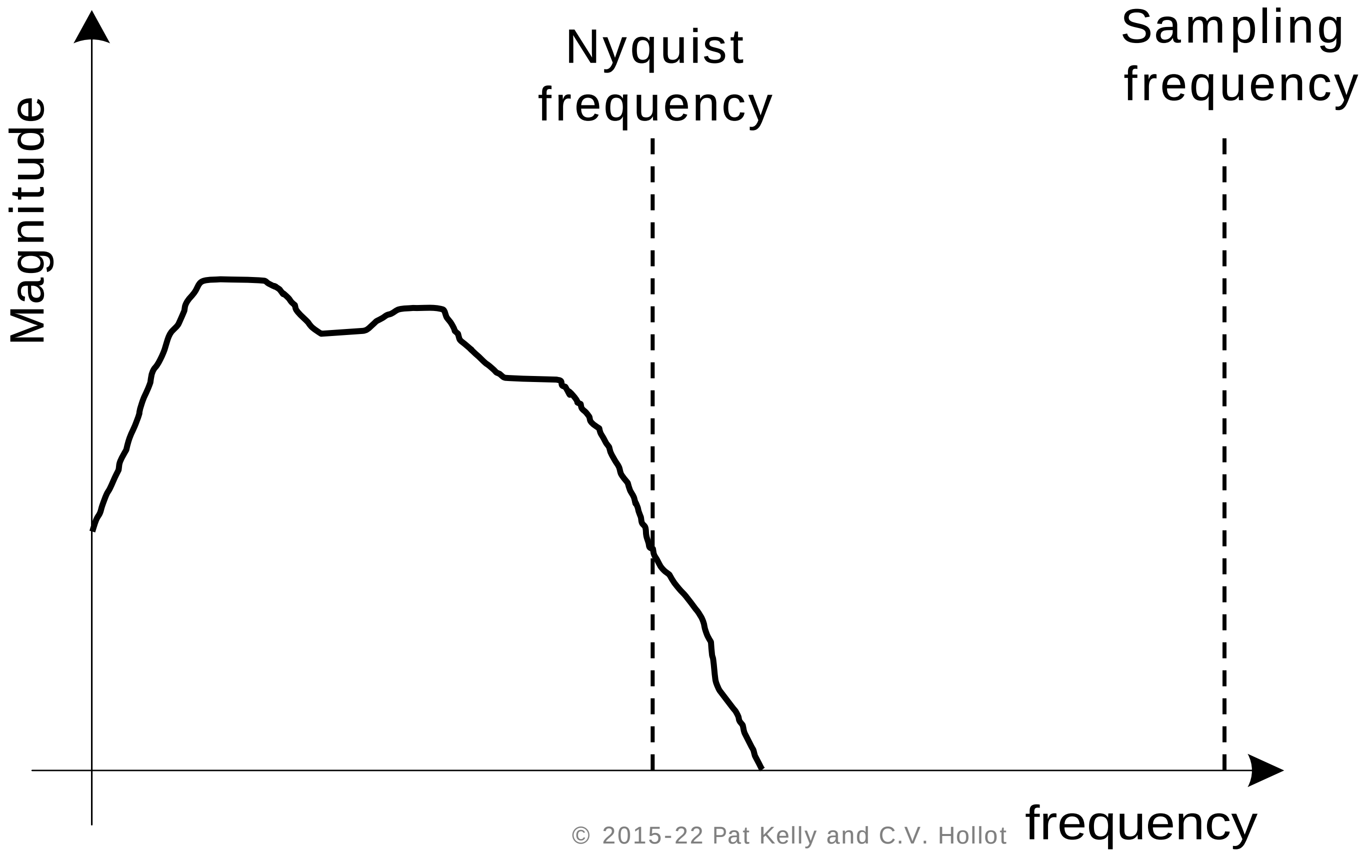
- Sampling (sample fast enough)
  - Sampling rate should be at least twice the “highest frequency” in the signal spectrum

- Quantization (need enough bits)
  - Every increase of 1 bit quadruples the quality of the quantized signal.

# Nyquist Sampling Theorem

**“Sampling rate must be at twice the highest frequency (Nyquist frequency) in the signal”**

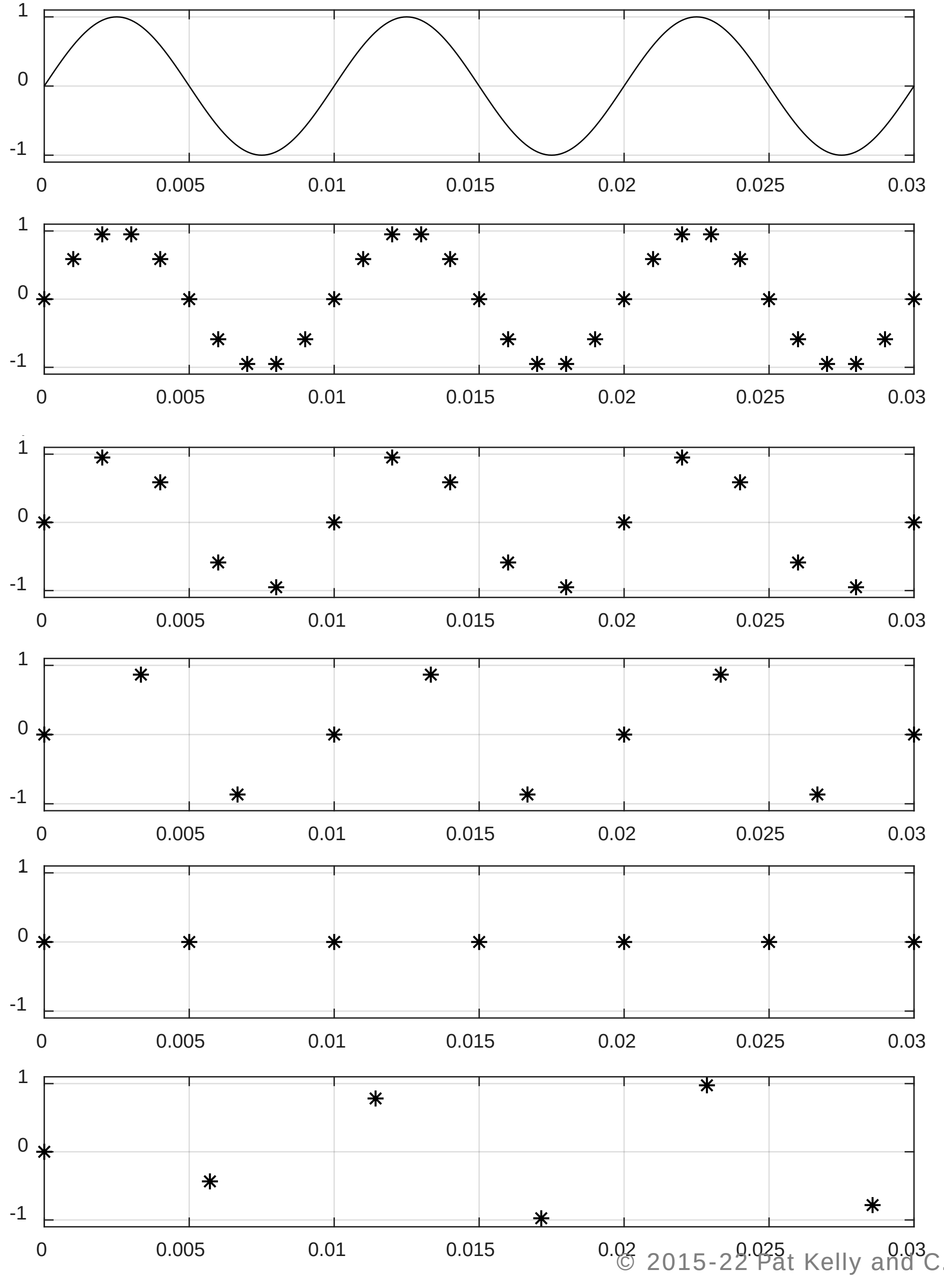
Signal Spectrum



| | Highest Freq | Nyquist Freq | Lowest sampling rate |
|---|---|---|---|
| Voice (Hello) | ~ 6KHz | ~ 6KHz | ~ 12KHz |
| Music | ~ 20KHz | ~ 20KHz | ~ 40KHz |

- Sampling rate in MP3 is 44.1KHz
- Why does sampling theory hold?
  - Fourier: enough to consider sampling sinusoids

# Sample a Sinusoid




f = 100 Hz, T = 0.01sec

Nyquist frequency = 100Hz

Sampling rate = 1000Hz

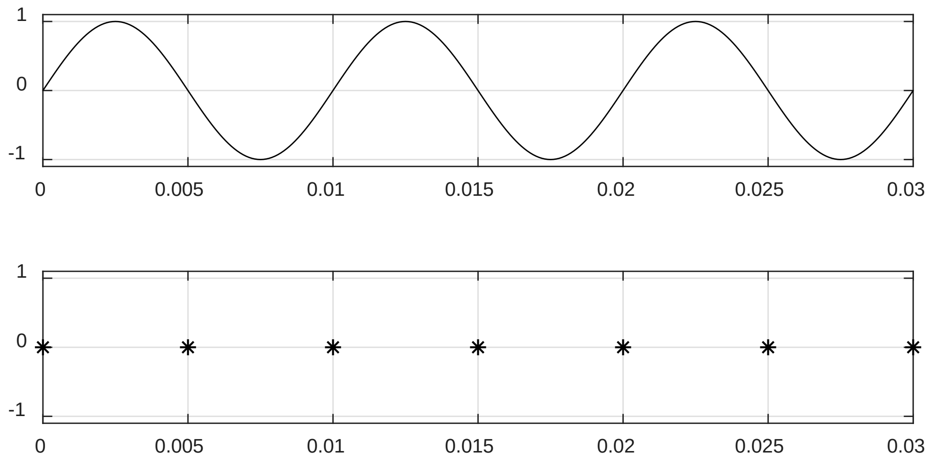
Sampling rate = 500Hz

Sampling rate = 250Hz

Sampling rate = 200Hz

Sampling rate = 175Hz

Aliasing, Aliases

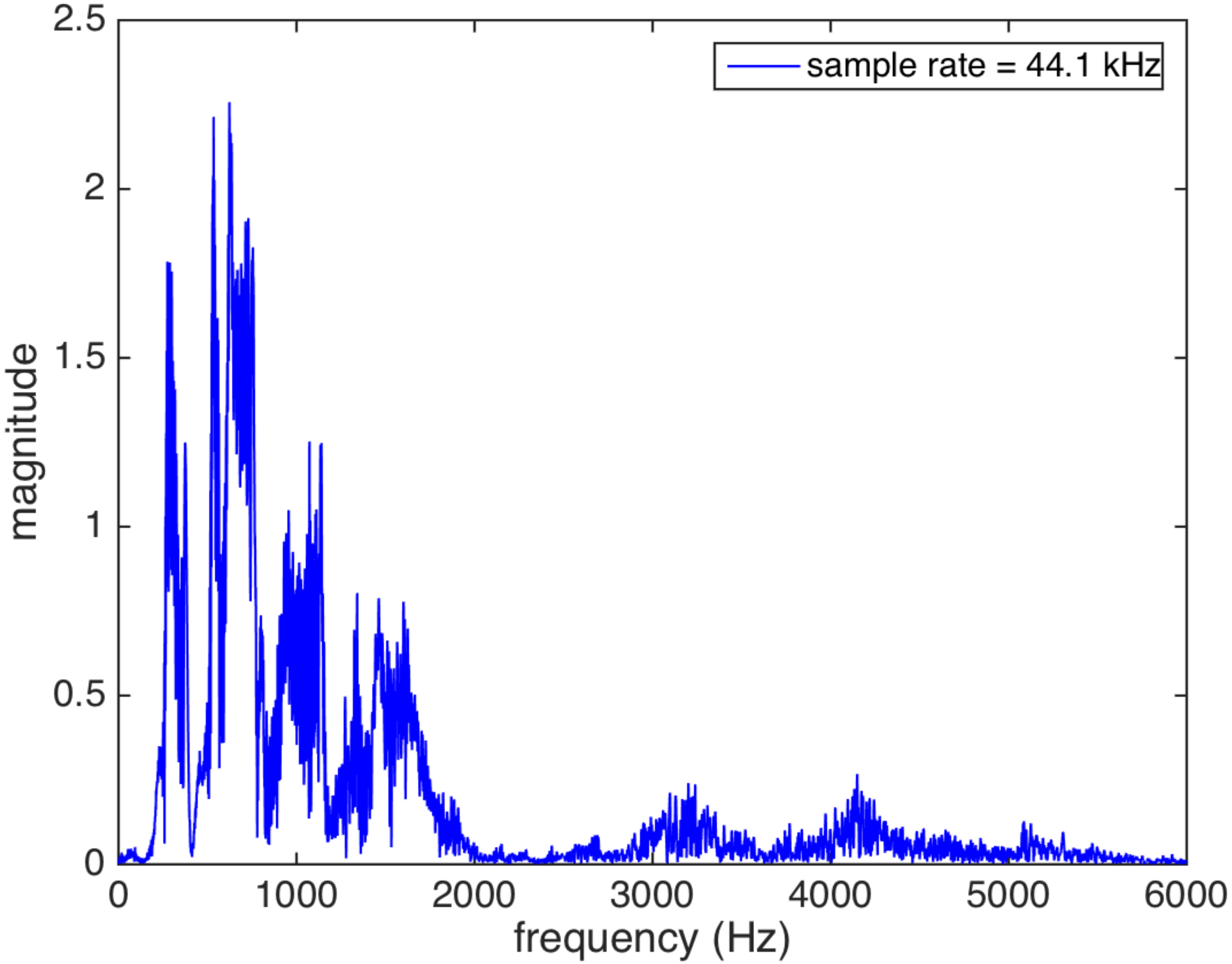



f = 100 Hz, T = 0.01sec

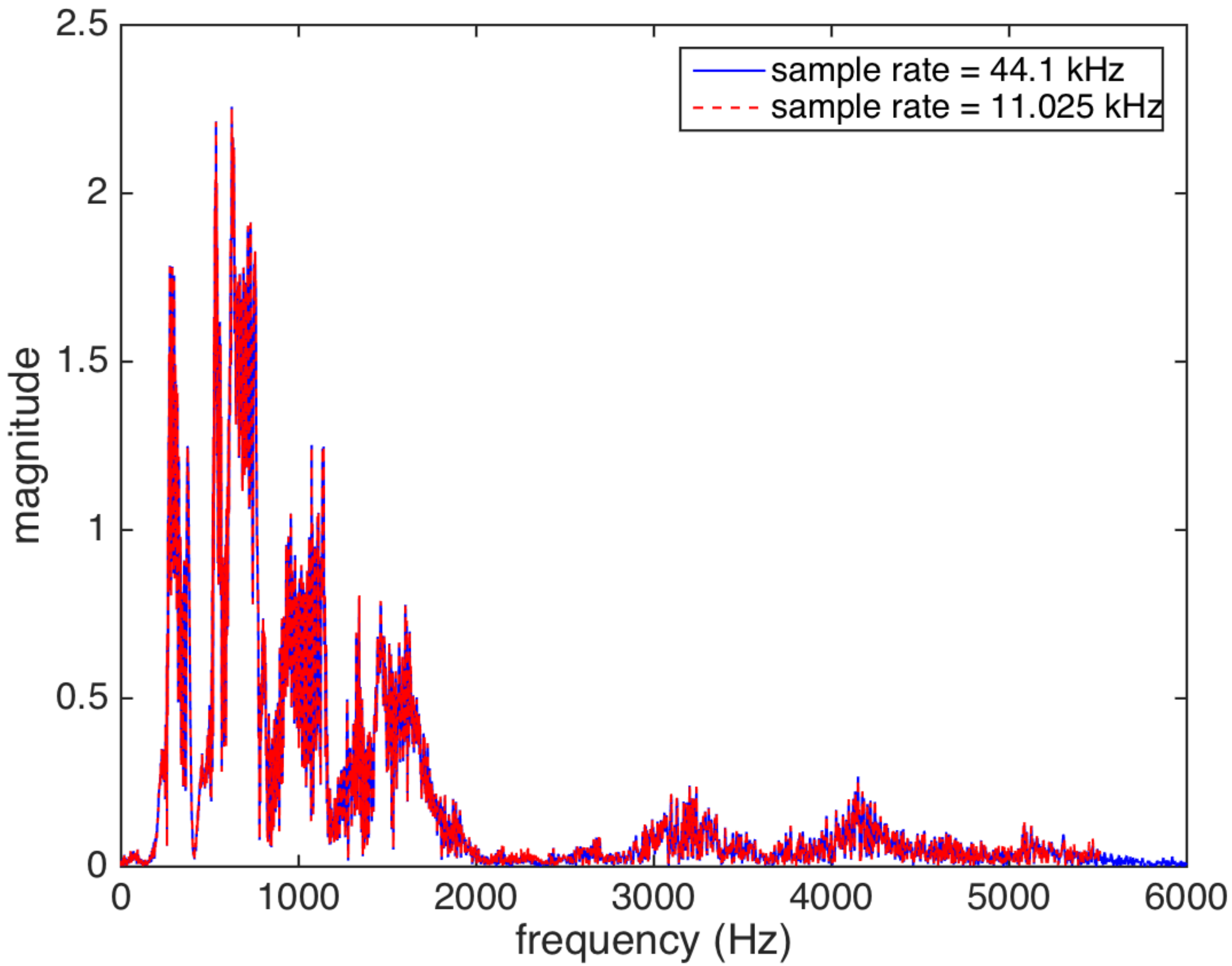
Nyquist frequency = 100Hz

Sampling rate = 200Hz

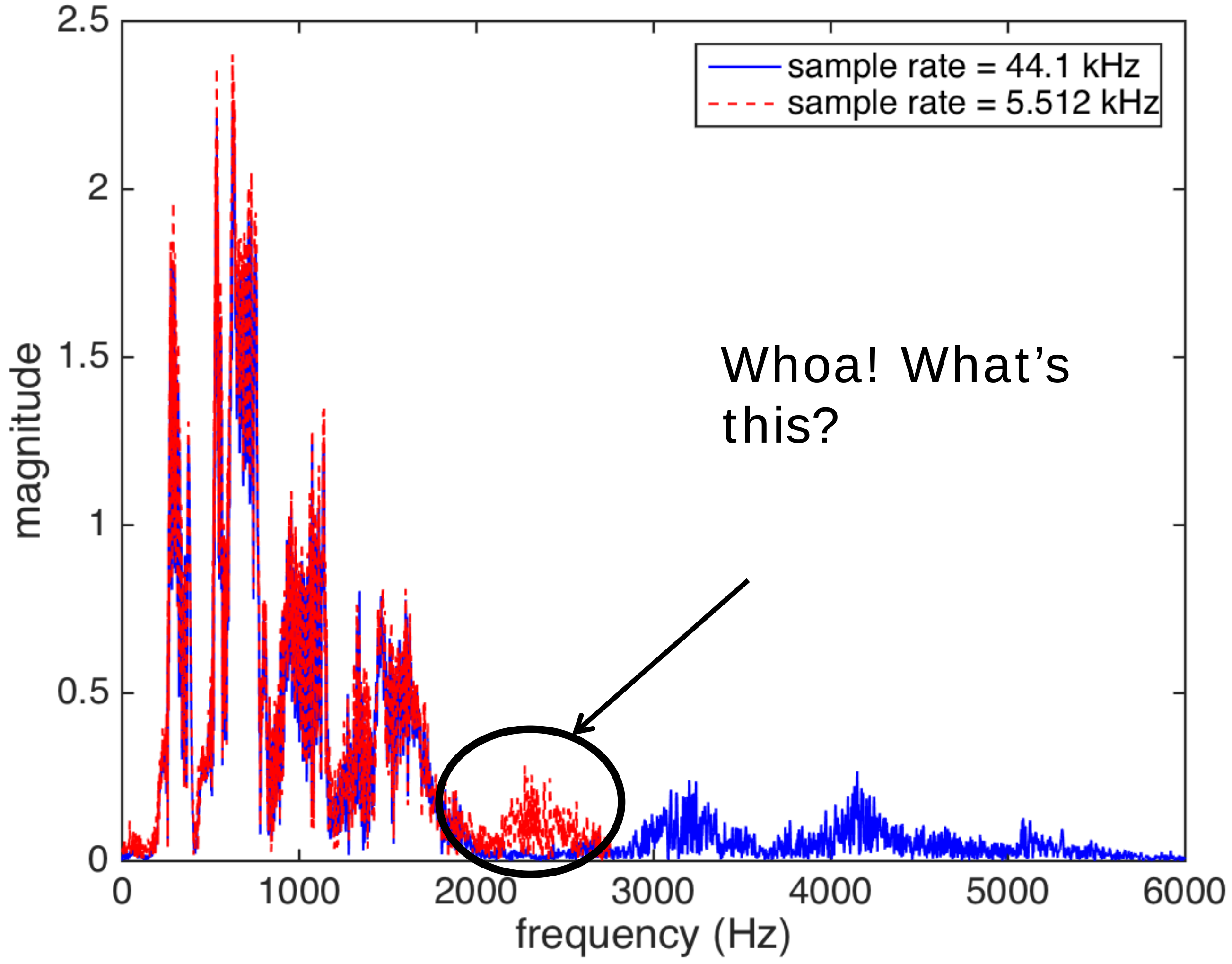
# Example: Spoken word "Hello"

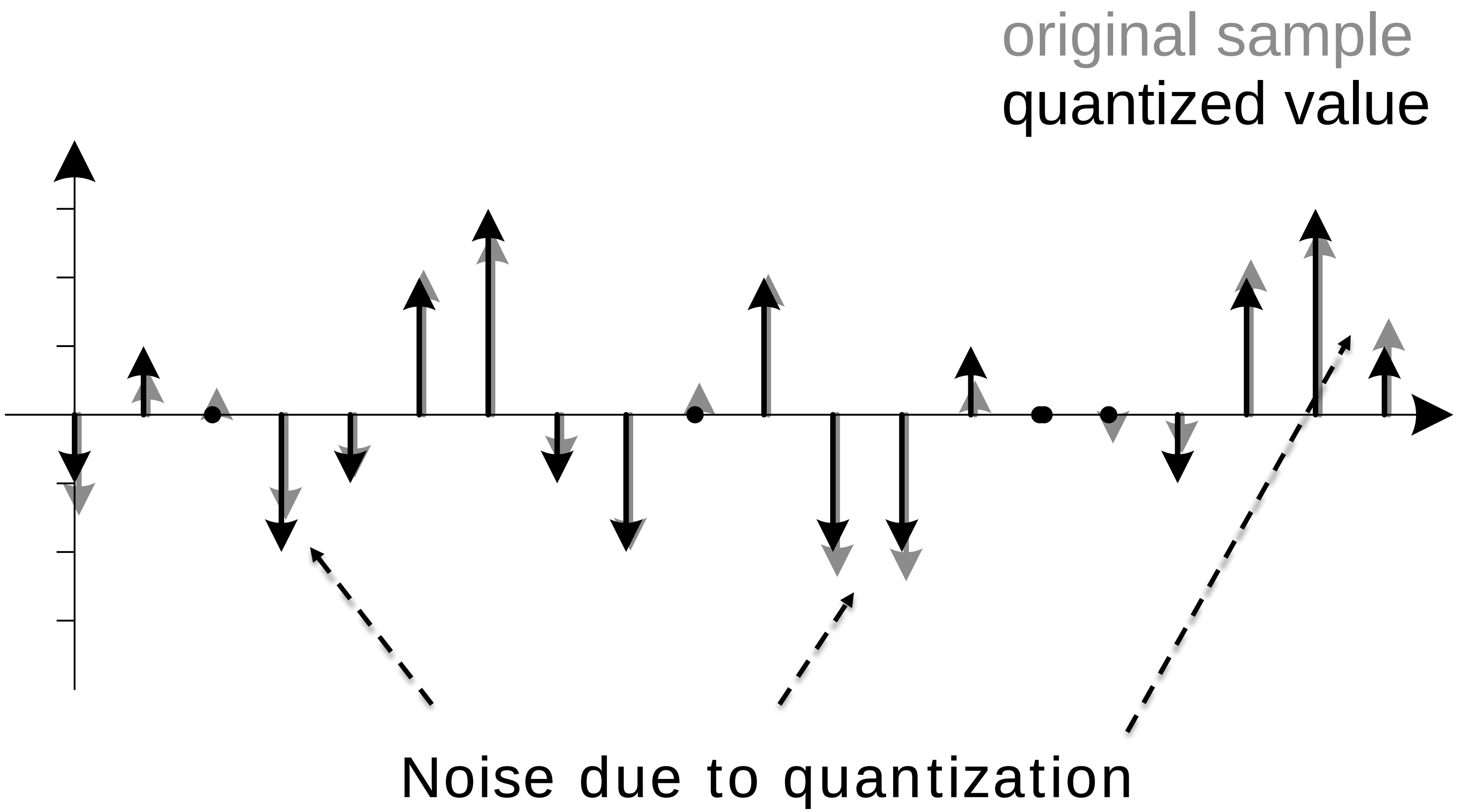

The highest significant frequency is about 6 kHz



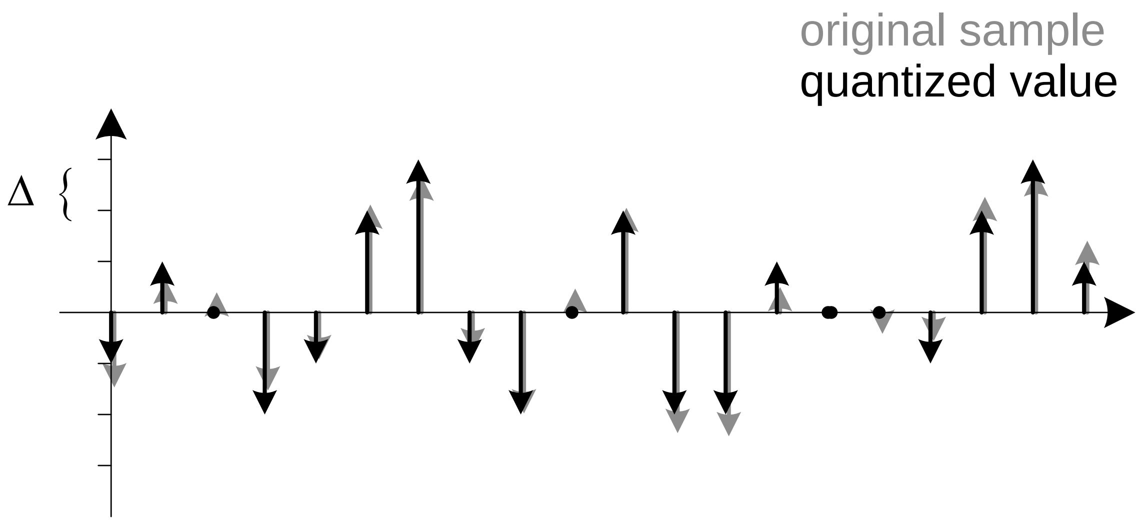
sample rate = 44.1 kHz
sample rate = 11.025 kHz
magnitude
frequency (Hz)
0
0.5
1
1.5
2
2.5
0
1000
2000
3000
4000
5000
6000



sample rate = 44.1 kHz
sample rate = 5.512 kHz
magnitude
frequency (Hz)
0
0.5
1
1.5
2
2.5
0
1000
2000
3000
4000
5000
6000
Whoa! What's this?

# Quantifying Quantization



# Power in Quantization Noise - 1




$$\Delta = \text{step level} = \frac{\text{voltage range of analog signal}}{2^n}; \quad n = \#\text{ bits}$$

$$\text{power in the quantization noise} = \frac{\Delta^2}{12}$$

$$= \frac{(\text{voltage range of analog signal})^2}{12 \cdot 2^{2n}}$$

$$= \frac{(\text{voltage range of analog signal})^2}{12 \cdot 4^n}$$

# Power in Quantization Noise - 2

$$\text{power in the quantization noise} = \frac{(\text{voltage range of analog signal})^2}{12 \cdot 4^n}$$

Increasing the number of bits from *n* to *n+1* decreases the quantization noise power by a factor or 4

# Signal to Quantization Noise Ratio (SQNR)

$$\text{SQNR} = \frac{\text{analog signal power}}{\text{quantized noise power}} = 4^n$$

$$\begin{aligned}\text{SQNR}_{db} &= 10\log_{10}(\text{SQNR}) \\ &= 10\log_{10}(4^n) \\ &= n \cdot 10\log_{10}(4) \\ &= 6n\end{aligned}$$

Increasing the number of bits from $n$ to $n+1$ increases the SQNR by a factor of 4 or $\text{SQNR}_{\text{db}}$ by a factor of 6 (6db rule).

# Main Concepts

- Signal ✓
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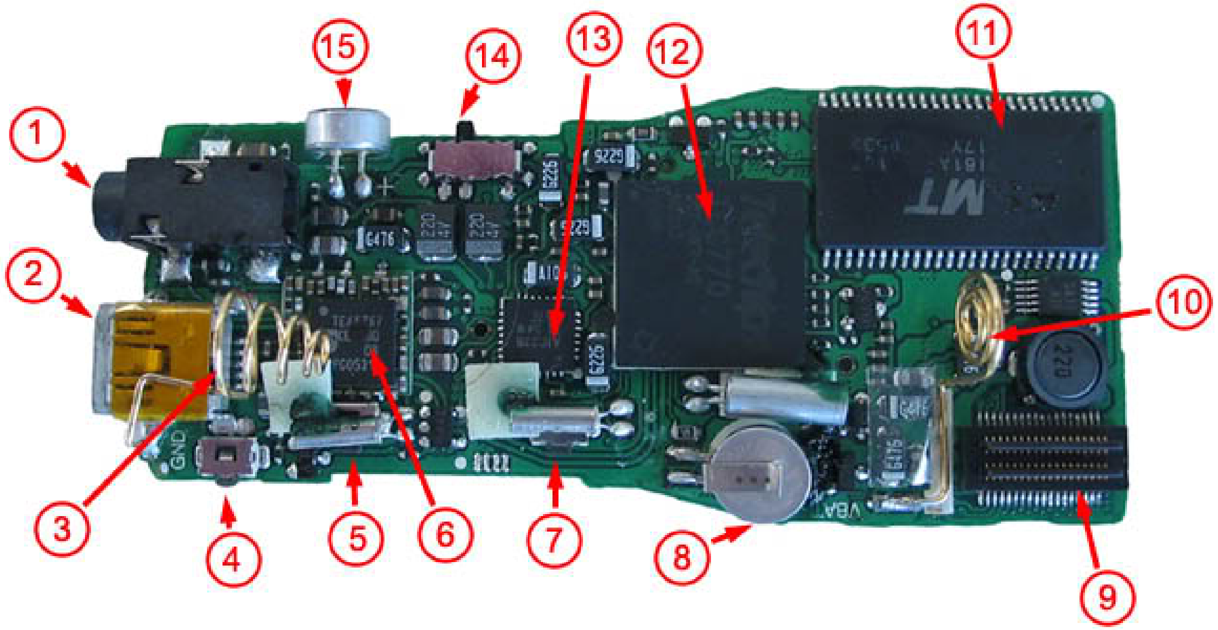
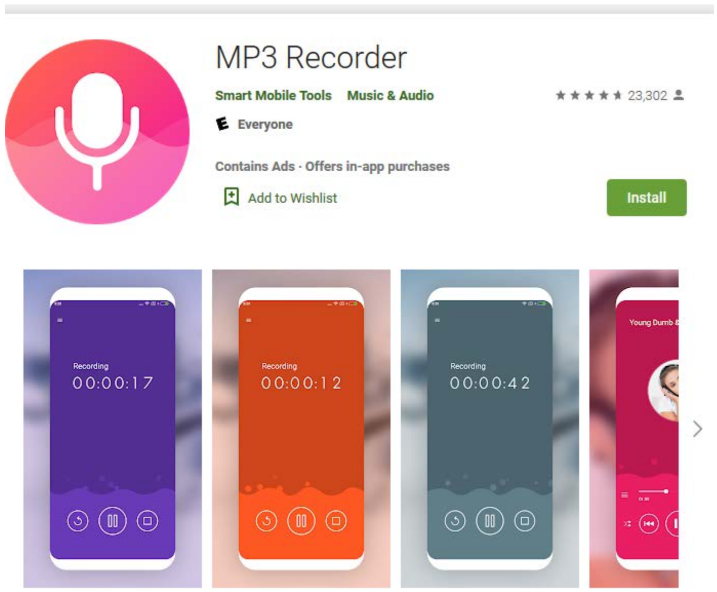
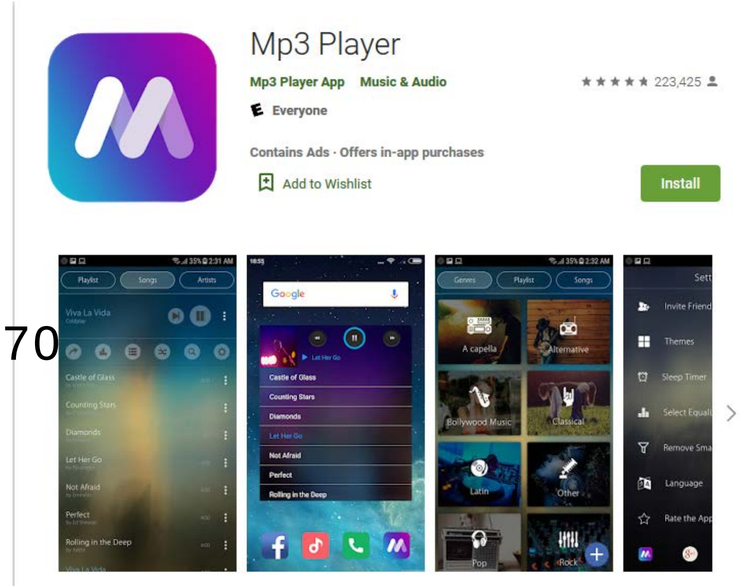
- Spectrum ✓
- Sampling ✓
- Quantization ✓
- Compression

# MP3 Recorder/Player

SanDisk Sansa m200 Series

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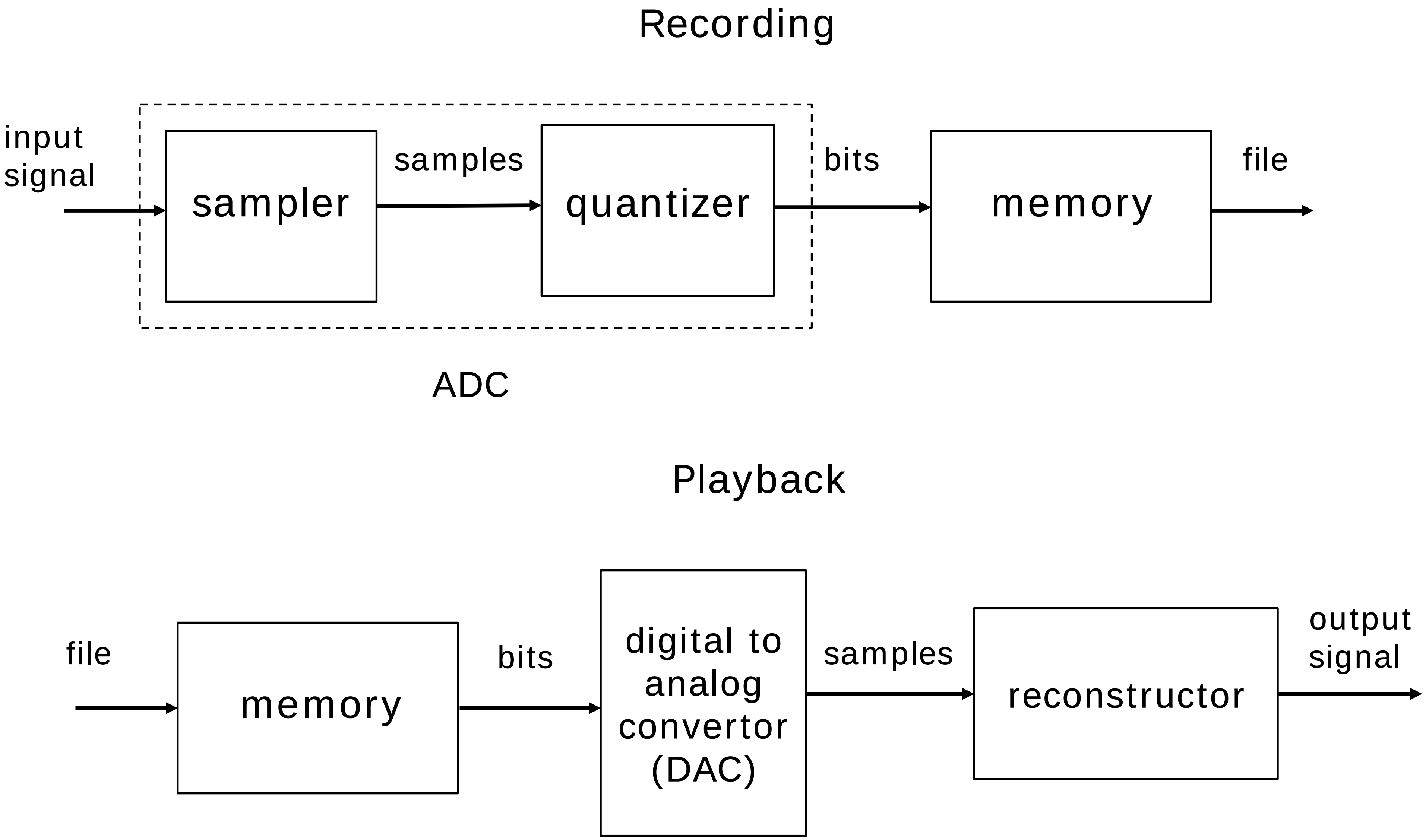
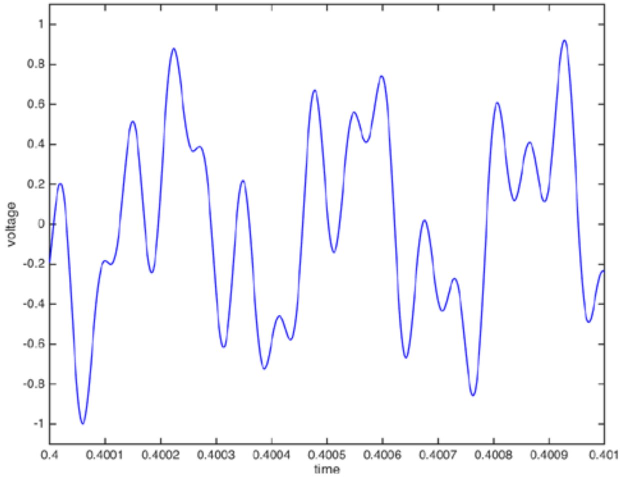
# Digital Recording/Playback



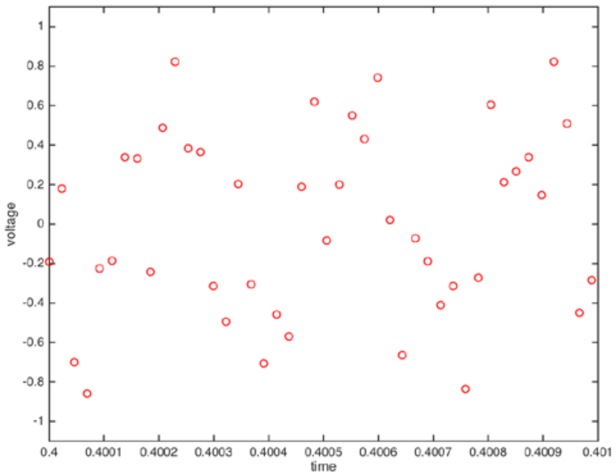
# Generating MP3 Recording - 1




Electrical
(voltage)
signal

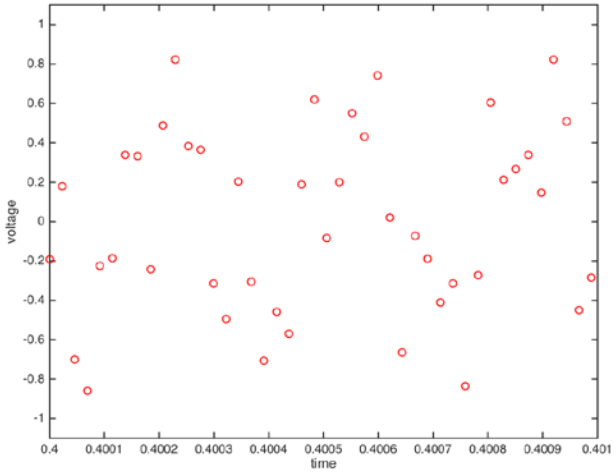
Sampler

44.1 kHz
sampling
rate




Voltage
Samples

# Generating MP3 Recording - 2




Voltage
Samples

Quantizer

16
bits/sample

0111000011110010
1000111101001001
0111100111100010
...

Digitized
Samples

# MP3's Secret Sauce (compression coding)

# MP3 Recording - Coding

$\Rightarrow$

from
ADC

0111000011110010
1000111101001001
0111100111100010
...

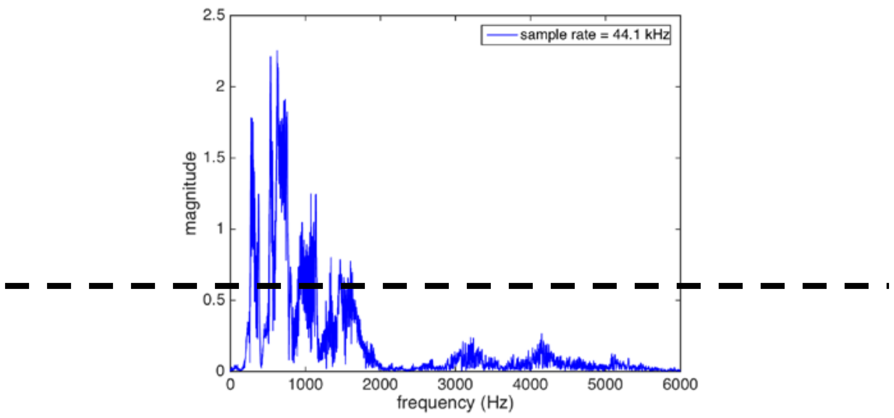
$\Rightarrow$ DTFT $\Rightarrow$

$$\begin{cases} (A_0, f_0, \theta_0) \\ (A_1, f_1, \theta_1) \\ (A_2, f_2, \theta_2) \end{cases}$$

$\vdots$

Discrete Time
Fourier Transform
(DTFT) or (FFT)

# MP3 Recording - Compress




$$\left\{\begin{array}{l}(A_0, f_0, \theta_0)\\(A_1, f_1, \theta_1)\\(A_2, f_2, \theta_2)\\\vdots\end{array}\right. \Rightarrow \boxed{\text{Compressor}} \Rightarrow \left\{\begin{array}{l}(A_k, f_k, \theta_k)\\\vdots\end{array}\right.$$

delete frequency components (memory) having $A_k$'s below threshold

# MP3 Playback - Decoding

$$\left\{ \left( A_k, f_k, \theta_k \right) \\ \vdots \right. \quad \Rightarrow \quad \boxed{\text{Inverse FFT}} \quad \Rightarrow$$

0111000011110110
1000111101001011
0111100111100000
...

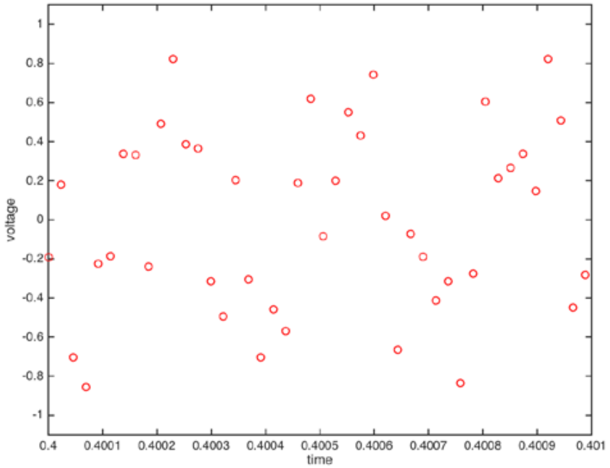
Stored coefficients for subset of frequency components kept after compression

Digitized output samples (with errors due to compression)

# MP3 Playback – Convert to Voltage Samples

0111000011110110
1000111101001011
0111100111100000
...

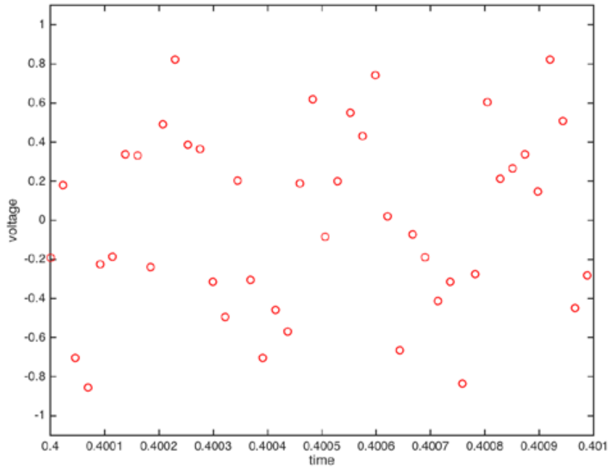
DAC



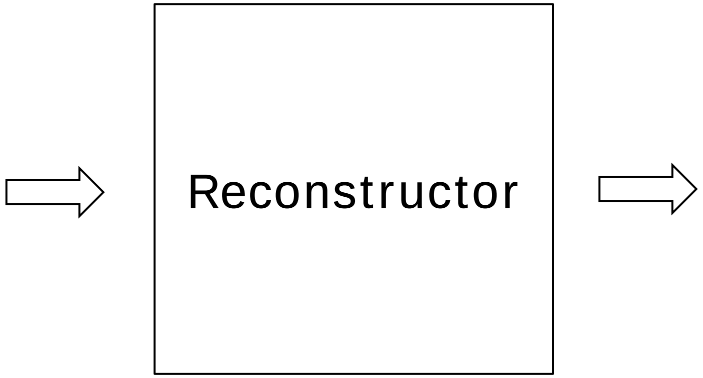

Digitized output samples

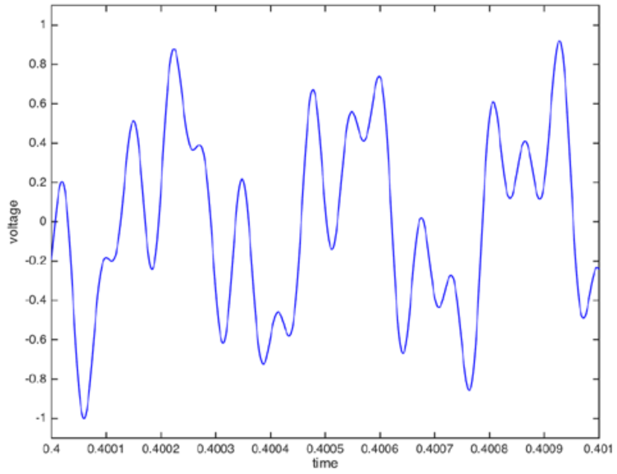
Output voltage samples

# MP3 Playback – Reconstruction



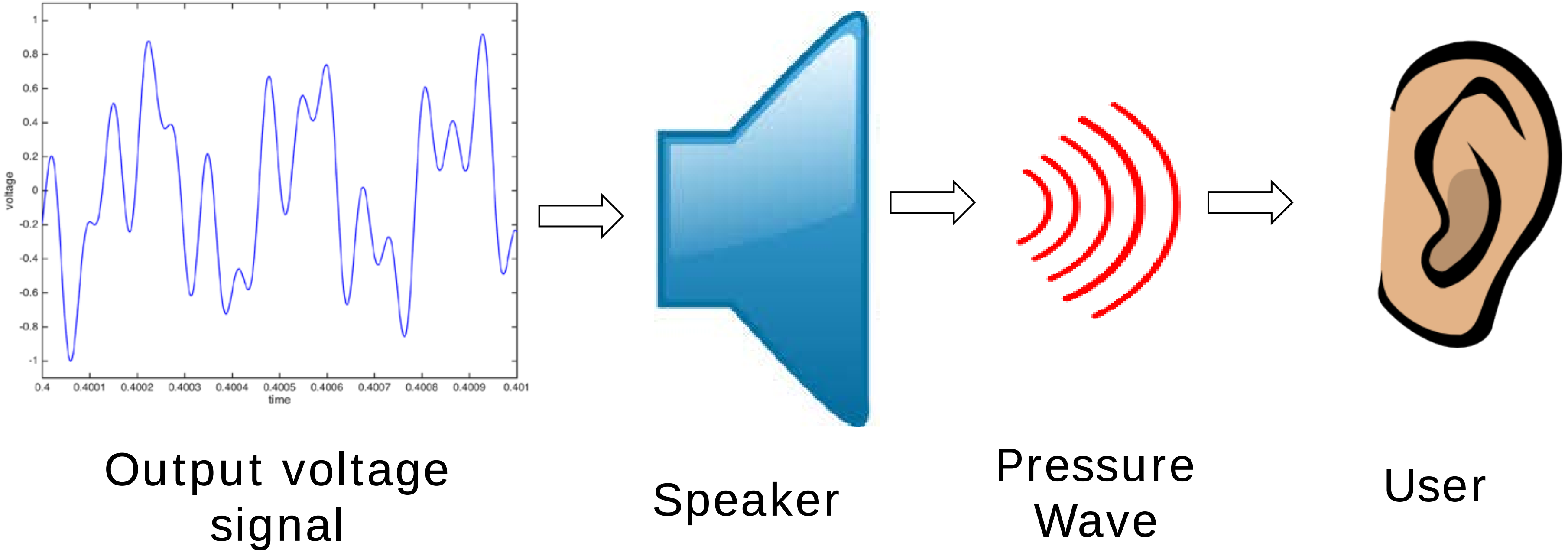

Output voltage
samples



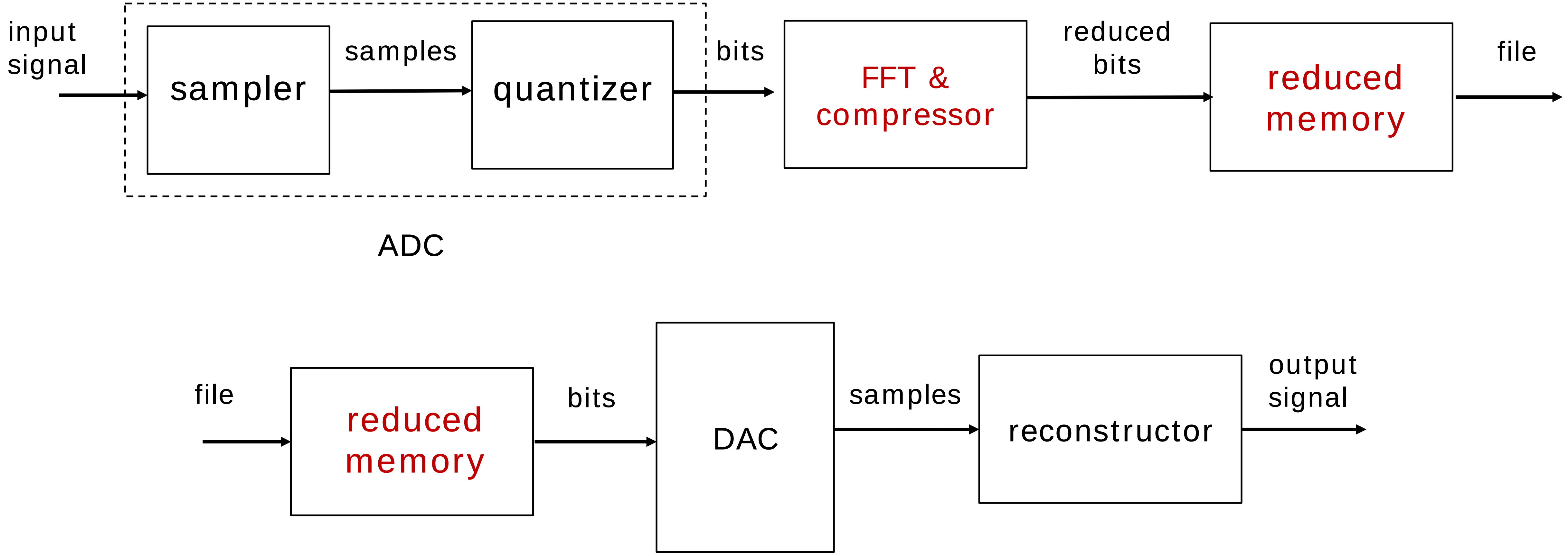




Output voltage
signal

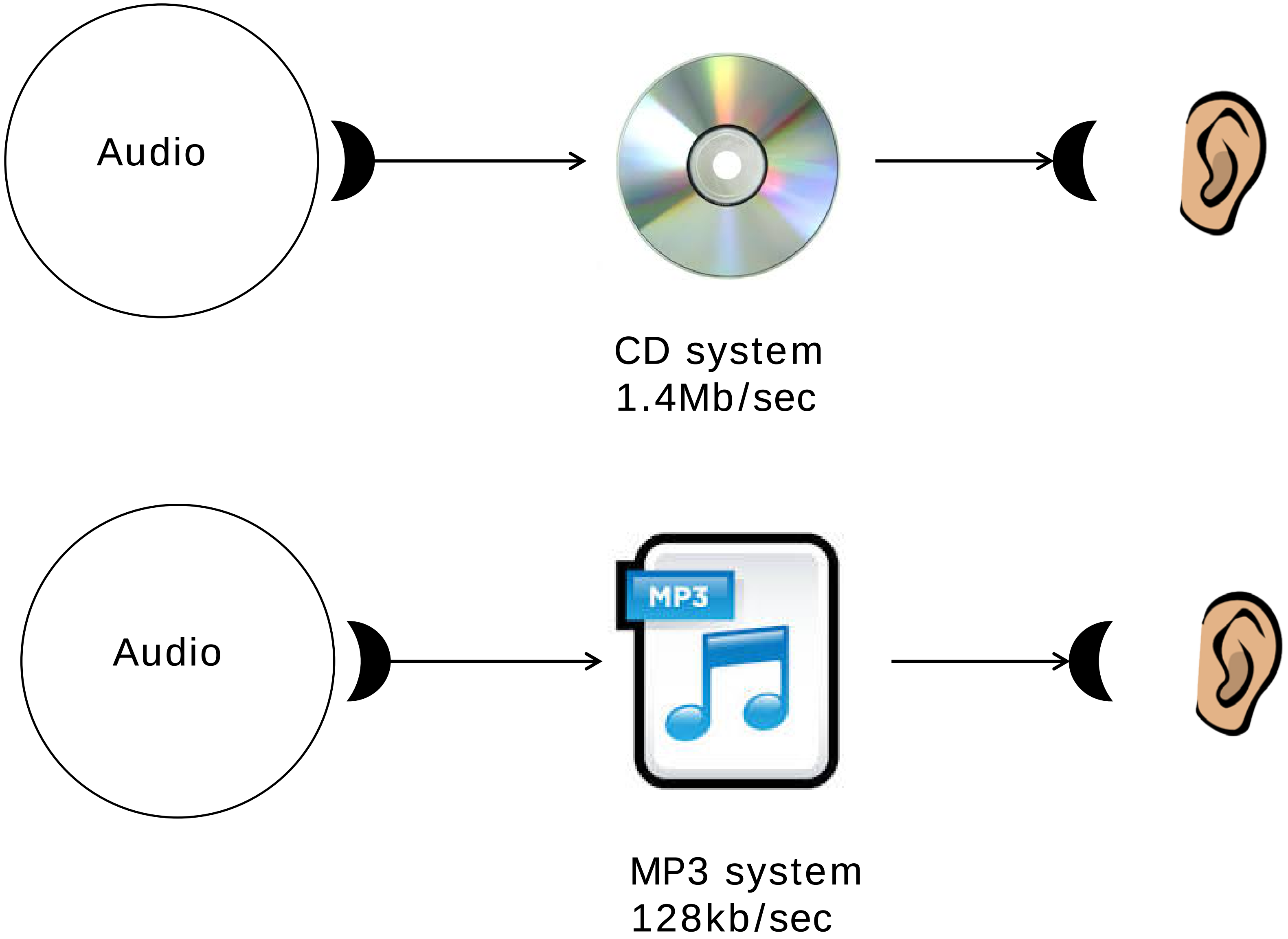
# MP3 Playing Back



# MP3 Payoff


input
signal
sampler
samples
quantizer
bits
ADC
FFT &
compressor
reduced
bits
reduced
memory
file
file
reduced
memory
bits
DAC
samples
reconstructor
output
signal

# 2. process input signals to put them in more useful forms



# Memory: Compact Disk (CD) recording -1

- music (20 KHz); sampling rate > 40 kHz.
- sampling rate = 44.1 kHz.
- quantization (16 bits/sample)
  - $(SQNR)_{dB}$ = 16 x 6 = 96 dB; SQNR = 4.3 x $10^9$
- bit rate for single channel cd recording
  - 44100 samples/sec x 16 bits/sample = 705,6000 bits/sec
  - stereo, 1.411 Mbits/sec

# Memory: Compact Disk (CD) recording -2

- song length ~ 227 sec
- bit rate = 1.411 Mbits/sec
- 227 sec x 1.411 Mb/sec = 320 Mb
- one song requires = 40 MB of memory on a CD
- album (15 songs) requires about 8GB
- great quality (large SQNR), but too much memory

# MP3 Payoff

- CD recordings, bit rate 1.411 Mbits/sec
- MP3, bit rate 128 Kbits/sec
- {uncompressed bit rate} / {compressed bit rate} = 1,411,000/128,000 = 11 = code rate
- 15 songs requires 8 GB (CD); 720 MB (MP3)

!!!

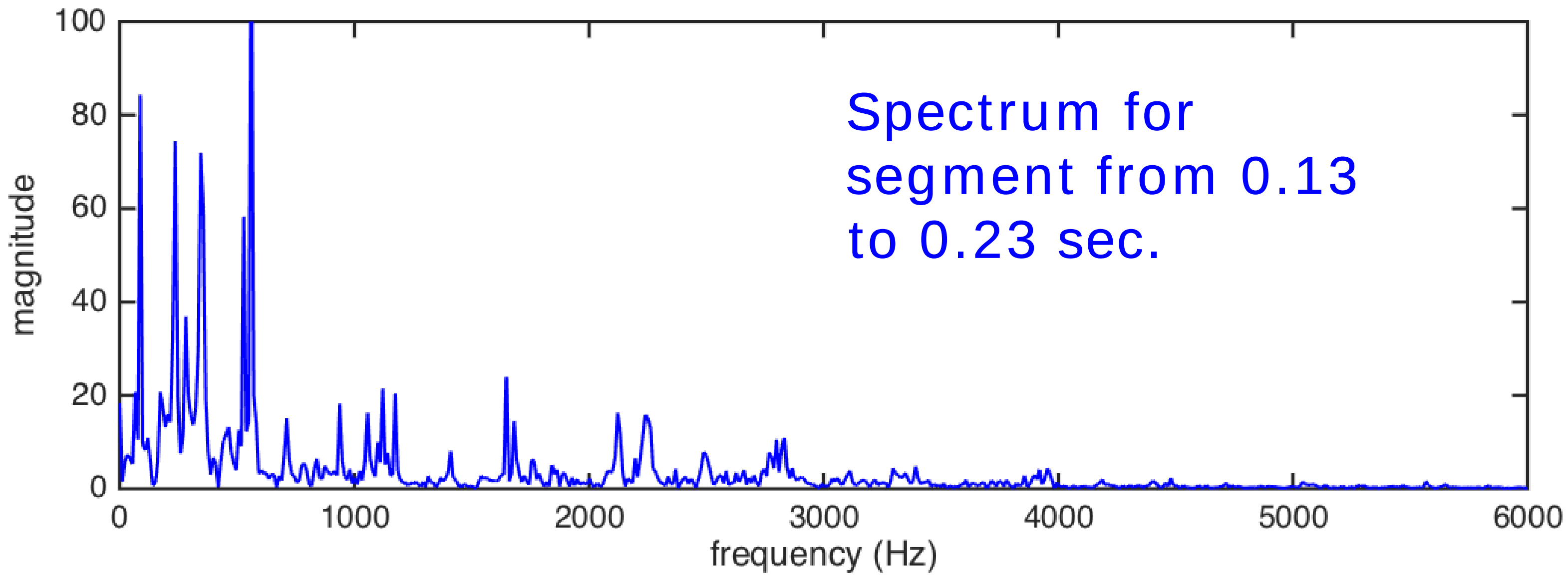
# Main Concepts

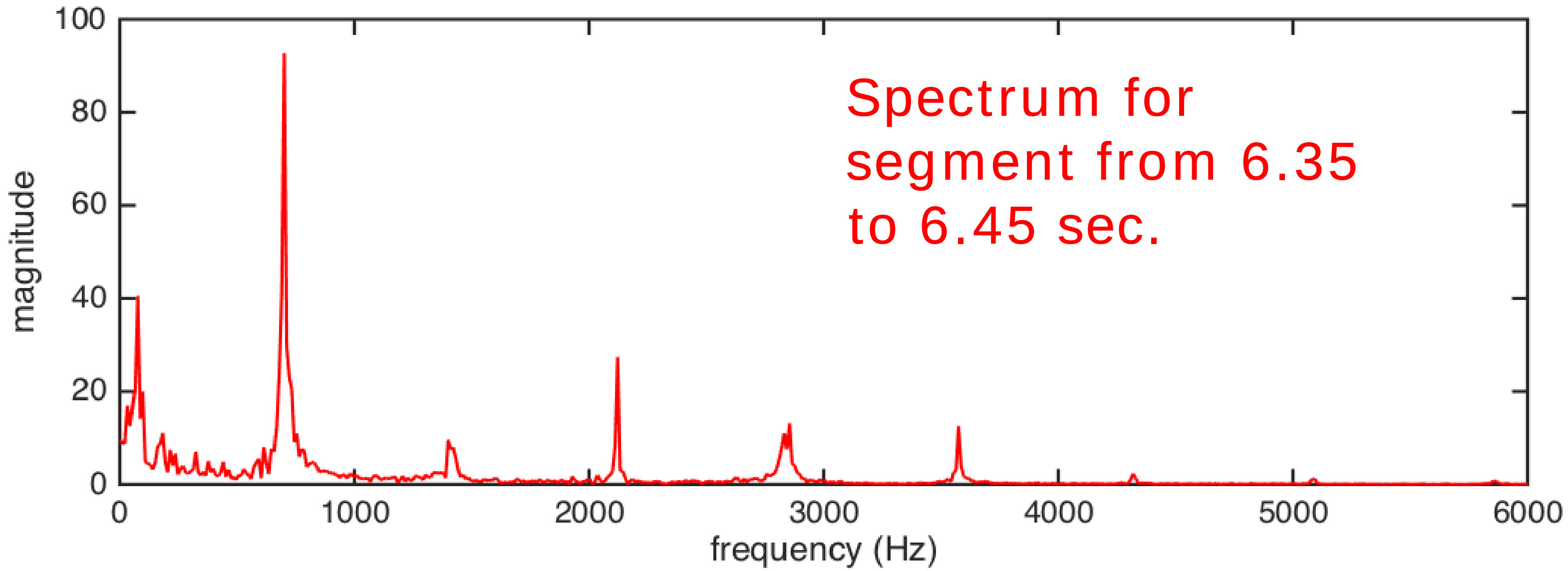
- Signal ✓
- Signal Processing ✓
- Spectrum ✓
- Sampling ✓
- Quantization ✓
- Compression ✓
- Spectrogram

# Spectrogram

- We typically don’t show the frequency content of a long audio signal (music or several spoken words) using a single spectrum – instead, we use a picture called a spectrogram.

- Example: 6.8seconds of long piece of recorded piano concerto:


Spectrum for segment from 0.13 to 0.23 sec.
magnitude
frequency (Hz)
0
20
40
60
80
100
0
1000
2000
3000
4000
5000
6000

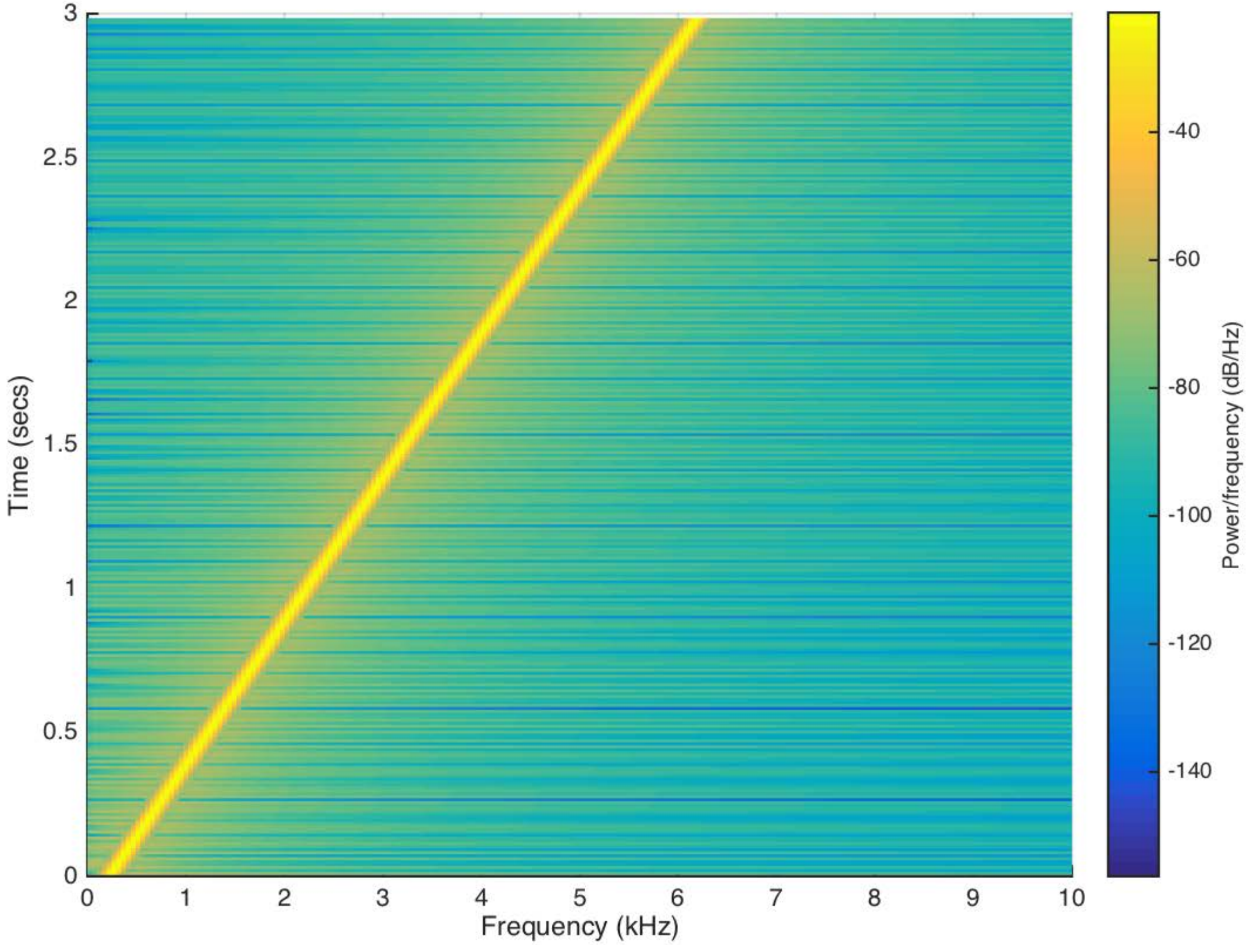

Spectrum for segment from 6.35 to 6.45 sec.
magnitude
frequency (Hz)
0
20
40
60
80
100
0
1000
2000
3000
4000
5000
6000

- We typically don’t show the frequency content of a long audio signal (music or several spoken words) using a single spectrum – instead, we use a picture called a ***spectrogram.***

- For a spectrogram: the (sampled and digitized) signal is divided into short segments. The frequency content for each segment is computed with an FFT (like in MP3).

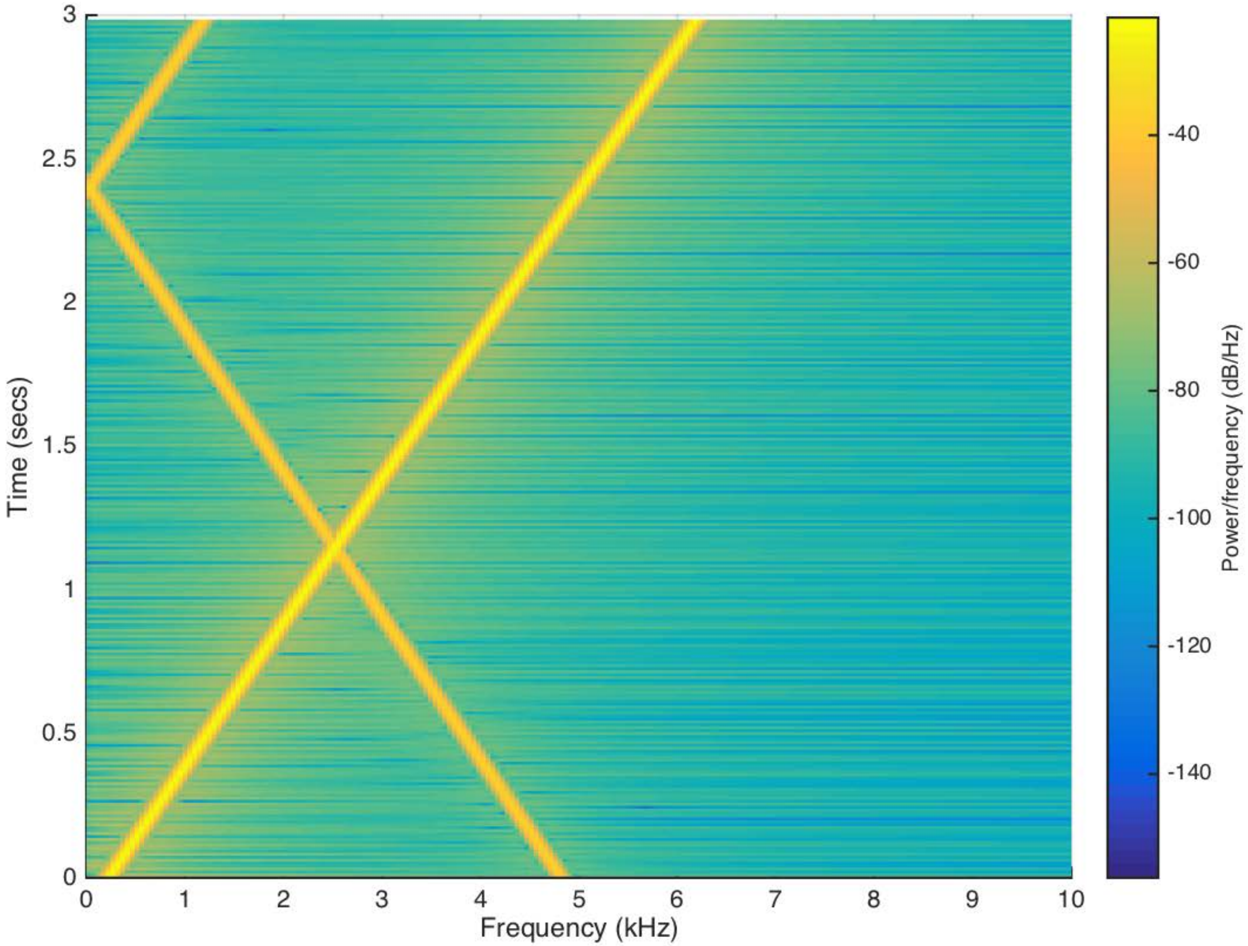
# Examples -1

(1) Rising Tone



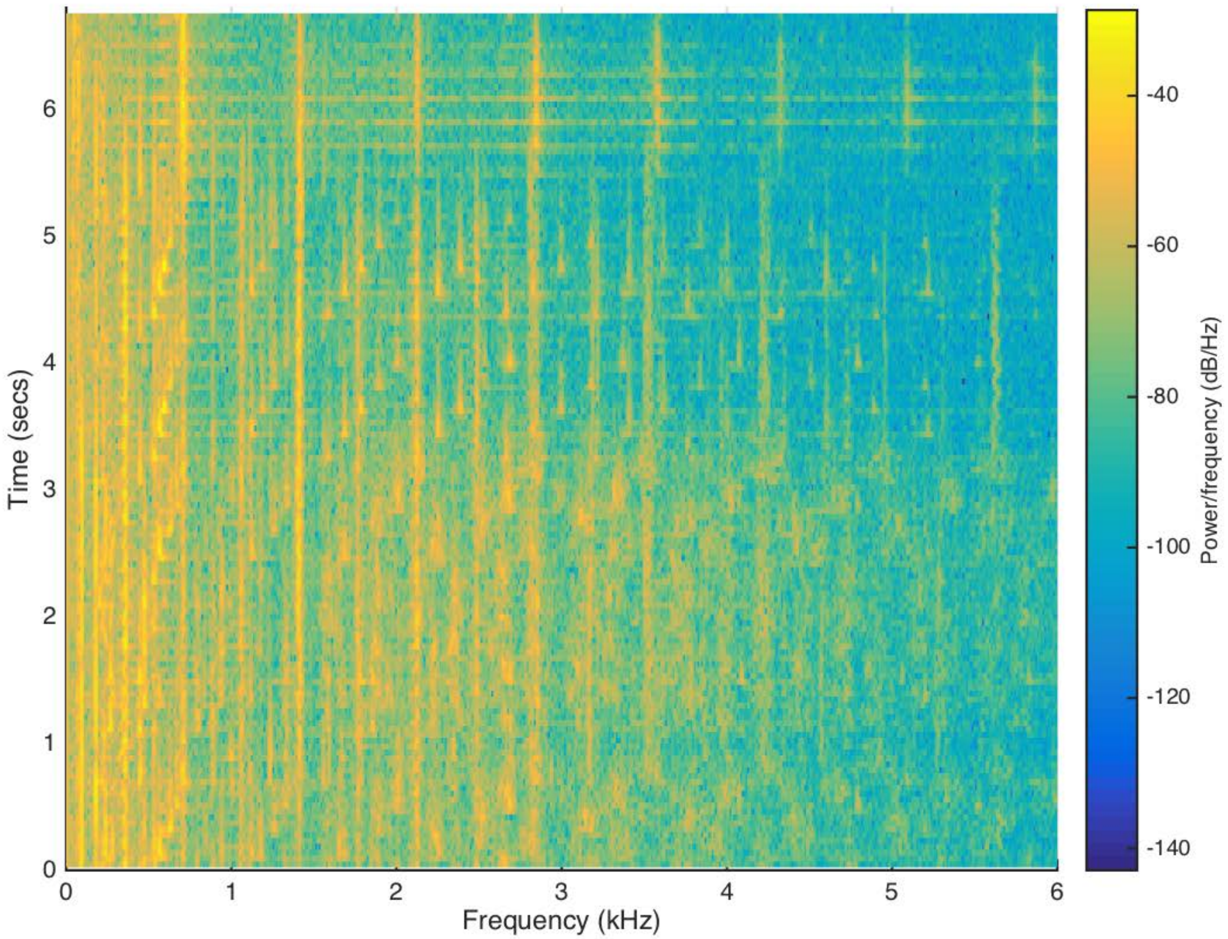
# Examples - 2

(2) Two changing tones:



# Examples - 3

(3) Piano concerto:



# Main Concepts

- Signal ✓
- Signal Processing ✓
- Spectrum ✓
- Sampling ✓
- Quantization ✓
- Compression ✓
- Spectrogram ✓