Announcements, Goals, and Reading

Announcements:

- HW08 due Tuesday 11/8
- MT2 will be **Tuesday 11/15.** Check your calendars now for conflicts.

Goals for Today:

- Dynamics in a plane

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 8: Dynamics II: Motion in a Plane

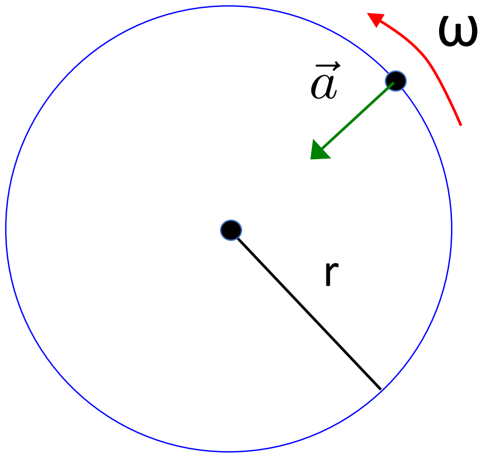
Midterm 2 **Tuesday November 15, 7-9PM**

- **Covers Chapters 5-8 from Knight textbook, HW5-8 (except circular motion). Hooke's law will also be fair game*. Basically: everything we have learned so far about forces and dynamics.**
- Location will depend on 1st letter of your last name. Room assignments will be set later this week or early next week.
- **Allowed a Calculator and an 8.5"×11" reference sheet with handwritten notes on both sides.** Be sure to bring a calculator as there are never enough spares.
- ~25 Multiple Choice Questions; **Bring a #2 pencil**
- Practice problems will be posted on Moodle and/or Mastering Physics
- SI/TA exam review sessions will be held on exam week and/or next week.
- Makeup Exams: If you have a conflict with another exam, please let us know as soon as possible. Wednesday 11/9 will be the last day to request a makeup exam without penalty. E-mail our TA, Joanna Wuko (jwuko@physics.umass.edu) and CC me.

**Questions about Hooke's law will be conceptual in nature.*

Centripetal Force

$$v = \omega r$$



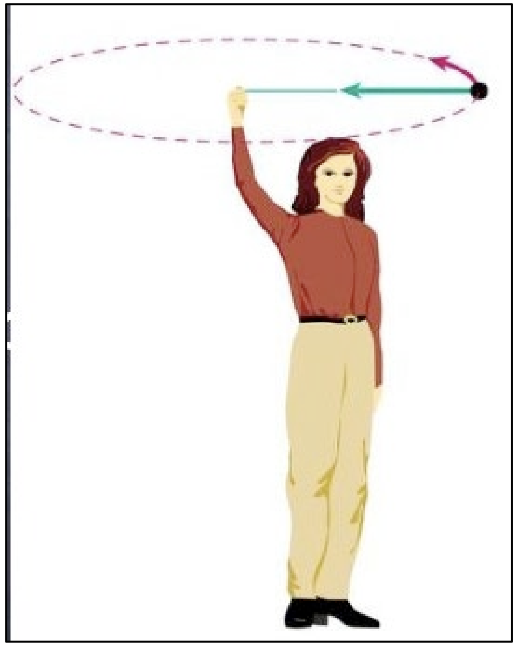
Particle of mass m moving with angular velocity ω around circle of radius r

$$F = m\omega^2 r = m\frac{v^2}{r}$$

Centripetal force

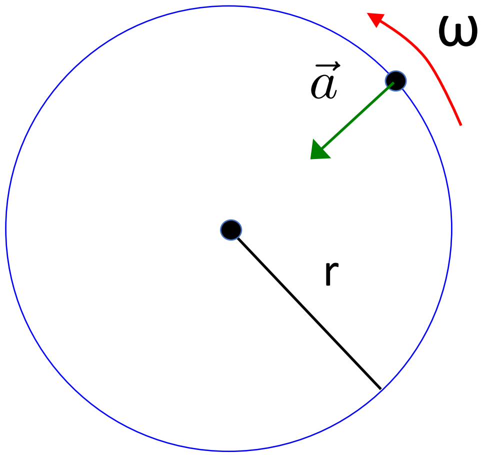
Must be something supplying centripetal force

Tension force of a string obviously points towards the center



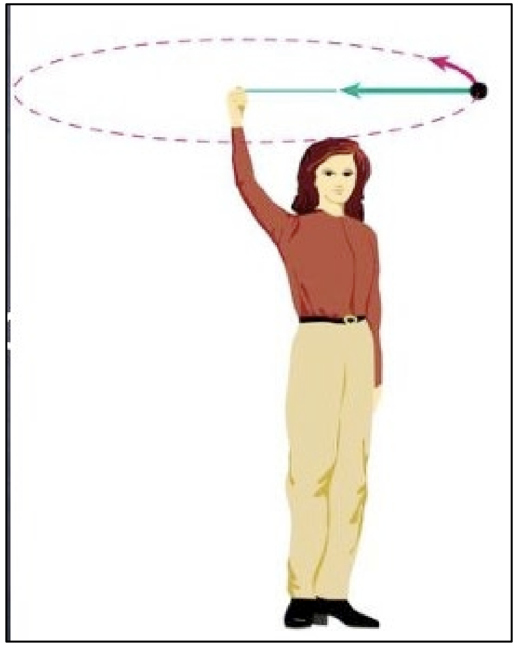
Centripetal Force Demo

Particle of mass m moving with angular velocity ω around circle of radius r



$$F = m\omega^2 r = m\frac{v^2}{r}$$

Tension force supplies centripetal force



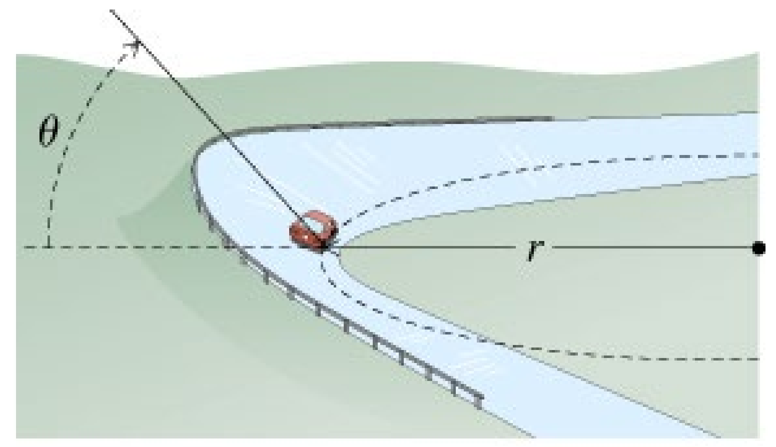
- Is the tension force from fixed mass constant?
- If r decreases, what happens to angular velocity?

Instructor's Choice (for chapter 8)

Item Type: Tutorial | Difficulty: 3 | Time: 13m | Co

± Banked Frictionless Curve, and Flat Curve with Friction

A car of mass M = 900 kg traveling at 55.0 $\mathrm{km/hour}$ enters a banked turn covered with ice. The road is banked at an angle θ, and there is no friction between the road and the car's tires as shown in (Figure 1) . Use g = 9.80 $\mathrm{m/s^2}$ throughout this problem.



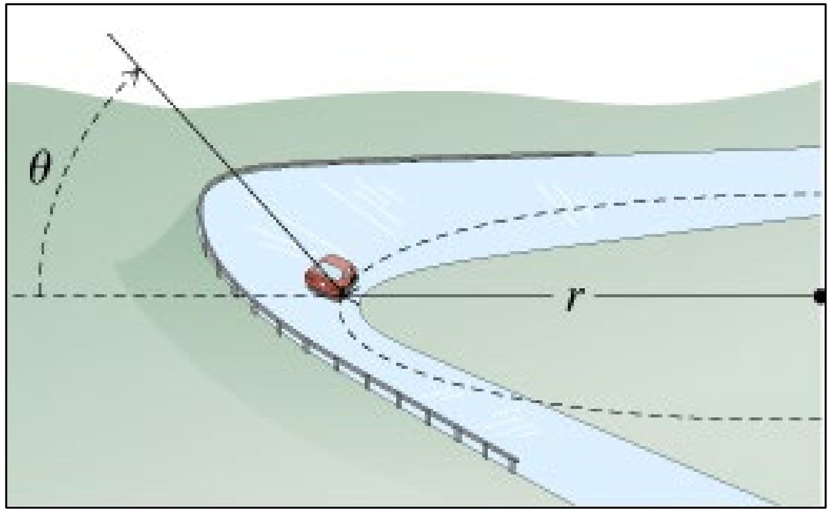
Part A

What is the radius r of the turn if θ = 20.0 $^\circ$ (assuming the car continues in uniform circular motion around the turn)?

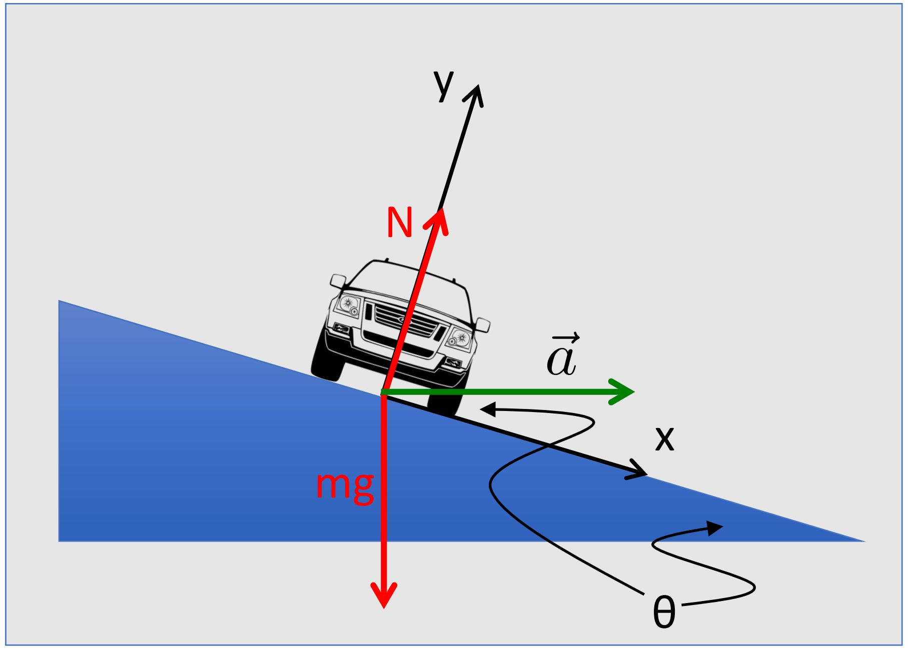
What force will supply centripetal force in this problem?

Normal force from road **has component** in horizontal direction, **perpendicular to gravity**

A car of mass M = 900 kg traveling at 55.0 km/hour enters a banked turn covered with ice. The road is banked at an angle θ, and there is no friction between the road and the car's tires as shown in (Figure 1) . Use g = 9.80 $\mathrm{m/s^2}$ throughout this problem.



for θ=20º, find r



Only 2 forces on car – gravity and normal force from road

$$\vec{F}_{net} = (mg\sin\theta)\,\hat{i} + (N - mg\cos\theta)\,\hat{j}$$

Centripetal acceleration points in horizontal direction

$$\vec{a} = \frac{v^2}{r}\cos\theta\,\hat{i} + \frac{v^2}{r}\sin\theta\,\hat{j}$$

$$\vec{F}_{net} = m\vec{a} \quad \Rightarrow \quad mg\sin\theta = \frac{mv^2}{r}\cos\theta \quad \Rightarrow \quad \boxed{r = \frac{v^2}{g\tan\theta}}$$

x-component

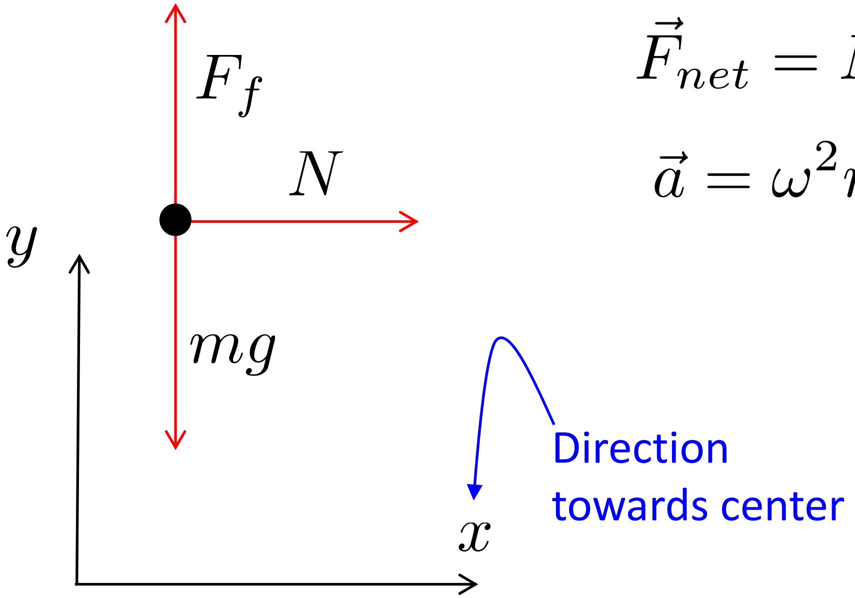
A 50 kg person rides the "Rotor" at an amusement park.

The coefficient of static friction of static friction is μ_s=0.4

Rotor radius is 5.0m

What is the minimum angular velocity for the rotor so that the person doesn't slide down the wall?

Draw free body diagram for person



$$\vec{F}_{net} = N\,\hat{i} + (F_f - mg)\,\hat{j}$$

$$\vec{a} = \omega^2 r\,\hat{i}$$ Centripetal acceleration

2nd law ➡ $N = m\omega^2 r$

➡ $F_f = \mu_s N = \mu_s m\omega^2 r$

Maximum static friction force

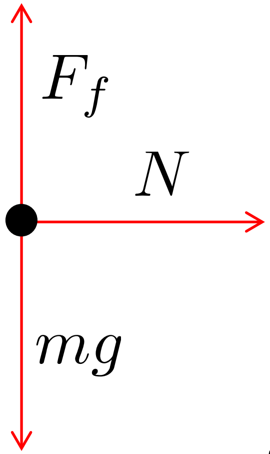
A 50 kg person rides the “Rotor” at an amusement park.

The coefficient of static friction of static friction is μ_s=0.4

Rotor radius is 5.0m

What is the minimum angular velocity for the rotor so that the person doesn’t slide down the wall?

Draw free body diagram



$$\vec{F}_{net} = N\,\hat{i} + (F_f - mg)\,\hat{j}$$

$$F_f = \mu_s N = \mu_s m\omega^2 r$$

2nd law ➡ $F_f - mg = 0$ Doesn’t fall down!

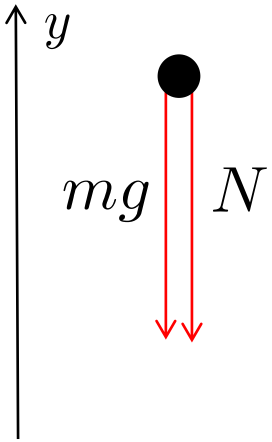
➡ $\mu_s m\omega^2 r = mg$

$$\omega = \sqrt{\frac{g}{\mu_2 r}} = \sqrt{\frac{9.8m/s^2}{(0.4)(5.0m)}} = 2.2rad/s = 21rpm$$

A driver attempts to go around a “loop the loop” of radius 20m in a 1000kg car

What is the minimum speed at the top for this to be successful?

Draw free body diagram for car at top



Normal force from the track and gravity both point down

$$\vec{F}_{net} = (-mg - N)\,\hat{j}$$

Correct even if car is not going around loop at precisely constant rate

Changing speed would be component of acceleration tangent to loop

Can ignore this for our purpose

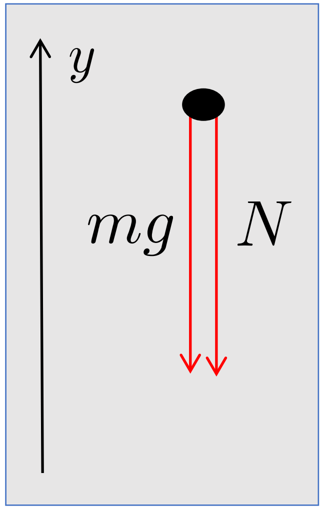
Assume car stays on track ➡ Centripetal acceleration $a=v^2/r$ pointing downward

Because gravity points toward center of circular motion, it acts to change the motion’s direction, not cause the car to fall off the track… if speed is sufficient.

A driver attempts to go around a “loop the loop” of radius 20m in a 1000kg car

What is the minimum speed at the top for this to be successful?

Draw free body diagram for car **at top**



$$\vec{F}_{net} = (-mg - N)\,\hat{j}$$

Assume car stays on track → Centripetal acceleration $a=v^2/r$ pointing downward

2nd law → $$\vec{F}_{net} = (-mg - N)\,\hat{j} = -\frac{mv^2}{r}\,\hat{j} = m\vec{a}$$

Together gravity and normal force provide centripetal force

Focus on normal force → $$N = \frac{mv^2}{r} - mg$$

A driver attempts to go around a “loop the loop” of radius 20m in a 1000kg car

What is the minimum speed at the top for this to be successful?

Draw free body diagram for car **at top**

Focus on normal force ➡ $$N = \frac{mv^2}{r} - mg$$

Not falling down!

$N > 0$ ➡ Track is pushing downward on car ⟷ (3rd law) Car is pushing upward on track

Gravity is not sufficient to provide all of centripetal force

$N < 0$ ➡ Car is no longer pushing upward on track. ➡ Car will fall down!

$N = 0$ ➡ Dividing line where car just barely makes it around

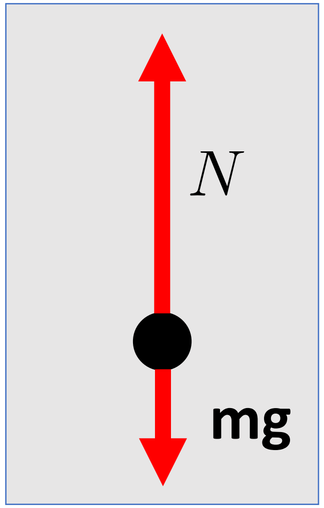
Gravity exactly equal to centripetal force

$$v = \sqrt{gr} = \sqrt{(9.8m/s^2)(20m)} = 14m/s = 31mph$$

A driver attempts to go around a “loop the loop” of radius 20m in a 1000kg car

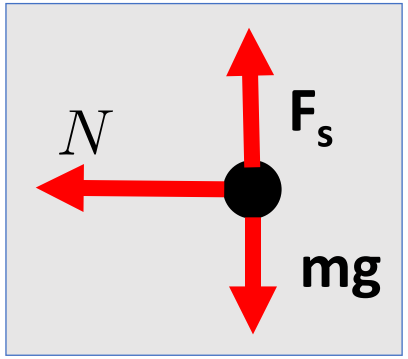
Bonus: consider car at other points in the loop

Car at bottom of loop



$F_c = N - mg \; = \frac{mv^2}{r}$ pointing **up**.
Normal force is greater here than at the top.

Car at side of loop



$F_c = N = \frac{mv^2}{r}$ in the radial direction.
(Tires push on road with $\mathbf{F_s}$ to maintain constant speed v)


JAGUAR

Centripetal Force Problem

Problem 8.39

A 900 g ball moves in a vertical circle on a 1.07 m -long string. If the speed at the top is 3.80 $\mathrm{m/s}$, then the speed at the bottom will be 7.51 $\mathrm{m/s}$.

What is the tension in the string when the ball is at the top?

What is the tension in the string when the ball is at the bottom?

Centripetal Force Problem

Problem 8.39

A 900 g ball moves in a vertical circle on a 1.07 m -long string. If the speed at the top is 3.80 $\mathrm{m/s}$, then the speed at the bottom will be 7.51 $\mathrm{m/s}$.

What is the tension in the string when the ball is at the top?

What is the tension in the string when the ball is at the bottom?

At top gravity and tension both point down
As does centripetal acceleration

$$F_{net} = -mg - T = -\frac{mv^2}{r}$$

ma

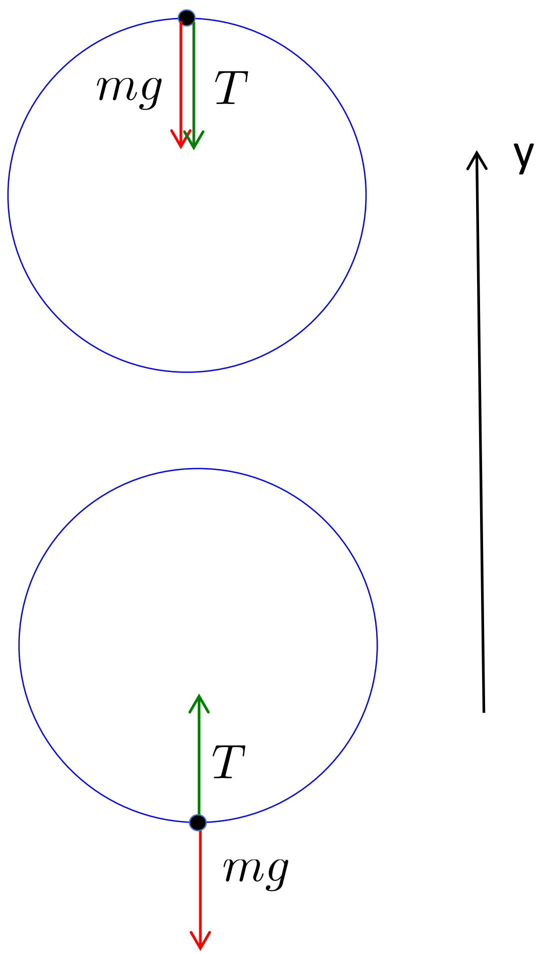
$$T = \frac{mv^2}{r} - mg$$

Tension in string at top

If T<0, then ball will fall down

At the top

At the bottom



Centripetal Force Problem

Problem 8.39

A 900 g ball moves in a vertical circle on a 1.07 m -long string. If the speed at the top is 3.80 $\mathrm{m/s}$, then the speed at the bottom will be 7.51 $\mathrm{m/s}$.

What is the tension in the string when the ball is at the top?

What is the tension in the string when the ball is at the bottom?

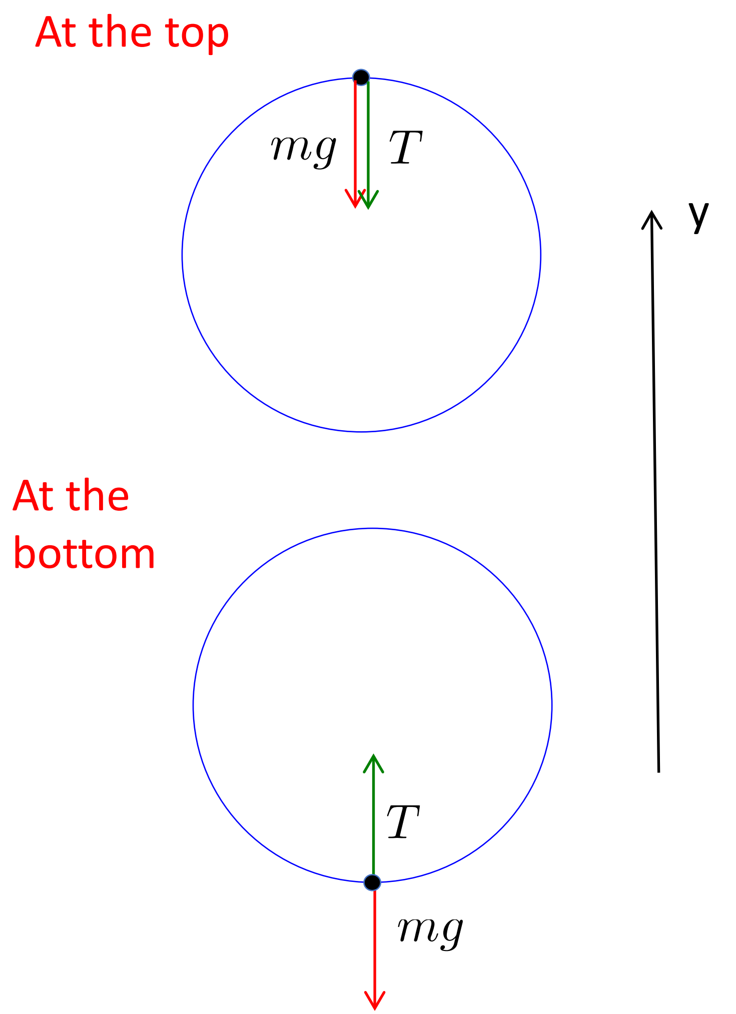
Bottom: gravity points down, while tension
And centripetal acceleration point up

$$F_{net} = T - mg = +\frac{mv^2}{r}$$

ma

$$T = \frac{mv^2}{r} + mg$$

Tension in string at bottom

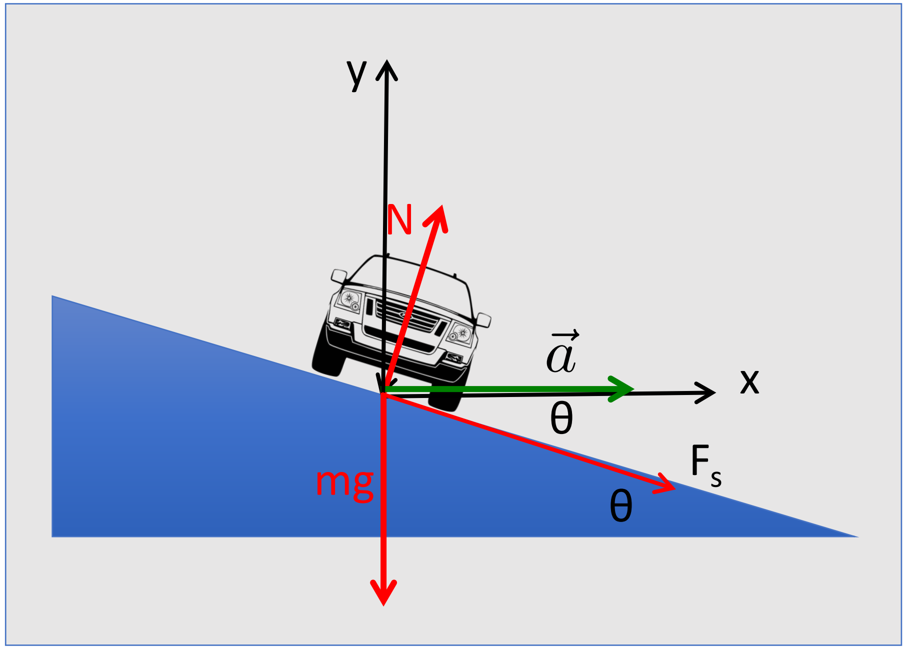


Centripetal Force Demo: Swinging Bucket of Water

Why doesn’t the water fall out?

Car in circular motion with friction:

If the car is not sliding/skidding, which frictional force is relevant: static or kinetic? Which way does it point?



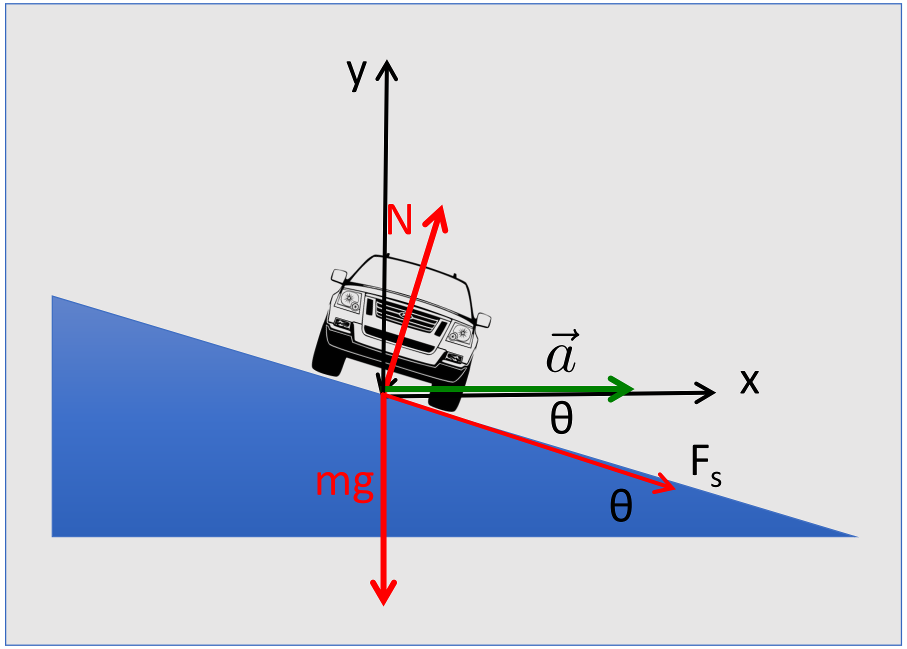
Relevant friction is **static**. The part of the tire in contact with the road is at rest with respect to the road at the instant of contact.

Which way does it point?

Which way does it point? It points down along the slope. Why? If there were no friction, the car would slide towards outside of the curve, friction opposes this.

Car in circular motion with friction:

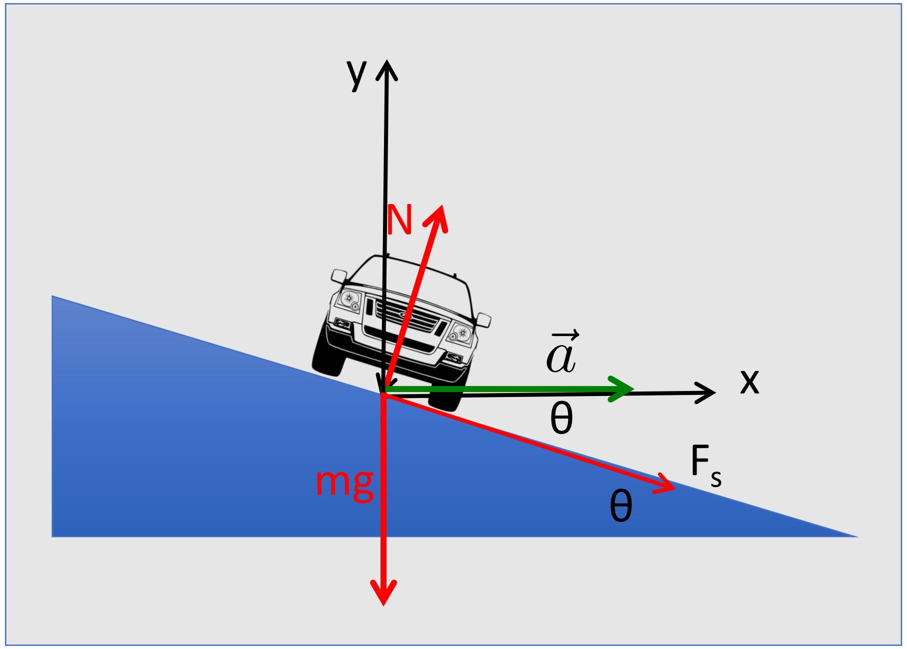
If the car is not sliding/skidding, which frictional force is relevant: static or kinetic? Which way does it point?



What is maximum velocity without sliding?

Car in circular motion with friction:

If the car is not sliding/skidding, which frictional force is relevant: static or kinetic? Which way does it point?



What is maximum velocity without sliding?

$$\Sigma F_{vertical} = F_N \cos\theta - mg - F_s \sin\theta = ma_{vertical} = 0\,.$$

$$\Sigma F_{horizontal} = F_N \sin\theta + F_s \cos\theta = ma_{horizontal} = mv^2/\mathrm{r}$$

$$F_N \cos\theta - \mu_s F_N \sin\theta = mg \quad so\ \ F_N = mg/(\cos\theta - \mu_s \sin\theta)$$

Sub $F_s=\mu_s F_N$ so $v_{max}^2 = [(\sin\theta + \mu_s \cos\theta)/(\cos\theta - \mu_s \sin\theta)]gr$

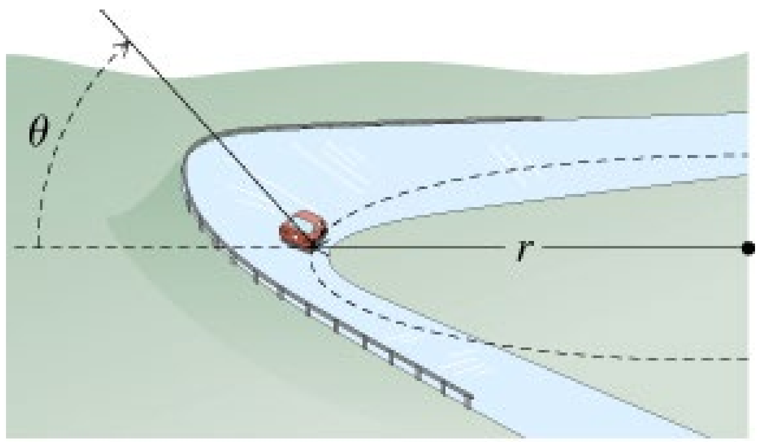
Note when $\mu_s=0$, then $\mathbf{v_{max}^2 = g\ r\ tan\theta}$

Example

Item Type: Tutorial | Difficulty: 3 | Time: 13m | Co

± Banked Frictionless Curve, and Flat Curve with Friction

A car of mass M = 900 kg traveling at 55.0 $\mathrm{km/hour}$ enters a banked turn covered with ice. The road is banked at an angle θ, and there is no friction between the road and the car's tires as shown in (Figure 1) . Use g = 9.80 $\mathrm{m/s^2}$ throughout this problem.



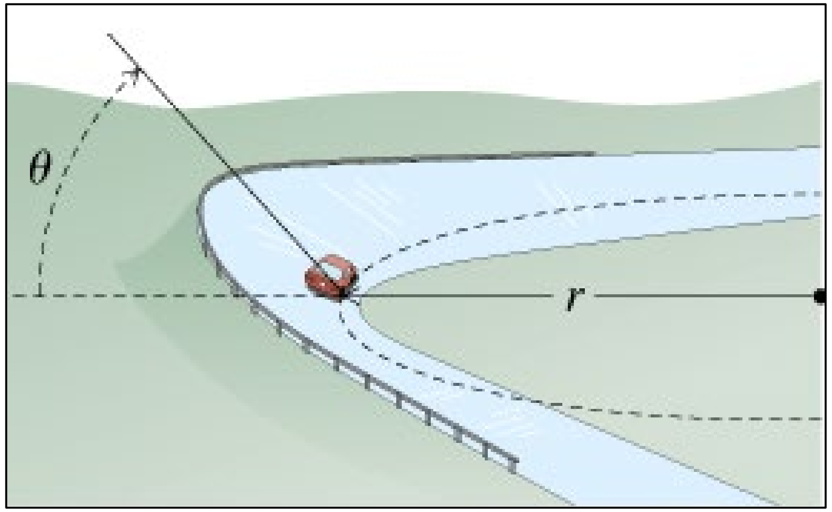
Part A

What is the radius r of the turn if θ = 20.0 $^\circ$ (assuming the car continues in uniform circular motion around the turn)?

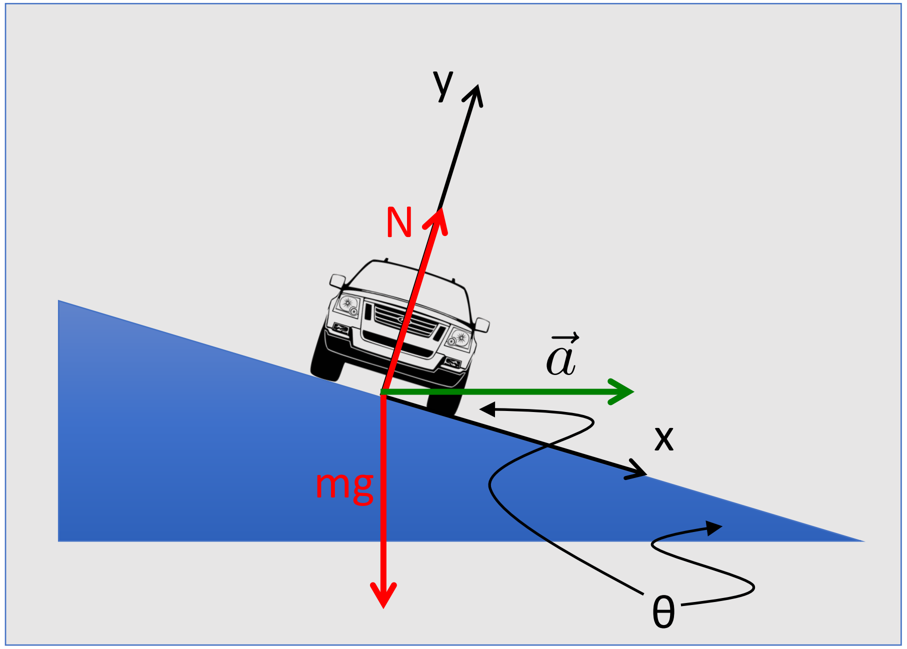
What force will supply centripetal force in this problem?

Normal force from road **has component** in horizontal direction, **perpendicular to gravity**

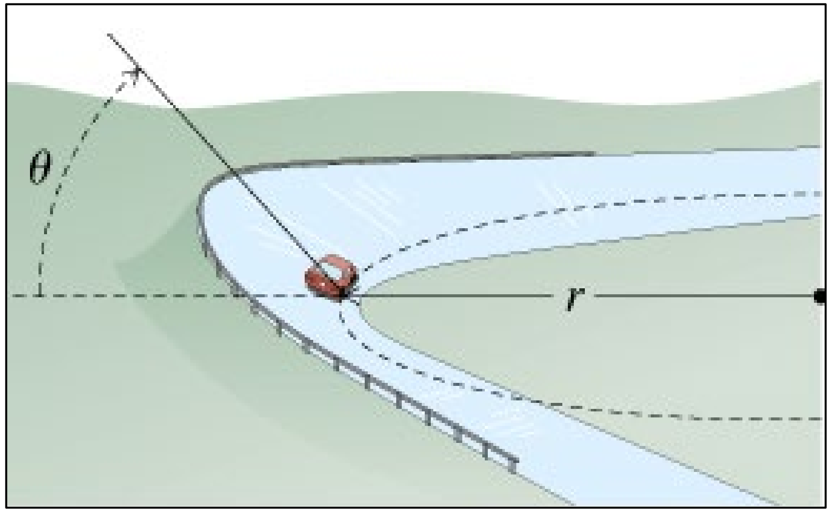
A car of mass M = 900 kg traveling at 55.0 km/hour enters a banked turn covered with ice. The road is banked at an angle θ, and there is no friction between the road and the car's tires as shown in (Figure 1) . Use g = 9.80 $\mathrm{m/s^2}$ throughout this problem.



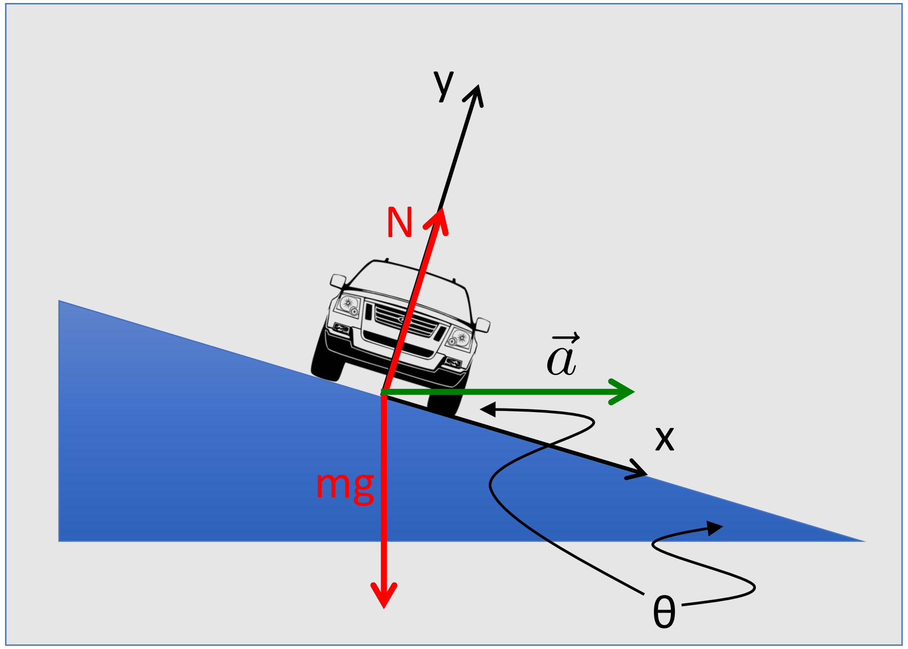
for θ=20°, find r



A car of mass M = 900 kg traveling at 55.0 km/hour enters a banked turn covered with ice. The road is banked at an angle θ, and there is no friction between the road and the car's tires as shown in (Figure 1) . Use g = 9.80 m/s^2 throughout this problem.



for θ=20º, find r



Only 2 forces on car – gravity and normal force from road

$$\vec{F}_{net} = (mg\sin\theta)\,\hat{i} + (N - mg\cos\theta)\,\hat{j}$$

Centripetal acceleration points in horizontal direction

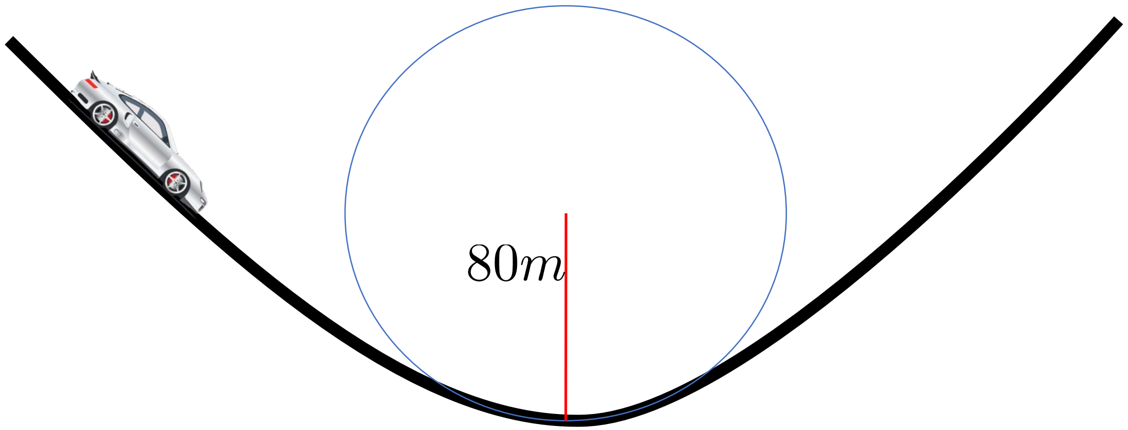
$$\vec{a} = \frac{v^2}{r}\cos\theta\,\hat{i} + \frac{v^2}{r}\sin\theta\,\hat{j}$$

$$\vec{F}_{net} = m\vec{a} \;\Rightarrow\; mg\sin\theta = \frac{mv^2}{r}\cos\theta \;\Rightarrow\; \boxed{r = \frac{v^2}{g\tan\theta}}$$

x-component

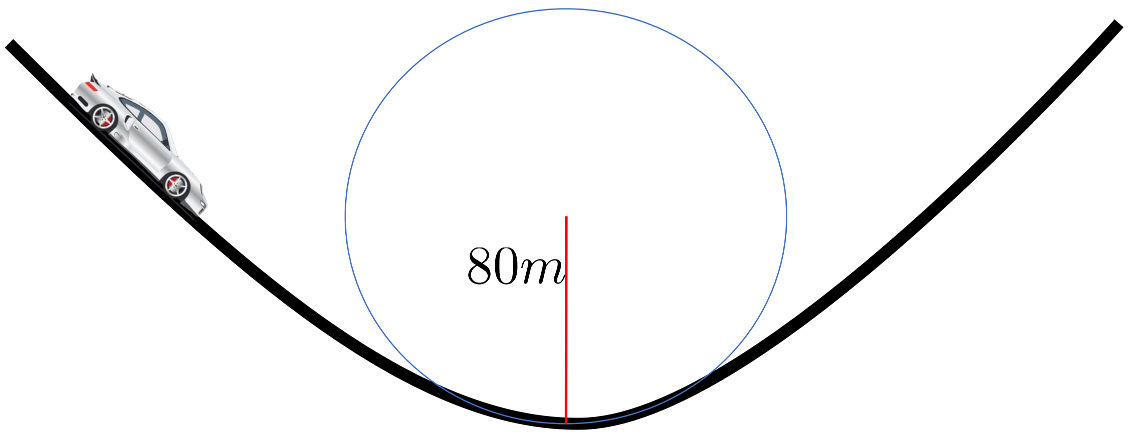
Problem on Centripetal Force

A sports car crosses the bottom of a valley with a radius of curvature equal to 80 m. At the very bottom, the driver feels heavier. Why is this?



Problem on Centripetal Force

A sports car crosses the bottom of a valley with a radius of curvature equal to 80 m. At the very bottom, the normal force on the driver is twice his weight. How fast was the car traveling?



At bottom, 2nd law gives

$$N - mg = \frac{mv^2}{r}$$

Problem states that $N = 2mg$

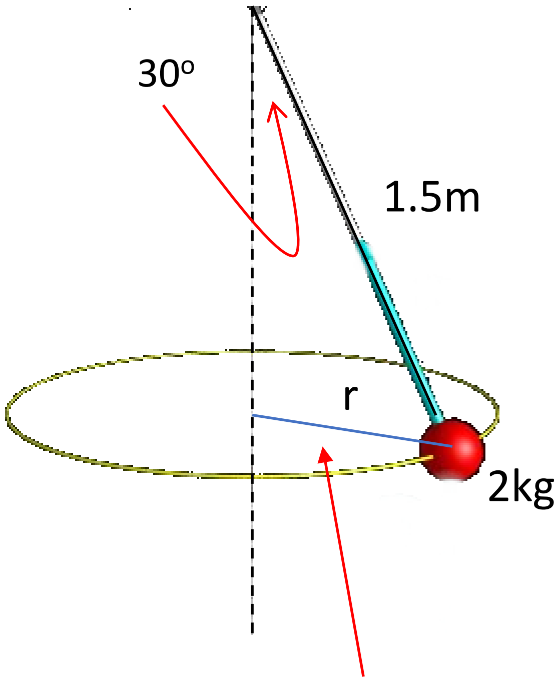
$$\Rightarrow \quad \frac{mv^2}{r} = mg \quad \Rightarrow \quad v = \sqrt{gr}$$

$$= 28m/s$$

Centrifugal Force Problem

A 2kg ball rotates at the end of a 1.5m string making an angle 30° with the vertical

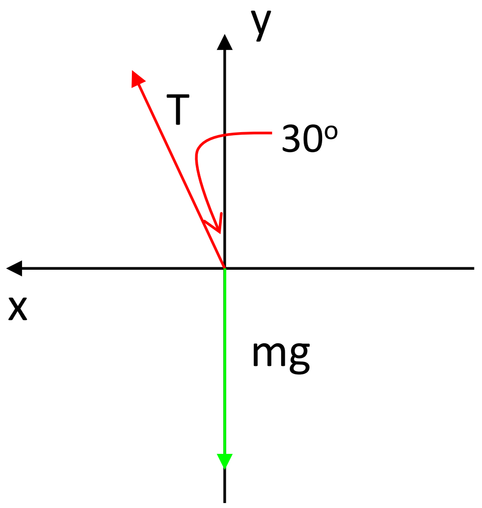
Find its angular velocity



$$r = (1.5m)\sin 30^o$$

Draw free body diagram for ball

Towards center of circle



2nd law y-component $\Rightarrow$ $$T\cos 30^o = mg \Rightarrow T = \frac{mg}{\cos 30^o}$$

x-component $\Rightarrow$ $$T\sin 30^o = m\omega^2 r$$

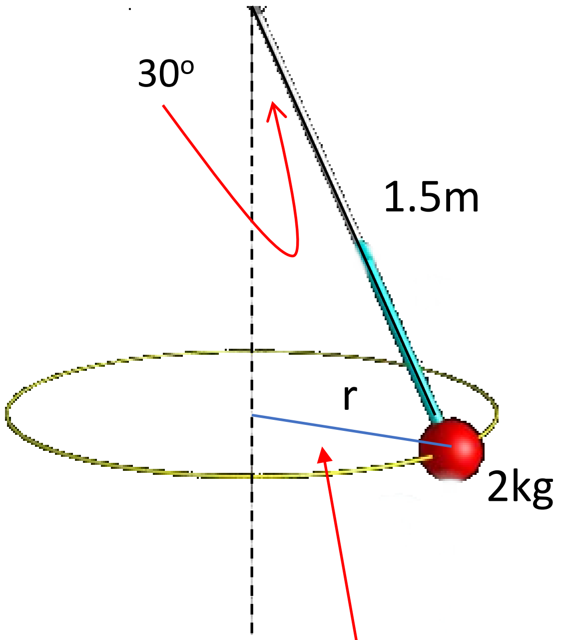
x-component of tension provides centripetal force

Now solve for ω

Centrifugal Force Problem

A 2kg ball rotates at the end of a 1.5m string making an angle 30º with the vertical

Find its angular velocity



$$r = (1.5m)\sin 30^o$$

$$T\sin 30^o = m\omega^2 r$$

$$T = \frac{mg}{\cos 30^o}$$

Gives…

$$\omega^2 = \frac{T\sin 30^o}{mr}$$

$$= \frac{mg}{\cos 30^o}\cdot\frac{sin30^o}{m}\cdot\frac{1}{(1.5m)\sin 30^o}$$

$$\omega = \sqrt{\frac{g}{(1.5m)\cos 30^o}} = 2.75rad/s$$