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1 %{
2 Aidan Chin ECE 213 Spring 2024
3 5/6/2024
4 --- Exercise SS-C1: Understanding Convolution ---
5 this code will solve for output voltage of convolution
6 between 2 functions h(t) and x(t)
7 %}
8
9 % ----- Initialize -----
10
11 clc % clear terminal
12 clf % clear all figures
13 clear % remove all variables from the workspace
14
15 % ----- Given -----
16
17 R = 10; % Resistor value in Ohms
18 C = 2; % Capacitor value in mF
19 V0 = 12; % Voltage (V)
20 tau = R*C; % Time constant
21 n = 4000; % Number of intervals
22 t0 = 0; % Start time
23 tf = 8 * tau; % End time
24 t = linspace(t0, tf, n+1); % Create overall time array, Because C is in mF,
25 % tau = RC is in ms
26 t2 = 2*tau; % the point in time where x becomes 0
27 t2i = (t2/tf)*(n); % the index in t where x becomes 0
28
29 h = (1/tau)*exp(-t/tau); % create function to that we will convolute
30
31 % ----- Calculations -----
32
33 % hand calculation
34
35 c1 = V0*(1-exp(-t/tau)); % convolution answer for 0<t<2tau
36 c2 = V0*(1-exp(-2))*exp(-(t-t2)/tau); % convolution answer t<2tau
37
38 % matlab calculation
39
40 ym = zeros(1, n+1); % initialize the convolution bounds
41 dr = (tf - t0)/n; % define step size scaler
42 for in = 1:n % iterate over whole time of convolution
43     for k = 1:length(h) % iterate over whole length of h
44         if in - k + 1 > 0 && in - k + 1 <= length(h) % represent x(t) using if
45 % statement with time shift
46             if in - k + 1 < t2i % only add to integration when v = V0
47                 ym(in) = ym(in) + V0 * h(k) * dr; % perform integration v = V0
48             end
49         end
50     end
51 end
```

```
52
53 % ----- Plotting -----
54
55 plot(t, c1, ':', t, c2, ':', t, ym, 'LineWidth', 2);
56 ylim([0 V0+1]); % adjust limits
57 legend('0<t<2\tau', 't>2\tau', 'y(t)', 'Location', 'east'); % create legend
58 title('Exercise SS-C1: Understanding Convolution Aidan Chin'); % create title
59 xlabel('time t (ms)'); % Custom x-axis label
60 ylabel('Voltage (V)'); % Custom y-axis label
61 xticks(0:tau:tf); % define x-axis steps
62 grid on;
63 % ----- Checks -----
64
65 % analytical check, should be near 0
66
67 analyticalCheck = c1(t2i) - c2(t2i)
68
69 % Numerical Checks
70
71 % Check if maximums are the same, if analytical check passes, this should
72 % also be near 0
73
74 [ymy, ymx] = max(ym); % find position of max of ym
75
76 yCheck1 = c1(t2i) - ymy
77 yCheck2 = c2(t2i) - ymy
78
79 % Check if maximums occur at the same time, should near 0
80 xCheck = t(ymx) - t2
```

```
analyticalCheck =
```

```
-0.0240
```

```
yCheck1 =
```

```
-0.0104
```

```
yCheck2 =
```

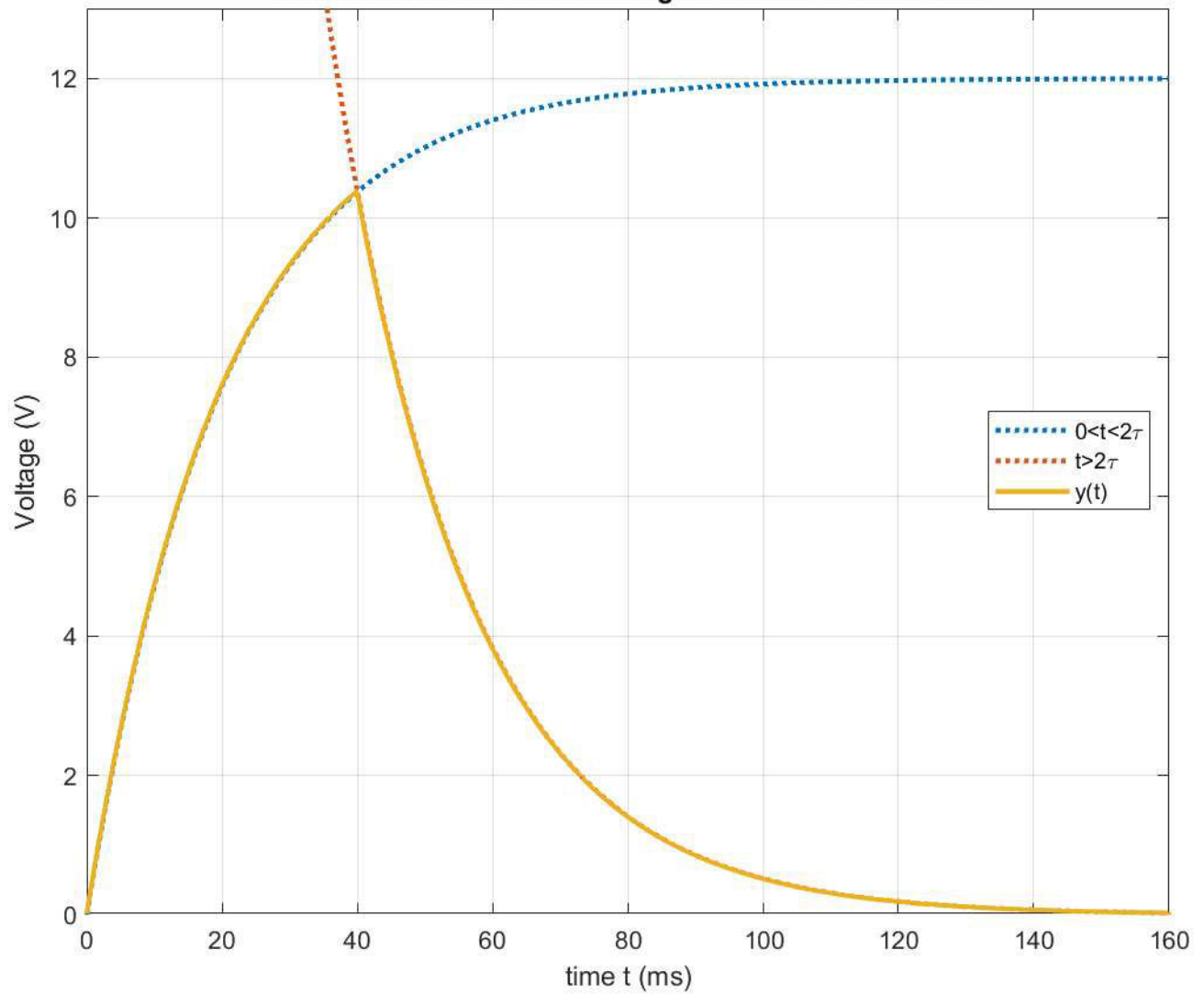
```
0.0136
```

```
xCheck =
```

```
-0.0800
```

```
>>
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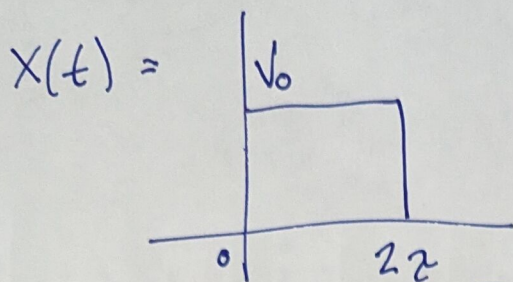
### Exercise SS-C1: Understanding Convolution Aidan Chin



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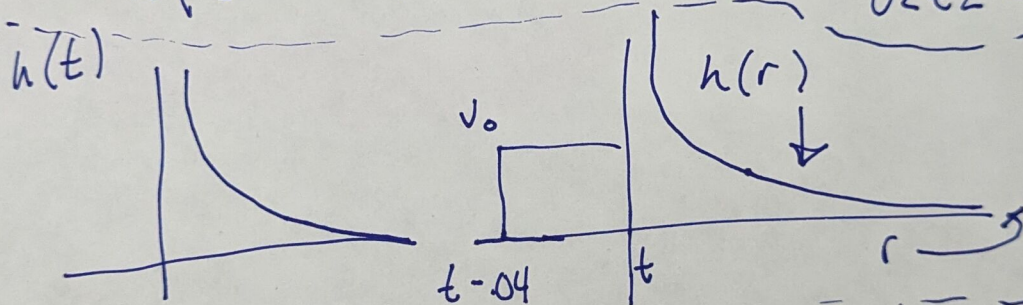
C-1



where  $0 < t < 2\tau$

$$\int_0^t V_0 \frac{1}{\tau} e^{-\frac{r}{\tau}} dr \Rightarrow \frac{V_0}{\tau} \left( -\tau \left[ e^{-\frac{r}{\tau}} \right]_0^t \right)$$

$$= -V_0 \left( e^{-t/\tau} - 1 \right) = V_0 \left( 1 - e^{-t/\tau} \right) \text{ where } 0 < t < 2\tau$$



$$\int_{t-2\tau}^t V_0 \cdot \frac{1}{\tau} e^{-\frac{r}{\tau}} dr \quad t > 2\tau \Rightarrow -V_0 \left[ e^{-\frac{r}{\tau}} \right]_{t-2\tau}^t$$

$$= -V_0 \left( e^{-t/\tau} - e^{-(t+2\tau)/\tau} \right)$$