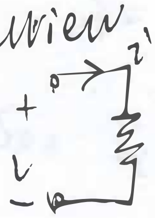
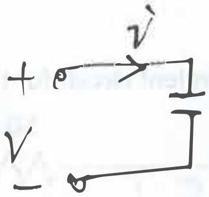
RC Circuits

review



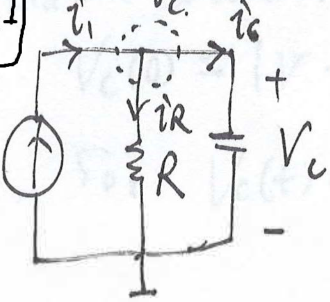
$i = \frac{1}{R} \cdot v \iff$



$i = C \cdot \frac{dv}{dt}$

Follow Asso. Var. Convension

$\boxed{\text{e.g.1}}$



$I_0 = 1A$. $V_{c(t=0)} = 1V$

$R = 3\Omega$, $C = 0.25F$.

find $V_{c(t)}$ $(t > 0)$.

① KCL @ node V_c.

$$i_1 - i_R - i_c = 0$$

$$I_0 - \frac{V_c}{R} - C \cdot \frac{dV_c}{dt} = 0 \Rightarrow \frac{dV_c}{dt} + \frac{1}{RC} \cdot V_c = \frac{I_0}{C}$$

② Generally... for 1^{st} order diff. eq.

$$\frac{d V_{(t)}}{dt} + \underbrace{A \cdot V_{(t)} = B}_{\text{constants}}$$

General Solution:

$$\text{So:} \quad \boxed{V_{(t)} = \frac{B}{A} + \underbrace{k}_{\text{Abritary Constant}} \cdot e^{-A \cdot t}}$$

(to be determined)

In this case:

$$A = \frac{1}{RC}, \quad B = \frac{I_0}{C}.$$

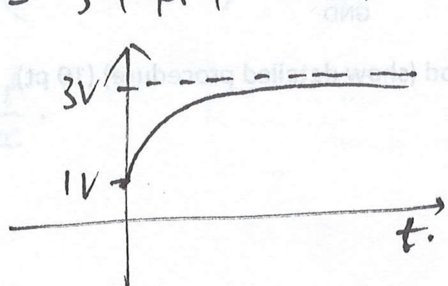
So: $V_C(t) = I_0 R + k \cdot e^{-\frac{t}{RC}}$

$= 1A \cdot 3\Omega + k \cdot e^{-\frac{2}{3}t} = 3 + k \cdot e^{-\frac{2}{3}t}.$

③ Use Initial Condition

$V_C(0) = 1V. \Rightarrow V_C(0) = 3 + k \cdot 1 \Rightarrow k = -2$

So: $V_C(t) = 3 - 2 \cdot e^{-\frac{2}{3}t}.$



Summarize: In general for RC Circuit.

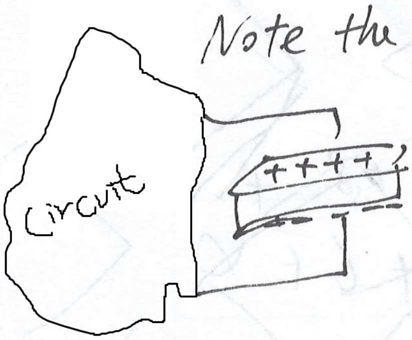
① KCL: find the differential equation

② find the general solution to the equation

③ Use initial conditions to determine the factors in General solution

How to determine the initial condition in a capacitor

Note the Basic property of a capacitor:



The charge amount stored in a capacitor can not be instantaneously changed.

That means. $Q_{(0^-)} = Q_{(0^+)}. \Rightarrow \frac{Q_{(0^-)}}{C} = \frac{Q_{(0^+)}}{C}.$

$\Rightarrow \boxed{V_C(0^-) = V_C(0^+)}$

or from mathematical perspective:

$i = C \cdot \frac{dV_C}{dt}. \Rightarrow V_C(0^-) = V_C(0^+).$

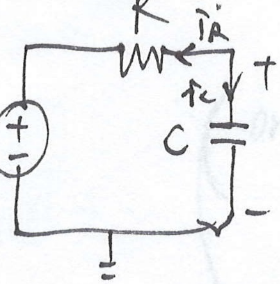
V_C needs to be continous.

So for capacitor:

$$\boxed{V_c(0^+) = V_c(0^-)}.$$

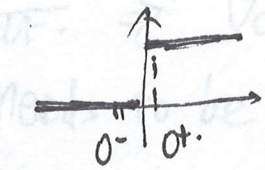
The voltage drop across a capacitor can not be instantaneous changed.

$\boxed{\text{e.g.2}}$



$V_s = (V_1 - V_0)\cdot u(t) + V_0$; find $V_c(t)$; $t > 0$

$u(t)$ is a step function:



$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0. \end{cases}$$

Or: $u(0^-) = 0$, $u(0^+) = 1$

Same as

$$\begin{cases} V_s(0^-) = V_0 \\ V_s(0^+) = V_1 \end{cases}$$

① KCL $(t>0)$

for $t>0$: $V_{s(t>0)} = V_1$

$$-i_c - i_R = 0 \Rightarrow -C\cdot\frac{dV_c}{dt} - \frac{V_c - V_1}{R} = 0$$

$$C\cdot\frac{dV_c}{dt} + \frac{V_c}{R} = \frac{V_1}{R}$$

$$\frac{dV_c}{dt} + \frac{V_c}{RC} = \frac{V_1}{RC}.$$

② So: $\dfrac{dV_c}{dt} + \dfrac{1}{RC}\cdot V_c = \dfrac{V_1}{RC}$ $\left(A = \dfrac{1}{RC},\ B = \dfrac{V_1}{RC}\right)$

So general solution:

$$V_c = V_1 + k\cdot e^{-\frac{t}{RC}}.$$

③ Initial Condition:

① to find $V_c(0^+) \rightarrow$ is to find $V_c(0^-)$

② $V_c(0^-) = V_0$. **(At steady state, a capacitor behavors like an open-circuit)**

So: $$V_c(0^+) = V_0 = V_1 + k \cdot e^{-0}$$
$$= V_1 + k \Rightarrow k = V_0 - V_1$$

So: $$V_c = V_1 + (V_0 - V_1) \cdot e^{-\frac{t}{RC}}$$