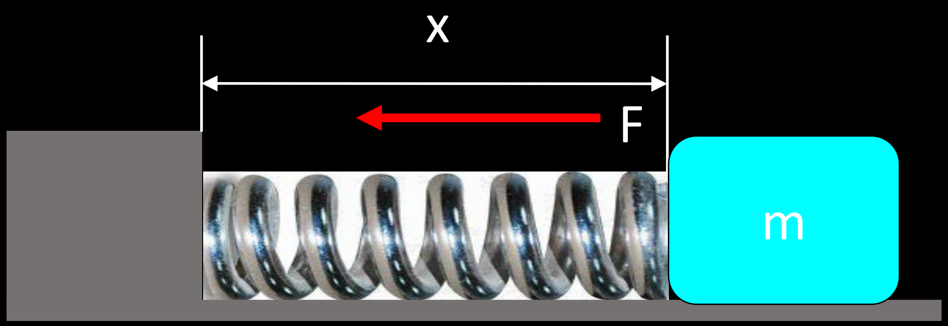
Hook[e’s law] brings you back...
[Toward the rest position]

$$F = -k(x - x_0)$$



Announcements, Goals, and Reading

Announcements:

- HW08 due Tuesday 11/8
- MT2 will be **Tuesday 11/15/**

Goals for Today:

- Hooke's law
- Energy
- Work

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 9: Work and Kinetic Energy

Midterm 2 Tuesday November 15, 7-9PM

- **Covers Chapters 5-8 from Knight textbook, HW5-8 (except circular *kinematics*). Newton's laws, forces, friction, circular dynamics. Hooke's law will also be fair game*. Basically: everything we have learned so far about forces and dynamics.**
- Location will depend on 1st letter of your last name. Room assignments will be set later this week or early next week.
- **Allowed a Calculator and an 8.5"×11" reference sheet with handwritten notes on both sides.** Be sure to bring a calculator as there are never enough spares.
- ~25 Multiple Choice Questions; **Bring a #2 pencil**
- Practice problems will be posted on Moodle and/or Mastering Physics
- SI/TA exam review sessions will be held on exam week and/or next week.
- Makeup Exams: If you have a conflict with another exam, please let us know as soon as possible. Wednesday 11/9 will be the last day to request a makeup exam without penalty. E-mail our TA, Joanna Wuko (jwuko@physics.umass.edu) and CC me.

**Questions about Hooke's law will be conceptual in nature.*

Restoring force of spring

Experiment → Restoring force is proportional to how far spring is stretched or compressed from its rest length

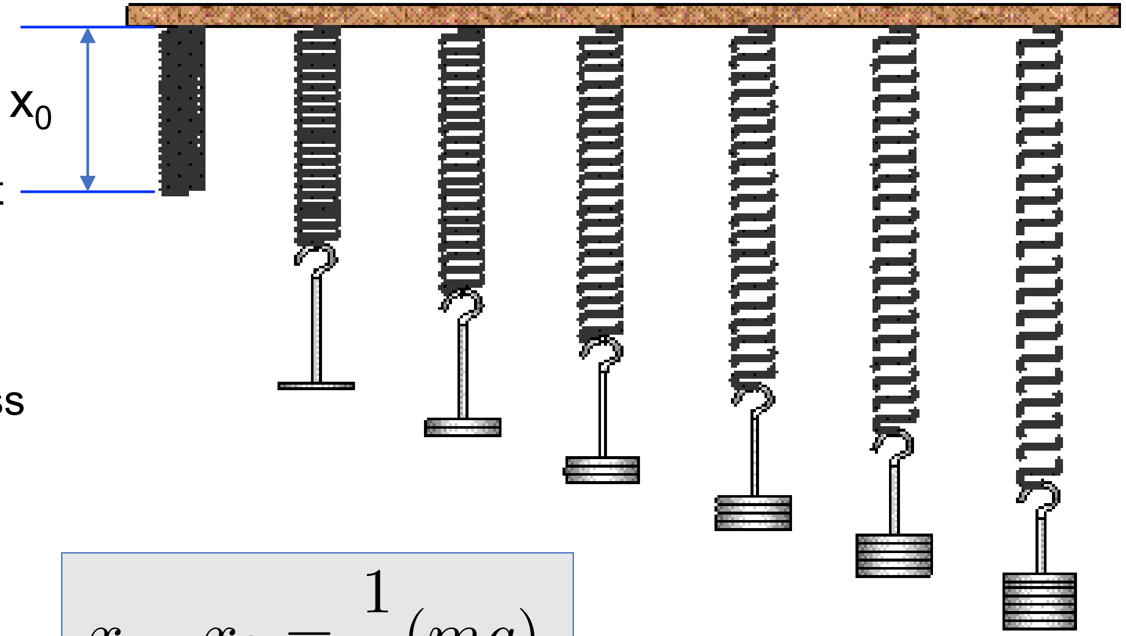
Known as Hooke's law

Not precise, but very good approximation

Can determine this by hanging weights from spring

Restoring force of spring must balance gravity

Observe stretching of spring proportional to amount of mass attached



x_0 = unstretched length

x = stretched length

$$\boxed{x - x_0 = \frac{1}{k}(mg)}$$

Amount of stretching

Spring constant

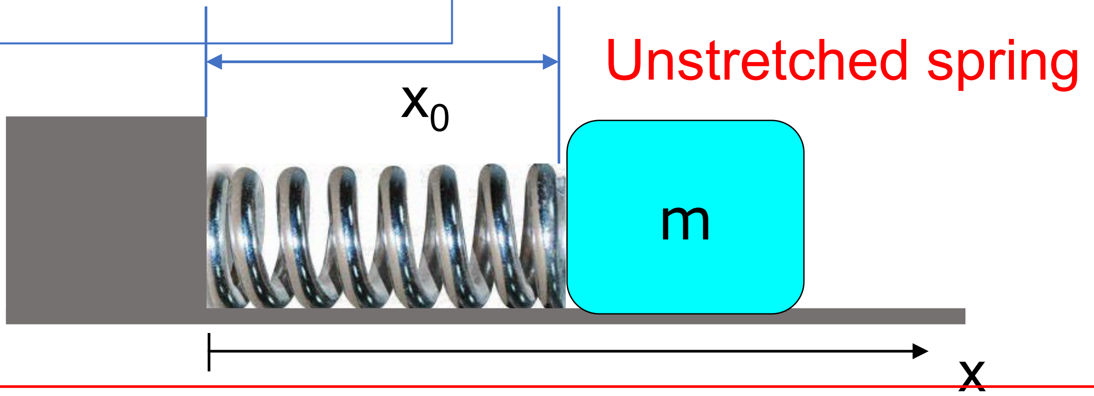
characterizes stiffness of spring

Restoring force of spring: Hooke's Law

$$F = -k(x - x_0)$$

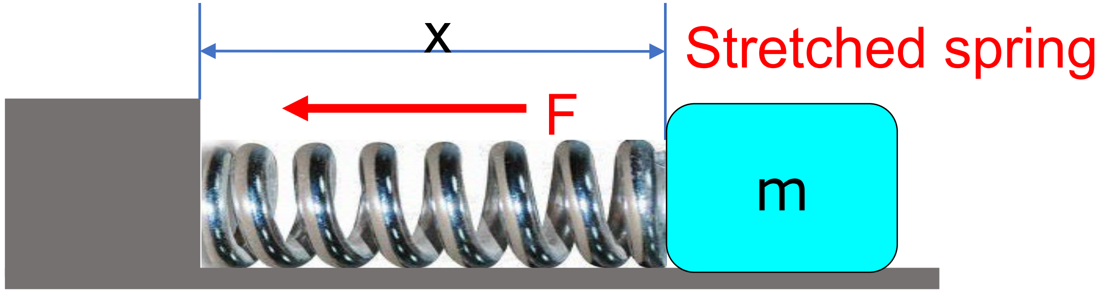
Restoring force proportional to how far spring is stretched or compressed from rest length: k = spring constant

➡ Force is directed to restore spring to rest length



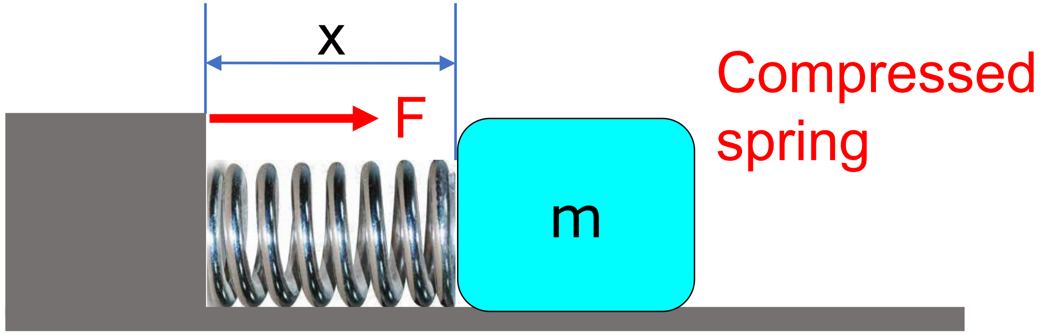
$$x - x_0 > 0$$

➡ $F < 0$ To the left



$$x - x_0 < 0$$

➡ $F > 0$ To the right



Spring Equations of Motion

An object of mass *m* moves back and forth due to a horizontal spring with spring constant *k*.

$$\begin{aligned} F &= ma \\ -k(x - x_0) &= m\frac{d^2x}{dt^2} \end{aligned}$$

"Fun" fact: This is a 2nd order differential equation
Solutions tend to be sin, cos functions.

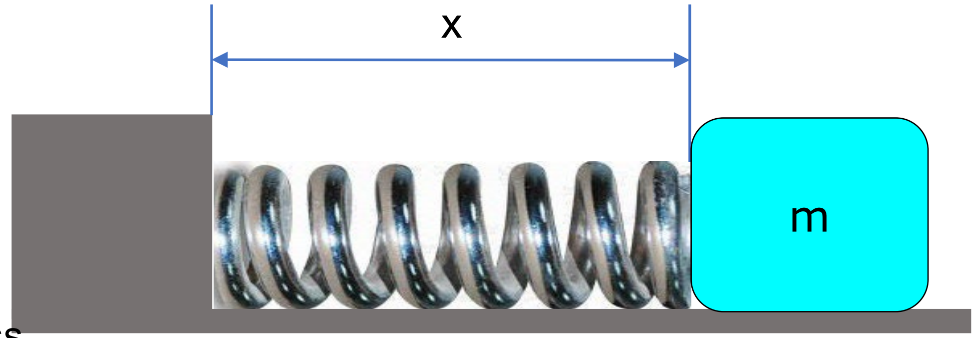
$$\begin{aligned} \text{Solution x(t)} &= x_0 + A\cos(\omega t + \phi) \\ \frac{dx}{dt} &= -\omega A \sin(\omega t + \phi) \\ \frac{d^2x}{dt^2} &= -\omega^2 A\cos(\omega t + \phi) = -\frac{k}{m}(x - x_0) \quad \text{if } \omega^2 = \frac{\text{k}}{\text{m}} \end{aligned}$$

$$\begin{aligned} -k(x - x_0) &= m\frac{d^2x}{dt^2} = -m\omega^2 A\cos(\omega t + \phi) \\ -k(x - x_0) &= -m\omega^2(x - x_0) \quad \Rightarrow \quad \omega^2 = \frac{k}{m} \end{aligned}$$

Hence frequency of oscillation depends on k, m only

Hooke's law

$$F = -k(x - x_0)$$



Use Newton's laws to analyze motion of mass attached to spring

Releasing mass attached to stretched spring leads to oscillatory behavior

$$x(t) = x_0 + A\cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

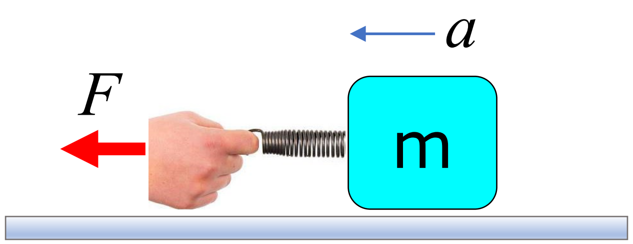
stiffer spring gives faster oscillation

larger mass gives slower oscillation

spring is stretched → accelerates left → overshoots rest position → spring is compressed, force changes direction → accelerates right → overshoots rest position → spring is stretched

An object attached to an ideal massless spring is pulled across a frictionless surface. If the spring constant is 45 N/m and the spring is stretched by 0.88 m when the object is accelerating at 2.0 m/s^2, what is the mass of the object?

A) 20 kg
B) 17 kg
C) 22 kg
D) 26 kg

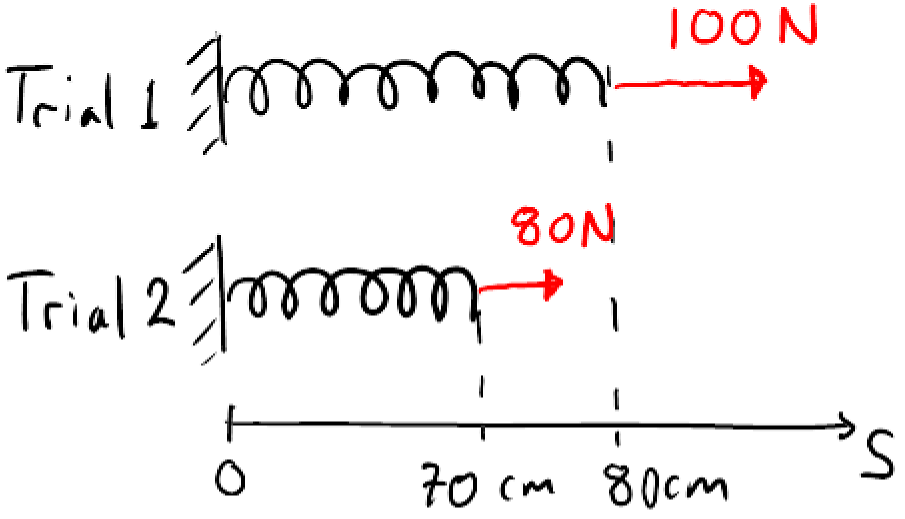


$$\text{Force exerted by spring on object:} \quad |F| = k\Delta x = ma$$

$$m = \frac{k\Delta x}{a} = \frac{45N/m \times 0.88m}{2m/s^2} = 20kg$$

Springs (Hooke's Law: $F_s = -k\Delta x$)

What is the spring constant k of the spring shown?



What is the unstretched length x_0 of the spring?

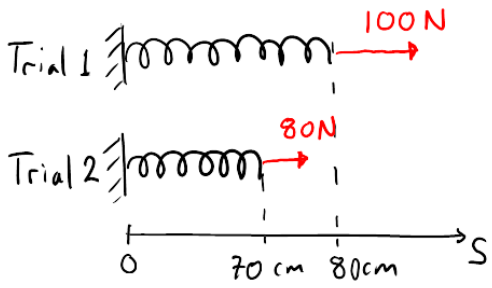
How far would a 130 N force stretch the spring?

Springs (Hooke's Law: $F_s = -k\Delta x$)

(1) What is the spring constant k of the spring shown?

(2) What is the unstretched length x_0 of the spring?

(3) How far would a 130 N force stretch the spring?



$$\begin{aligned} -100 &= -k(0.8 - x_0) \\ -80 &= -k(0.7 - x_0) \\ \Rightarrow\ 20 &= k(0.1) \quad \Rightarrow \quad k = 200\ \frac{N}{m} \end{aligned}$$

$$\begin{aligned} 100 &= 200(0.8 - x_0) \\ 0.5 &= (0.8 - x_0) \\ x_0 &= 0.3m \end{aligned}$$

$$\begin{aligned} -130 &= -200\Delta x \\ \Delta x &= 0.65m = x - x_0 = x - 0.30m \\ x &= 0.95m \end{aligned}$$

Chapter 9 – Work and Kinetic Energy
Chapter 10: Interactions and Potential Energy

Book order is slightly different! We start with **Kinetic Energy** and **Potential Energy**

Collectively known as "**Mechanical Energy**"

Natural to start by defining energy. However, we won't yet…

Energy is easy to define in this or that physical context, but it takes so many **different forms** that a general definition is difficult to formulate

Standard textbook definition

"Energy measures the ability of a system to do work on another system"

"Work" will have a technical definition soon

Inadequate in various ways

"ability" is vague and referring to "work" applies to some forms of energy better than other. Why not to use "heat" instead …

Standard textbook definition

“Energy measures the ability of a system to do work on another system”

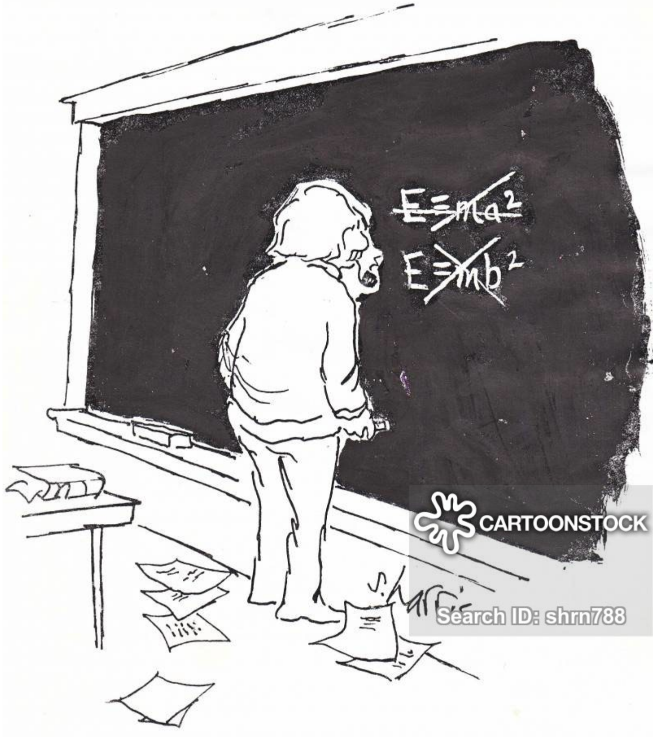
Not the best description of some forms of energy

$$E = mc^2$$

For example…

Einstein tells us that rocks at famous Ryoan-ji temple in Kyoto have energy

However, they are unlikely to do any “work” with their ***mc*²** in either the physics or more usual senses

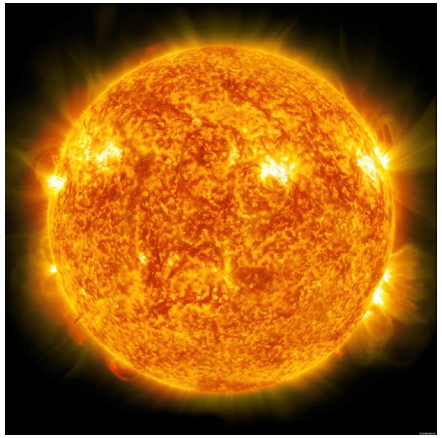


Explore key properties of energy rather than dwelling on precise definition

Examine key properties of energy rather than developing a precise definition

Energy exists in many forms

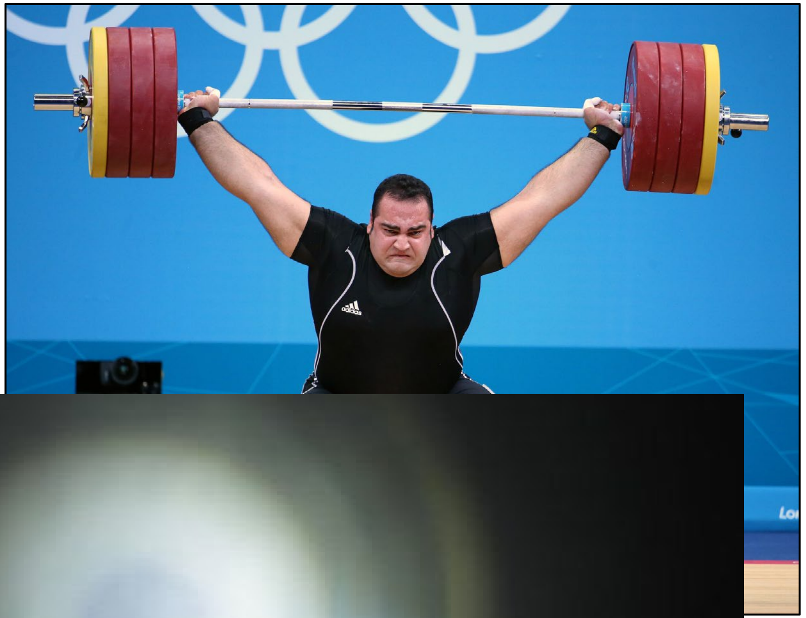
- Kinetic Energy: energy of motion
- Potential Energy: associated with position under influence of some forces, e.g. gravity
- Thermal Energy: energy stored in motion of molecules
- Electromagnetic (E&M) field energy: light carries energy
- Electricity: energy of electrons and E&M fields
- Chemical energy: e.g. in fossil fuels
- Nuclear Energy: from relativity theory ($E=mc^2$)



Energy can move between different forms

In accordance with laws of physics

A weightlifter converts chemical energy of food to mechanical energy in muscles into potential energy



A flashlight converts chemical energy in batteries into electricity and then into light

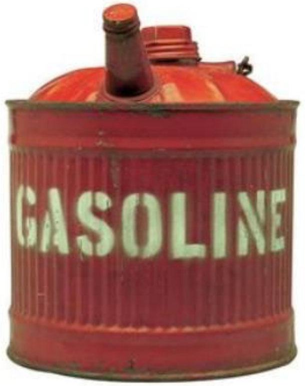
On the beach, energy in sunlight is converted to thermal energy

Energy in sunlight converted into chemical energy in leaves

As energy changes from one form to another, the total amount of energy always stays the same

Fundamental physical principle

Conservation of energy



For example

Chemical energy

Fill a car with gasoline

Drive until the car runs out of gas and stops

Where has all the energy gone?

- Hot exhaust out tailpipe of the car
- Heating the engine
- Heating the road via friction
- Moving around air as car passes

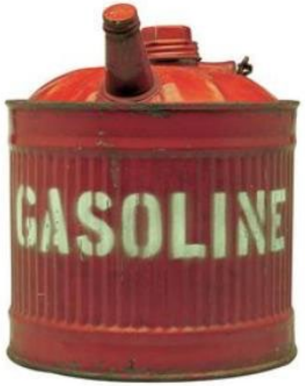
Difficult in practice to do the detailed accounting

But since laws of physics conserve energy, we know all the energy is still there in some form

Mostly in thermal energy

As energy changes from one form to another, the total amount of energy always stays the same

Conservation of energy



As energy changes from one form to another it always becomes less useful

2^{nd} law of thermodynamics

Technical statement is in terms of entropy encrease

In car example…

Even though energy is conserved and all the energy originally in the gas is somewhere

It can never be reassembled back into a form as useful as its original form

“It's like when you’re standing on a rooftop and you shake all the feathers out of a pillow, and then the next day you say,
‘I want to get all those feathers back.’”

-Eduardo from NBC’s AP Bio, paraphrasing the movie Doubt (2008)

Focus on “Mechanical Energy”:
Kinetic Energy and Potential Energy

Kinetic energy K: of object with velocity $\vec{v}$

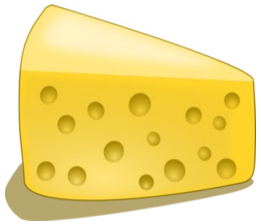
$$K = \frac{1}{2}mv^2$$

$$v = |\vec{v}|$$

Example

The 100kg block of cheese is moving along at 5m/s

What is its kinetic energy?



100kg

5 m/s

$$K = \frac{1}{2}(100kg)(5m/s)^2 = 1250\frac{kg\,m^2}{s^2} = 1250J$$

SI unit of energy

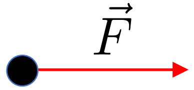
$$1J = 1Joule = 1\frac{kg\,m^2}{s^2}$$

Dimensions of energy

$$\frac{(mass)(distance)^2}{(time)^2}$$

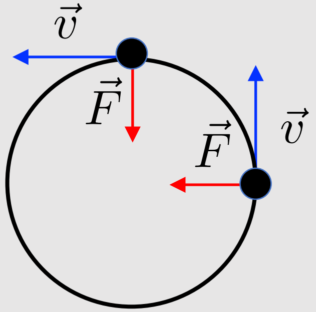
Work: Related to changes in kinetic energy (Chapter 9)

Consider an object acted on by a single force



The object will accelerate and its kinetic energy *can* change

Recall that acceleration can be a change in direction, with no change in speed or kinetic energy



uniform circular motion

"Work" done by force equals change in kinetic energy of object $$W = \Delta K$$

Units of work = units of energy, Joules

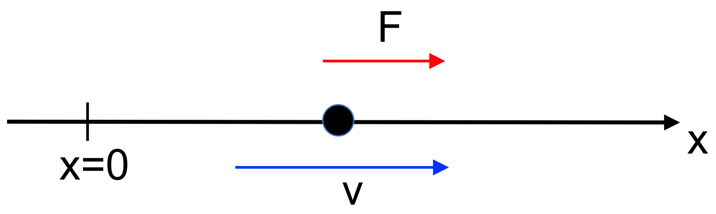
"Work" depends on relative orientation of force and direction of motion

Circular motion: force perpendicular to direction of motion does no work (KE constant)

Only component of force in direction of motion will do work

Compute **work** for object moving in 1D

Object is acted on by constant force F and is initially moving with velocity v



Compute work done in small time interval Δt over which speed changes to

$$v' = v + \Delta v$$

and kinetic energy changes to

$$K' = K + \Delta K$$

Assume Δt small enough so that

$$|\Delta v| \ll |v|$$

Ignore $(\Delta v)^2$ terms

Work done by force equals change in kinetic energy

$$W = \Delta K$$

$$K' = \frac{1}{2}m(v+\Delta v)^2 \simeq \frac{1}{2}m(v^2 + 2v\Delta v) \simeq K + mv\Delta v$$

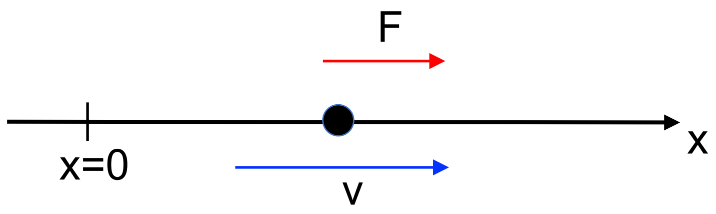
$$W = mv\Delta v = mv(a\Delta t) = mv(\frac{F}{m}\Delta t) = F\Delta x$$

acceleration

using F=ma

using Δx=vΔt

Compute **work** for object moving in 1D



Object is acted on by force F and is initially moving with velocity v

Compute work done in small time interval Δt over which speed changes to

$$v' = v + \Delta v$$

Assume Δt small enough so that

$$|\Delta v| \ll |v|$$

Ignore $(\Delta v)^2$ terms

and kinetic energy changes to

$$K' = K + \Delta K$$

Work done by force equals change in kinetic energy

$$W = \Delta K$$

1D Work done over small time Δt equals force times displacement

$$W = F\Delta x$$

Work is positive if force is in same direction as displacement => System is doing work on object

Work is negative if force is in opposite direction to displacement => Object doing work on system

Work in 3D

1D Work done over small time Δt equals force times displacement

$$W = F\Delta x$$

Result in 3D simply adds in y and z components of force and displacement

$$W = F_x\Delta x + F_y\Delta y + F_z\Delta z$$
$$= \vec{F}\cdot\Delta\vec{x}$$

Each component of force vector multiplies corresponding of displacement vector

Can be written more compactly in terms of the "dot product"

Take a few slides to focus on dot product

Dot product of vectors

"Cross product" is way of multiplying two vectors to get another vector (e.g. angular momentum in chapter 12)

Mathematically "multiplying" two vectors to get a number

$$\vec{u} = u_x\hat{x} + u_y\hat{y} + u_z\hat{z}$$

$$\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

Dot product $\boxed{\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z}$

provides a measure of how well two vectors are aligned

Examples

$$\vec{u} = 3\hat{x} - 4\hat{y} + 9\hat{z}$$

$$\vec{v} = 5\hat{x} + 5\hat{y} - 5\hat{z}$$

$$\Rightarrow \quad \vec{u}\cdot\vec{v} = 15 - 20 - 45 = -50$$

$$\vec{u} = 5\hat{x}$$

$$\vec{v} = -2\hat{y}$$

$$\Rightarrow \quad \vec{u}\cdot\vec{v} = 0$$

dot product of orthogonal vectors is zero

Dot product

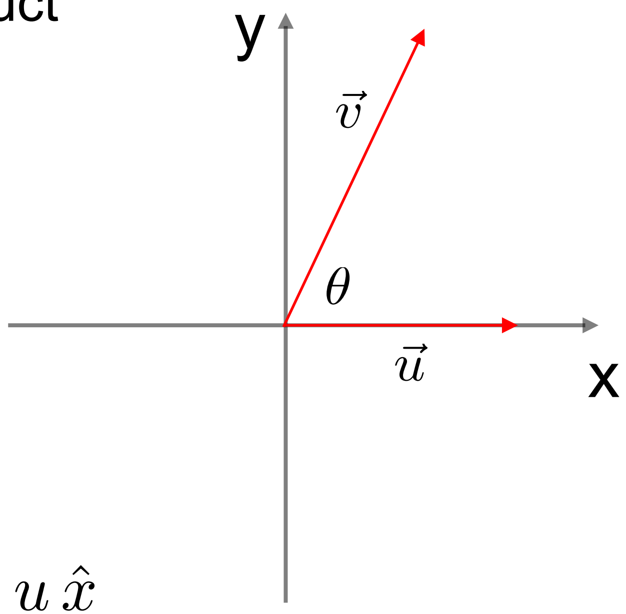
$$\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z$$

Fundamental property → Dot product doesn't change if we rotate both vectors in same way

Get a more intuitive expression for dot product by using this property

Rotate both vectors into xy-plane with one vector aligned in x-direction

Can always do this



Let $u = |\vec{u}|$

$v = |\vec{v}|$

Component forms of vectors are then $\vec{u} = u\,\hat{x}$

$$\vec{v} = v\cos\theta\,\hat{x} + v\sin\theta\,\hat{y}$$

→ $$\boxed{\vec{u}\cdot\vec{v} = uv\cos\theta}$$

Involves the component of one vector in the direction of the other

Alternative expressions for dot product

$$\boxed{\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z}$$

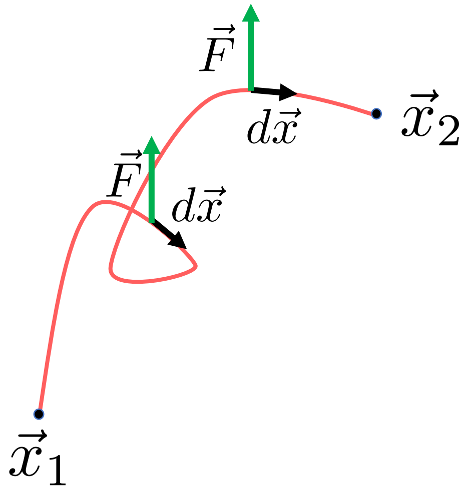
$$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}|\,\cos\theta}$$

Back to “Work”

Work done by a single force on an object that move through a small displacement

$$W = \vec{F} \cdot \Delta\vec{x}$$

Only involves component of force in the direction of motion



To compute work for a general force and path, need to integrate

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} \cdot d\vec{x}$$

known as a line integral in vector calculus

Deal with simple cases

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} \cdot d\vec{x}$$

Motion in a fixed direction (e.g. x-direction) with a ***constant force***

$$d\vec{x} = (dx)\hat{x}$$

$$\vec{F} \cdot d\vec{x} = F_x \, dx$$

$$W = \int_{x_1}^{x_2} F_x \, dx = F_x(x_2 - x_1)$$

Multiple forces acting on object $\vec{F}_i$ $\quad i = 1, 2, \ldots, n$

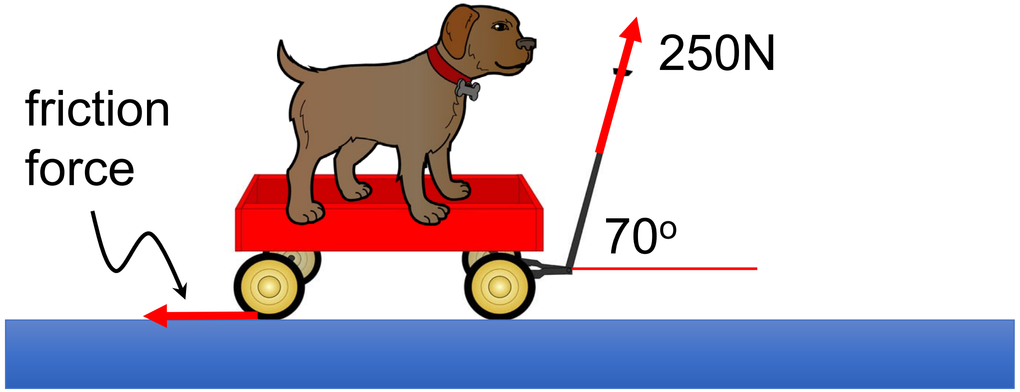
Work associated with each force $W_i = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F}_i \cdot d\vec{x}$

Change in kinetic energy equals total work done on object by all forces

$$\Delta K = W_1 + \cdots + W_n = W_{tot}$$

Example

A dog in a wagon is pulled through a distance of 50m by a constant force of 250N applied at 70° with respect to horizontal



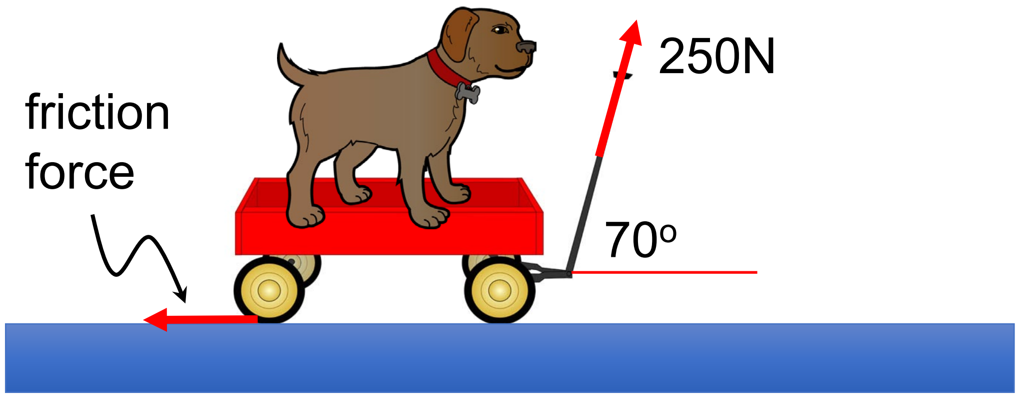
How much work is done by the applied force?

$$W_1 = F_x(x_2 - x_1) \quad *$$

If the wagon is moving at constant velocity, how much work is done by friction in this process?

Example

A dog in a wagon is pulled through a distance of 50m by a constant force of 250N applied at 70° with respect to horizontal



How much work is done by the applied force?

$$W_1 = F_x(x_2 - x_1) = (250N)(\cos 70^o)(50m) = 4300J$$

work has the same units as energy

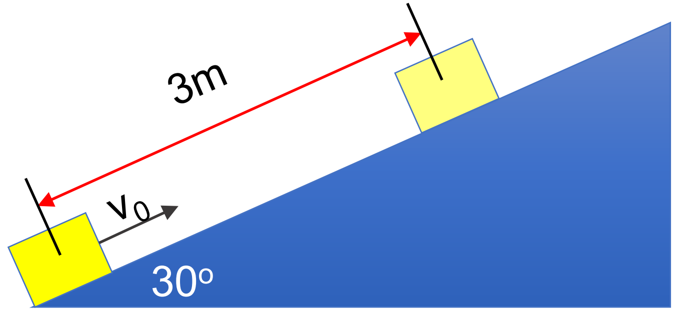
If the wagon is moving at constant velocity, how much work is done by friction in this process?

No change in kinetic energy $\Rightarrow$ work from all forces on wagon must total zero

Work done by friction $W_2 = -4300J$ so that $W_1 + W_2 = 0$

Example

A 2kg block is given a shove up a frictionless inclined plane and comes to rest after sliding 3 m along the plane



How much work does gravity do on the block in bringing it to rest?

Work done by gravity must equal (minus) the initial kinetic energy of the block

Initial kinetic energy $$K_i = \frac{1}{2}mv_0^2$$

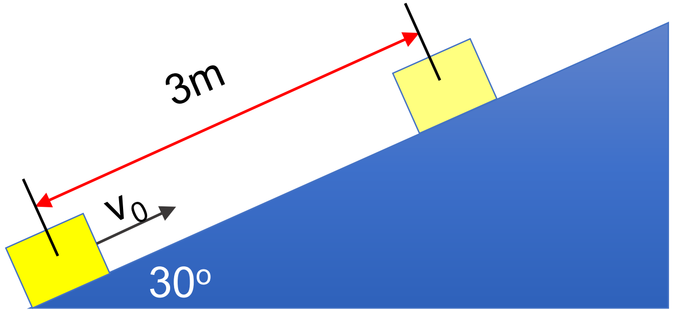
Need to find v_0

Final kinetic energy $$K_f = 0$$

Work $$W = \Delta K = K_f - K_i = -\frac{1}{2}mv_0^2$$

Example

A 2kg block is given a shove up a frictionless inclined plane and comes to rest after sliding 3 m along the plane



How much work does gravity do on the block in bringing it to rest?

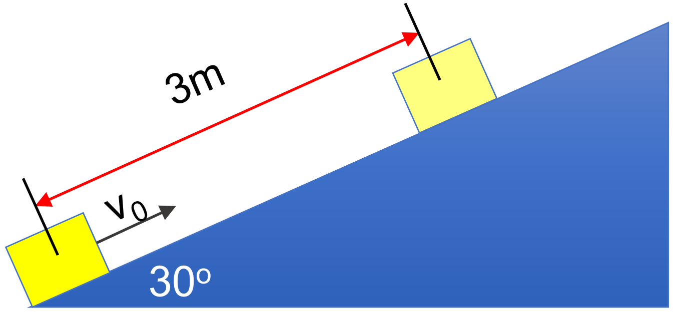
Work done by gravity must equal (minus) the initial kinetic energy of the block

$$W = -\frac{1}{2}mv_0^2$$

Use conservation of energy to find initial kinetic energy *

Example

A 2kg block is given a shove up a frictionless inclined plane and comes to rest after sliding 3 m along the plane



How much work does gravity do on the block in bringing it to rest?

Work done by gravity must equal (minus) the initial kinetic energy of the block

$$W = -\frac{1}{2}mv_0^2$$

Use conservation of energy to find initial kinetic energy

$$E_i = \frac{1}{2}mv_0^2 = mgh = E_f$$

final height

$$h = (3m)\sin 30^o = 1.5m$$

$$\frac{1}{2}mv_0^2 = (2kg)(9.8m/s^2)(1.5m) = 29.4J$$

$$W = -29.4J$$

=work done by gravity
=$F_{gravity}$ x(x_{final}-$x_{initial}$) = -mgsin(30) x (3 m) = -24.9 J

Conceptual Problem 11.4

Part A

Three cars with identical engines and tires start from rest, and accelerate at their maximum rate. Car X is the most massive, and car Z is the least massive. Which car needs to travel the farthest before reaching a speed of 60 mi/h?

Idea is that all 3 cars can exert the **same force** to accelerate the car

Heaviest car (X) will have the biggest final kinetic energy

Will need the most work done by engine to reach this kinetic energy

Work = change in kinetic energy
= (force) x (distance)

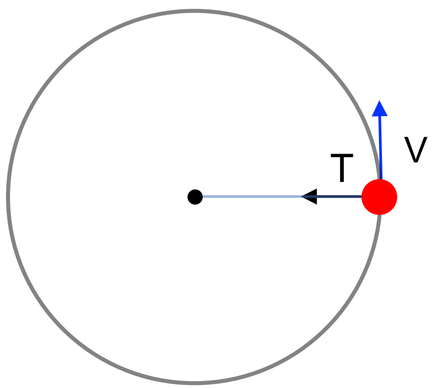
→ Car X needs to travel the farthest distance to reach speed of 60mph

A ball of mass m swings around a circle of radius R with constant angular velocity ω

A rope attached to the center provides the centripetal force

a) What is the tension in the rope?

b) How much work does the rope do as the ball swings through ¼ of a revolution?



Tension ➡ $T = m\omega^2 R$

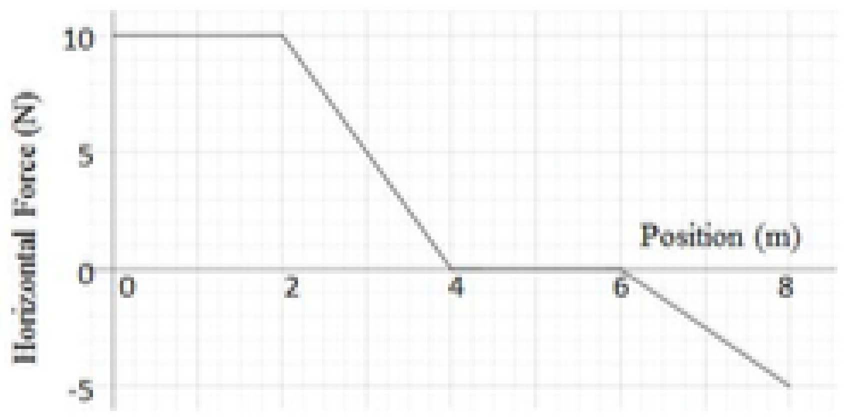
Work ➡ $W = 0$

Because tension force is always perpendicular to direction of motion

-KE is constant, no net work done

Non-constant force

A 5.0 kg toolbox is initially at rest on the floor in the back of a large truck as the truck starts driving straight along a flat road. The only horizontal force that the toolbox feels comes from static friction, due to static friction with the truck bed. That horizontal force (from the truck) is indicated on the graph for the first 8 m of a very short trip. How fast is the toolbox moving after 8 m?



Go back to expression for work in 1D

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Work = area under force vs position curve

Need to add up areas for different parts of curve. Then determine speed from KE.

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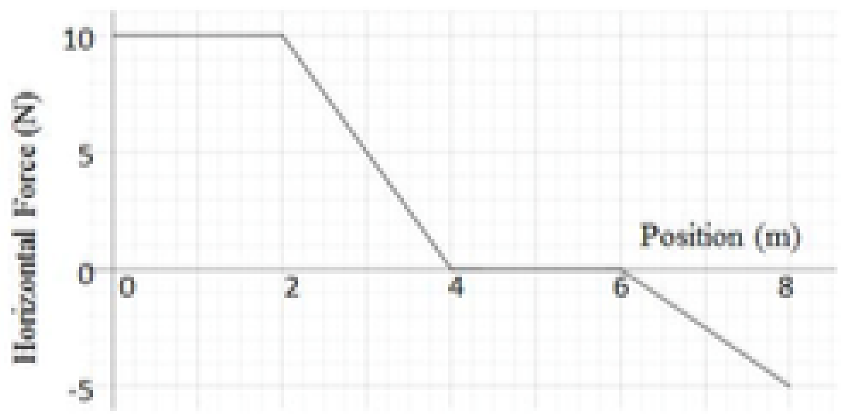

Go back to expression for work in 1D

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Work = area under force vs position curve

Need to add up areas for different parts of curve

$$W = (10N)(2m) + \frac{1}{2}(10N)(2m) + 0 + \frac{1}{2}(-5N)(2m)$$

$$= 25Nm = 25J$$

Non-constant force

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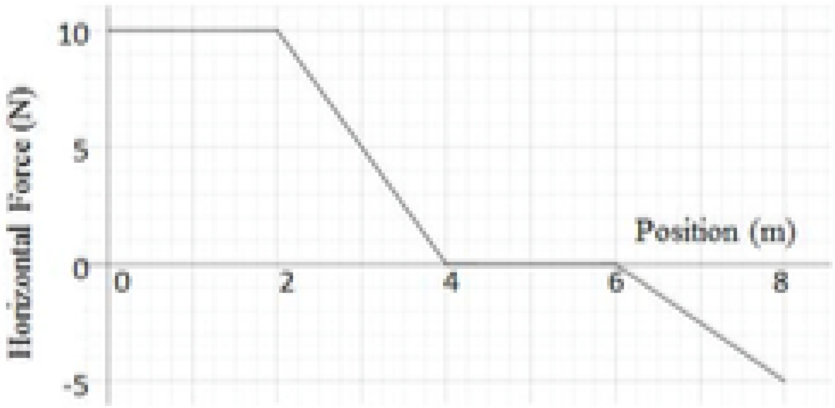


$$W = 25J$$

Work gives change in kinetic energy

$$\Delta K = K_f - K_i = W \quad \Rightarrow \quad K_f = \frac{1}{2}mv_f^2 = 25J$$

Gives $$v_f = \sqrt{\frac{2(25J)}{5kg}} = 3.2m/s$$

12) Three cars (car *F*, car *G*, and car *H*) are moving with the same velocity when the driver suddenly slams on the brakes, locking the wheels. The most massive car is car *F*, the least massive is car *H*, and all three cars have identical tires.

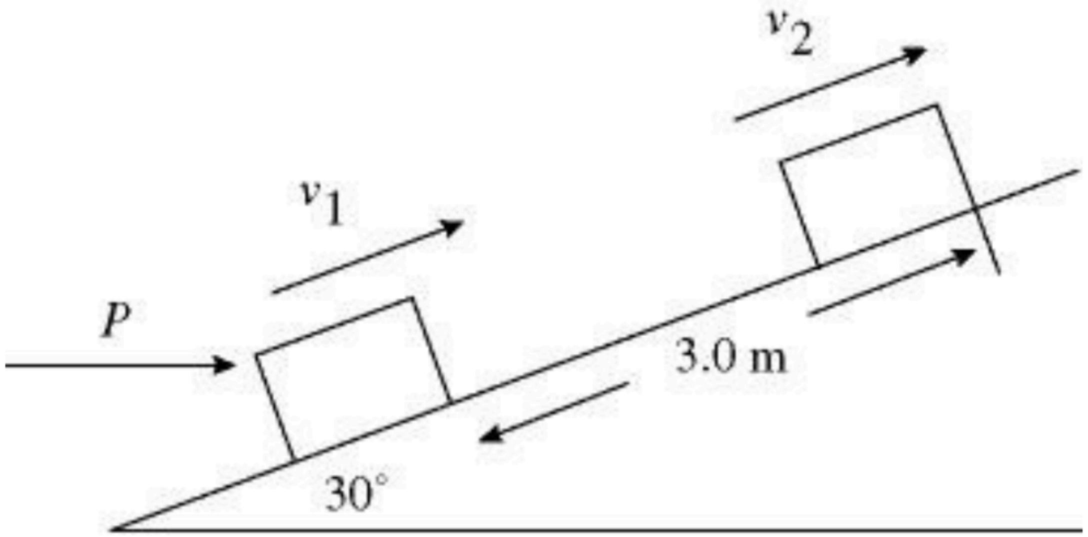
(a) Which car travels the longest distance to skid to a stop?

A) Car *F*
B) Car *G*
C) Car *H*
D) They all travel the same distance in stopping.

(b) For which car does friction do the largest amount of work in stopping the car?

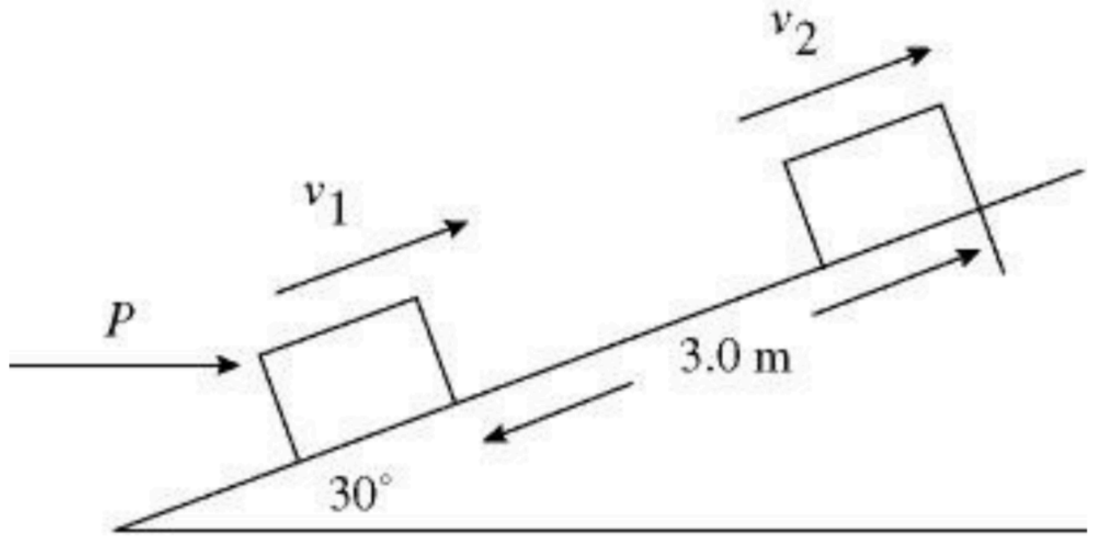
A) Car *F*
B) Car *G*
C) Car *H*
D) The amount of work done by friction is the same for all cars.

19) In the figure, a 700-kg crate is on a rough surface inclined at 30°. A constant external force P = 5600 N is applied horizontally to the crate. As the force pushes the crate a distance of 3.00 m up the incline, the speed changes from 1.40 m/s to 2.50 m/s. How much work does gravity do on the crate during this process?



A) -10,300 J
B) -3400 J
C) +10,300 J
D) +3400 J
E) zero

19) In the figure, a 700-kg crate is on a rough surface inclined at 30°. A constant external force $P = 5600$ N is applied horizontally to the crate. As the force pushes the crate a distance of 3.00 m up the incline, the speed changes from 1.40 m/s to 2.50 m/s. How much work does gravity do on the crate during this process?



A) -10,300 J
B) -3400 J
C) +10,300 J
D) +3400 J
E) zero

$$\begin{aligned} W &= \vec{F} \cdot \Delta\vec{x}, \quad \Delta\vec{x} = 3.0m \times \sin(30°)\hat{y} \\ &= (-mg\hat{y}) \cdot (3.0m \sin(30°)\hat{y}) \\ &= -700kg \times (9.8m/s^2) \times 1.5\, m \\ &= -10,300\, N \end{aligned}$$

Summary: Work in 3D, Dot Product, Relation to Kinetic Energy

Work:

Work done over small time Δt equals force times displacement in 1D

$$\boxed{W = F\Delta x}$$

In 3D add in y and z components

$$W = F_x\Delta x + F_y\Delta y + F_z\Delta z$$
$$= \vec{F}\cdot\Delta\vec{x}$$

Each component of force vector multiplies the corresponding component of displacement vector

Dot Product:

$$\boxed{\vec{u}\cdot\vec{v} = u_x v_x + u_y v_y + u_z v_z}$$

$$\boxed{\vec{u}\cdot\vec{v} = |\vec{u}|\,|\vec{v}|\,\cos\theta}$$

Work from many forces i:

$$W_i = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F}_i\cdot d\vec{x}$$

Change in Kinetic Energy from Work :

Change in kinetic energy equals total work done on object by all forces

$$\boxed{\Delta K = W_1 + \cdots + W_n = W_{tot}}$$