Names: Aidan Chin
Thinh Truong
Mion fujie

COMPSCI 250 Discussion #1: What is a Proof?
Group Response Sheet

David Mix Barrington and Mordecai Golin
9 February 2024

Writing Exercises:

1. Prove that if x is an even natural, x's predecessor (if it has one) is odd.

$$x = 2y$$
$$x \neq \emptyset$$
$$x - 1 = 2y - 1$$
$$= 2(y-1) + 1$$
$$x - 1 \text{ is odd}$$

2. Prove that if x is an odd natural, x's predecessor is even.

$$x = 2y + 1 \qquad x \neq \emptyset$$
$$x - 1 = 2y + 1 - 1$$
$$x - 1 = 2y$$
$$x - 1 \text{ is even}$$

3. Prove that if x's predecessor is odd, then x is even.

$$x - 1 = 2y + 1 \qquad x \neq \emptyset$$
$$x = 2y + 2$$
$$x = 2(y+1)$$
$$x \text{ is even}$$

4. Prove that if x's predecessor is even, then x is odd.

$$X - 1 = 2y \qquad X = \emptyset$$
$$X = 2y + 1$$
$$X \text{ IS odd}$$

5. Prove that every natural is either odd or even. (**Hint**: By the Least Number Axiom, if any natural is *neither* odd *nor* even, there's a least such natural. Could it be 0? If not, what about its predecessor? Use the results of (1) – (4) to get a contradiction.)

$1 = $ odd

predecessor of odd must be even

therefore $1 - 1 = $ even , $0 = $ even smaller

Assume natural neither odd or even $\leftarrow x$ / even

x cannot be 0 because $0 = 2(0)$ is even.

predecessor $x - 1 \rightarrow$ must be odd / even

$\downarrow x$ is the least number that is neither

$x - 1 < x$ by (2) if $x - 1$ is even then x is odd

and vice versa Contradiction no natural exists

6. Prove that no natural is both odd and even. (Similar to (5) – get a contradiction by assuming some natural is both.)

x cannot be 0 because 0 is not odd

if it were equal to $2z + 1$ (definition of odd)

then $2z$ is the predecessor which doesn't

exist.

x has predecessor $x - 1$

By (2) $x - 1$ is even because x is odd and

$x - 1$ is also odd because x is even

$\downarrow x - 1$ both even and odd therefore

contradicts