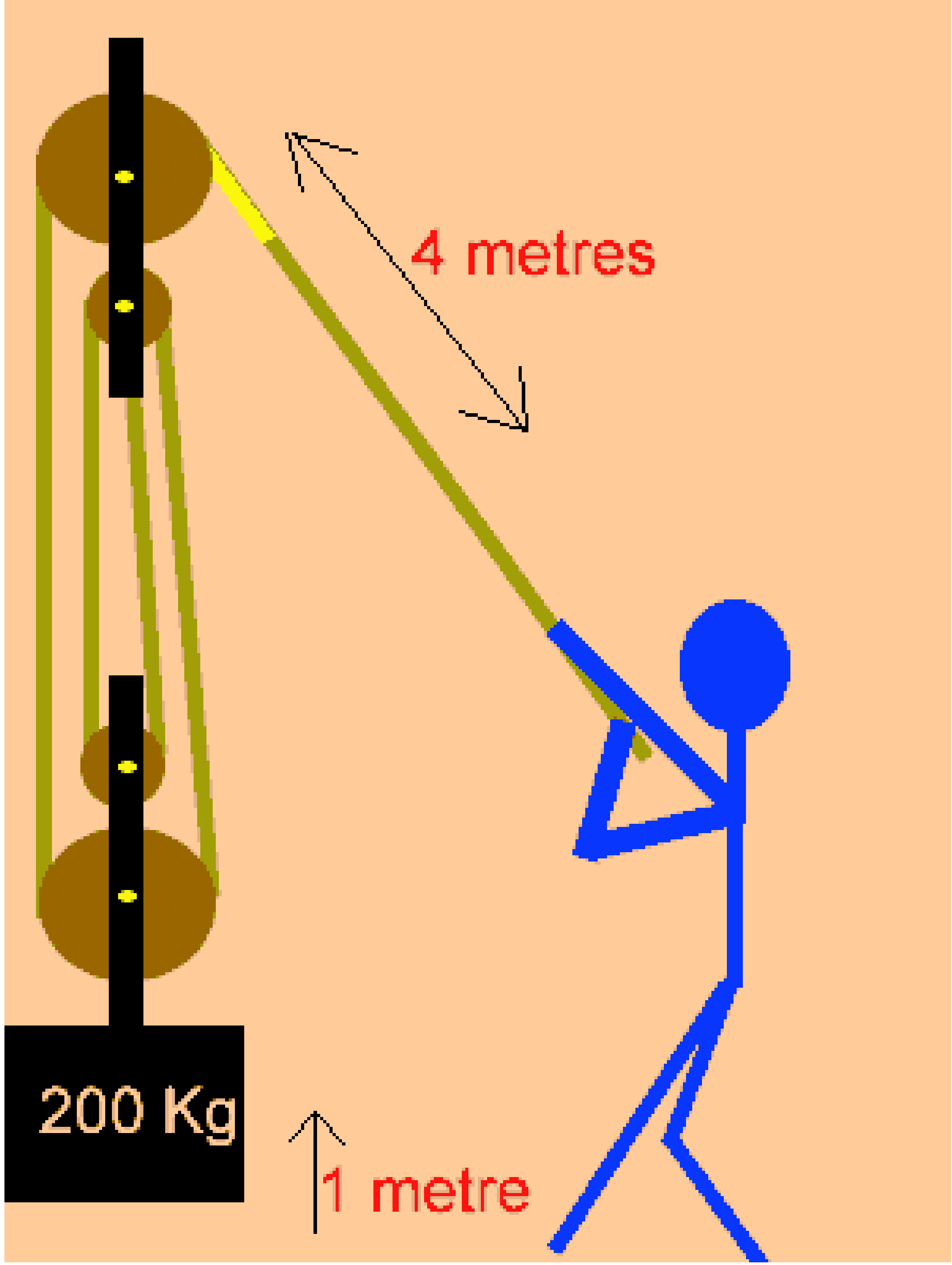

4 metres
200 Kg
1 metre

Announcements, Goals, and Reading

Announcements:

- HW06 due ***yesterday.*** 72hr Grace period applies.
- HW07 due Tuesday 11/1
- MT1 solutions posted to Moodle under week 8.

Goals for Today:

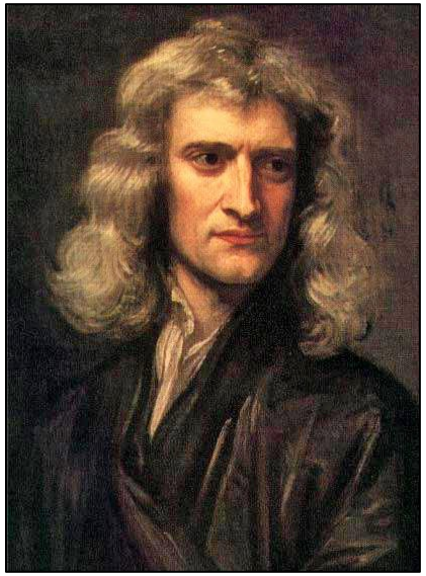
- Newton's 3rd law
- Pulleys

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 7: Newton's 3rd Law

Chapter 6: Newton's Third Law

3rd Law – When two objects interact, they exert equal and opposite forces on each other

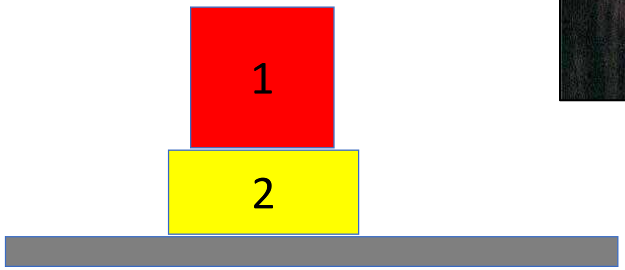


Focus on interactions between objects

Simple examples

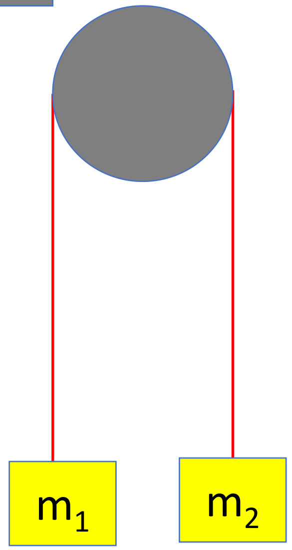
- Contact interactions

  Force of block 1 on block 2 and vice-versa



- Interactions via ropes and pulleys

  e.g. "Atwood's machine"



$$F_{12} = -F_{21}$$

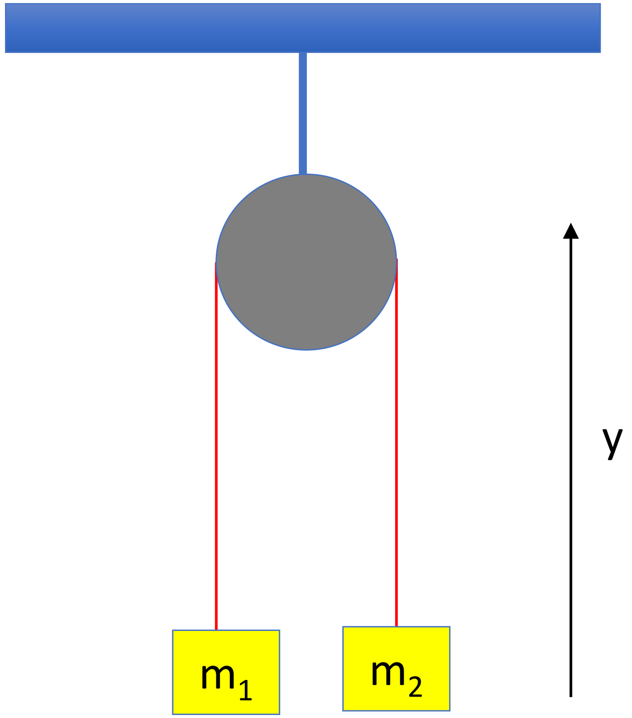
Atwood's machine

Two masses are connected by a massless rope over a frictionless pulley

Find the tension in the rope and the acceleration of the masses

Note that if m_1 is accelerating down, then m_2 is accelerating up at the same rate, and vice-versa*

$$a_2 = -a_1$$



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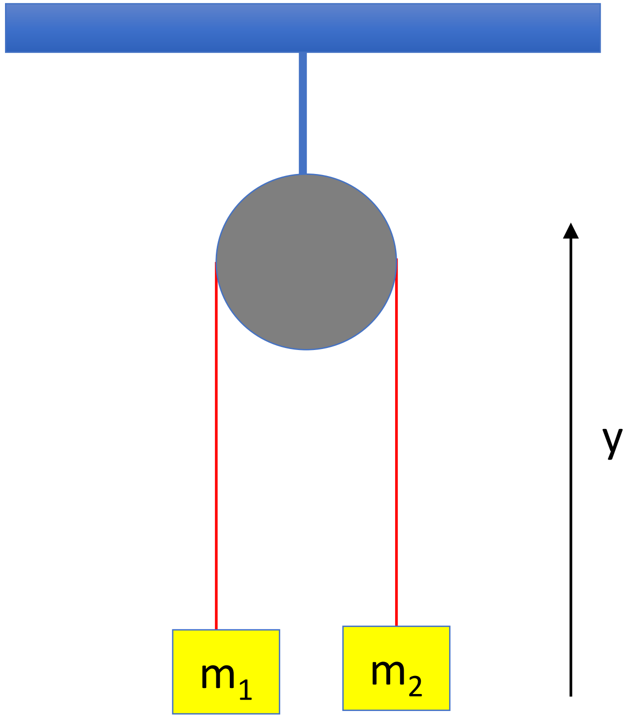
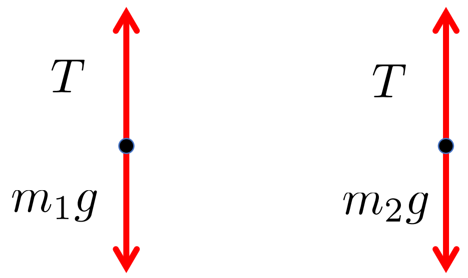
Find the tension in the rope and the acceleration of the masses



Note that if m_1 is accelerating down, then m_2 is accelerating up at the same rate, and vice-versa

$$a_2 = -a_1$$

Draw free body diagrams for both masses



2nd law…

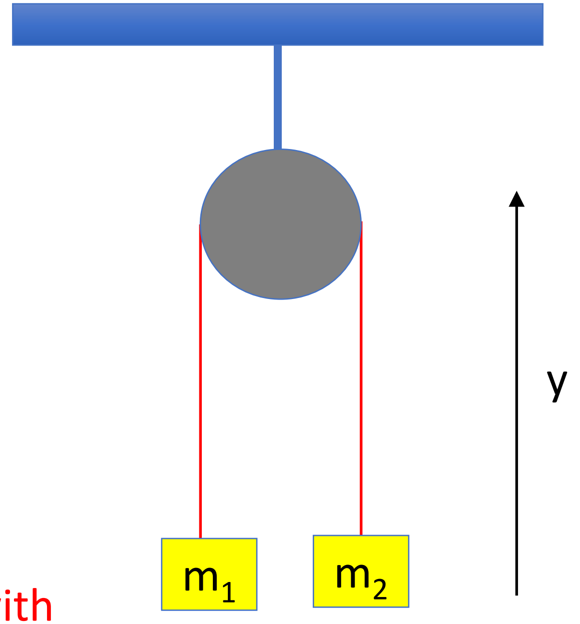
$$T - m_1 g = m_1 a_1$$

$$T - m_2 g = m_2 a_2 = m_2(-a_1)$$

Atwood's machine

Two masses are connected by a massless rope over a frictionless pulley

Find the tension in the rope and the acceleration of the masses



$$a_2 = -a_1$$

2nd law…

$$\boxed{\begin{aligned} T - m_1 g &= m_1 a_1 \\ T - m_2 g &= -m_2 a_1 \end{aligned}}$$

2 equations with 2 unknowns

Solve 1st equation for T $\rightarrow$ $T = m_1(g + a_1)$

Substitute into 2nd equation (and rearrange) $\rightarrow$ $(m_1 - m_2)g = -(m_1 + m_2)a_1$

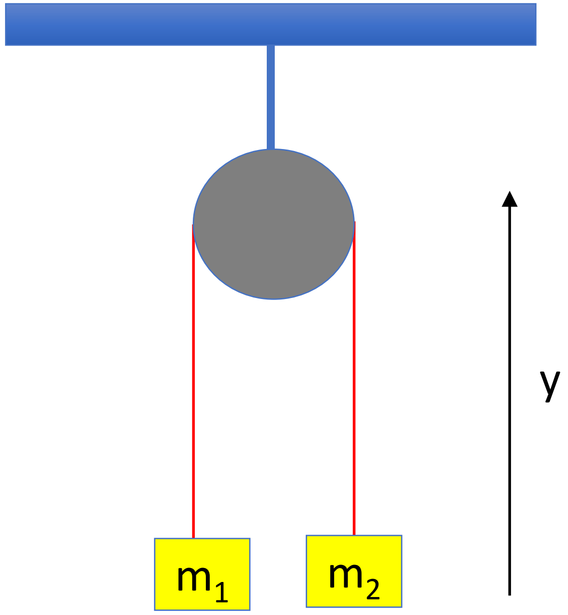
$$\rightarrow \boxed{a_1 = \frac{(m_2 - m_1)g}{m_2 + m_1}}$$

m_1 goes up if m_2 is bigger

Atwood's machine

Two masses are connected by a massless rope over a frictionless pulley

Find the tension in the rope and the acceleration of the masses



$$a_2 = -a_1$$

$$T = m_1(g + a_1) \qquad a_1 = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

Plug back in to get tension…

$$\boxed{T = \frac{2m_1 m_2}{m_1 + m_2} g}$$

Check for balanced masses

$$m_1 = m_2 = m$$

$$\Rightarrow \quad T = mg \ \checkmark$$

Friction and Tension

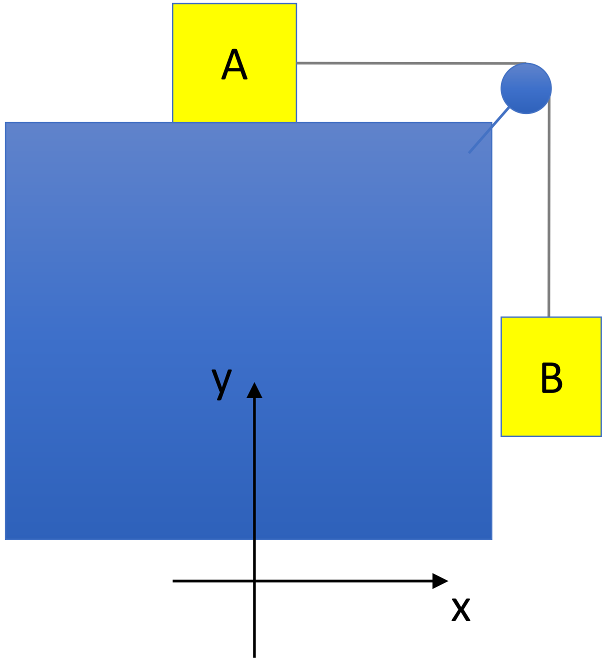
Blocks A and B are connected by a massless rope over a frictionless pulley

Blocks have masses m_A and m_B

Block A is on table with coefficient of kinetic friction μ_k

Find acceleration of the blocks and tension in the rope

Draw free body diagrams and use 2nd law



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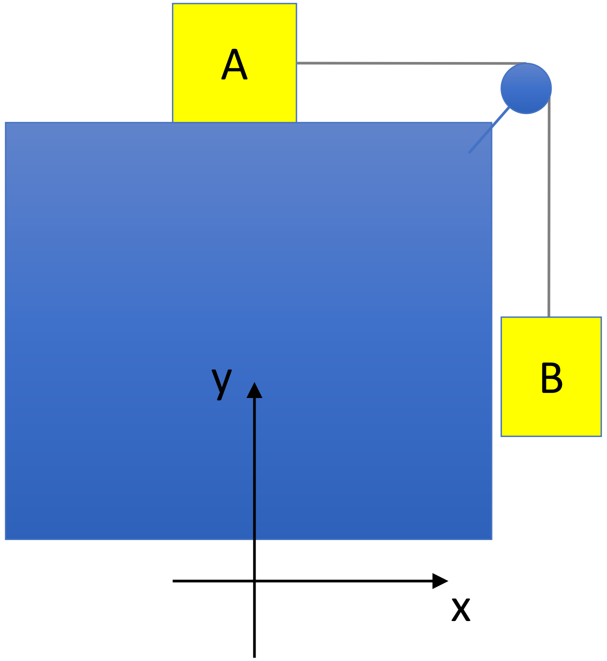
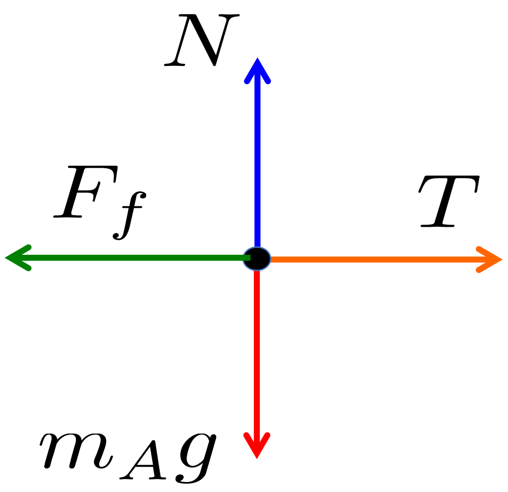


Find acceleration of the blocks and tension in the rope

Draw free body diagrams and use 2nd law

Block A $\vec{F}_{net} = (T - F_f)\,\hat{i} + (N - m_A g)\hat{j} = m_A a_A\,\hat{i}$

No acceleration in y direction for block A



y-component $\Rightarrow$ $N = m_A g$

Friction force $\Rightarrow$ $F_f = \mu_k N = \mu_k m_A g$

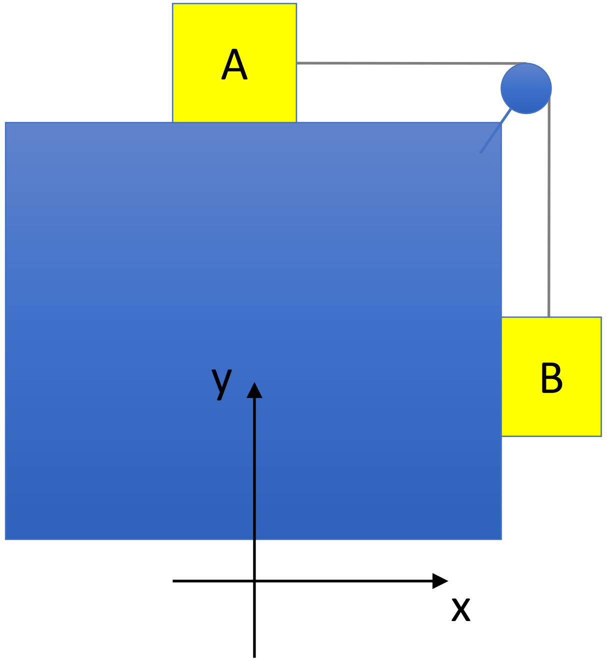
x-component $\Rightarrow$ $T - \mu_k m_A g = m_A a_A$

Friction and Tension

Blocks A and B are connected by a massless rope over a frictionless pulley

Blocks have masses m_A and m_B

Block A is on table with coefficient of kinetic friction μ_k

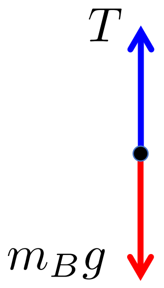


Find acceleration of the blocks and tension in the rope

Draw free body diagrams and use 2nd law

Block A $\quad T - \mu_k m_A g = m_A a_A$

Block B $\quad \vec{F}_{net} = (T - m_B g)\,\hat{j} = m_B a_B\,\hat{j}$

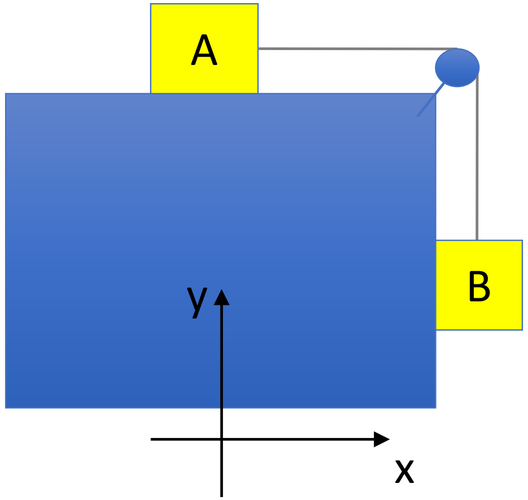


Connection of A to B via rope $\quad a_B = -a_A$

If A moves to the right, then B moves down

$$T - m_B g = -m_B a_A$$

Friction and Tension



Blocks A and B are connected by a massless rope over a frictionless pulley

Blocks have masses m_A and m_B

Block A is on table with coefficient of kinetic friction μ_k

Find acceleration of the blocks and tension in the rope

Draw free body diagrams and use 2nd law

Block A ➡ $$T - \mu_k m_A g = m_A a_A$$

Block B ➡ $$T - m_B g = -m_B a_A$$

2 equations in
2 unknowns

Solve 1st equation for T in terms of a_A and substitute into 2nd equation

➡ $$T = m_A(\mu_k g + a_A)$$

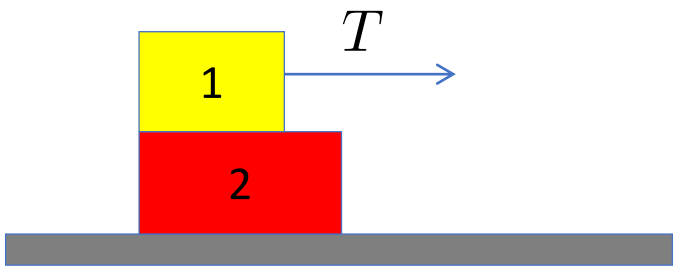
➡ $$\boxed{a_A = \frac{m_B g - \mu_k m_A g}{m_A + m_B}}$$

$$\boxed{T = \frac{m_A m_B}{m_A + m_B}(1 + \mu_k)g}$$

As if *a* = *F*$_{net}$ / *m*$_{net}$

Friction and Newton's 3rd Law

2 blocks with masses
m_1 = 4kg and m_2 = 3kg



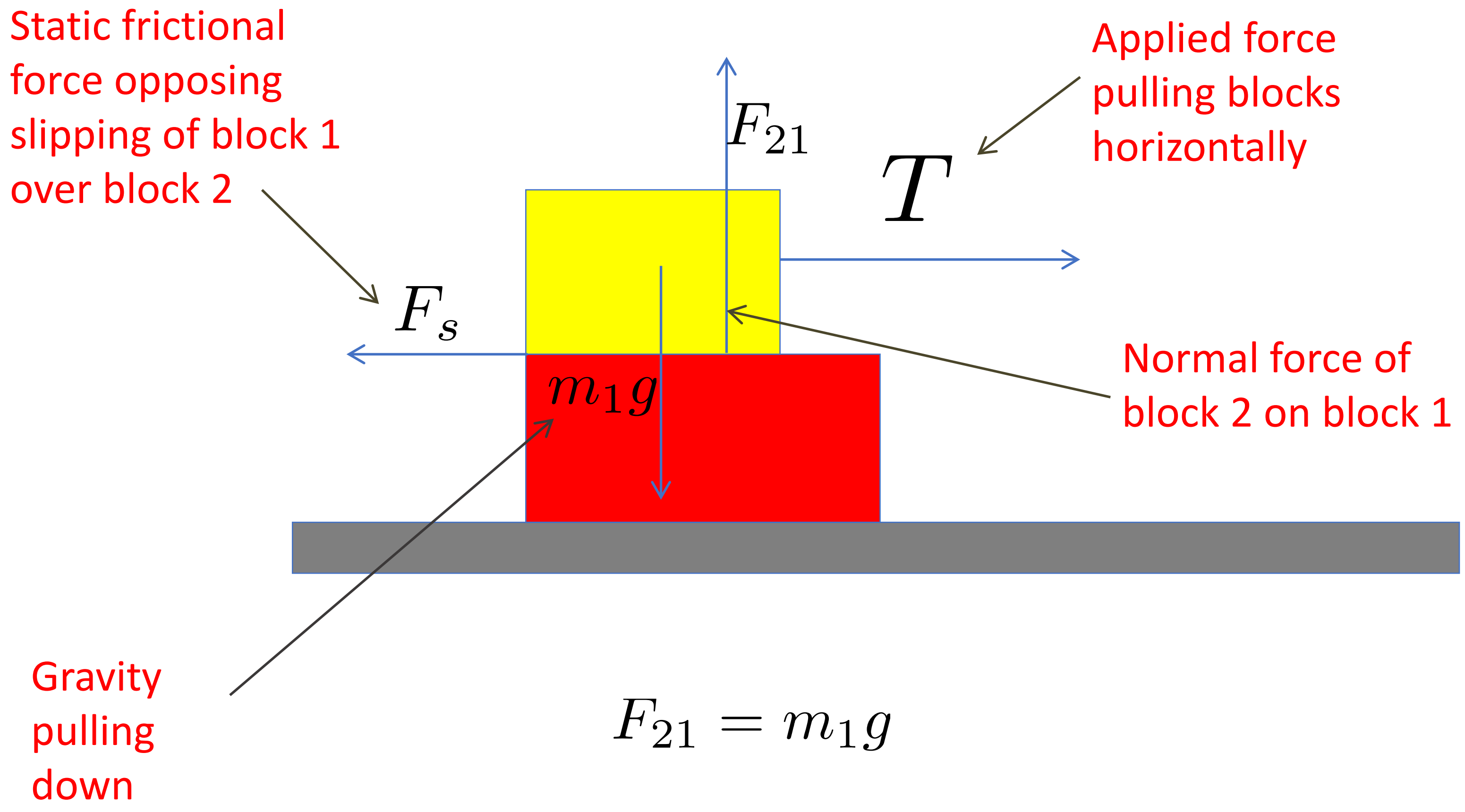
Coefficient of static friction between blocks μ_s=0.6

Coefficient of kinetic friction between block 2 and floor μ_k=0.2

What is maximum force T that can be applied horizontally to block 1 without slipping between the blocks? What is the acceleration of the blocks?

Look at forces on block 1

Look at forces on block 1



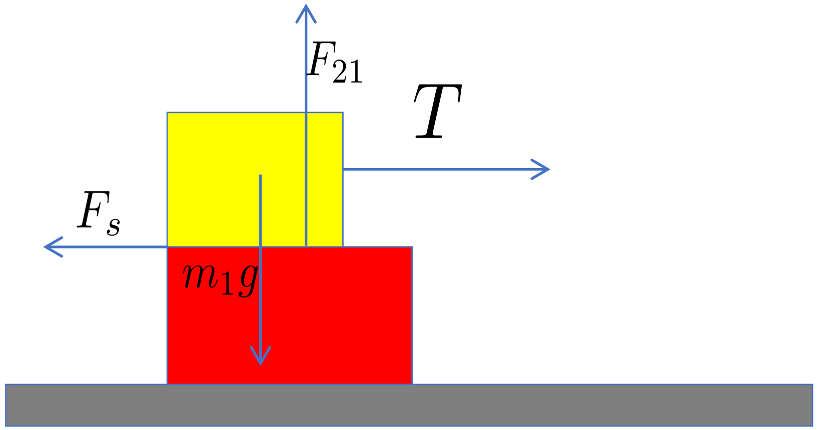
$$F_{21} = m_1 g$$

Friction and Newton's 3rd Law

2 blocks with masses
m_1 = 4kg and m_2 = 3kg

Coefficient of static friction between blocks μ_s=0.6

Coefficient of kinetic friction between block 2 and floor μ_k=0.2



What is maximum force T that can be applied horizontally to block 1 without slipping between the blocks? What is the acceleration of the blocks?

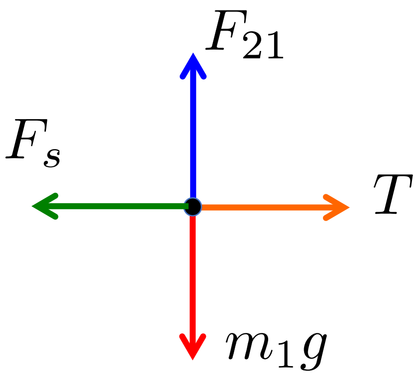
Draw free body diagrams and use 2nd law

Make use of earlier results from stacked blocks!

Block 1

F_{ab}= force of block a on block b

Static friction



y-direction ➡ $F_{21} = m_1 g$ Normal force on block 1 from block 2

➡ $F_s = \mu_s N = \mu_s m_1 g$ Assume maximum static friction

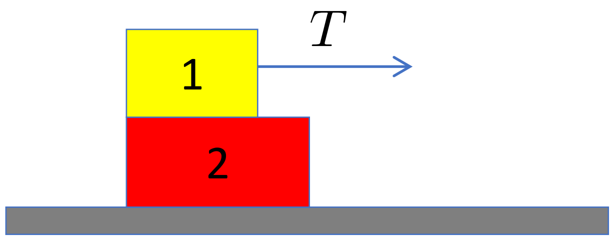
x-direction ➡ $T - \mu_s m_1 g = m_1 a$

Friction and Newton's 3rd Law

2 blocks with masses m_1 = 4kg and m_2 = 3kg

Coefficient of static friction between blocks μ_s=0.6

Coefficient of kinetic friction between block 2 and floor μ_k=0.2



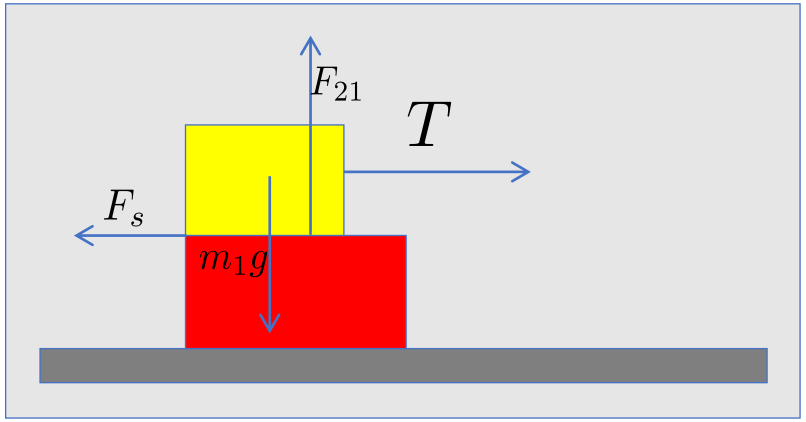
What is maximum force T that can be applied horizontally to block 1 without slipping between the blocks? What is the acceleration of the blocks?

F_{ab}= force of block a on block b

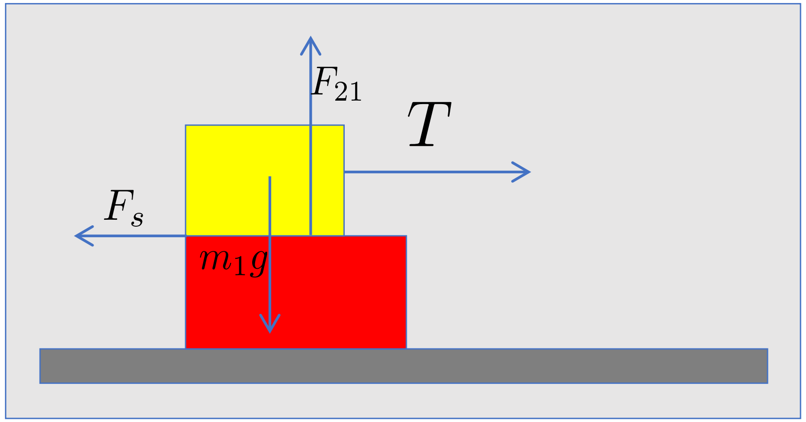
Block 1 $$T - \mu_s m_1 g = m_1 a$$

Block 2 – look at forces in more detail

Question: Forces on block one are *external.* *What exerts the frictional force on block 1?*



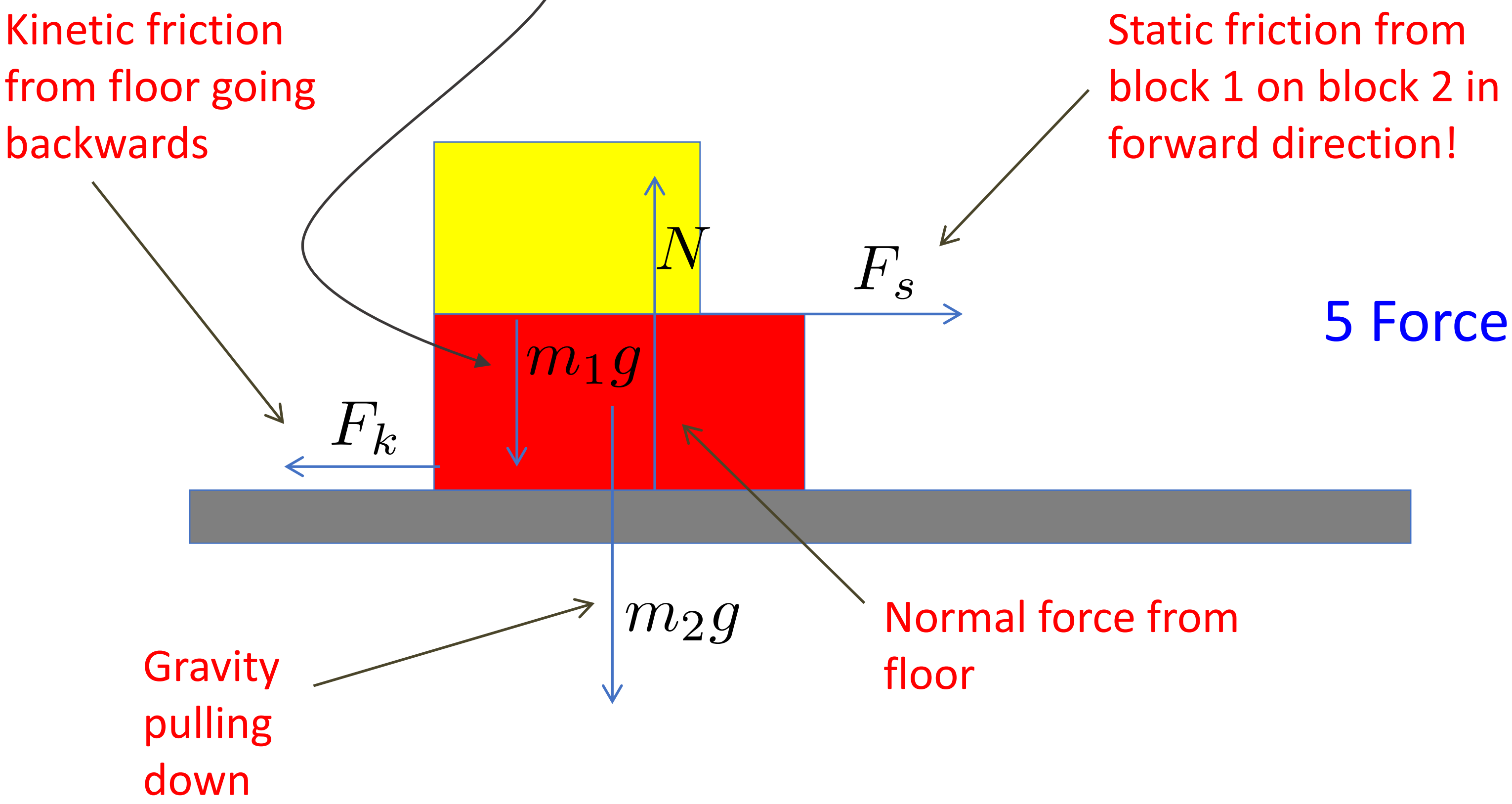
Block 2 – look at forces in more detail



Newton's 3rd law implies…

Block 2 exerts static friction force on block 1 in backward direction ⇔ Block 1 exerts friction force on 2 in forward direction!

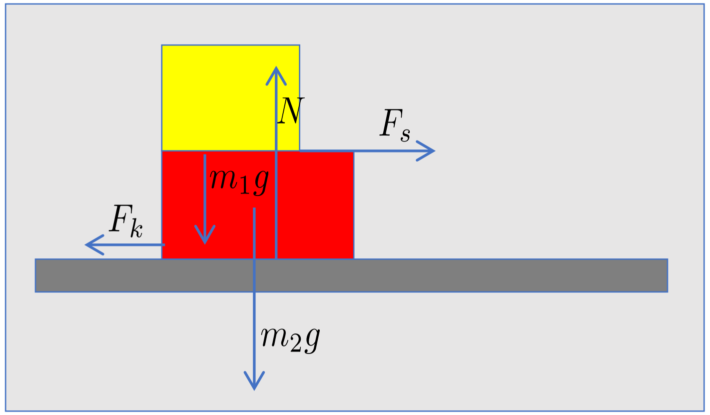
And normal force of magnitude m_1g downwards



5 Forces!

Free body diagram and 2nd law for block 2

$$F_{net} = (F_s - F_k)\,\hat{i} + (N - m_1 g - m_2 g)\,\hat{j}$$

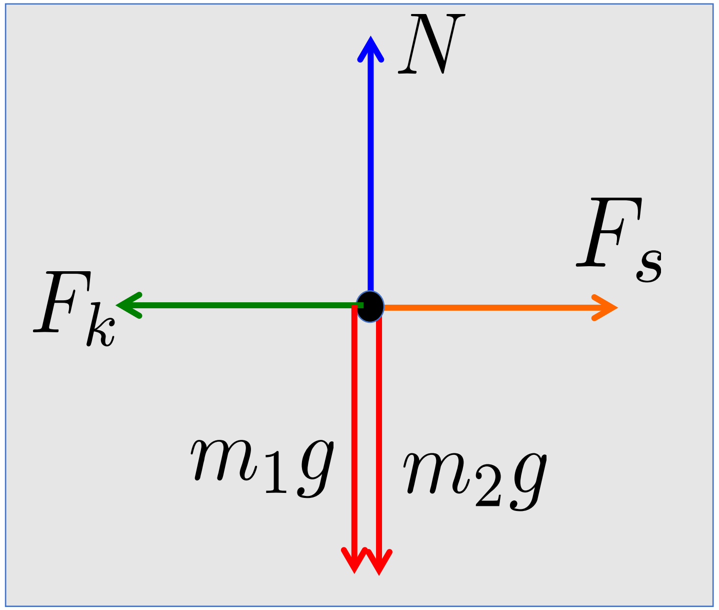


No acceleration in y-direction

$$N = (m_1 + m_2)g$$

Kinetic friction force

$$F_k = \mu_k N = \mu_k (m_1 + m_2)g$$



Recall static friction

$$F_s = \mu_s m_1 g$$

x-direction

$$F_s - F_k = m_2 a$$

Solve for acceleration

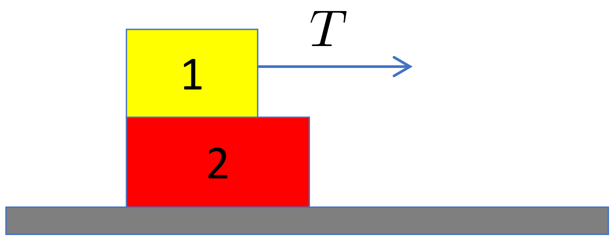
$$a = \frac{(\mu_s - \mu_k) m_1 g - \mu_k m_2 g}{m_2}$$

Friction and Newton's 3rd Law

2 blocks with masses m_1 = 4kg and m_2 = 3kg

Coefficient of static friction between blocks μ_s=0.6

Coefficient of kinetic friction between block 2 and floor μ_k=0.2



What is maximum force T that can be applied horizontally to block 1 without slipping between the blocks? What is the acceleration of the blocks?

F_{ab}= force of block a on block b

Block 1 $$T - \mu_s m_1 g = m_1 a$$

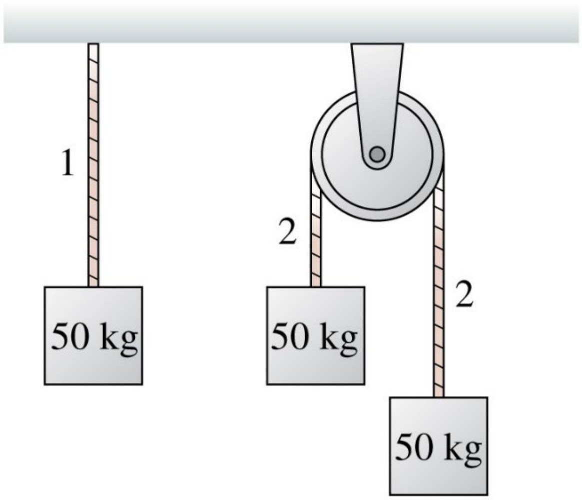
Block 2 $$\boxed{a = \frac{(\mu_s - \mu_k) m_1 g - \mu_k m_2 g}{m_2}} = 3.3 m/s^2$$

Plug in to find T

$$\boxed{T = (\mu_s - \mu_k) \frac{m_1 (m_1 + m_2)}{m_2} g} = 37N$$

All three 50-kg blocks are at rest.
The tension in rope 2 is:

A. greater than the tension in rope 1.
B. equal to the tension in rope 1.
C. less than the tension in rope 1.



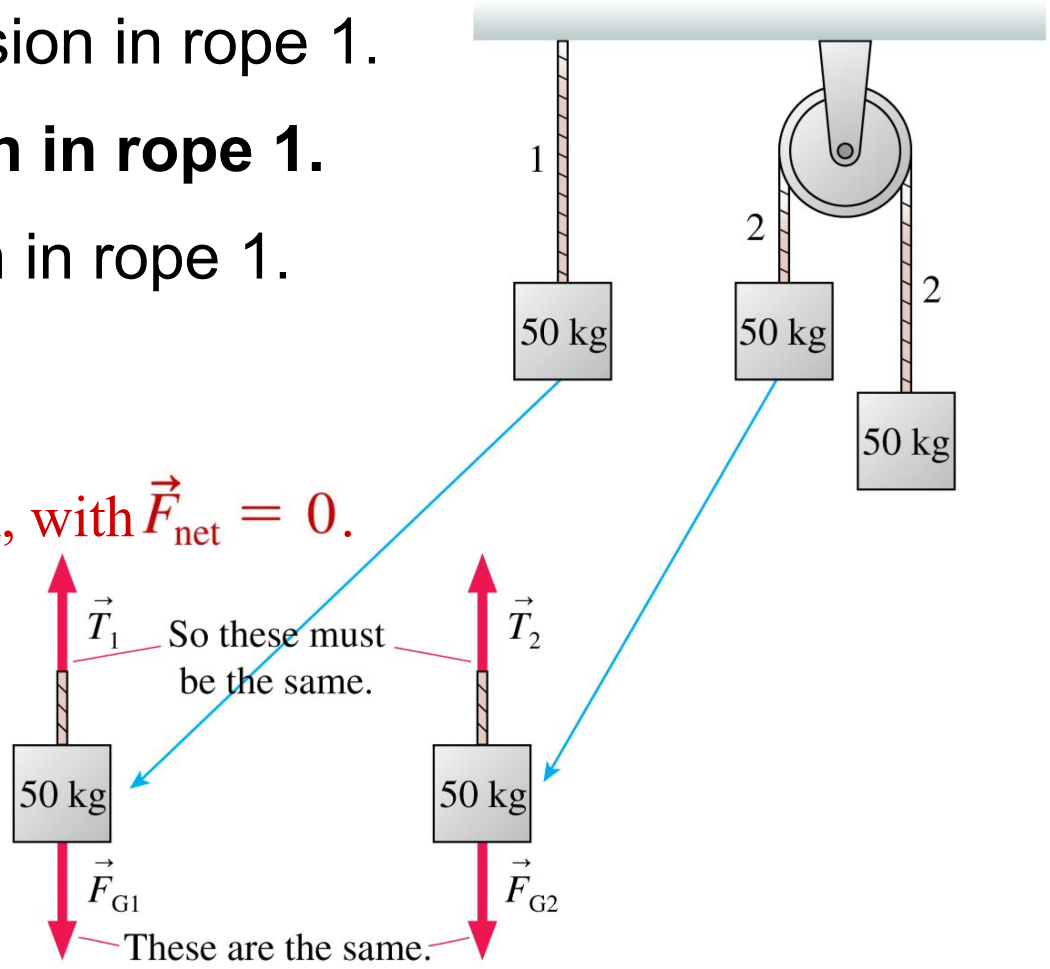
All three 50-kg blocks are at rest. The tension in rope 2 is

A. greater than the tension in rope 1.

B. equal to the tension in rope 1.

C. less than the tension in rope 1.

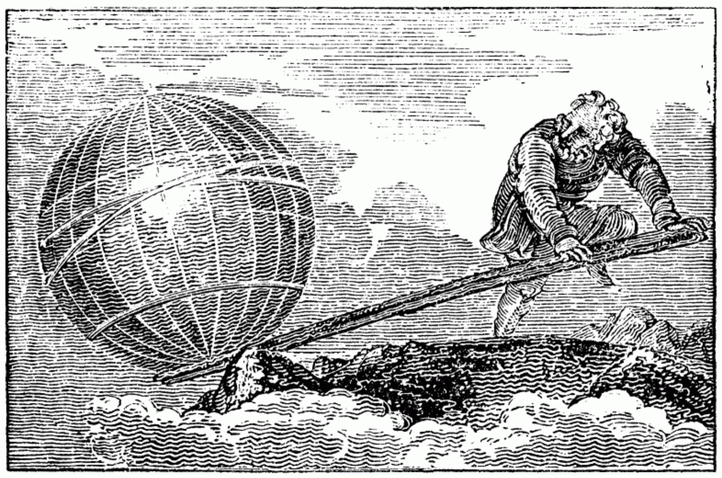
Each block is in static equilibrium, with $\vec{F}_{\text{net}} = 0$.

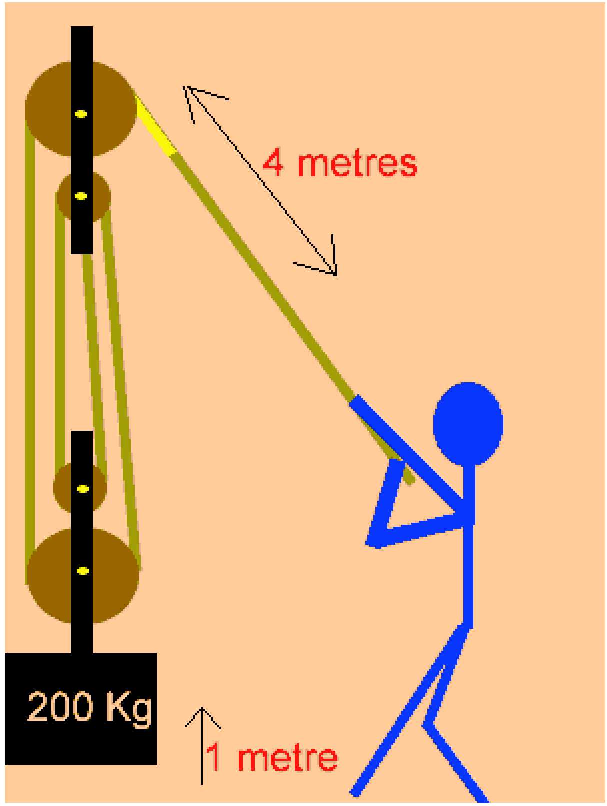


Pulleys

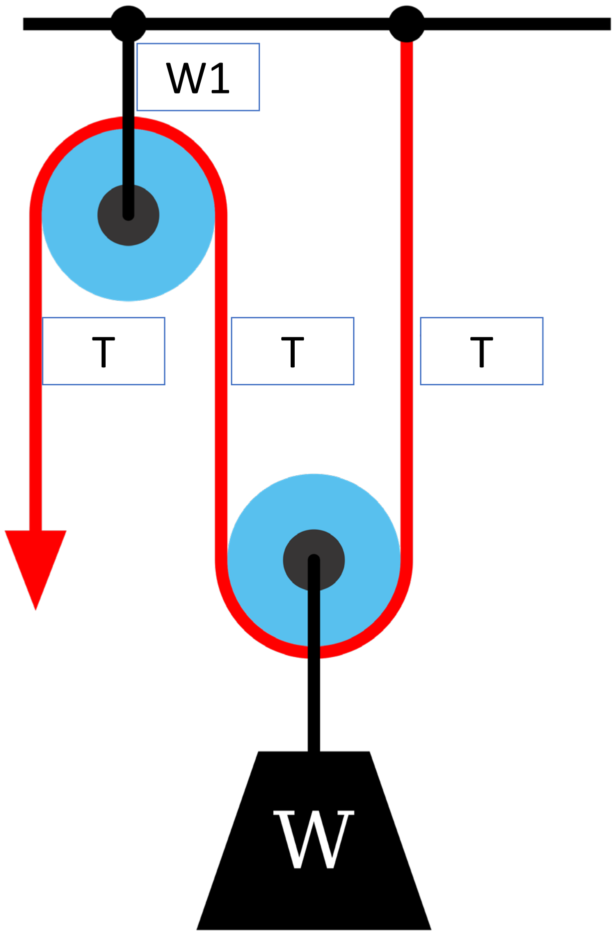
- A pulley is a wheel that carries a flexible rope or other type of cable
- Pulleys are used singly or in combination to transmit energy and motion.
- A pulley is a type of **simple machine** which can effectively multiply an applied force and allow lifting heavy weights with less effort.
- This amplification of force is called **mechanical advantage**.
- "Give me a lever long enough, and I will move the world."

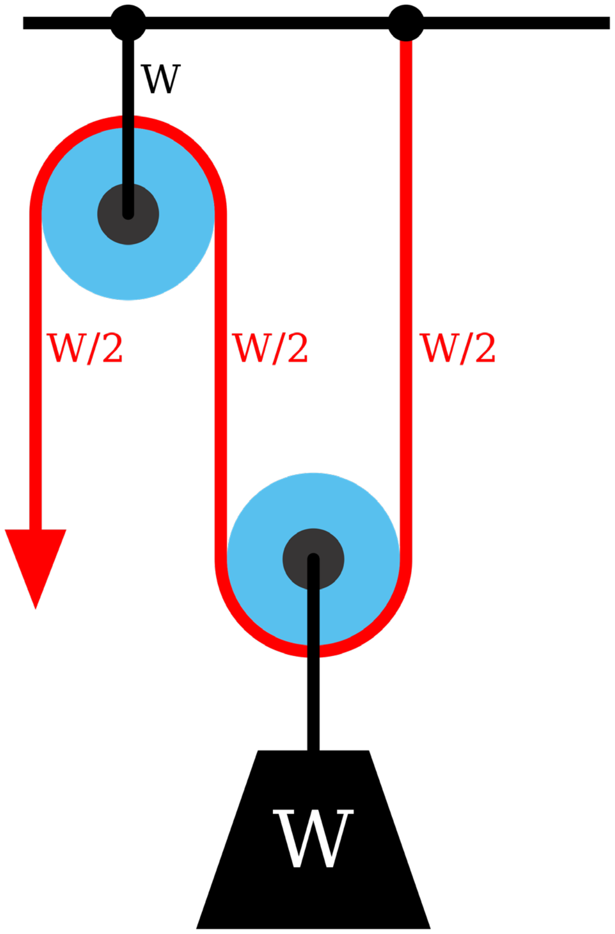
  -Archimedes on mechanical advantage





Pulleys and Compound Pulleys



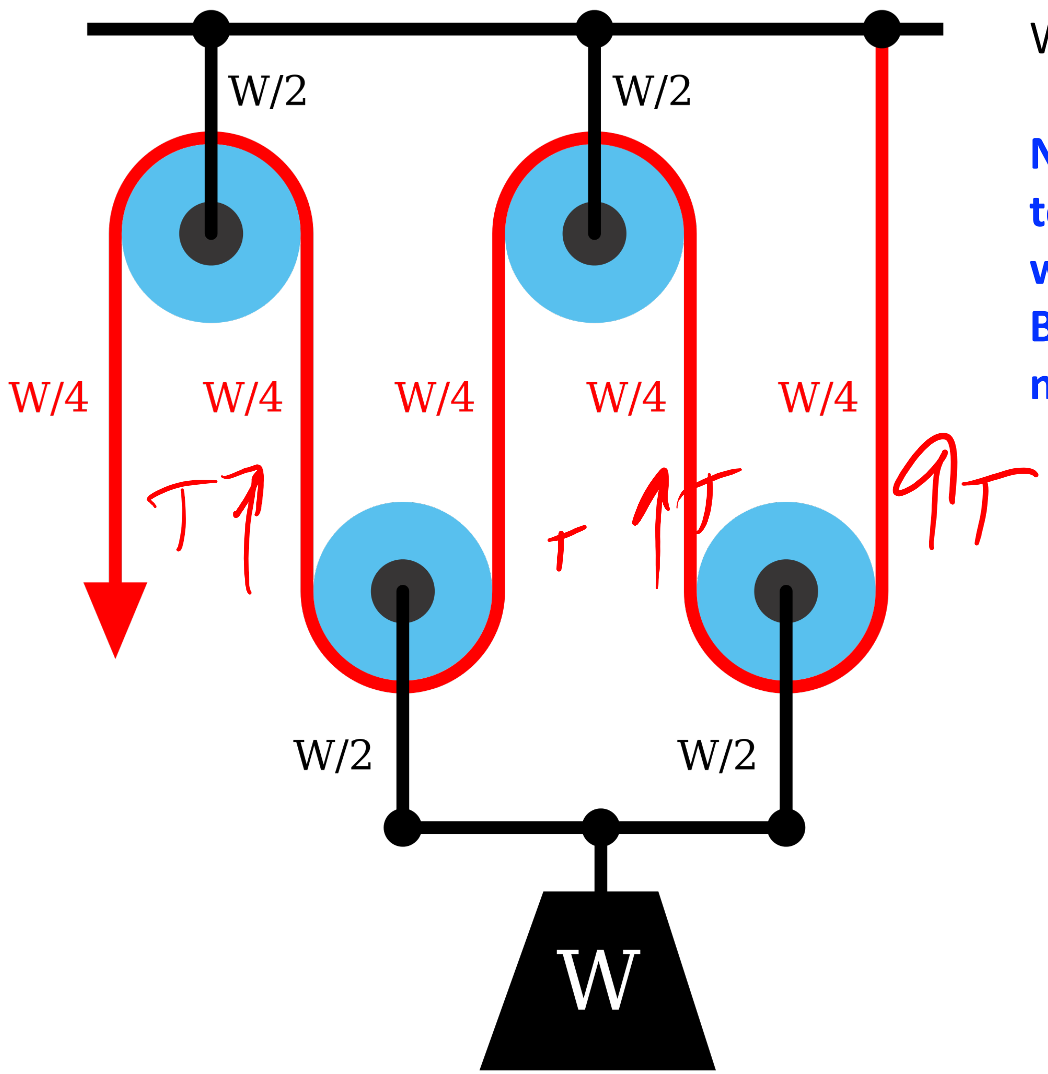


What is the tension in the rope T? What is the force W1 on the pulley?

Notice that you only need to apply a tension force T=W/2 to support the weight.

But to pull weight up 1 meter, you need to pull rope down by 2 meters

Pulleys and Compound Pulleys



What is the tension in the rope T?

Notice that you only need to apply a tension force T=W/4 to support the weight.
But to pull weight up 1 meter, you need to pull rope down by 4 meters

The painter + chair have a combined mass M.
What force must the painter apply to keep the chair at rest?

$$2T - Mg = Ma = 0$$

$$\Rightarrow 2T = Mg$$

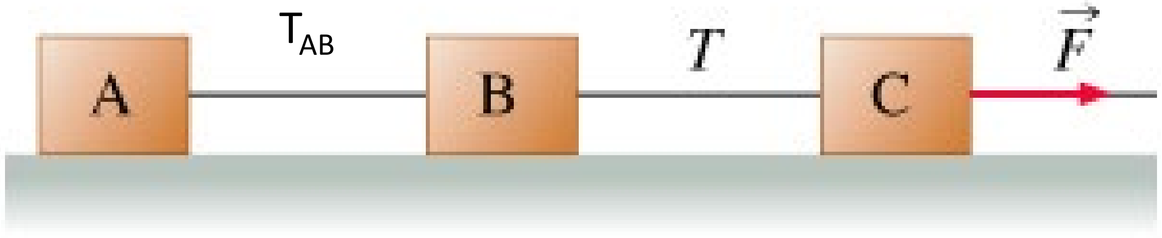
$$\Rightarrow T = Mg/2$$

Remember: we only care about the forces acting on the painter + chair.

We don't care if the force is applied by the painter or an assistant on the ground.

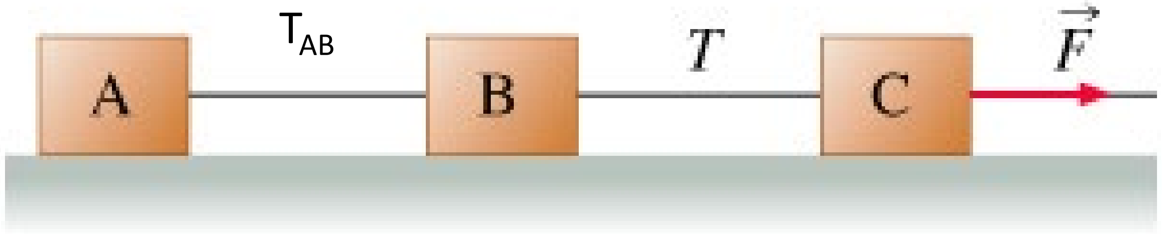
Can also solve for non-zero acceleration

Three identical blocks, connected by massless strings, are pulled along a horizontal frictionless surface by a horizontal force F. The magnitude of the tension in the string between blocks B and C is $T = 10.00$ N . Each block has mass $m = 2.0$ kg . What is the force F? What is the tension T_{AB} in the string between block A and B?



-Draw free body diagrams for each block
-Acceleration, mass of each block is the same
-Need F and T_{ab}, given a known $T=T_{\mathbf{bc}}$

Three identical blocks, connected by massless strings, are pulled along a horizontal frictionless surface by a horizontal force F. The magnitude of the tension in the string between blocks B and C is T = 10.00 N . Each block has mass m = 2.0 kg . What is the force F? What is the tension T_{AB} in the string between block A and B?



-Draw free body diagrams for each block

-Acceleration, mass of each block is the same

-Need F and T_{ab}, given a known $T = T_{bc}$

$$a_A = \frac{T_{AB}}{m_A} = a_B = \frac{T - T_{AB}}{m_B} = a_C = \frac{F - T}{m_B}$$

$$\frac{T_{AB}}{m} = \frac{T - T_{AB}}{m} \Rightarrow T = 2T_{AB} \Rightarrow T_{AB} = \frac{T}{2} = 5.0N$$

$$\frac{F - T}{m} = \frac{T_{AB}}{m} \Rightarrow F = T + T_{AB} = 15N$$