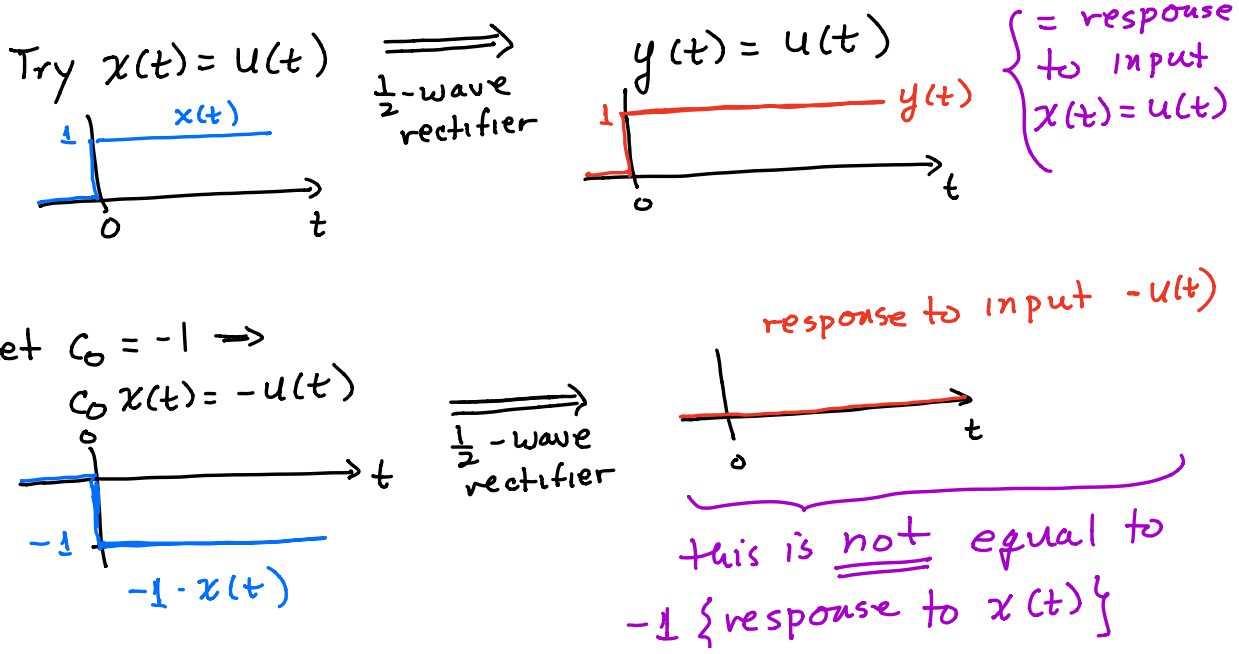
Example 2.1: Test if (a) integrator, (b) 1/2-wave rectifier, (c) modulator have the scalability property.

Solution:
(i) Integrator
Let $x_1(t) = c_0 x(t)$. Its response $y_1(t)$ is

$$y_1(t) = \int_{-\infty}^{t} c_0 x(\tau) d\tau = c_0 \int_{-\infty}^{t} x(\tau) d\tau = c_0 y(t). \quad \text{(E1)}$$

The integrator is scalable. To show a property, the property has to be verified for general input $x(t)$.

(ii) 1/2-wave rectifier
Try $x(t) = u(t)$. The output is $y(t) = u(t)$.



Let $c_0 = -1$ for a scaled input $x_1(t) = c_0 x(t) = -u(t)$.

Its response is $y_1(t) = 0$ at all t.
So, the 1/2-wave rectifier is *not* scalable.

(iii) Modulator
Consider an input $x_1(t) = c_0 x(t)$. Its output $y_1(t)$ is

$$y_1(t) = [c_0 x(t)] \cos \omega_0 t = c_0 [x(t) \cos \omega_0 t] = c_0 y(t). \tag{E2}$$

The modulator is has the scalability property.