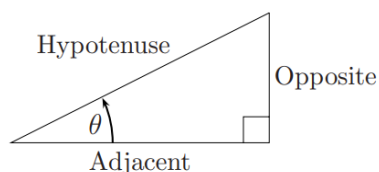


****You may detach this page from the test booklet****

PLEASE READ THESE INSTRUCTIONS CAREFULLY

- **Allowed:** Calculator and an 8.5"×11" reference sheet with handwritten notes on both sides
- **Not Allowed:** phones, laptops, tablets, headphones, music players, cameras, anything with internet connectivity. Put these away while the exam is in progress.
- **PRINT YOUR NAME AND SPIRE ID ON THE EXAM BOOKLET AND ANSWER SHEET**
- **>>> USE #2 PENCIL TO FILL IN THE CIRCLES ON ANSWER SHEET WITH YOUR NAME (last name first) and SPIRE ID. <<<**
- Please go to the restroom before the midterm starts.
- Unless friction or air resistance are mentioned, you can assume that they are negligible.
- Use #2 Pencil to fill the circles with your answers in spaces 1 through 27. Each question is worth 1 point. Only bubble in one circle per answer, or you may not receive credit. Erase pencil marks cleanly.
- When done, hand in ANSWER SHEET, EXAM BOOKLET, and show your UMass ID.
- There are 27 questions but the exam will be graded out of 25. This means you can get one question incorrect and still get a perfect score.

<i>Math</i>	<i>1D Kinematics</i>	<i>Rotational Dynamics</i>
$\frac{d}{dx}(x^n) = nx^{n-1}$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $ax^2 + bx + c = 0$ $\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$ $v(t) = v_0 + at$ $\Delta x = \frac{v_1^2 - v_0^2}{2a}$	$\vec{x}_{CM} = \sum m_i \vec{r}_i / \sum m_i$ $I = \sum m_i r_i^2$ $I_{parallel} = I_{CM} + Md^2$ $\vec{\tau} = \vec{r} \times \vec{F}$ $ \vec{\tau} = \vec{r} \vec{F} \sin\phi$ $\sum \vec{\tau} = I\alpha$ $v_{CM} = R\omega$ (rolling without slipping) $\vec{L} = \vec{r} \times \vec{p}$ $\vec{L} = I\omega$ (fixed axis)
Angular Motion $a = v^2/r = \omega^2 r$ $v = \omega r$ $\omega = 2\pi/T$ $L = r\theta$; $v = r\omega$; $a = r\alpha$ $\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega(t) = \omega_0 + \alpha t$ $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$	Projectile Motion Range: $D = \frac{v_0^2 \sin(2\theta)}{g}$ [Same initial/final height only] Dynamics $\sum \vec{F}_{ext} = m\vec{a}$ $\vec{F}_{AB} = -\vec{F}_{BA}$ $\vec{F}_c = \frac{mv^2}{r}$ (toward $-\hat{r}$) $\vec{F}_{spring} = -k\Delta\vec{x}$ Friction $f_k = \mu_k N$ $f_s \leq \mu_s N$	Conversion Factors and Constants 1 minute = 60s 1 hour = 3600s 1 mile = 1.60934 km 1 mile = 5280 feet 1 foot = 0.3048 meters 1 foot = 12 inches 1 inch = 2.54 cm $g = 9.8 \text{ m/s}^2$



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Name _____
SPIRE ID _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

- 1) One of the below answers corresponds to your test version. Select the answer choice 1) _____
highlighted in bold below. **PLEASE SELECT THE CORRECT ANSWER OR YOU MAY LOSE ALL CREDIT FOR THE EXAM.**

A) [SELECT THIS ONE PLEASE]

B) ---

C) ---

D) ---

E) ---

- 2) A ball is tied to the end of a cable of negligible mass. The ball is spun in a circle with a radius 2.00 m making 7.00 revolutions every 10.0 seconds. What is the magnitude of the angular velocity of the ball? 2) _____

A) 2.2 rad/s B) 4.4 rad/s C) 7 rad/s D) 0.7 rad/s E) zero.

$$\omega = \frac{7 \text{ rev}}{10 \text{ s}} = \frac{7 \cdot 2\pi \text{ rad}}{10 \text{ s}} = 4.4 \text{ rad/s}$$

- 3) The ball from the previous question continues to spin in a circle with a radius 2.00 m making 7.00 revolutions every 10.0 seconds. What is the magnitude of the acceleration of the ball? 3) _____

A) 38.7 m/s²

B) 74.2 m/s²

C) 29.3 m/s²

D) 67.9 m/s²

E) 14.8 m/s²

$$a = \omega^2 r = (4.4 \text{ rad/s})^2 (2 \text{ m}) = 38.7 \text{ m/s}^2$$

- 4) An object moves in a circle of radius R at constant speed with a period T . If you want to change only the period in order to cut the object's acceleration in half, the new period should be 4) _____

A) $T/4$.

B) $T\sqrt{2}$.

C) $T/\sqrt{2}$.

D) $T/2$.

E) $4T$.

$$a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$$

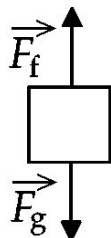
$$a' = \frac{a}{2} = \frac{4\pi^2 R}{(T')^2} = \frac{4\pi^2 R}{2T^2}$$

$$T'^2 = 2T^2$$

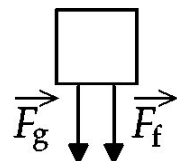
$$T' = T\sqrt{2}$$

- 5) Which one of the following free-body diagrams best represents the free-body diagram, with correct relative force magnitudes, of a person in an elevator that is traveling upward but is gradually slowing down at a rate of 9 m/s^2 ? \vec{F}_f is the force of the floor on the person and \vec{F}_g is the force of gravity on the person.

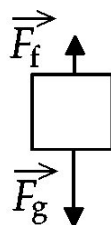
A)



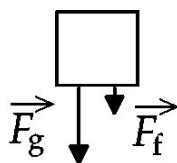
B)



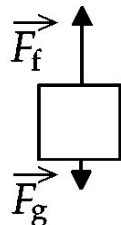
C)



D)



E)



Moving up & decreasing speed
 \rightarrow Acceleration is downward
 \rightarrow Net force is in the downward direction
 \vec{F}_g does not change, but \vec{F}_f can.
 so $|\vec{F}_f| < |\vec{F}_g|$, \vec{F}_f points up.

6) Two particles, A and B, are in uniform circular motion about a common center. The acceleration of particle A is 8.5 times that of particle B. The period of particle B is 2.0 times the period of particle A. The ratio of the radius of the motion of particle A to that of particle B is closest to

6) _____

A) $r_A/r_B = 4.3$.

B) $r_A/r_B = 2.1$.

C) $r_A/r_B = 18$.

D) $r_A/r_B = 0.24$.

E) $r_A/r_B = 17$.

$$a_A = 8.5 a_B$$

$$\omega_A^2 R_A = 8.5 \omega_B^2 R_B$$

$$\frac{R_A}{R_B} = 8.5 \frac{\omega_B^2}{\omega_A^2} = 8.5 \left(\frac{\frac{2\pi}{T_B}}{\frac{2\pi}{T_A}} \right)^2 = 8.5 \frac{T_A^2}{(2T_A)^2} = \frac{8.5}{4} = 2.1$$

7) Suppose you are playing hockey on a new-age ice surface for which there is no friction between the ice and the hockey puck. You wind up and hit the puck as hard as you can. After the puck loses contact with your stick, the puck will

7) _____

A) speed up a little, and then move at a constant speed.

B) start to slow down.

C) not slow down or speed up.

D) speed up a little, and then slow down.

$F = 0$ horizontally on the ice
By 1st law, it continues to move @ constant speed

8) A 7.0-kg object is acted on by two forces. One of the forces is 10.0 N acting toward the east. Which of the following forces is the other force if the acceleration of the object is 1.0 m/s^2 toward the east?

8) _____

A) 12 N east

B) 6.0 N east

C) 7.0 N west

D) 3.0 N west

E) 9.0 N west

$\Rightarrow a$

$F_1 = 10 \text{ N}$

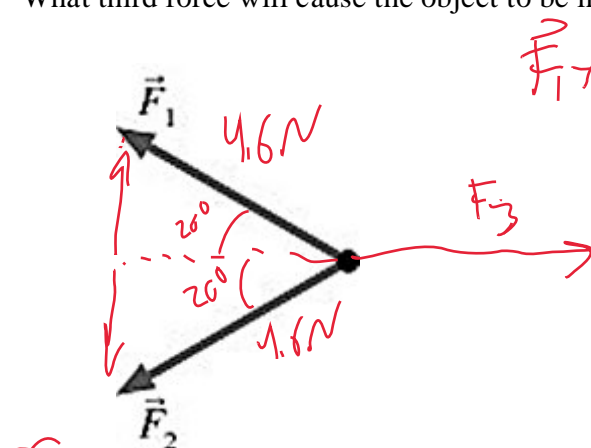
F_2

$$F_1 + F_2 = ma$$

$$F_2 = ma - 10 \text{ N} = (7 \text{ kg})(1 \text{ m/s}^2) - 10 \text{ N} = -3 \text{ N (west)}$$

- 9) The figure shows two forces, each of magnitude 4.6 N, acting on an object. The angle between these forces is 40° , and they make equal angles above and below the horizontal. What third force will cause the object to be in equilibrium (acceleration equals zero)?

9) _____



$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$$

$$F_{1y} + F_{2y} = 0$$

so $F_{3y} = 0$

$$F_{1x} + F_{2x} + F_{3x} = 0$$

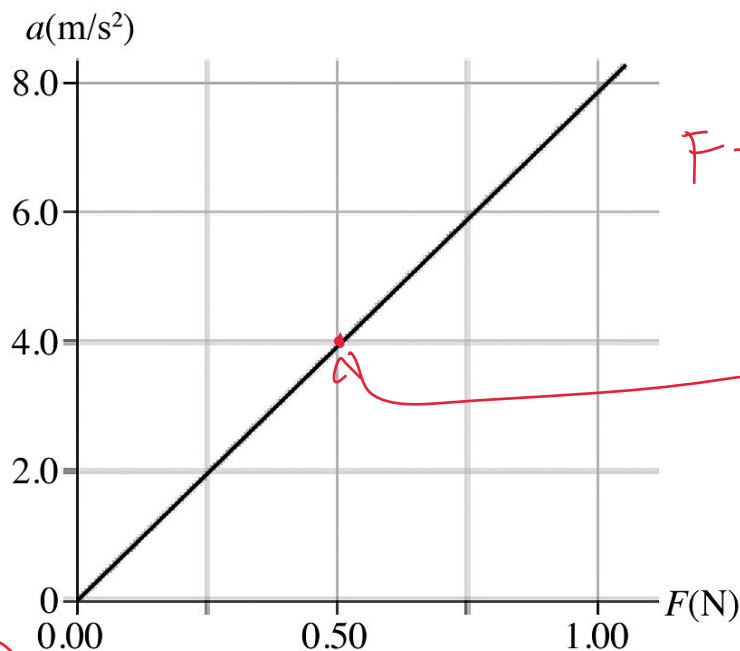
$$F_{3x} = -2(4.6\text{ N})\cos(20^\circ) = +8.6\text{ N}$$

- A) 8.6 N pointing to the right
C) 3.5 N pointing to the right

- B) 7.0 N pointing to the right
D) 4.3 N pointing to the right

- 10) The figure shows a graph of the acceleration of an object as a function of the net force acting on it. The mass of this object, in grams, is closest to

10) _____



$$F = ma$$

$$m = \frac{F}{a}$$

$$= \frac{0.5\text{ N}}{4\text{ m/s}^2} = 0.125\text{ kg} = 125\text{ g}$$

- A) 130.

- B) 11.

- C) 8000.

- D) 89.

- 11) Two forces act on a 55-kg object. One force has magnitude 65 N directed 59° clockwise from the positive x-axis, and the other has a magnitude 35 N at 32° clockwise from the positive y-axis. What is the magnitude of this object's acceleration? 11) _____



A) 1.3 m/s^2 B) 1.7 m/s^2 C) 1.1 m/s^2 D) 1.5 m/s^2

$$\begin{aligned}\Sigma F_x &= 65 \text{ N} \cos 59^\circ + (35 \text{ N}) \sin 32^\circ = 52 \text{ N} \\ \Sigma F_y &= 35 \text{ N} \cos 32^\circ - 65 \text{ N} \sin 59^\circ = -26 \text{ N} \\ |F| &= \sqrt{F_x^2 + F_y^2} = 58 \text{ N}, \quad a = \frac{|F|}{m} = \frac{58 \text{ N}}{55 \text{ kg}} \approx 1.1 \text{ m/s}^2\end{aligned}$$

- 12) A block is on a frictionless horizontal table, on earth. This block accelerates at 3.6 m/s^2 when a 90 N horizontal force is applied to it. The block and table are then set up on the moon where the acceleration due to gravity is 1.62 m/s^2 . A horizontal force of 45 N is applied to the block when it is on the moon. What acceleration does this force impart to the block? 12) _____

A) 1.8 m/s^2 B) 2.0 m/s^2 C) 2.2 m/s^2 D) 2.3 m/s^2 E) 1.6 m/s^2



$$\begin{aligned}F &= ma \\ 90 \text{ N} &= m(3.6 \text{ m/s}^2), \quad m = 25 \text{ kg} \\ 45 \text{ N} &= ma \\ a &= \frac{45 \text{ N}}{25 \text{ kg}} = 1.8 \text{ m/s}^2\end{aligned}$$

- 13) A 10,000-kg rocket blasts off from earth with a uniform upward acceleration of 2.00 m/s^2 and feels no air resistance. The upward thrust force its engines must provide during this acceleration is closest to 13) _____

A) 20,000 N. B) 118,000 N. C) 980,000 N. D) 78,000 N.

$$\Sigma F = ma = F_T - mg$$

$$\begin{aligned}F_T &= m(a + g) \\ &= (10,000 \text{ kg})(11.8 \text{ m/s}^2) = 118 \text{ kN}\end{aligned}$$

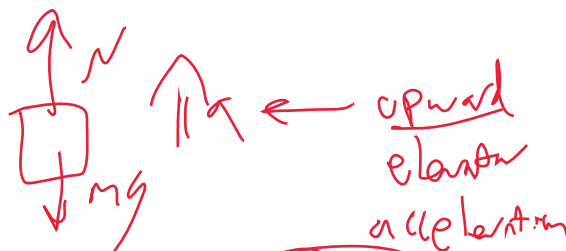


- 14) A 60.0-kg person rides in elevator while standing on a scale. The elevator is traveling downward but slowing down at a rate of 2.00 m/s^2 . The reading on the scale is closest to 14) _____

A) 708 N. B) 349 N. C) 589 N. D) 120 N. E) 469 N.

$$N - mg = ma$$

$$\begin{aligned}N &= m(g + a) \\ &= (60 \text{ kg})(9.8 + 2 \text{ m/s}^2) = 708 \text{ N}\end{aligned}$$



- 15) You push downward on a box at an angle 25° below the horizontal with a force of 750 N. If the box is on a flat horizontal surface for which the coefficient of static friction with the box is 0.76, what is the mass of the heaviest box you will be able to move?

A) 68 kg B) 54 kg C) 82 kg D) 59 kg

Handwritten solution for Q15:

$$F_s \leq \mu N = \mu N_{\text{at limit}}$$

$$\sum F_y = N - mg - 750 \sin(25^\circ) = 0$$

$$N = mg + 317 \text{ N}$$

$$\sum F_x = \mu N - 750 \cos(25^\circ) = 0$$

$$\mu(mg + 317 \text{ N}) = 680 \text{ N}$$

$$m = \frac{680 \text{ N}}{g} - 317 \text{ N} = 59 \text{ kg}$$

- 16) A 6.0 kg box slides down an inclined plane that makes an angle of 39° with the horizontal. If the coefficient of kinetic friction is 0.19, at what rate does the box accelerate down the slope?

A) 5.5 m/s^2 B) 6.2 m/s^2 C) 5.2 m/s^2 D) 4.7 m/s^2

Handwritten solution for Q16:

$$F_k = \mu N$$

$$\sum F_y = N - mg \cos \theta = 0$$

$$N = mg \cos \theta$$

$$\Rightarrow F_k = \mu mg \cos \theta$$

$$\sum F_x = mg \sin \theta - \mu mg \cos \theta = ma$$

$$a_x = g(\sin \theta - \mu \cos \theta) = 4.7 \text{ m/s}^2$$

- 17) An object weighing 4.00 N falls from rest subject to a frictional drag force given by $F_{\text{drag}} = bv^2$, where v is the speed of the object and $b = 3.00 \text{ N} \cdot \text{s}^2/\text{m}^2$. What terminal speed will this object approach?

A) 2.25 m/s B) 0.75 m/s C) 1.15 m/s D) 1.78 m/s E) 3.42 m/s

Handwritten solution for Q17:

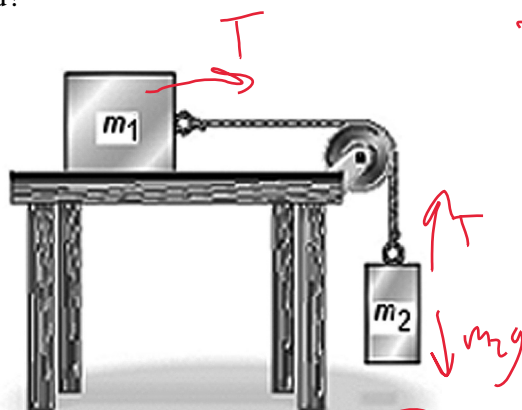
terminal velocity: $a = 0$

$$\sum F = F_d - mg = 0$$

$$bv^2 = mg = 4 \text{ N}$$

$$v = \sqrt{\frac{mg}{b}} = 1.15 \text{ m/s}$$

- 18) Two objects having masses m_1 and m_2 are connected to each other as shown in the figure and are released from rest. There is no friction on the table surface or in the pulley. The masses of the pulley and the string connecting the objects are completely negligible. What must be true about the tension T in the string just after the objects are released?



A) $T > m_2 g$ B) $T = m_1 g$ C) $T < m_2 g$ D) $T > m_1 g$ E) $T = m_2 g$

Handwritten solution for Q18:

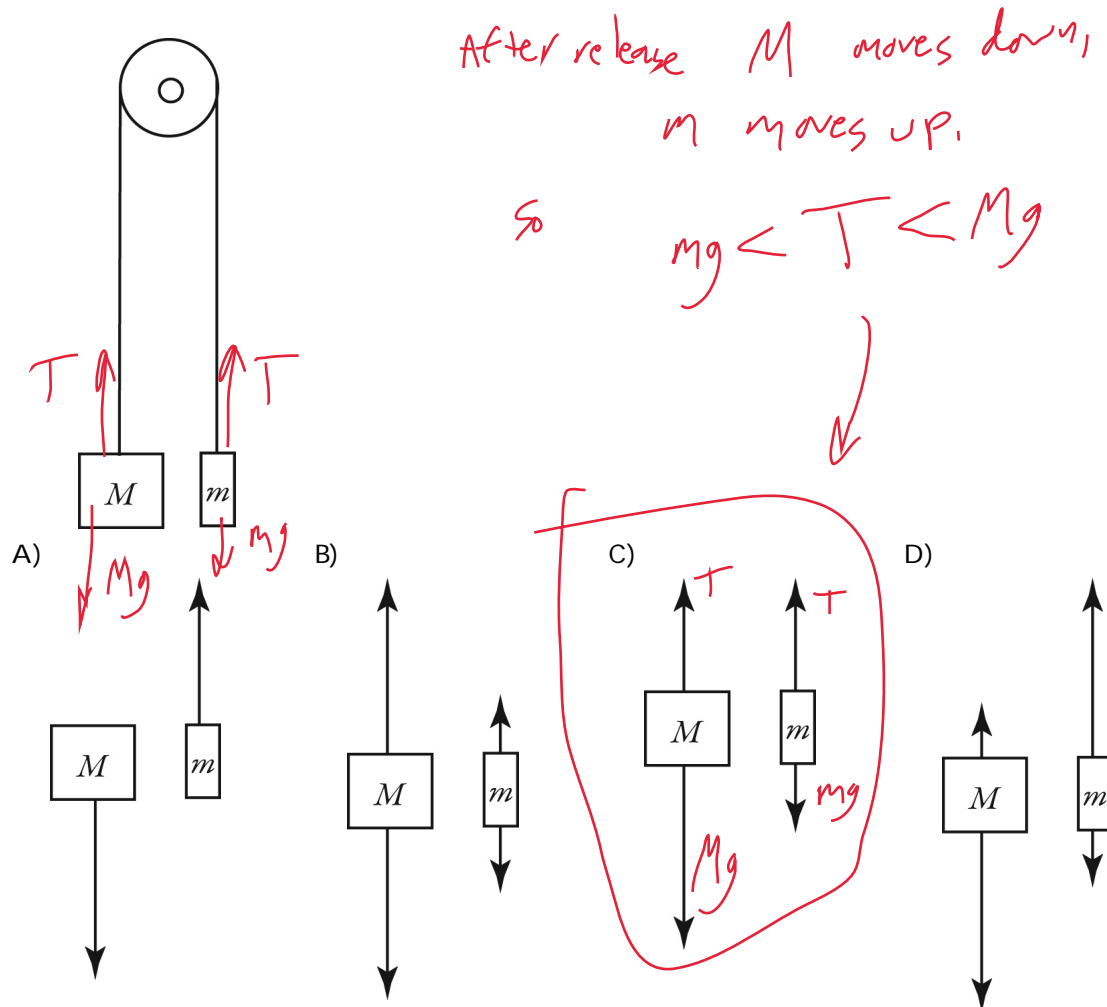
The weight accelerates downward. so

$$m_2 a = T - m_2 g < 0$$

$$T < m_2 g$$

- 19) Two unequal masses M and m ($M > m$) are connected by a light cord passing over a pulley of negligible mass, as shown in the figure. When released, the system accelerates. Friction is negligible. Which figure below gives the correct free-body force diagrams for the two masses in the moving system?

19) _____



- 20) The International Space Station has a mass of 1.8×10^5 kg. A 70.0-kg astronaut inside the station pushes off one wall of the station so she accelerates at 1.50 m/s^2 . What is the magnitude of the acceleration of the space station as the astronaut is pushing off the wall? Give your answer relative to an observer who is space walking and therefore does not accelerate with the space station due to the push.

20) _____

- A) zero
B) $4.7 \times 10^{-4} \text{ m/s}^2$
C) $3.9 \times 10^{-3} \text{ m/s}^2$
D) $5.8 \times 10^{-4} \text{ m/s}^2$
E) 1.50 m/s^2



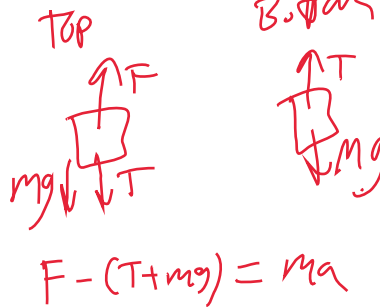
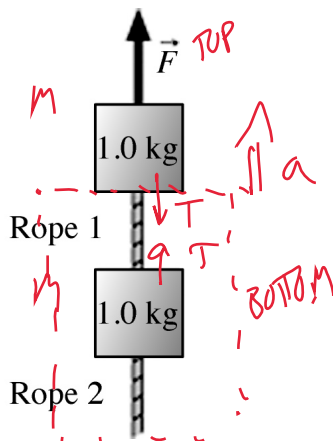
$| \text{Force on station} |$
 $= | \text{Force on astronaut} |$
(3rd Law)

$$|F| = m|a_m| = M|a_M|$$

$$|a_M| = \frac{m}{M} |a_m| = \left(\frac{70}{1.8 \times 10^5} \right) (1.5 \text{ m/s}^2) = 5.8 \times 10^{-4} \text{ m/s}^2$$

- 21) The figure shows two 1.0 kg-blocks connected by a rope. A second rope hangs beneath the lower block. Both ropes have a mass of 250 g. The entire assembly is accelerated upward at 2.3 m/s^2 by force \vec{F} . What is the tension at the top end of rope 1?

21) _____



$(M = \text{mass of block} + 2 \text{ ropes})$

$$T - Mg = Ma$$

$$T = Mg + Ma$$

$$T = (1 \text{ kg} + 2 \times 0.25 \text{ kg})(9.8 + 2.3)$$

$$= 18 \text{ N}$$

A) 15 N

B) 3.5 N

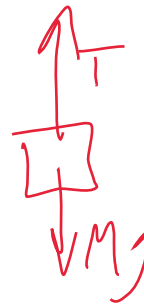
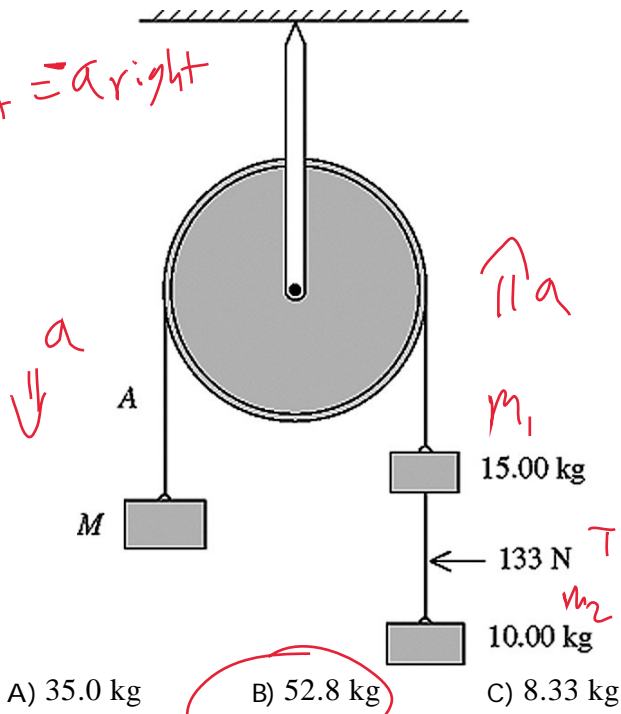
C) 18 N

D) 2.9 N

22) Three objects are connected by massless wires over a massless frictionless pulley as shown in the figure. The tension in the wire connecting the 10.0-kg and 15.0-kg objects is measured to be 133 N. What is the mass M ?

22) _____

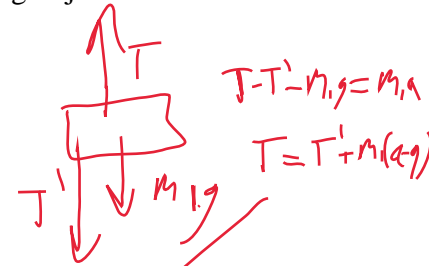
$a_{\text{left}} = a_{\text{right}}$



$$T - Mg = -Ma$$

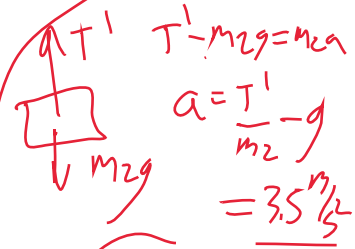
$$T = M(g - a)$$

$$M = \frac{T}{g - a}$$



$$T - T' - m_1g = m_1a$$

$$T = T' + m_1(a + g)$$



$$T' - m_2g = m_2a$$

$$a = \frac{T'}{m_2} - g = 3.5 \text{ m/s}^2$$

$$T = 133 \text{ N} + 15 \times (9.8 + 3.5) = 333 \text{ N}$$

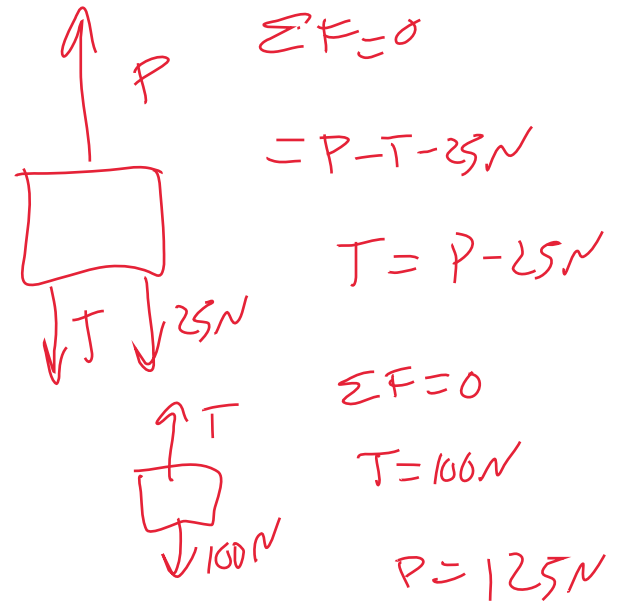
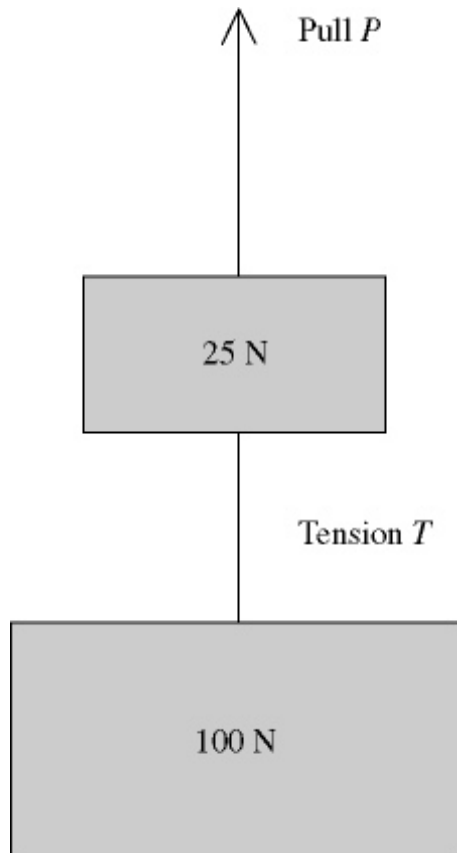
D) 95.0 kg

E) 33.9 kg

$$M = \frac{333 \text{ N}}{(9.8 - 3.5) \text{ m/s}^2} = 53 \text{ kg}$$

- 23) Two weights are connected by a massless wire and pulled upward with a constant speed of 1.50 m/s by a vertical pull P . The tension in the wire is T (see figure). P is closest to

23) _____



A) 125 N.

B) 1225 N.

C) 245 N.

D) 25 N.

E) 187.5 N.

- 24) A string is attached to the rear-view mirror of a car. A ball is hanging at the other end of the string. The car is driving around in a circle, at a constant speed. Which of the following lists gives all of the forces directly acting on the ball?

24) _____

- A) tension
 B) tension, gravity, the centripetal force, and friction
 C) tension and gravity
 D) tension, gravity, and the centripetal force

Note: "the centripetal force" is not a type of force. Any force can act as a centripetal force to create circular motion. See Knight 8.2

- 25) A 23 kg mass is connected to a nail on a frictionless table by a massless string 1.3 m long. There is no appreciable friction between the nail and the string. If the tension in the string is 51 N while the mass moves in a uniform circle on the table, how long does it take for the mass to make one complete revolution?

25) _____

A) 4.5 s

B) 5.2 s

C) 4.8 s

D) 3.8 s



$$F_T = \frac{mv^2}{r} = \frac{m}{r} \left(\frac{2\pi r}{T} \right)^2 = \frac{4\pi^2 m r}{T^2}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{mr}{F_T}} = 4.8 \text{ s}$$

- 26) A car travels at a steady 40.0 m/s around a horizontal curve of radius 200 m. What is the minimum coefficient of static friction between the road and the car's tires that will allow the car to travel at this speed without sliding?

26) _____

A) 0.816

B) 1.23

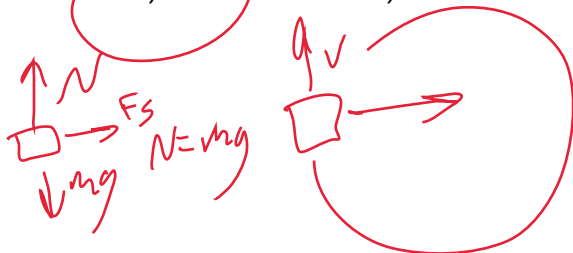
Top

C) 0.952

D) 0.736

E) 0.662

side



$$\frac{mv^2}{r} = F_s = \mu N \quad \text{at min speed}$$

$$\mu = \frac{mv^2}{rN} = \frac{mv^2}{mgr} = \frac{v^2}{gr} = 0.816$$

- 27) A new roller coaster contains a loop-the-loop in which the car and rider are completely upside down. If the radius of the loop is 13.2 m, with what minimum speed must the car traverse the loop so that the rider does not fall out while upside down at the top? Assume the rider is not strapped to the car.

27) _____

A) 12.5 m/s

B) 10.1 m/s

C) 11.4 m/s

D) 14.9 m/s

Minimum speed? $N=0$

$$N + mg = \frac{mv^2}{r}$$

$$v = \sqrt{gr} = 11.4 \text{ m/s}$$



