so sad

Whos fault is that?

Ex sheet 2

1.1.) Image of

$$\begin{pmatrix} 1 & -2 & 0 \\ 6 & -6 & -12 \\ 1 & 1 & -6 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -2 & 0 \\ 1 & -1 & -2 \\ 1 & 1 & -6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & +1 & -2 \\ 0 & 3 & -6 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 0 & 0 \\ 6 & -6 & -12 & 0 \\ 1 & 1 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$-4x_1 + -2x_2 = \vec{0}$ therefore, basis exists

$$\text{Base} = \text{Span}\left\{ \begin{bmatrix} 1 \\ 6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -6 \\ 1 \\ 0 \end{bmatrix} \right\}$$

# of pivot columns = # of colums

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{Linearly independent}$$

1.2.) Kernel of $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_1 + x_2 \end{pmatrix}$

$$\begin{bmatrix} 1 & X_1 + X_2 \\ 2 & X_1 + X_2 \\ 3 & 0 \end{bmatrix}$$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \cdot \begin{pmatrix} X_1 + X_2 \\ X_1 + X_2 \end{pmatrix} = \vec{0} \quad \begin{bmatrix} X_1(X_1+X_2) & X_1(X_1+X_2) \\ X_2(X_1+X_2) & X_2(X_1+X_2) \\ X_3(X_1+X_2) & X_3(X_1+X_2) \end{bmatrix}$$

$$\begin{bmatrix} X_1 & X_1 & 0 \\ X_2 & X_2 & 0 \\ X_3 & X_3 & 0 \end{bmatrix}$$

$X_2 = -X_1$

$X_3 = 0$

$$\begin{matrix} 1 & -1 \\ -1 & 1 \\ 0 & 0 \end{matrix}$$

$$\text{Kernel} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1/5 & -1/5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & 3/5 & 7/5 & 0 \\ 0 & 1 & 0 & 1/5 & -1/5 & 0 \end{bmatrix}$$

1.3) $\begin{bmatrix} \boxed{1} & 2 & -3 & 1 & 1 & 0 \\ 0 & \boxed{5} & 0 & 1 & -1 & 0 \end{bmatrix}$

basis = $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$

$\begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = linearly independent

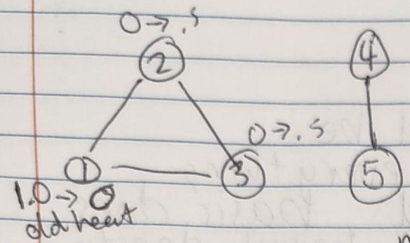
# of pivot columns = # of columns = linearly independent

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 3x_3 - \frac{3}{5}x_4 - \frac{7}{5}x_5 \\ \frac{1}{5}x_4 - \frac{1}{5}x_5 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -3/5 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -7/5 \\ -1/5 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

1.3) basis = $\left\{ \begin{bmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3/5 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -7/5 \\ -1/5 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

3 pivots, 3 columns, linearly independent

2.2



$$\{\vec{v} \mid T(\vec{v}) = \vec{v}\}$$

$$T(\vec{v}) = \vec{v}$$
$$T(\vec{v}) = \{\vec{0}\}$$

$$\begin{bmatrix} h_1 \\ \vdots \\ h_5 \end{bmatrix} \cdot T = \begin{bmatrix} y_1 \\ \vdots \\ y_5 \end{bmatrix}$$ new heat

Define

$$G(\vec{v}) = T(\vec{v}) - \vec{v}$$

now we just want kernel of G

Standard matrix

$$T(\vec{v}) = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

standard matrix of G

$$G(\vec{v}) = \begin{bmatrix} -1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & -1 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & .5 & .5 & 0 & 0 \\ 0 & -.75 & .75 & 0 & 0 \\ 0 & .75 & -.75 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \to \begin{bmatrix} -1 & .5 & .5 & 0 & 0 \\ 0 & -.75 & .75 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 - x_3 \\ x_2 - x_3 \\ x_4 - x_5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad x_2 \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad x_4 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$x_5 \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{Basis} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$$V_{1/2} = \begin{bmatrix} 1/2 & -1/4 & -1/4 & 0 & 0 \\ -1/4 & 1/2 & -1/4 & 0 & 0 \\ -1/4 & -1/4 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 0 & 1/2 & -1/2 \end{bmatrix}$$

ther kernel will be the same b/c everything is just scaled, basis does not change if kernel doestn change

when you use vectors of $-\frac{1}{2}\vec{v}$, the values will just be the inverse of 1/2 of each heat value