ECE 213 Continuous-Time Signals and Systems - Midterm Exam 2

Wednesday, April 26 2023, 7:00pm-9:00pm, 104 Thompson Hall

Name: ____ **Student ID:** ____________________

Overview

Before you take the exam, read the following exam taking conditions:

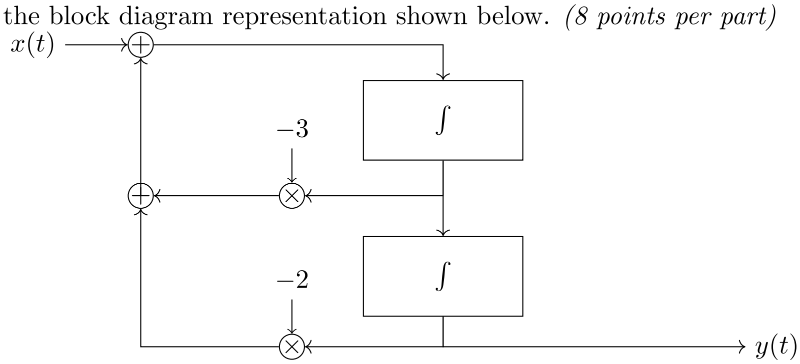
- The exam consists of five problems for 100 points. The points are partitioned among the different part of each problem; you should spend your **two hours** accordingly.
- When you complete your work, you will be asked scan your exam into a PDF file using your smartphone (e.g., with an app such as CameraScan, Adobe Scan, etc.) and send it to the instructor as a single PDF file in an email (mduarte@ecs.umass.edu) before you submit your completed work. Your scan should include the cover sheet, the question sheet, all your work, and the handwritten formula sheet at the end; each page of your exam should be scanned as a separate page in the PDF file. Let the instructor know when submitting your exam if you are not able to scan.
- You are allowed **one handwritten page** of notes (one side). Calculators are **not** allowed. You will be provided with all necessary blank paper; please number each of the pages you submit in order.

Instructions

- **Full credit will be given only to fully justified answers.**
- Giving the steps along the way to the answer will not only earn full credit but also maximize the partial credit should you make a mistake or get stuck. If you get stuck, attempt to neatly define your approach to the problem and why you are stuck.
- If part of a problem depends on a previous part that you are unable to solve, explain the method for doing the current part and, if possible, give the answer in terms of the quantities of the previous part that you were unable to obtain.
- Start each problem on a new page. Not only will this facilitate grading but also make it easier for you to jump back and forth between problems.
- If you get to the end of the problem and realize that your answer must be wrong, be sure to write "*this answer must be wrong because...*" so that I will know you recognized such a fact.
- Academic dishonesty **will be dealt with harshly** - the *minimum penalty* will be an "F" for the course.

1. *What An Exciting Life You Lead*: Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation $\frac{d^2y(t)}{dt^2} + 11\frac{dy(t)}{dt} + 24y(t) = 5\frac{dx(t)}{dt} + 3x(t)$.
   (a) Determine the system's transfer function $H(s)$ as the ratio of two polynomials in s. *(7 points)*
   (b) Sketch the zero-pole plot of $H(s)$. Is this system stable? *(7 points)*

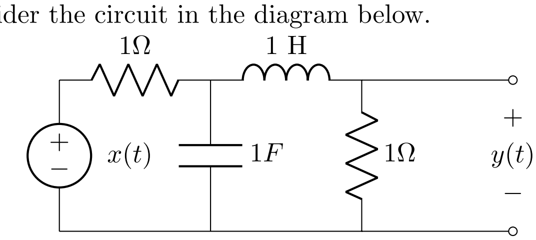
2. *Et Tu, Direct Form II?* The input $x(t)$ and the output $y(t)$ of a causal linear time-invariant system are related through the block diagram representation shown below. *(8 points per part)*



   (a) Determine the transfer function of the system, $H(s)$.
   (b) Draw a Direct-Form II representation for the system $\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} + 5x(t)$.

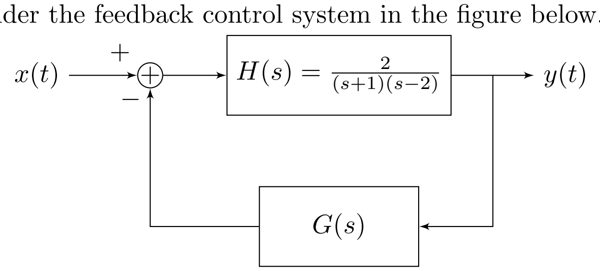
3. *How The Turntables*: Assume a linear time-invariant system has transfer function $H(s) = \frac{2s+2}{s^2+2s+5}$.
   (a) Find the system output $y(t)$ if the input is $x(t) = 10u(t)$. *(15 points)*
   (b) Find the system output $y(t)$ if the input is $x(t) = 3\cos(2t - 4)$. *(10 points)*

4. *Resistance is Futile*: Consider the circuit in the diagram below.



   (a) Find the transfer function $H(s)$ and the differential equation relating $x(t)$ and $y(t)$. *(15 points)*
   (b) Find the output voltage $y(t)$ if the input voltage is $x(t) = e^{-t}u(t)$. *(10 points)*

5. *Hold Your Horses*: Consider the feedback control system in the figure below.



   (a) If we use proportional feedback, $G(s) = K$, for what values of K will the closed loop system be stable? *(5 points)*
   (b) If we use proportional-derivative feedback, $G(s) = K_1 + K_2 s$, for what values of K_1 and K_2 will the closed loop system be critically damped? *(10 points)*

6. *We Need a Vacation:* You will be awarded points for meeting the following requirements for your submitted scans: (*i*) cover sheet (the other side of this paper): 2 points; (*ii*): question sheet (this page): 1 point; (*iii*): handwritten formula sheet: 1 point; (*iv*) single PDF file: 1 point.

ECE 213 Midterm Exam 2 (B) Spring 2023 Solutions

① What An Exciting Life You Lead

$$\frac{\partial^2 y(t)}{dt^2} + 11\frac{\partial y(t)}{dt} + 24y(t) = 5\frac{dx(t)}{dt} + 3x(t)$$

(a) $H(s) = ?$ Since $Y(s) = X(s)H(s)$ we find $H(s) = \frac{Y(s)}{X(s)}$

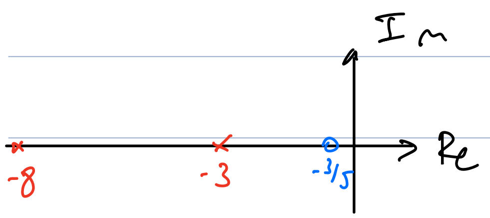
Apply Laplace transform above:

$$s^2 Y(s) + 11sY(s) + 24Y(s) = 5sX(s) + 3X(s)$$

Factorize: $Y(s)\,(s^2+11s+24) = X(s)\,(5s+3)$

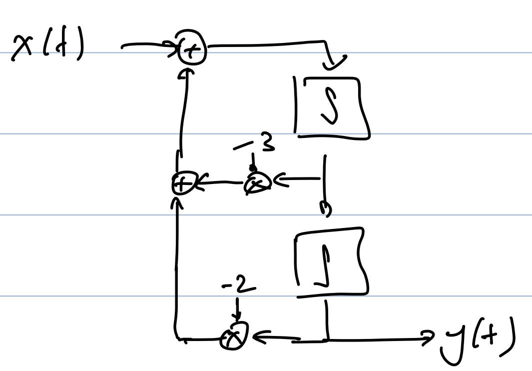
Divide: $\frac{Y(s)}{X(s)} = \frac{5s+3}{s^2+11s+24}$ and so $H(s) = \frac{5s+3}{s^2+11s+24}$

(b) Factorize: $H(s) = \frac{5(s+3/5)}{(s+8)(s+3)}$ $\Rightarrow$ zero at $s=-3/5$, Poles at $s=-8, s=-3$

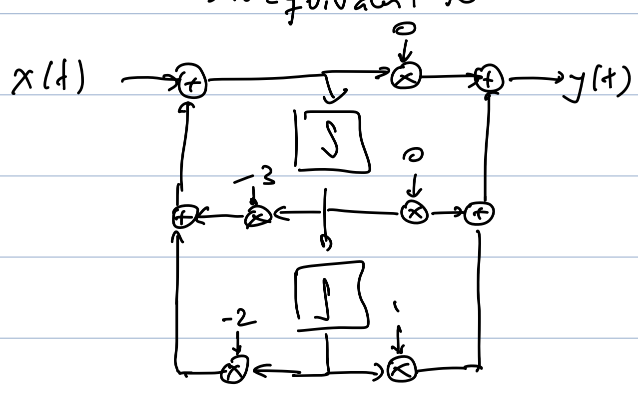


All poles are to the left of the axis (negative real part) and so the system is stable.

② Ef tu, direct form II?



(a) This is equivalent to



Comparing to the sketch from the notes we have

$b_0/a_0 = 0 \qquad b_1/a_0 = 0 \qquad b_2/a_0 = 1 \qquad -a_1/a_0 = -3 \qquad -a_2/a_0 = -2$

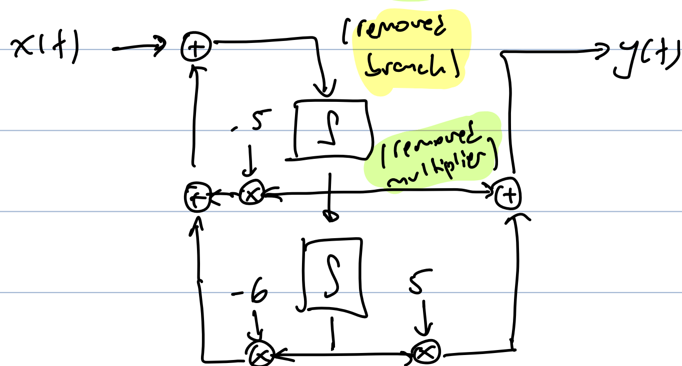
Set $a_0 = 1$ for simplicity $\rightarrow$ $b_0 = 0$ $b_1 = 0$ $b_2 = 1$ $a_0 = 1$ $a_1 = 3$ $a_2 = 2$

So we write $\quad H(s) = \dfrac{0s^2 + 0s + 1}{s^2 + 3s + 2} = \dfrac{1}{s^2+3s+2}$.

(b) $\quad \dfrac{\partial^2 y(t)}{dt^2} + 5\dfrac{dy(t)}{dt} + 6y(t) = \dfrac{\partial x(t)}{dt} + 5x(t)$

Copying from sketch: $a_0 = 1$ $a_1 = 5$ $a_2 = 6$ $b_0 = 0$ $b_1 = 1$ $b_2 = 5$

$\dfrac{b_0}{a_0} = 0 \quad \dfrac{b_1}{a_0} = 1 \quad \dfrac{b_2}{a_0} = 5 \quad -\dfrac{a_1}{a_0} = -5 \quad -\dfrac{a_2}{a_0} = -6$



③ $H(s) = \dfrac{2s+3}{s^2+2s+5}$ (a) Input $x(t) = 10u(t)$

$\mathcal{L} \downarrow$

$X(s) = \dfrac{10}{s}$

$Y(s) = X(s) \cdot H(s) = \dfrac{10(2s+2)}{s(s^2+2s+5)} = \dfrac{10(2s+2)}{s(s+1-2j)(s+1+2j)}$

Roots $\uparrow$

$p = -\dfrac{2 \pm \sqrt{4-20}}{2} = -1 \pm \dfrac{\sqrt{-16}}{2} = -1 \pm \dfrac{4j}{2}$

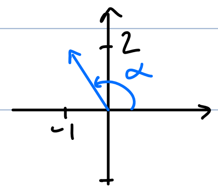
$= -1 \pm 2j$

Partial Fraction Expansion: $\quad Y(s) = \dfrac{A}{s} + \dfrac{B}{s+1-2j} + \dfrac{B^*}{s+1+2j}$

$$A = Y(s)\cdot s\big|_{s=0} = \frac{20s+20}{s^2+2s+5}\Big|_{s=0} = \frac{20}{5} = 4$$

$$B = Y(s)(s+1-2j)\big|_{s=-1+2j} = \frac{20s+20}{s(s+1+2j)}\Big|_{s=-1+2j} = \frac{\cancel{-20}+40j\cancel{+20}}{(-1+2j)(\cancel{4j})}$$

$$= \frac{10}{-1+2j} = \frac{10}{\sqrt{5}\cdot e^{j\alpha}} = 2\sqrt{5}\, e^{-j\alpha}$$



Polar form: $|-1+2j| = \sqrt{1^2+2^2} = \sqrt{5}$

$\angle(-1+2j) = \operatorname{atan}(-2)+\pi = \alpha$

So $$Y(s) = 4\cdot\frac{1}{s} + \frac{2\sqrt{5}\,e^{-j\alpha}}{s+1-2j} + \frac{2\sqrt{5}\,e^{j\alpha}}{s+1+2j}$$

$\mathcal{L}^{-1}\downarrow$ $\downarrow$ Pair 2 $\downarrow$ Pair 15

$$y(t) = 4\cdot u(t) + 4\sqrt{5}\cdot e^{-t}\cos(2t-\alpha)u(t)$$

(b) Input $x(t) = 3\cos(2t-4)$

Use Frequency response: $H(\omega) = H(j\omega) = H(s)\big|_{s=j\omega}$

$$= \frac{2\cdot 2j+2}{(2j)^2+2(2j)+5} = \frac{4j+2}{-4+4j+5}$$

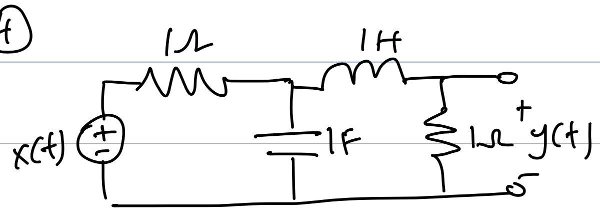
$$= \frac{4j+2}{4j+1}$$

$$|H(\omega)| = \frac{|4j+2|}{|4j+1|} = \frac{\sqrt{4^2+2^2}}{\sqrt{4^2+1^2}} = \frac{\sqrt{20}}{\sqrt{17}} = \sqrt{\frac{20}{17}} = 2\sqrt{\frac{5}{17}}$$

$$\angle H(\omega) = \operatorname{atan}(2) - \operatorname{atan}(4) = \beta$$

So $$y(t) = |H(2j)|\cdot x(t-\angle H(2j)) = 6\sqrt{\frac{5}{17}}\cos(2t-4+\beta)$$

④

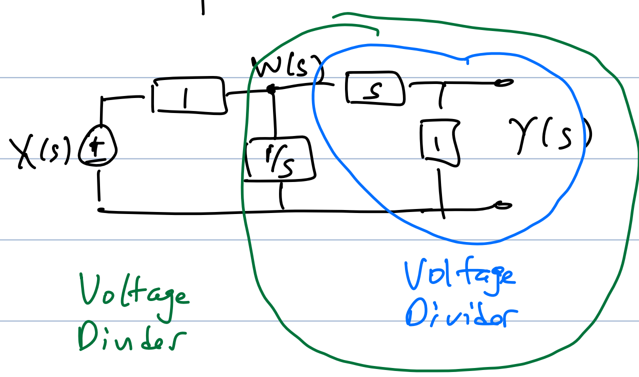


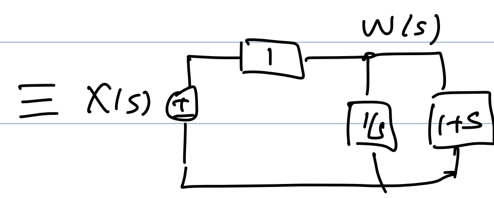
(a) $H(s) = ?$

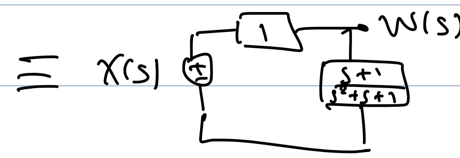
$H(s) = Y(s)/X(s)$:

Double Voltage Divider.

Equivalent impedance circuit:





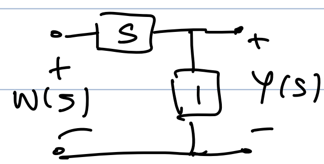


$$z = \frac{\cancel{s}(1+s)}{\frac{1}{s}+1+s} \cdot \frac{\cancel{s}}{s} = \frac{s+1}{s^2+s+1}$$

$$W(s) = X(s) \cdot \frac{\frac{s+1}{s^2+s+1}}{\frac{s+1}{s^2+s+1}+1}$$

$$W(s) = X(s) \cdot \frac{s+1}{s+1+s^2+s+1} = \frac{s+1}{s^2+2s+2} X(s)$$

Second voltage divider:



$$Y(s) = \frac{1}{s+1} \cdot W(s)$$

$$= \frac{1}{s^2+2s+2} X(s)$$

and so $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{1}{s^2+2s+2}$. and $\dfrac{\partial^2 y(t)}{dt^2} + 2\dfrac{\partial y(t)}{dt} + 2y(t) = x(t)$.

(b) Output for $x(t) = e^{-t}u(t)$? $\longrightarrow$ $X(s) = \dfrac{1}{s+1}$

$$Y(s) = X(s)H(s) = \frac{1}{(s+1)(s^2+2s+2)} = \frac{1}{(s+1)(s+1+j)(s+1-j)}$$

roots $p = \dfrac{-2 \pm \sqrt{4-8}}{2} = -1 \pm j$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+1+j} + \frac{B^*}{s+1-j}$$

$$A = Y(s)(s+1)\Big|_{s=-1} = \frac{1}{s^2+2s+2}\Big|_{s=-1} = \frac{1}{1-\cancel{2}+\cancel{2}} = 1$$

$$B = Y(s)(s+1+j)\Big|_{s=-1-j} = \frac{1}{(s+1)(s+1-j)}\Big|_{s=-1-j} = \frac{1}{(\cancel{-1}-j\cancel{+1})(\cancel{-1}-j\cancel{+1}-j)}$$

$$= \frac{1}{(-j)(-2j)} = -\frac{1}{2}$$

$$Y(s) = \frac{1}{s+1} - \frac{1/2}{s+1+j} - \frac{1/2}{s+1-j}$$

$\mathcal{L}^{-1}$ (Pair 3 Pair 15

$$y(t) = e^{-t}u(t) - \frac{1}{\cancel{2}}\cdot\cancel{2}\, e^{-t}\cos(t-0)\,u(t)$$

$$= e^{-t}u(t) - e^{-t}\cos t\; u(t)$$

⑤ Hold Your Horses

$$H(s) = \frac{2}{(s+1)(s-2)} = \frac{2}{s^2-s-2} \qquad N(s) = 2 \qquad D(s) = s^2-s-2$$

(a) Case $G(s) = K$: Closed loop equivalent

$$Q(s) = \frac{N(s)}{D(s)+N(s)G(s)} = \frac{2}{s^2-s-2+2\cdot K} = \frac{2}{s^2-s+2K-2}$$

Because s has a negative coefficient, the closed loop equivalent is unstable regardless of the value of K.

(b) Case $G(s) = K_1 + K_2 s$: Closed loop equivalent

$$Q(s) = \frac{N(s)}{D(s)+N(s)G(s)} = \frac{2}{s^2-s-2+2(K_1+K_2 s)} = \frac{2}{s^2+s(2K_2-1)+2K_1-2}$$

$2\alpha = 2K_2 - 1$ $\quad \beta = 2K_1 - 2$

We have $\alpha = K_2 - \frac{1}{2}$, $\beta = 2K_1 - 2$. For a critically damped system we need $\alpha^2 = \beta > 0$ or $(K_2 - \frac{1}{2})^2 = 2K_1 - 2 > 0$

So we could set $K_1 = \frac{3}{2}$, $K_2 = 4$ to get $(\frac{4}{2} - 1)^2 = 2 \cdot \frac{3}{2} - 2$ ✓