1.1

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 8 & 1 & 0 & 0 \\ -1 & 0 & -4 & 0 & 1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|ccc} 1 & 0 & 5 & 1 & 0 & -1 \\ 1 & 0 & 4 & 0 & -1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 1 & -1 \\ 1 & 0 & 4 & 0 & -1 & 0 \\ 1 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 1 & -1 \\ 1 & 0 & 0 & -4 & 5 & 4 \\ 1 & 1 & 0 & -3 & -3 & 4 \end{array}\right] \left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 1 & -1 \\ 1 & 0 & 0 & -4 & -3 & 4 \\ 0 & 1 & 0 & 1 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -5 & 4 \\ 0 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 1 & 1 & -1 \end{array}\right] \qquad \begin{bmatrix} 4 & -5 & 4 \\ 1 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 4 & -5 & 4 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right]$$ do these steps backwards $= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

Kernel is $\{\vec{0}\}$

because matrix has a pivot in every column, all columns are linearly independent thus spans all of $\mathbb{R}^3$

2

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 1 & 1 & 0 & 0 \\ 2 & -2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 2 & -2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{4}{3} & \frac{1}{3} & -\frac{2}{3} & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{4}{3} & \frac{1}{3} & -\frac{2}{3} & 1 & 0 \\ 0 & \frac{5}{3} & -\frac{2}{3} & -\frac{2}{3} & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{3}{4} & 0 \\ 0 & \frac{5}{3} & -\frac{2}{3} & -\frac{2}{3} & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{3}{4} & 0 \\ 0 & 0 & -\frac{1}{4} & -\frac{2}{3} & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{4} & -1 & 1 & 1 \\ 0 & 1 & -\frac{1}{4} & \frac{1}{2} & -\frac{3}{4} & 0 \\ 0 & 0 & 1 & 6 & -5 & -4 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 2 & -2 & -1 \\ 0 & 0 & 1 & 6 & -5 & -4 \end{array}\right]$$

do backwards

$$\begin{bmatrix} -1 & 1 & 1 \\ 2 & -2 & -1 \\ 6 & -5 & -4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 2 & -2 & -1 & 0 \\ 6 & -5 & -4 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

because the matrix has a pivot in every column so all columns are linearly independent & therefore spans all of $\mathbb{R}^3$

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

Kernel is $\{\vec{0}\}$

3

the matrix for linear transformation $T_{E \leftarrow A}$

is $\begin{bmatrix} 2 & 1 & 8 \\ -1 & 0 & -4 \\ 1 & 1 & 3 \end{bmatrix}$ because you are just takin the identity matrix to the other matrix, this uses just the original matrix

4 the matrix for linear transformation $T_{E \leftarrow B}$

$\begin{bmatrix} 3 & -1 & 1 \\ 2 & -2 & 1 \\ 2 & 1 & 0 \end{bmatrix}$ for the same reason stated above

5 $T_{B \leftarrow A}$ $T_{B \leftarrow E}$ $T_{E \leftarrow A}$

$$\cancel{\begin{bmatrix} -1 & 1 & 1 \\ 2 & -2 & -1 \\ 6 & -5 & -4 \end{bmatrix}} \cancel{\begin{bmatrix} 2 & 1 & 8 \\ -1 & 0 & -4 \\ 1 & 1 & 3 \end{bmatrix}}$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 8 & 3 & -1 & 1 \\ -1 & 0 & -4 & 2 & -2 & 1 \\ 1 & 1 & 3 & 2 & 1 & 0 \end{array}\right] \left[\begin{array}{ccc|ccc} 1 & 1/2 & 4 & 3/2 & -1/2 & 1/2 \\ -1 & 0 & -4 & 2 & -2 & 1 \\ 1 & 1 & 3 & 2 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 4 & 3/2 & -1/2 & 1/2 \\ 0 & 1/2 & 0 & 7/2 & -5/2 & 3/2 \\ 1 & 1 & 3 & 2 & 1 & 0 \end{array}\right] \left[\begin{array}{ccc|ccc} 1 & 1/2 & 4 & 3/2 & -1/2 & 1/2 \\ 0 & 1/2 & 0 & 7/2 & -5/2 & 3/2 \\ 0 & 1/2 & -1 & 1/2 & 3/2 & -1/2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 4 & -2 & 2 & -1 \\ 0 & 1 & 0 & 7 & -5 & 3 \\ 0 & 1/2 & -1 & 1/2 & 3/2 & -1/2 \end{array}\right] \left[\begin{array}{ccc|ccc} 1 & 0 & 4 & -2 & 2 & -1 \\ 0 & 1 & 0 & 7 & -5 & 3 \\ 0 & 0 & -1 & -3 & 4 & 2 \end{array}\right]$$

5 cont

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & -2 & 2 & -1 \\ 0 & 1 & 0 & 7 & -5 & 3 \\ 0 & 0 & 1 & 3 & -4 & 2 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -14 & 18 & -9 \\ 0 & 1 & 0 & 7 & -5 & 3 \\ 0 & 0 & 1 & 3 & -4 & 2 \end{array}\right]$$

$$\begin{bmatrix} -14 & 18 & -9 \\ 7 & -5 & 3 \\ 3 & -4 & 2 \end{bmatrix}$$

6.)

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 1 & 2 & 1 & 8 \\ 2 & -2 & 1 & -1 & 0 & -4 \\ 2 & 1 & 0 & 1 & 1 & 3 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & -1/3 & 1/3 & 2/3 & 1/3 & 8/3 \\ 2 & -2 & 1 \cancel{4} & -1 & 0 & 4 \\ 2 & 1 & 0 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -1/3 & 1/3 & 2/3 & 1/3 & 8/3 \\ 0 & -4/3 & 1/3 & -7/3 & -2/3 & -28/3 \\ 2 & 1 & 0 & 1 & 1 & 3 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & -1/3 & 1/3 & 2/3 & 1/3 & 8/3 \\ 0 & -4/3 & 1/3 & -7/3 & -2/3 & -28/3 \\ 0 & 5/3 & -2/3 & -1/3 & 1/3 & -7/3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{4} & 5/4 & 1/2 & 5 \\ 0 & 1 & -1/4 & 7/4 & 1/2 & 7 \\ 0 & 0 & 1 & 13 & 2 & 56 \end{array}\right] \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & -9 \\ 0 & 1 & 0 & 5 & 1 & 21 \\ 0 & 0 & 1 & 13 & 2 & 56 \end{array}\right]$$

$$\begin{matrix} -2 & 0 & -9 \\ 5 & 1 & 21 \\ 13 & 2 & 56 \end{matrix}$$