
"What's our vector, Victor?"

Announcements, Goals, and Reading

Announcements:

- HW02 due Tuesday Sep 27^{th}, 11:59 pm on Mastering Physics
- *Reminder: SEC01 ends at 12:05pm, SEC02 ends at 1:10pm*
- **Help Resources: See moodle**

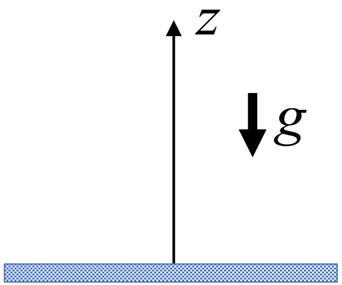
Goals for Today:

- Free Fall
- Vectors

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 2: Kinematics in 1 Dimension
- Chapter 3: Vectors

Free Fall



Let z-coordinate measure height above ground

Basic Equations…

Height
$$z(t) = z_0 + v_0 t - \frac{1}{2} g t^2$$

Initial height

Initial velocity in z-direction

Constant gravitational acceleration $g=9.8 m/s^2$ is a (nearly) constant positive quantity.
a=-g, acceleration acting along the -z direction

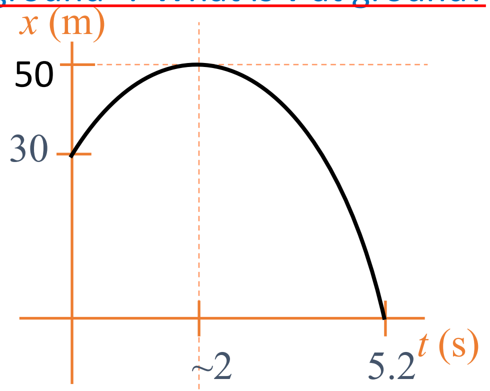
Velocity
$$v(t) = \frac{dz}{dt} = v_0 - gt$$

Can tailor these formulas to numerous physics problems!

Motion in 1 Dimension: Free Fall

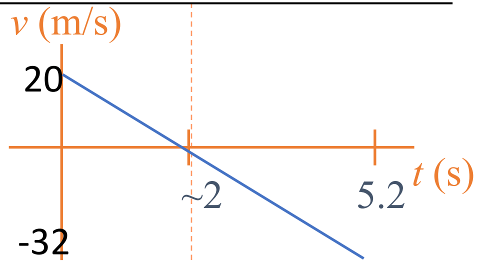
Example: A ball is thrown vertically upward at 20 m/s from the edge of cliff 30 m high. Time to reach top=? Max height? Time to reach ground=? What is v at ground?

Position: $x(t) = 30 + 20t - 0.5gt^2$

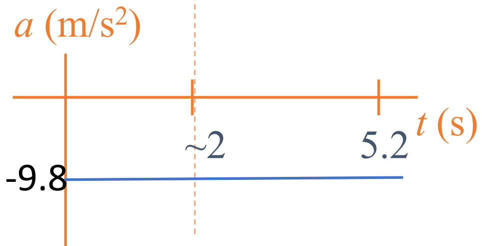


T_{gd} is where x=0

Velocity = dx/dt= v(t) = 20 – 9.8t



Acceleration a(t) = dv/dt = -9.8 m/s^2



[Acceleration is constant in this case.]

Summary of kinematics along a line:

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$
$$v(t) = v_0 + a t$$

$\forall a =$ for any a

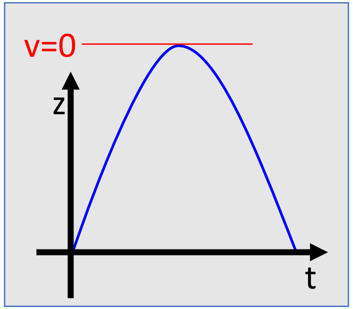
$$z(t) = z_0 + v_0 t - \frac{1}{2} g t^2$$
$$v(t) = v_0 - g t$$

Gravity on Earth case g=9.8m/s^2

$$D = \frac{1}{2a}(v_1^2 - v_0^2)$$

final velocity

Initial velocity



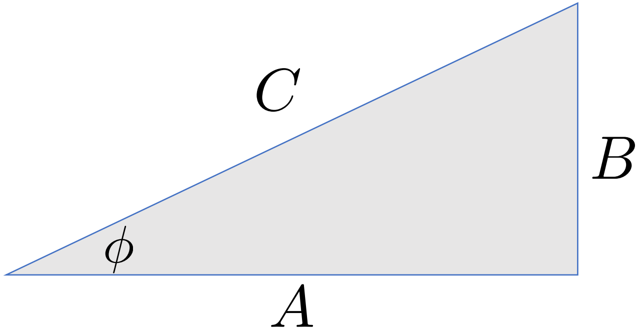
Trigonometry review

(this will be important soon)

A Length adjacent to angle

B Length opposite to angle

C Hypotenuse



$$\sin\phi = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{B}{C}$$

$$\cos\phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{C}$$

$$\tan\phi = \frac{\text{opposite}}{\text{adjacent}} = \frac{B}{A}$$

Length of $C = \sqrt{A^2 + B^2}$

Vectors and Coordinate Systems

Recall…

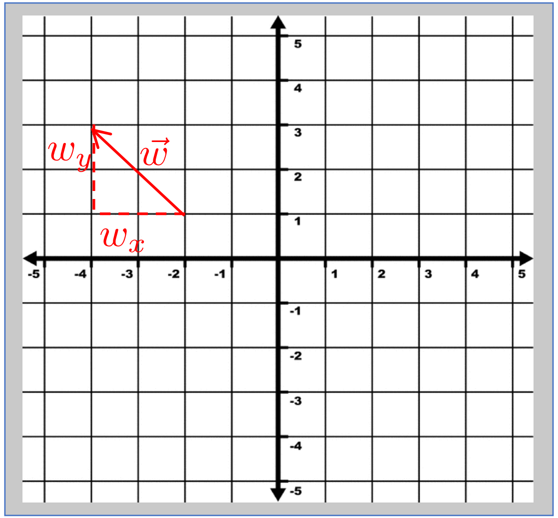
Scalars & Vectors

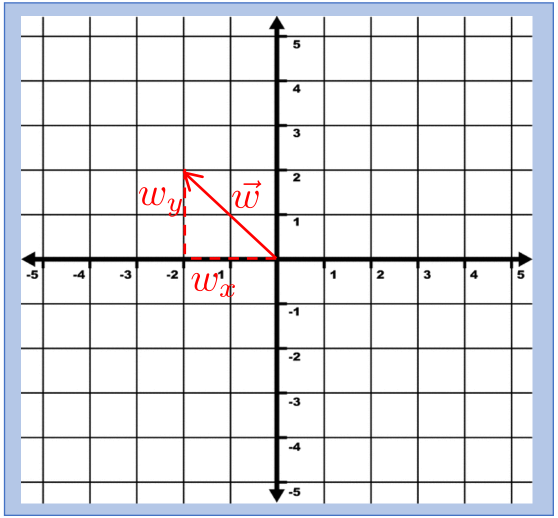
- A **scalar** is a quantity that only has a magnitude, such as…

  Mass, length, distance, speed, temperature

- A **vector** is a quantity that has both magnitude and direction, such as…

  Displacement, velocity, acceleration, force,
  momentum, angular momentum, wind





Vectors and Components

- 2D vector: Pair of numbers

(x,y) coordinates of tip, when tail is placed at origin

For a vector not drawn with tail at origin…

2 equivalent ways to find components

1. Translate tail over to origin

2. Measure x and y components of tip relative to tail of vector left in place

For this vector both methods give…

$$\vec{w} = (-2, 2)$$

What are the x- and y-components of vector $\vec{C}$?

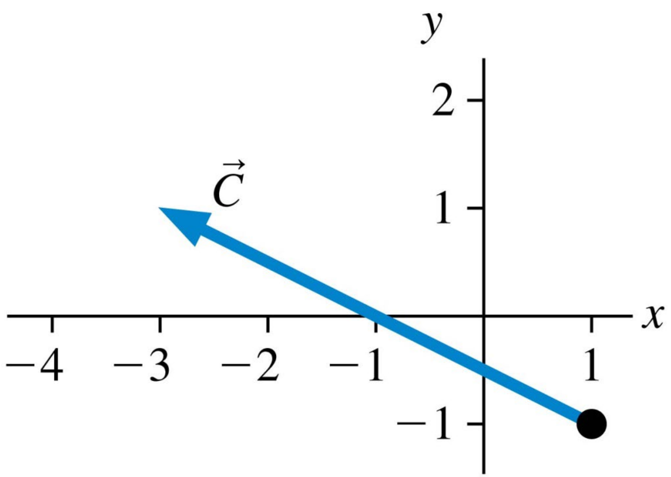
A. 1, –3

B. –3, 1

C. 1, –1

D. –4, 2

E. 2, –4



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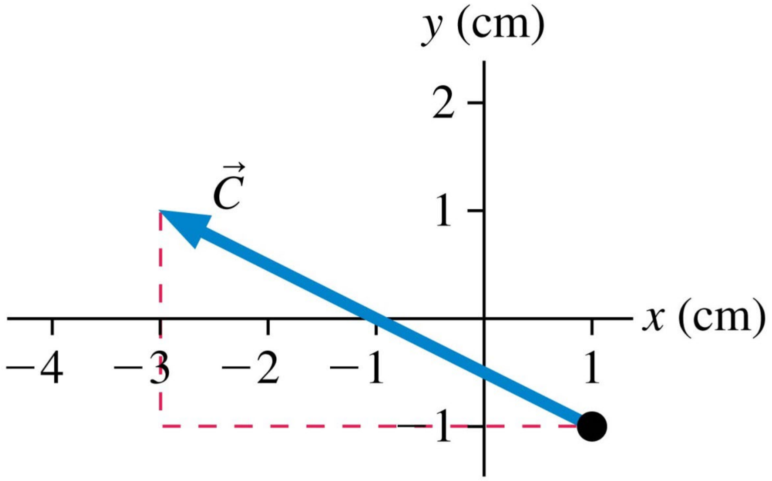


Vectors

Work in 2D for convenience.

Straightforward to generalize to 3D

Represent in several equivalent ways

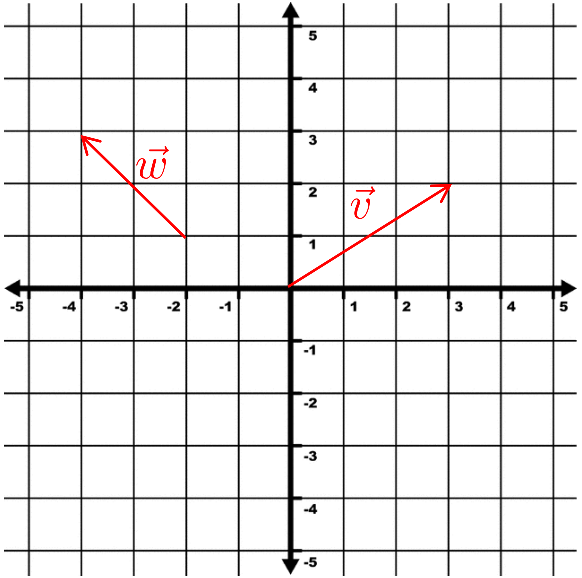
(1) Graphically

An arrow on a plane with fixed length and direction

(2) Pair of numbers

(x,y) coordinates of tip, when tail placed at origin

$$\vec{v} = (3, 2)$$
$$\vec{w} = (-2, 2)$$



Called x & y components of vector

Generic vector in terms of x & y components:

$$\vec{v} = (v_x , v_y)$$

(2) Pair of numbers

(x,y) coordinates of tip, when tail placed at origin

Called x & y components of vector

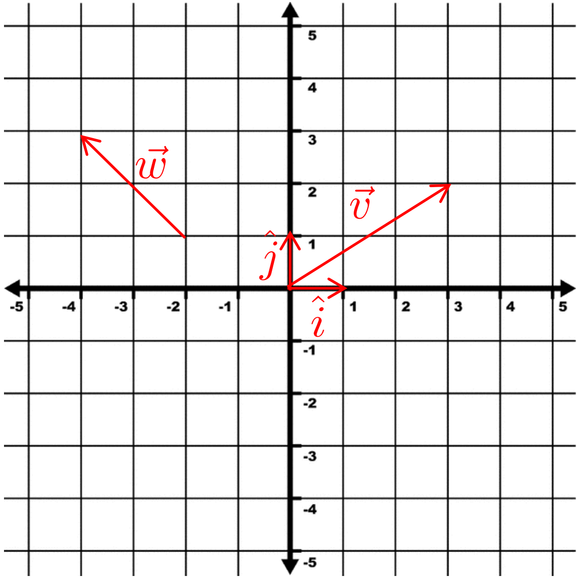
$$\vec{v} = (v_x, v_y)$$

(3) Write vector as sum of two vectors in perpendicular directions

$\hat{i} \longleftarrow$ Vector of length 1 in x-direction

$\hat{j} \longleftarrow$ Vector of length 1 in y-direction

General vector becomes $\vec{v} = v_x\hat{i} + v_y\hat{j}$



For the specific vectors in the graphic

$$\vec{v} = 3\hat{i} + 2\hat{j}$$

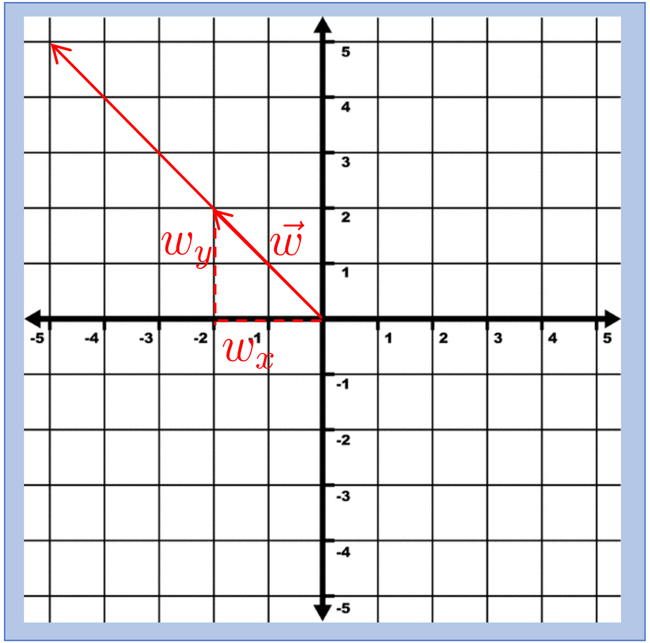
$$\vec{w} = -2\hat{i} + 2\hat{j}$$

Multiplying vectors by numbers

For any vector $\vec{w} = (wx, wy, wz)$

and real number r:

$$r\vec{w} = (rw_x, rw_y, rw_z)$$



Vector points in same direction, but is now r times longer (r>1) or shorter (r<1) …

Can we divide vectors by numbers?

Yes! $$\frac{\vec{w}}{r} = \left(\frac{w_x}{r}, \frac{w_y}{r}, \frac{w_z}{r}\right)$$

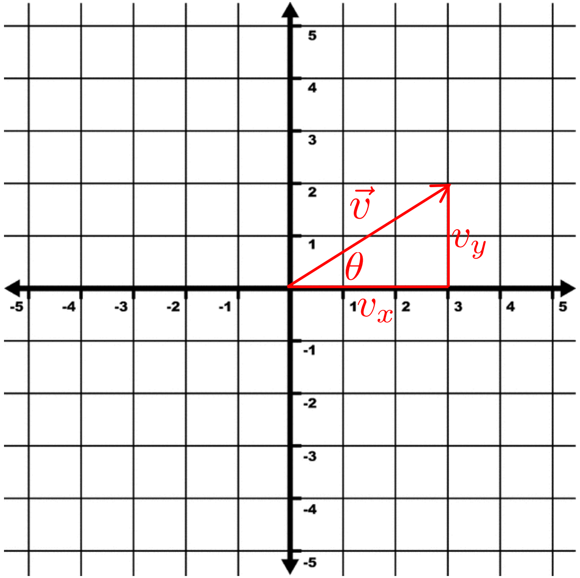
Magnitude and Direction of vector

Magnitude for general vector

$$\vec{v} = (v_x, v_y)$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

Magnitude is length of vector given by Pythagorean theorem



Direction

Angle vector makes with positive x-axis

$$\tan\theta = \frac{v_y}{v_x}$$

Recall tangent is opposite over adjacent

(find inverse tangent of both sides)

$$\tan\theta = \frac{2}{3} \Rightarrow \theta = 34^o$$

For vector in graphic…

$$\vec{v} = (\underset{v_x}{3}, \underset{v_y}{2}) \Rightarrow |\vec{v}| = \sqrt{(3)^2 + (2)^2} = \sqrt{13}$$

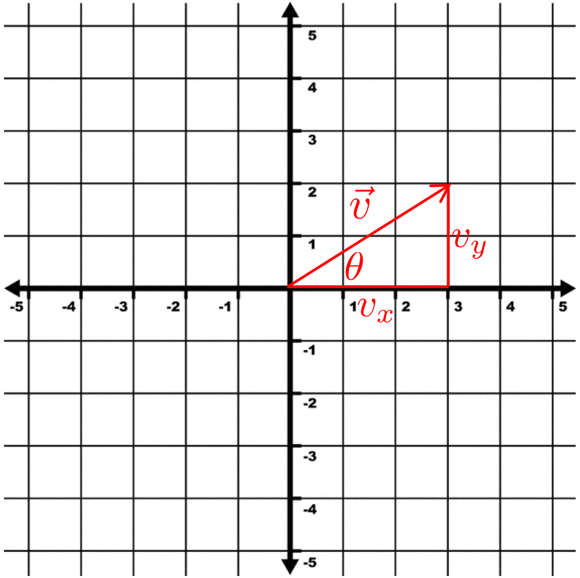
Magnitude and Direction of vector

Magnitude for general vector

$$\vec{v} = (v_x, v_y)$$

$$\boxed{|\vec{v}| = \sqrt{v_x^2 + v_y^2}}$$

$$\boxed{\tan\theta = \frac{v_y}{v_x}}$$



What if we know the magnitude but need the components!?

Express x & y components in terms of magnitude and direction

From trigonometry:
$$\boxed{\begin{aligned} v_x &= |\vec{v}|\cos\theta \\ v_y &= |\vec{v}|\sin\theta \end{aligned}}$$

Example…

A vector has length 5.0 and makes angle 49° with the positive x-axis

$$v_x = 5\cos(49^o) = 3.3$$

$$v_y = 5\sin(49^o) = 3.8$$

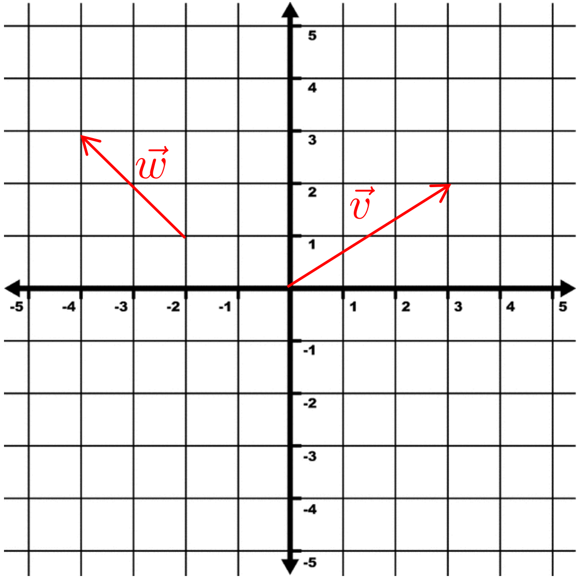
Vector Notation Summary

$$
\begin{aligned}
\text{Scalar} &\quad : \quad v \\
\text{Vector} &\quad : \quad \vec{v} \\
\text{Unit Vector} &\quad : \quad \hat{v} \\
\text{Decomposition} &\quad : \quad \vec{v} = |v| \cos\theta\, \hat{i} + |v| \sin\theta\, \hat{j} = v_x \hat{i} + v_y \hat{j}
\end{aligned}
$$

Adding and subtracting vectors in terms of components

Two general vectors $\vec{v} = (v_x, v_y)$

$\vec{w} = (w_x, w_y)$



Sum

$$\boxed{\vec{v} + \vec{w} = (v_x + w_x, v_y + w_y)}$$

x & y components add

Difference

$$\boxed{\vec{v} - \vec{w} = (v_x - w_x, v_y - w_y)}$$

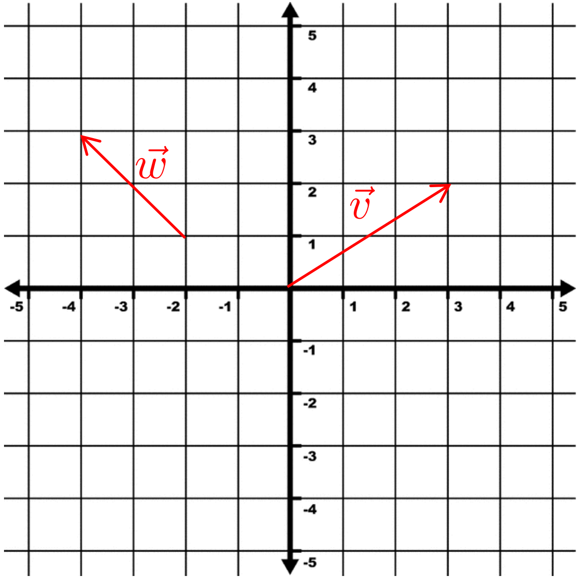
x & y components subtract

Adding and subtracting vectors in terms of components

Particularly clear in terms of basis vectors…

$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{w} = w_x\hat{i} + w_y\hat{j}$$



$$\vec{v} + \vec{w} = (v_x\hat{i} + v_y\hat{j}) + (w_x\hat{i} + w_y\hat{j})$$

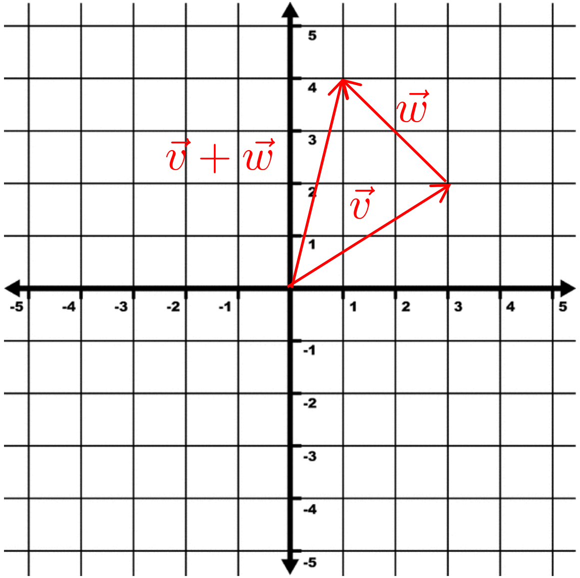
$$= (v_x + w_x)\hat{i} + (v_y + w_y)\hat{j}$$

Gather together everything that multiplies the same basis vector

$$\vec{u} = \vec{v} + \vec{w} \quad \Rightarrow \quad \begin{aligned} u_x &= v_x + w_x \\ u_y &= v_y + w_y \end{aligned}$$

Adding and subtracting vectors in terms of components

Corresponds to graphical "tip to tail" addition prescription



Reading from graph...

$$\vec{v} + \vec{w} = 1\hat{i} + 4\hat{j}$$

Using algebra...

$$\vec{v} = 3\hat{i} + 2\hat{j}$$

$$\vec{w} = -2\hat{i} + 2\hat{j}$$

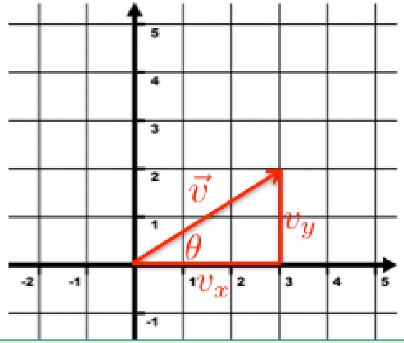
$$\vec{v} + \vec{w} = (3-2)\hat{i} + (2+2)\hat{j}$$

$$= 1\hat{i} + 4\hat{j} \qquad \checkmark$$

Trigonometry & Vectors

Fairly standard setup…

θ Angle of vector with respect to positive x axis; positive angle is in counterclockwise direction

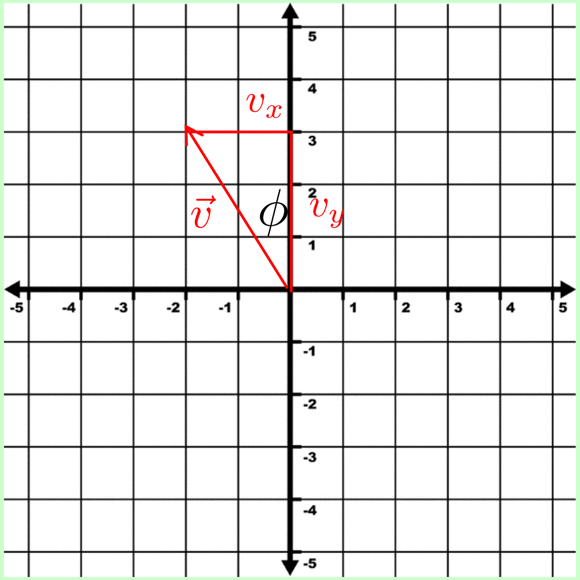


$$v_x = |\vec{v}| \cos\theta$$
$$v_y = |\vec{v}| \sin\theta$$

However, sometimes natural to use a different angle…

ϕ Angle of vector with respect to positive y axis with positive angle in counterclockwise direction

$$v_x = -|\vec{v}| \sin\phi$$
$$v_y = +|\vec{v}| \cos\phi$$



Vector addition problem

- Alice's plane first flies 175 km to airport A located in the direction 30^o north of east.
- Next it flies 153 km, 20^o west of north to town B.
- Finally, it flies 195 km due west to city C.

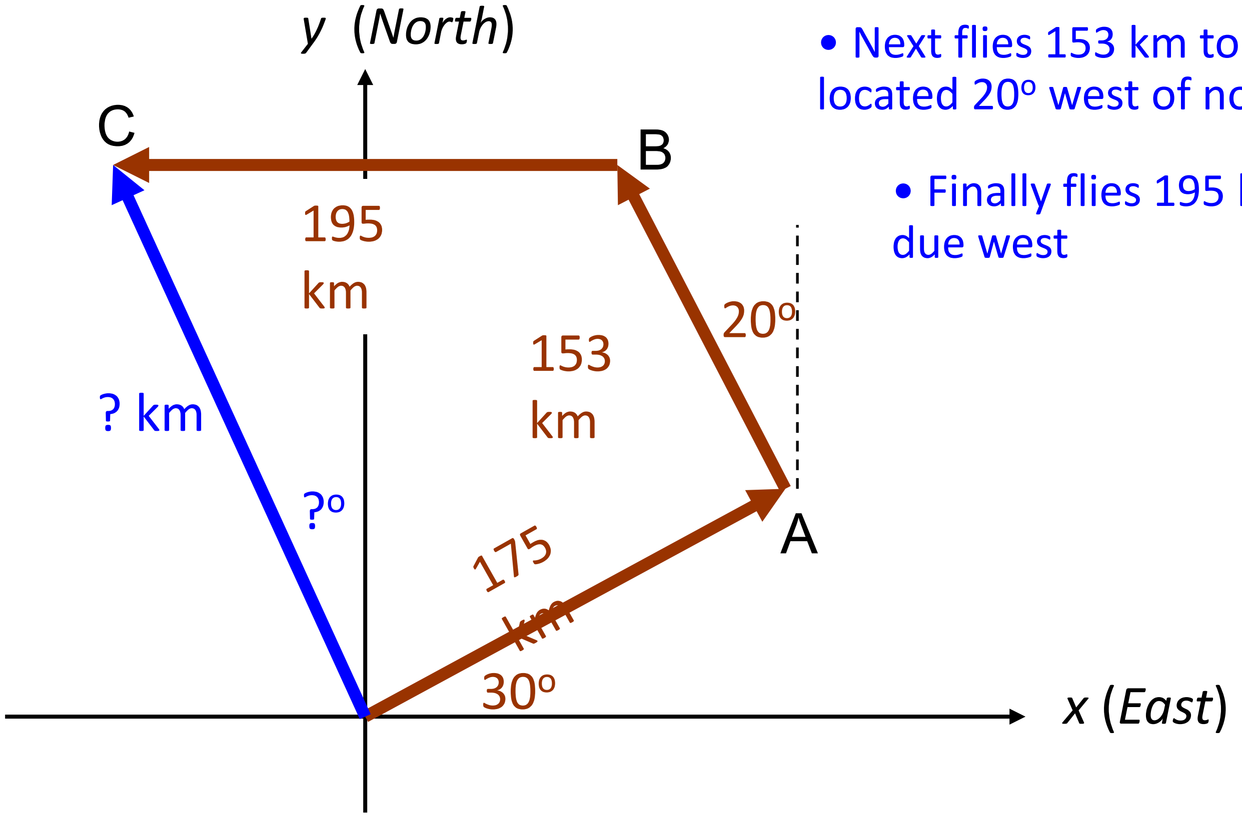
- What is the location of city C relative to the starting point? (How far away and in what direction?)

Strategy:

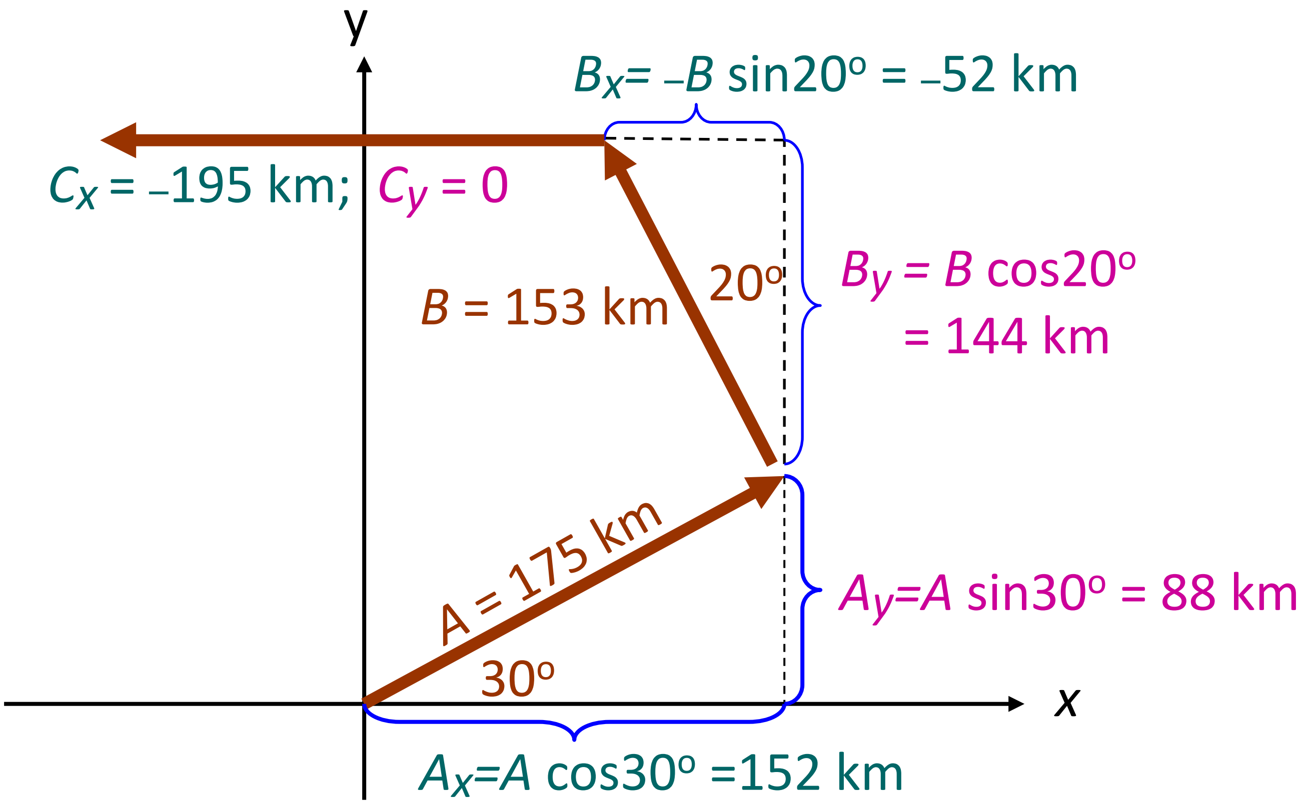
1. It's a vector problem; draw a diagram—a map.
2. Find *x*- (east) and *y*- (north) components of all vectors.
3. Add components to find total displacement vector.
4. Determine magnitude and angle of total displacement.

Step 1 – Draw diagram

- First flies 175 km to A located 30^o north of east
- Next flies 153 km to B located 20^o west of north
- Finally flies 195 km due west

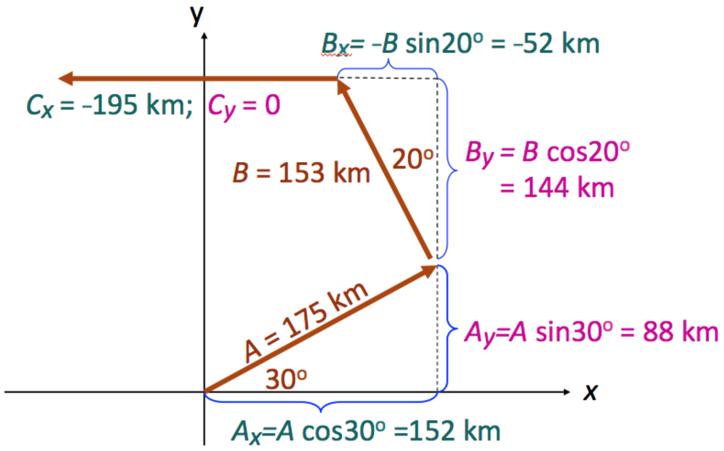


Step 2 – Find components of vectors



Step 2 – Find total displacement vector

$$\vec{A} = (152km)\hat{i} + (88km)\hat{j}$$
$$\vec{B} = (-52km)\hat{i} + (144\ km)\hat{j}$$
$$\vec{C} = (-195km)\hat{i}$$



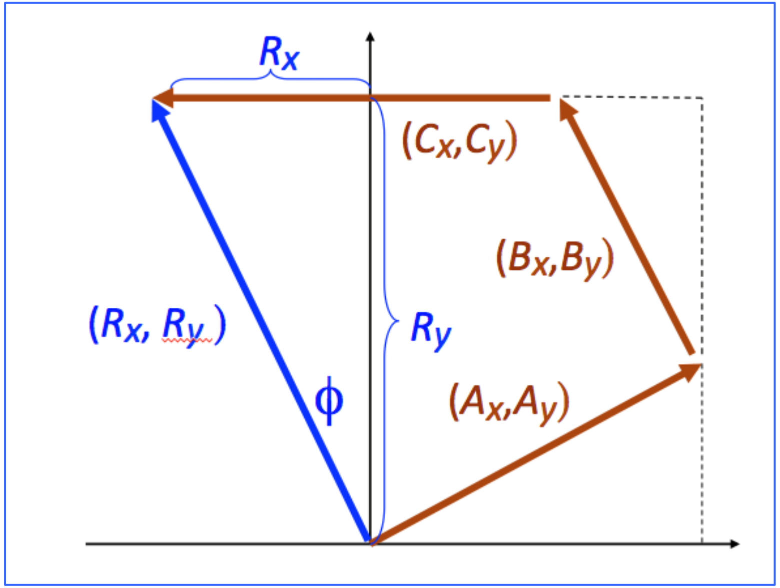
Displacement vector

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$
$$= (152km - 52km - 195km)\hat{i} + (88km + 144km)\hat{j}$$
$$= (-95km)\hat{i} + (232km)\hat{j}$$

Components of displacement vector

$$R_x = -95km$$
$$R_y = 232km$$

Find magnitude and angle of displacement vector



$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

$$\boxed{\begin{aligned} R_x &= -95km \\ R_y &= 232km \end{aligned}}$$

$$R = |\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= 251km$$

How far from starting point
Alice's plane ends up

$$\tan\phi = \frac{|R_x|}{|R_y|} = \frac{95km}{232km} = 0.41 \quad \Rightarrow \quad \phi = \tan^{-1}(0.41) = 22^o$$

West of North

Finishing Kinematics in One Dimension

Motion on an Inclined Plane = motion with tilted axes

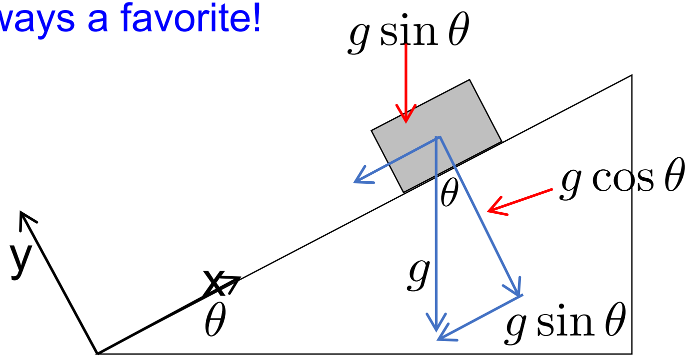
Block on Inclined Plane

Always a favorite!

Use coordinate x to measure distance up the plane

Use trigonometry to figure out acceleration of block down plane

$$a = -g\sin\theta$$



y-component of gravitational acceleration is "blocked" by the plane
NO FRICTION HERE!

Motion along the plane has constant acceleration

$$x = x_0 + v_0 t - \frac{1}{2}(g\sin\theta)\,t^2$$

$$v = v_0 - (g\sin\theta)\,t$$

$$D = x - x_0 = \frac{1}{2a}(v_1^2 - v_0^2)$$

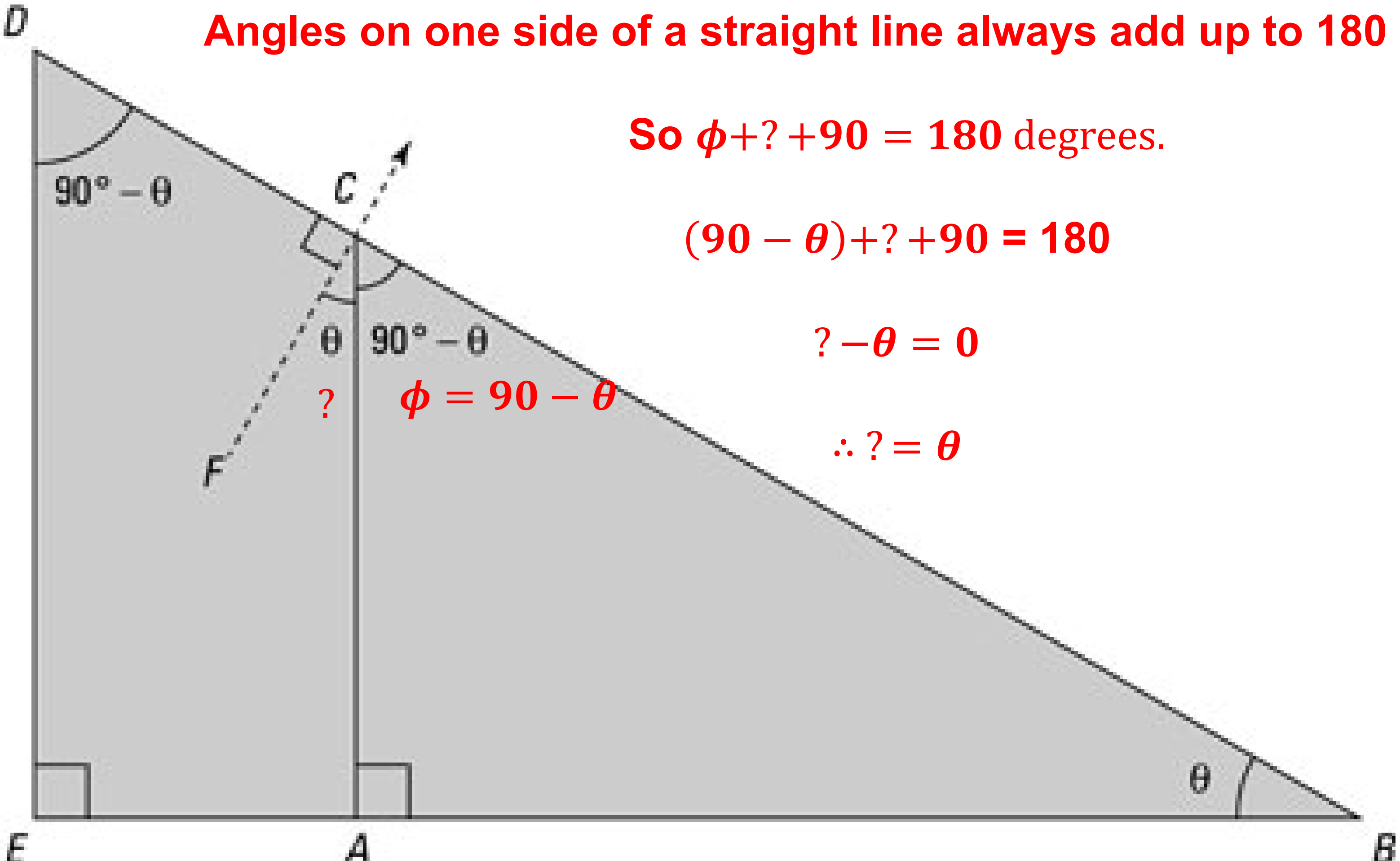
Note

$\theta = 0 \Rightarrow g\sin\theta = 0$

No acceleration for block on flat surface!

If a right triangle is drawn such that the hypotenuse is // to the side of the triangle opposite to θ, and the adjacent side of the new triangle is normal to the hypotenuse of the old triangle, and the hat is the unknown angle ? of this new triangle?

Angles on one side of a straight line always add up to 180 degrees.



So $\phi + ? + 90 = 180$ degrees.

$(90 - \theta) + ? + 90 = 180$

$? - \theta = 0$

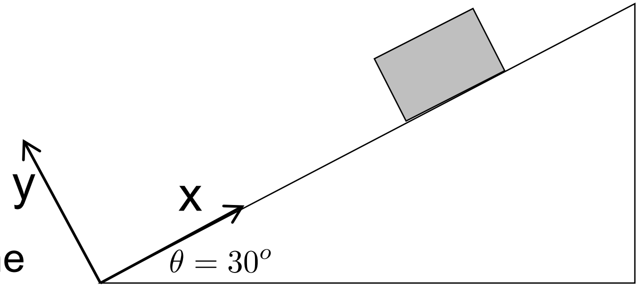
$\therefore ? = \theta$

Block on Inclined Plane

Constant acceleration

$a = -g \sin\theta$

Work out example…



- Block starts out 100m up the plane moving upwards with velocity 25m/s
- How far up the plane does it go?
- How fast is it moving when it slides back down to the bottom of the plane?

Not interested in times…

$$D = x - x_0 = \frac{1}{2a}(v_1^2 - v_0^2)$$

Initial conditions $x_0 = 100m$

$$v_0 = 25m/s$$

Top $v_1 = 0$

Also need $\sin(30^o) = 0.5$

$$D = \frac{-(25m/s)^2}{2(-9.8m/s^2)(0.5)}$$

$$= 63m$$

$\sin 30^o$

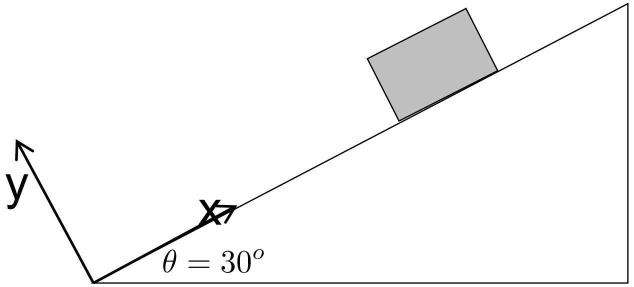
How much further block goes up plane

Block on Inclined Plane

$$\sin(30^o) = 0.5$$

Constant acceleration

$$a = -g \sin\theta$$



Work out example…

- Block starts out 100m up plane moving upwards with velocity 25m/s
- How far up the plane does it go?
- How fast is it moving when it slides back down to the bottom of the plane?

Bottom ➡ $x = 0$

Need to find v_1

Rearrange formula

Not interested in times…

➡ $$D = x - x_0 = \frac{1}{2a}(v_1^2 - v_0^2)$$

$$v_1 = \sqrt{v_0^2 + 2a(x - x_0)}$$

$$= \sqrt{(25m/s)^2 + 2(-9.8m/s^2)(0.5)(0 - 100m)}$$

$$= 40m/s$$

Initial conditions $x_0 = 100m$

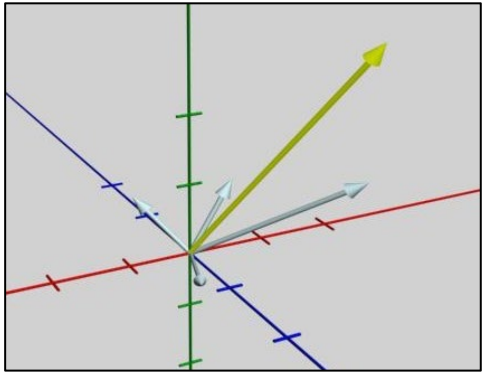
$v_0 = 25m/s$

$\sin 30^o$

Vectors in 3D

Have x, y and z components

Different representations



Graphical → Generally too awkward to be useful

Component form

$$\vec{v} = (v_x, v_y, v_z)$$

In terms of basis vectors

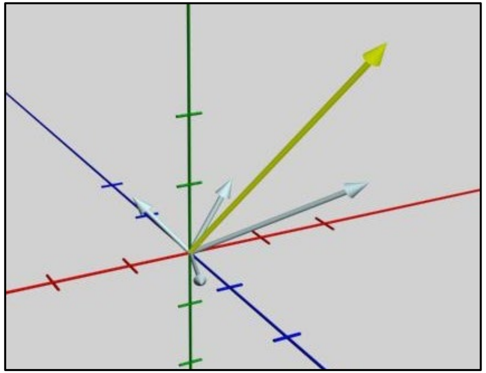
Unit basis vector in z-direction

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

Magnitude of vector

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Adding vectors in 3D



If we have two vectors…

$$\vec{u} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{w} = \vec{u} + \vec{v}$$

$$= (u_x + v_x)\hat{i} + (u_y + v_y)\hat{j} + (u_z + v_z)\hat{k}$$

$$\boxed{w_x = u_x + v_x, \quad w_y = u_y + v_y, \quad w_z = u_z + v_z}$$