YOU'RE THE CREEP WHO'S GOING TO PAY!
I'M GOING TO GET YOU, GOBLIN! I'M GOING TO DESTROY YOU SLOWLY-- AND WHEN YOU START BEGGING FOR ME TO END IT--
--I'M GOING TO REMIND YOU OF ONE THING--
--YOU KILLED THE WOMAN I LOVE--
AND FOR THAT, YOU'RE GOING TO DIE!
THIS NEVER WOULD HAVE HAPPENED IF I UNDERSTOOD THE CONCEPT OF IMPULSE!

Announcements, Goals, and Reading

Announcements:

- HW10 due Tuesday 11/29 – Usual 72hr grace period will apply.
- MT2 solutions will be posted by end of *this* week.
- Forward FOCUS survey is now open—please provide feedback.

Goals for Today:

- Impulse
- Momentum
- Demonstrations: Tablecloth pull, pen and hoop, egg in sheet

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 11: Impulse and Momentum

New Topic: Linear Momentum and Collisions

Start with **momentum**…

$$\vec{p} = m\vec{v}$$

Momentum of an object is the product of its mass and velocity

Momentum is a vector

It has both magnitude and direction

Momentum is a conserved quantity

Total momentum of an isolated system is constant

Follows from Newton's 3rd Law

We'll come back to this..

A large object moving slowly can have the same momentum as a small object moving fast

$\vec{p} = m\vec{v}$ Momentum

A 100g chipmunk wants to run fast enough to have as much momentum as a 6000kg elephant moving at 25 mph.

How fast does the chipmunk need to run?

Only dealing with the magnitude of momentum here

$$m_c v_c = m_e v_e \quad \Rightarrow \quad v_c = \frac{m_e}{m_c} v_e = \left(\frac{6000kg}{0.1kg}\right) 25mph$$

$$= 1.5 \times 10^6 \, mph$$

This was just a dream …

Linear Momentum and Impulse

What happens if variable external force acts on object for some time?

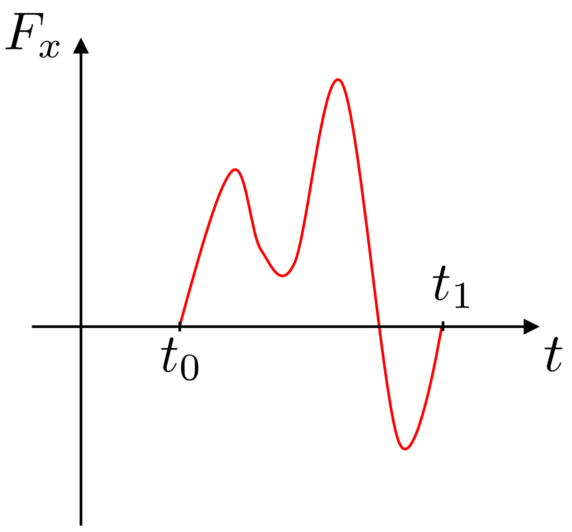
If we have a force that is applied on an object for a certain amount of time

$$\vec{F}(t) = F_x(t)\hat{i} + F_y(t)\hat{j} + F_z(t)\hat{k}$$

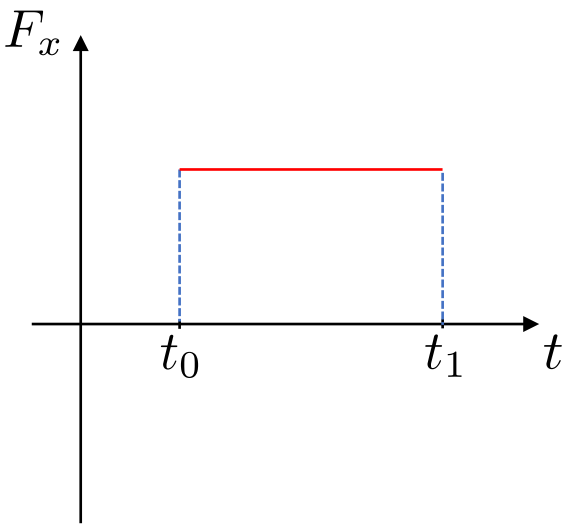
Assume force vanishes before time t_0 and after time t_1

So that graphs of components of force may look for example like…

Or perhaps like…



Complicated time dependent force



Constant force is turned on, then turned off

<u>Impulse vector</u> determines how much momentum of object changes under action of time dependent force.

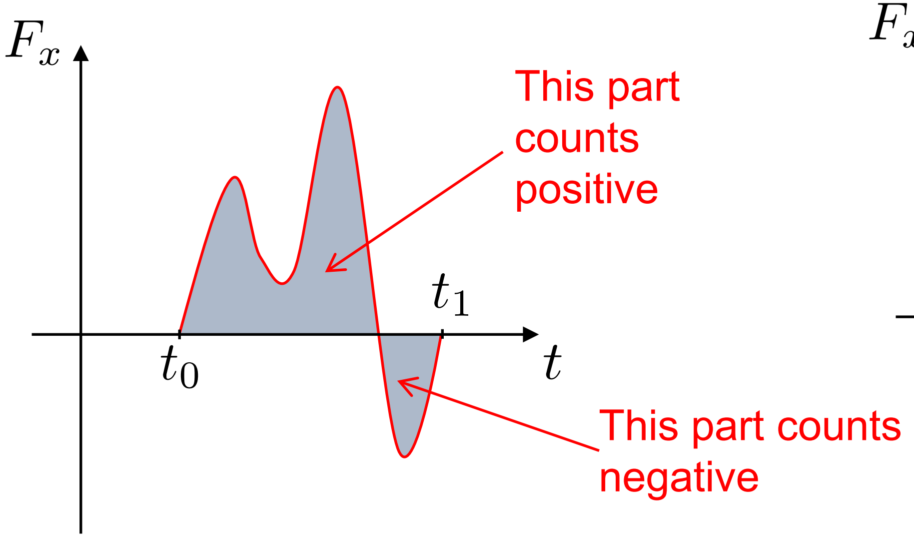
Adds up impact of force on object from t_0 to t_1

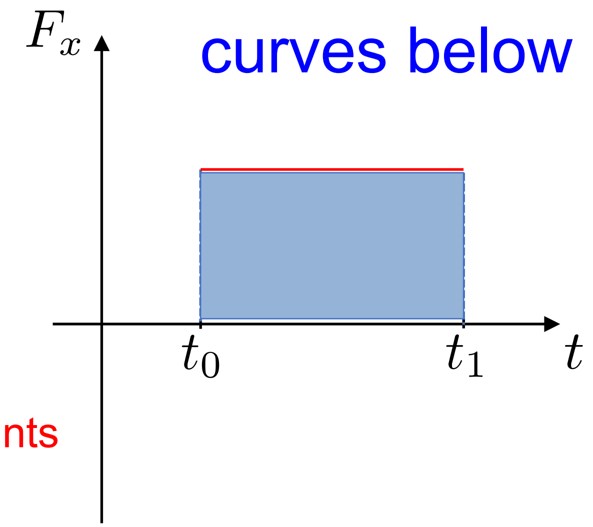
Define **Impulse vector** $\vec{J} = J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$

In words -

Components of impulse vector are the areas under graphs of components of force vector

In our examples the J_x would be given by the areas under the curves below





(Force is pointing in negative x-direction)

Impulse vector determines how much momentum of object changes under action of time dependent force.

Define **Impulse vector** $\vec{J} = J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$

In words -

Components of impulse vector are the areas under graphs of components of force vector

In math formula –

Impulse is the integral of force over time

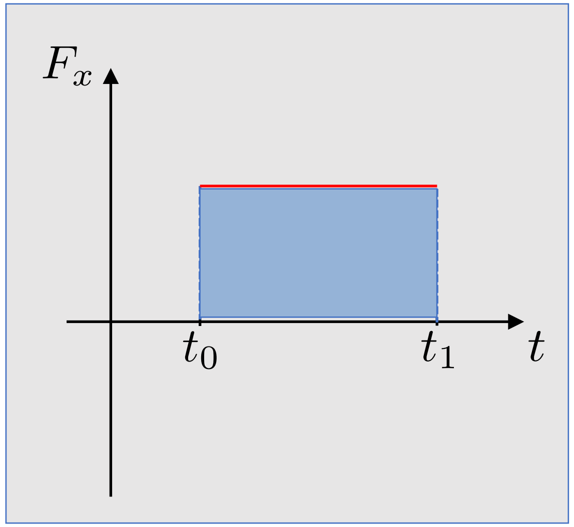
$$\vec{J} = \int_{t_0}^{t_1} dt\, \vec{F}(t)$$

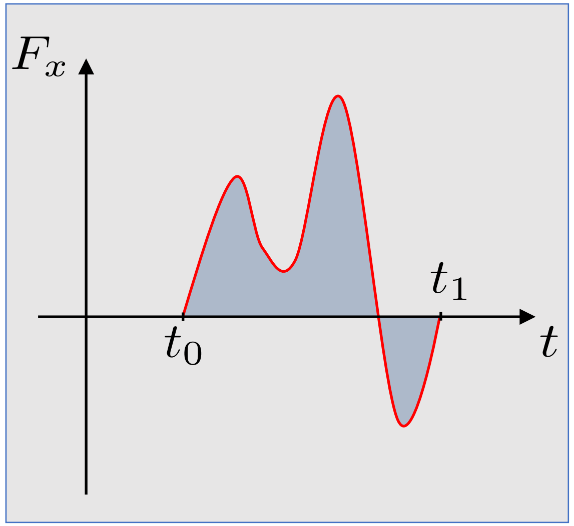
$$J_x = \int_{t_0}^{t_1} dt\, F_x(t)$$

$$J_y = \int_{t_0}^{t_1} dt\, F_y(t)$$

$$J_z = \int_{t_0}^{t_1} dt\, F_z(t)$$

This vector expression means that to get the components of impulse we integrate the components of force





Impulse has units of Newton-seconds

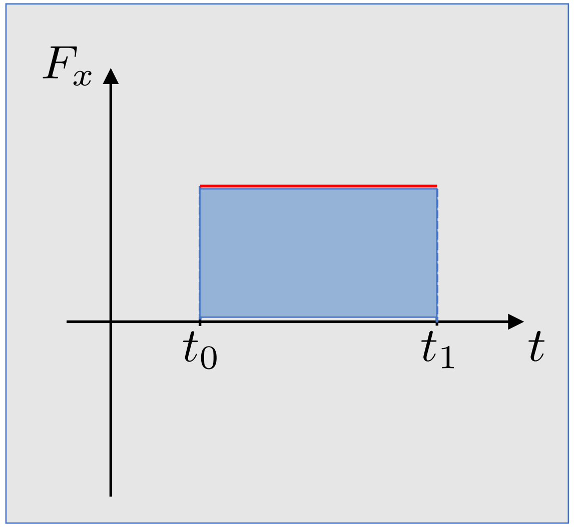
Impulse determines how much momentum of object changes under action of time dependent force.

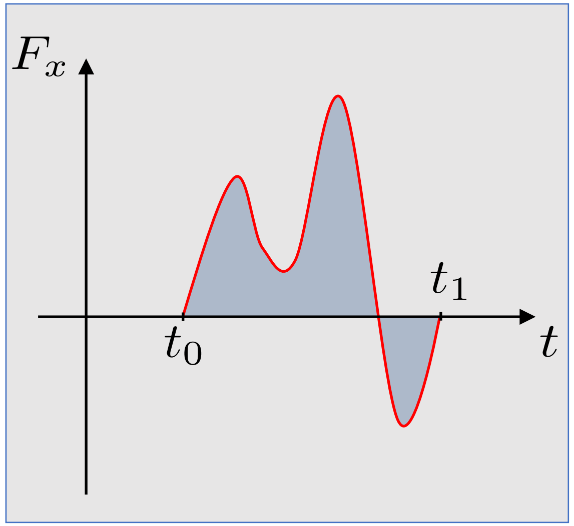
$$\Delta\vec{p} = \vec{p}(t_1) - \vec{p}(t_0)$$

Change in momentum between t_0 and t_1

$$\Delta\vec{p} = \vec{J}$$

Change in momentum equals impulse!





This does not involve any new principles of dynamics beyond Newton's laws

It comes from adding up the impact of Newton's 2nd law on an object's velocity over time

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \quad \Delta\vec{p} = \int_{t_0}^{t_1} dt\, \vec{F} = \vec{J}$$

Using fundamental theorem of calculus

Demo: Tablecloth Pull

Should one pull tablecloth fast or slow? Why?

-what type of friction is most relevant?
-how does impulse relate to the movement of the dishes?

Demo: Pen and Hoop

If I pull hoop away – what determines if pen falls in the bottle?

-which of Newton’s Laws are relevant?
-is concept of impulse important here?

For pen, impulse = Δp_x = J = $F_{friction}$ x Δt. Make Δt small to make Δp_x small

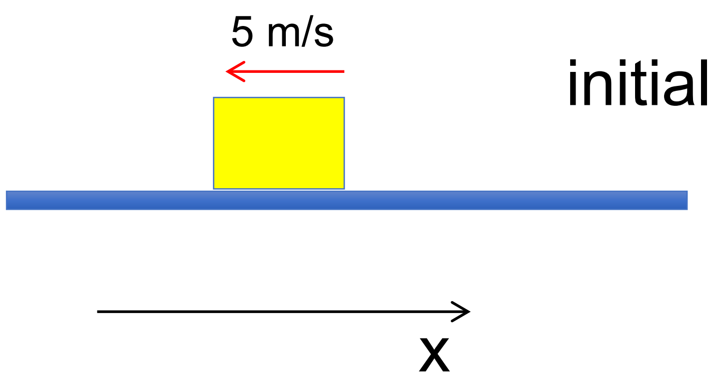
A 2 kg block starts sliding to the left along the floor with speed 5 m/s

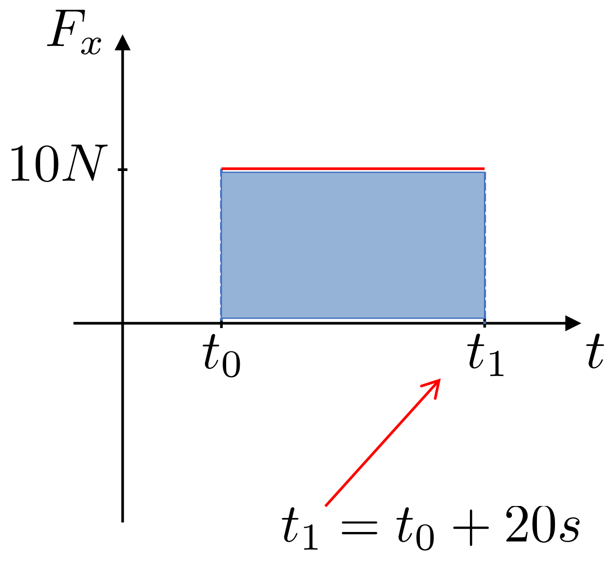
A force of 10 N directed to the right acts on the block for 20 s

What are the initial and final momenta of the block?

What is the impulse delivered by the force?

What is its final velocity?





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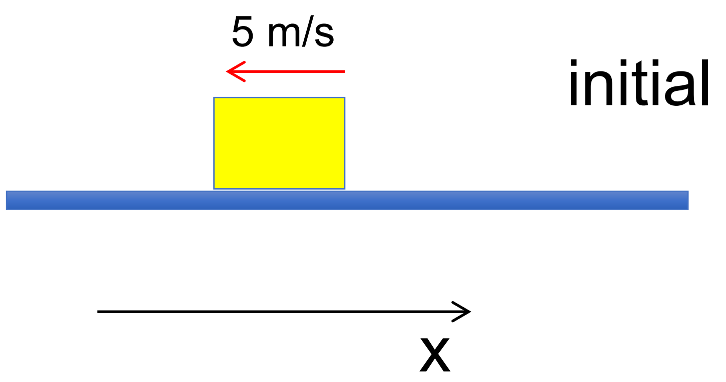


What are the initial and final momenta of the block?

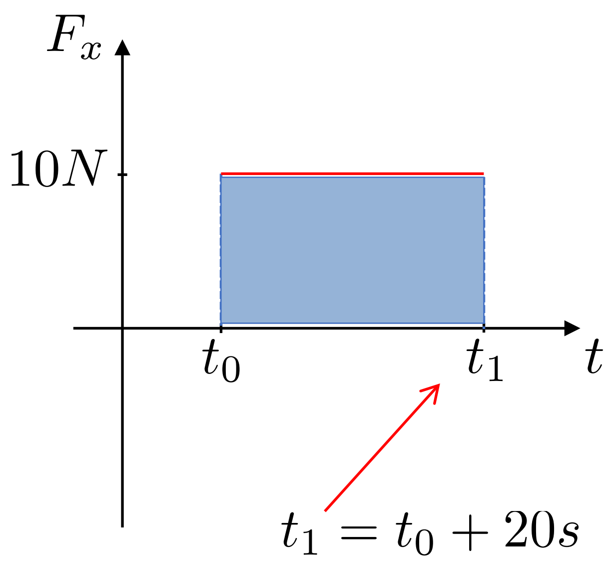
What is the impulse delivered by the force?

What is its final velocity?

$p_i = -10N\,s$

Find impulse

Rectangular area under force curve



$$J = (10N)(20s) = 200Ns$$

Final momentum

$$\Delta p = p_f - p_i = J$$

$$p_f = p_i + J$$

$$= -10Ns + 200Ns = +190Ns$$

Final velocity $v_f = \frac{p_f}{m} = +95m/s$

To the right

Impulse: change in momentum of object under action of force.

$$Average\ Force = \frac{Change\ in\ momentum}{Change\ in\ time}$$

- To increase momentum of object by a lot: apply large force for a large time
- Why have crumple zones in cars?
- Why have airbags? Padded dashboards?
- Why bend your knees when you land from a jump?
- Collision between 1959 Chevy Bel Air & 2009 Chevy Malibu

Popular Science: 600 MPH pumpkin air cannon

Can also figure out average force from change in momentum

0.5 kg hammer is moving at 10m/s when it hits a nail

Hammer comes to rest in 8 milliseconds

What impulse does the hammer deliver to the nail?

What is the average force exerted?

Change in hammers momentum gives the force acting on it.

By Newton’s 3rd law, this is the same as the force the hammer exerts on the nail.

Impulse

Can also figure out average force from change in momentum

0.5 kg hammer is moving at 10m/s when it hits a nail

Hammer comes to rest in 8 milliseconds

What impulse does the hammer deliver to the nail?

What is the average force exerted?

Change in hammers momentum gives the force acting on it.

By Newton's 3rd law, this is the same as the force the hammer exerts on the nail.

$$J = \Delta p = (0.5kg)(10m/s) = 5Ns$$ Impulse

$$J = F_{avg}\Delta t \Rightarrow F_{avg} = \frac{J}{\Delta t} = \frac{5Ns}{0.008s} = 625N$$

Average force is simply impulse divided by the time over which the force is exerted

Can be quite large, if time duration is short

0.13 kg baseball is pitched at 33 m/s and hit directly back towards the pitcher at 50 m/s

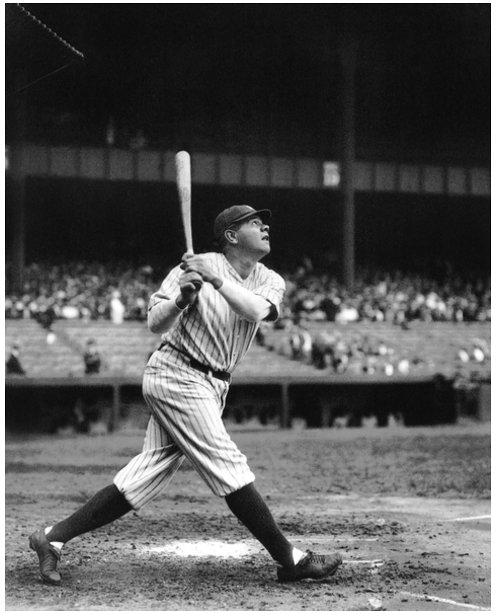
Contact time between bat and ball is 5.4 milliseconds

What is the average force of the bat on the ball?

$$v_i = -33m/s$$

$$v_f = +50m/s$$

x



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$$v_i = -33m/s$$

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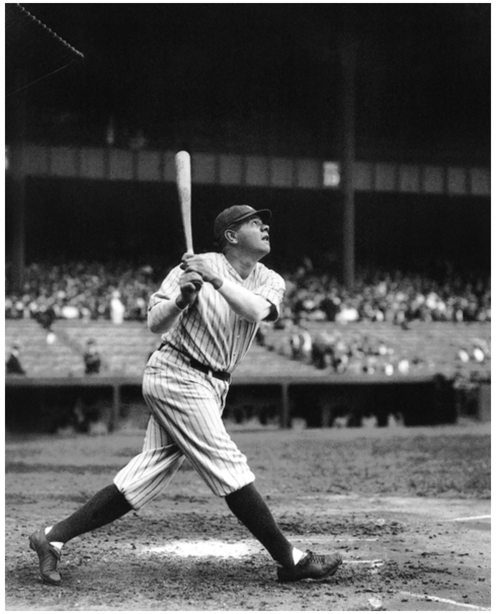
x

$$J = \Delta p = p_f - p_i = mv_f - mv_i$$

Impulse

$$= (0.13kg)(50m/s - (-33m/s)) = 10.8Ns$$

$$F_{avg} = \frac{J}{\Delta t} = \frac{10.8Ns}{0.0054s} = 2000N$$



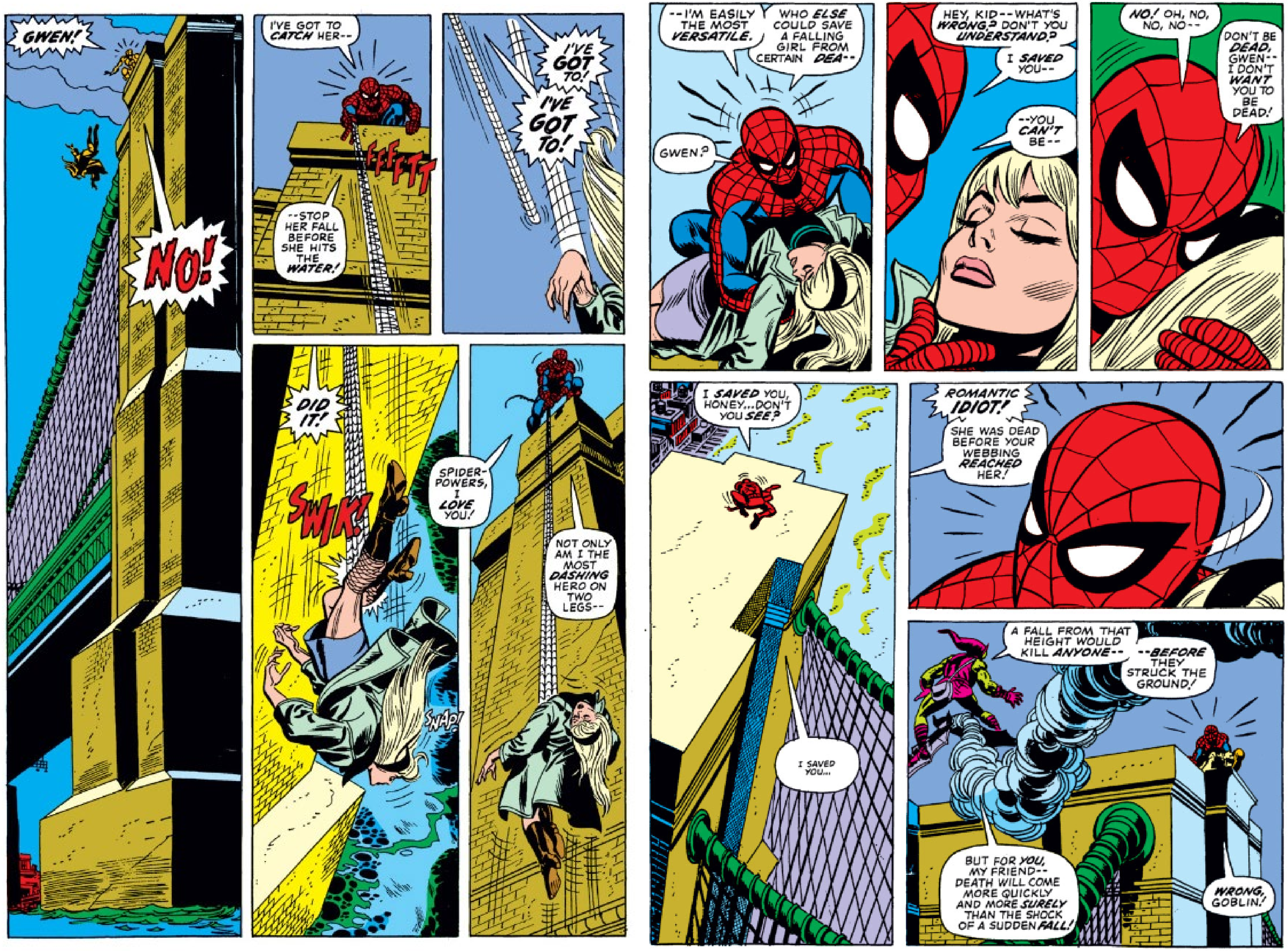
Demo: Egg in Sheet

-Will you be able to break the egg?
-Why/why not?

Spider-Man Didn't understand impulse..

-Gwen falls and accumulates large downward momentum
-Spider man shoots a very stiff web to quickly reduce her momentum to 0
-$F_{avg} = \Delta p/\Delta t$, so small Δt means a large (Fatal) upward force

-Pro tip: maybe try shooting a more elastic web next time?



Practical question…

You need to close a heavy door that is partway open, but you are on the couch across the room watching your favorite movie, The Matrix (1999).

You have two different things you could throw at the door to try to close it all the way

A bouncy Superball or a ball of clay

Which one should you throw?

Because superball bounces back, its momentum change is double that of clay ball.

Door has to deliver twice the impulse to it.

By Newton's 3rd law, the Superball must also deliver twice the impulse to the door

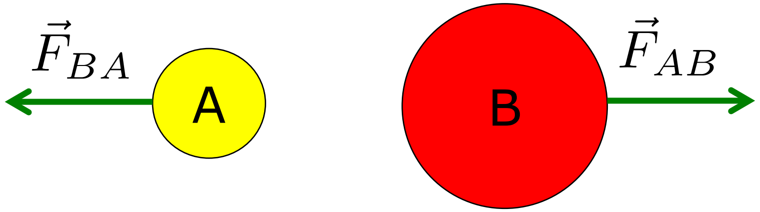
Conservation of momentum

The total momentum of an isolated system does not change in time

Means no external forces

Very important basic principle of physics!

Consequence of Newton's 3rd law



For example…

Consider system made of two objects – A & B

$\vec{F}_{AB}$ = force of A on B

$\vec{F}_{BA}$ = force of B on A

Recall for a single object

$$\vec{F} = m\vec{a} = m\frac{dv}{dt} = \frac{d\vec{p}}{dt}$$

Newton's 3rd law $\Rightarrow$ $\vec{F}_{BA} = -\vec{F}_{AB}$

Total momentum of system $\Rightarrow$ $\vec{p}_{tot} = \vec{p}_A + \vec{p}_B$

Force on object A

Force on object B

Rate of change of total momentum

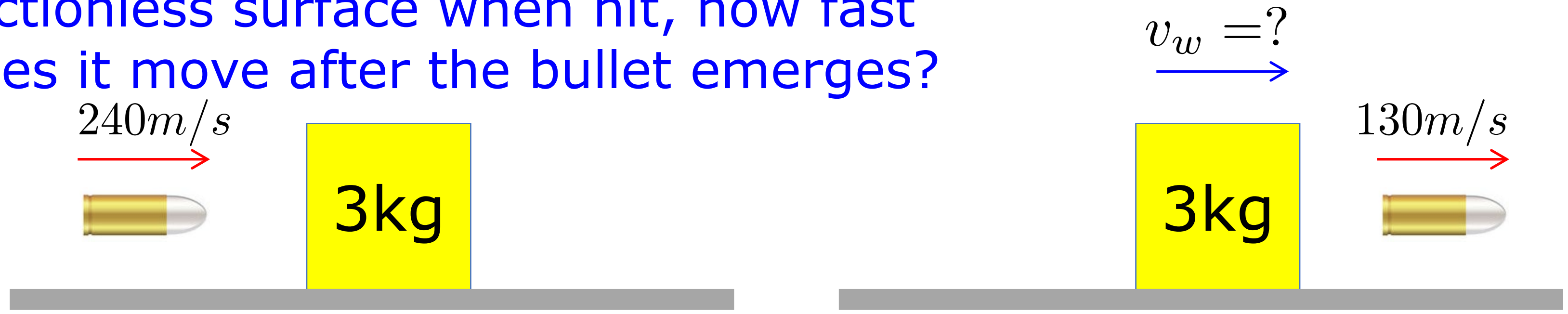
$$\frac{d\vec{p}_{tot}}{dt} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \vec{F}_{BA} + \vec{F}_{AB} = 0$$

Total momentum is conserved!

Conservation of momentum problem

A 22 g bullet traveling 240 m/s penetrates a 3.0 kg block of wood and emerges going 130 m/s.

If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?



Initial total momentum $p_i = (.022kg)(240m/s) = 5.3Ns$

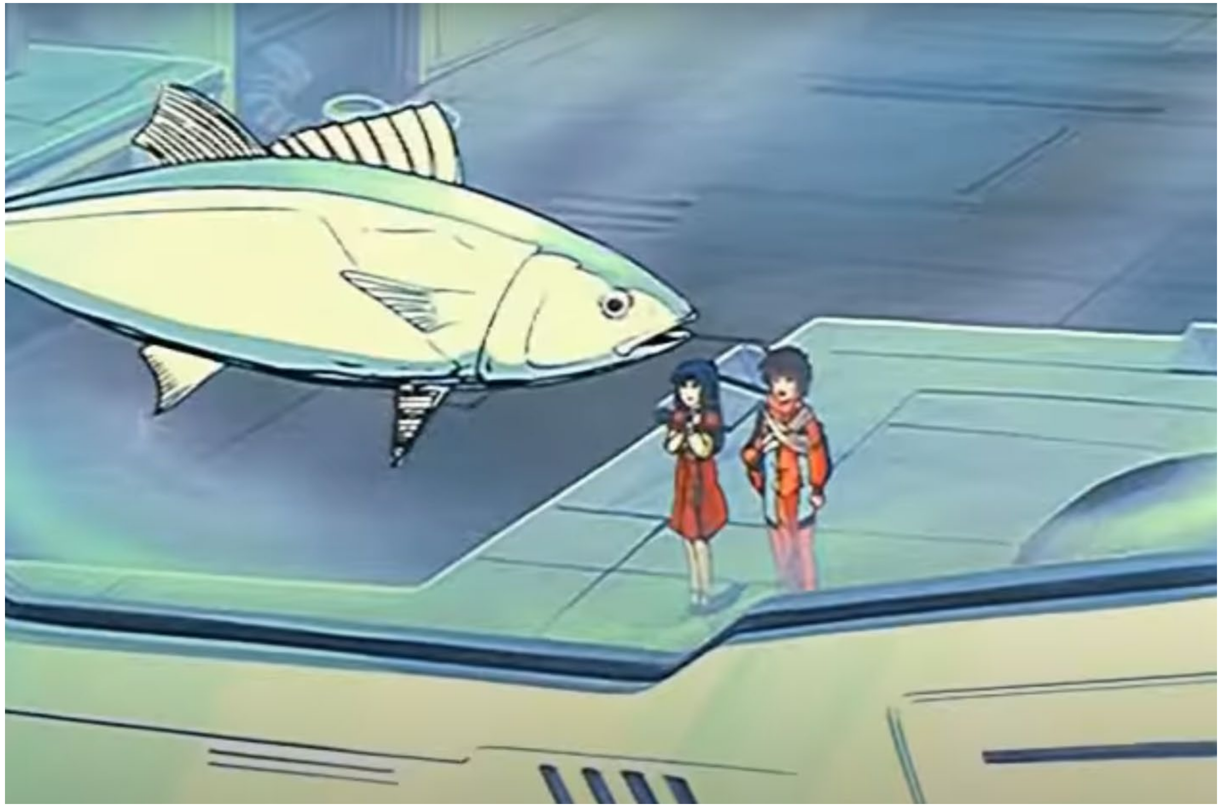
Final total momentum $p_f = (.022kg)(130m/s) + (3kg)v_w$

$$= 2.9Ns + (3kg)v_w$$

Conservation of momentum ➡ $p_f = p_i$ ➡ $v_w = \frac{5.3Ns - 2.9Ns}{3kg} = 0.8m/s$

Using Momentum to ‘Walk’ in Space

- Ichijyo and Minmei are trapped on a space station without food.
- Inexplicably they see a tuna fish floating in the vacuum of space.
- To catch the fish and save them from starvation, Ichijyo exits the airlock holding two weights.



Using Momentum to 'Walk' in Space

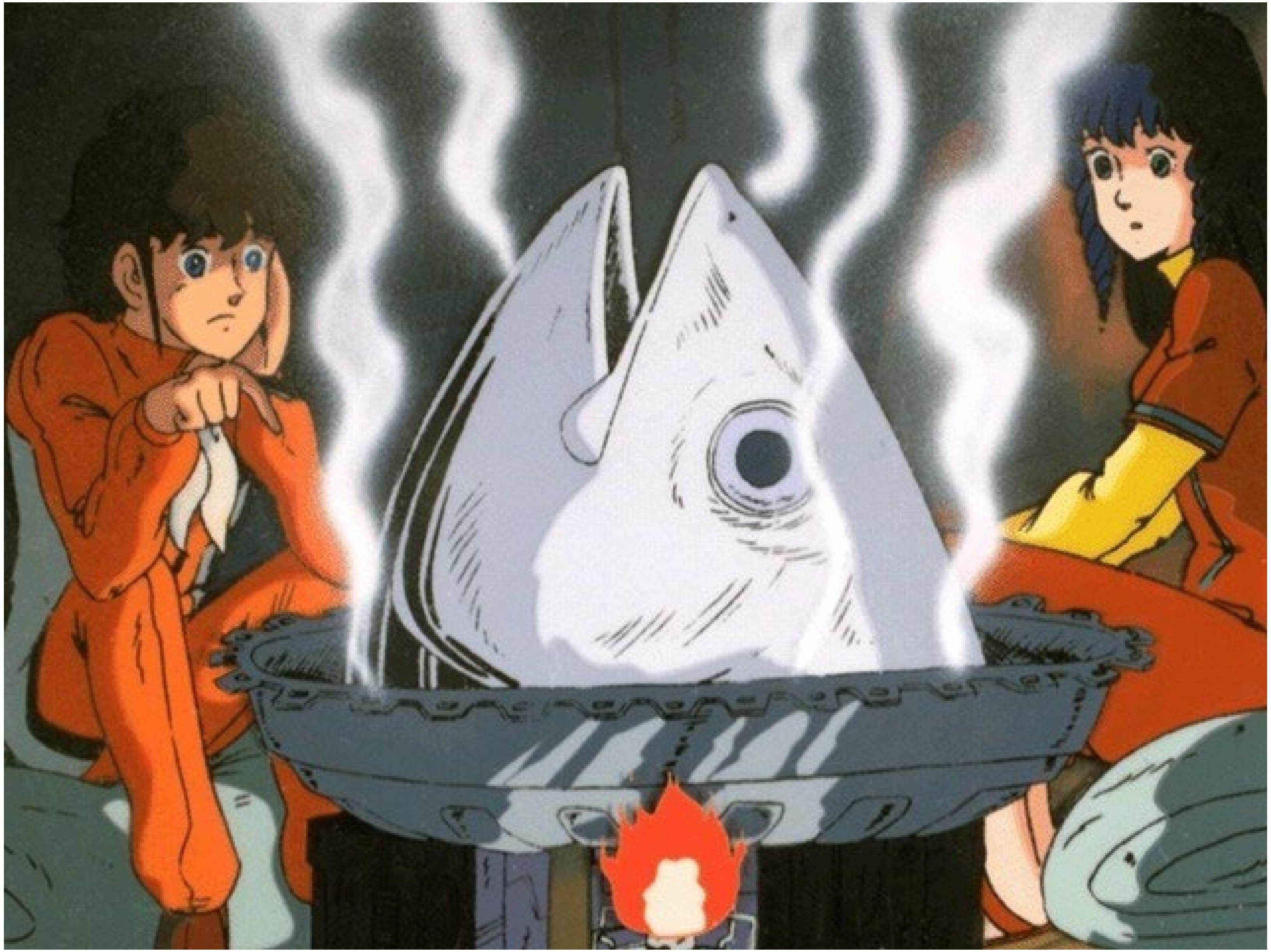
- Ichijyo exits the airlock holding two weights.
- **Why??**

- To <u>move</u>!
- Suppose total momentum of weight + Ichijyo is 0
- Ichijyo throws the weight <u>behind</u> him with speed ***v***
- By 3rd law, Ichijyo is propelled <u>forward</u> with speed **v'**
- Conservation of momentum tells us $m_{ichijyo}v' + m_{weight}v = 0$;

$$v' = -v\left(\frac{m_{ichijyo}}{m_{weight}}\right)$$

- 1st weight propels him toward the fish, 2nd weight sends him back toward the airlock.
- Note 2nd weight will need to be thrown with a greater speed than the 1st.

Success!



A child in a boat throws a 4.80 kg package out horizontally with a speed of 10.0 m/s.

The mass of the child is 22.0 kg and that of the boat is 45.0 kg.

Calculate the velocity of the boat immediately after, assuming it was initially at rest.

*

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Calculate the velocity of the boat immediately after, assuming it was initially at rest.

Initial total momentum of system

$$p_{tot} = 0$$

Before child throws package nothing is moving

Final total momentum of system

$$p_{tot} = 0$$

By conservation of momentum

But now different parts must cancel

Child + Boat

Package

$$0 = p_{tot} = m_{(c+b)} v_{(c+b)} + m_p v_p$$

Recoil from throwing package

$$v_{(c+b)} = -\frac{m_p}{m_{(c+b)}} v_p = -\frac{4.80kg}{67.0kg}(10.0m/s) = 0.716m/s$$

Rocket science!

Child in boat problem is essentially the same as rocket propulsion

In order to move yourself in one direction, you throw stuff out the other direction as hard as you can

This is what the explosive burning of rocket fuel accomplishes

Conservation of momentum does the rest

The amount of momentum gained by the rocket equals the amount ejected through the exhaust gas

Mathematics becomes more complicated because mass of rocket is decreasing as mass is ejected

We won’t cover this in detail….