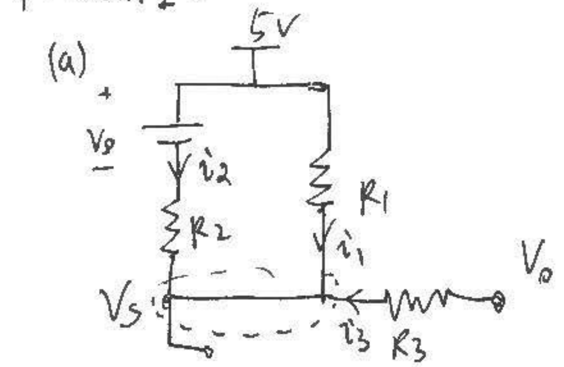
Problem 1.

(a)



At the boundary. $V_D = V_{th} = 0.7\,V$. $i_D = i_2 = 0$

$$\text{So: } V_s = 5 - 0.7 - \underbrace{i_2 \cdot R_2}_{\to 0} = 4.3\,(V)$$

KCL:

$$i_2 + i_1 + i_3 = 0$$

$$0 + \frac{5 - V_s}{R_1} + \frac{V_o - V_s}{R_3} = 0$$

$$\frac{5 - 4.3}{1.4k} + \frac{V_o - 4.3}{2k} = 0 \Rightarrow V_o = 3.3\,(V)$$

(b) Suppose: Diode "ON" when $V_s \leq 4.3\,V$.

KCL:

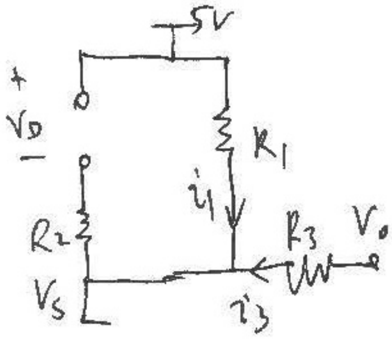
$$i_2 + i_1 + i_3 = 0$$

$$\frac{5 - 0.7 - V_s}{R_2} + \frac{5 - V_s}{1.4k} + \frac{V_o - V_s}{R_3} = 0 \Rightarrow \frac{4.3 - V_s}{3k} + \frac{5 - V_s}{1.4k} + \frac{V_o - V_s}{2k} = 0$$

$$\Rightarrow V_o = \frac{13V_s - 42.04}{4.2}$$

Check: $i_D = i_2 = \frac{4.3 - V_s}{3k} \geq 0 \Rightarrow$ "ON"

Assume: Diode "OFF" when $V_s > 4.3\,V$.

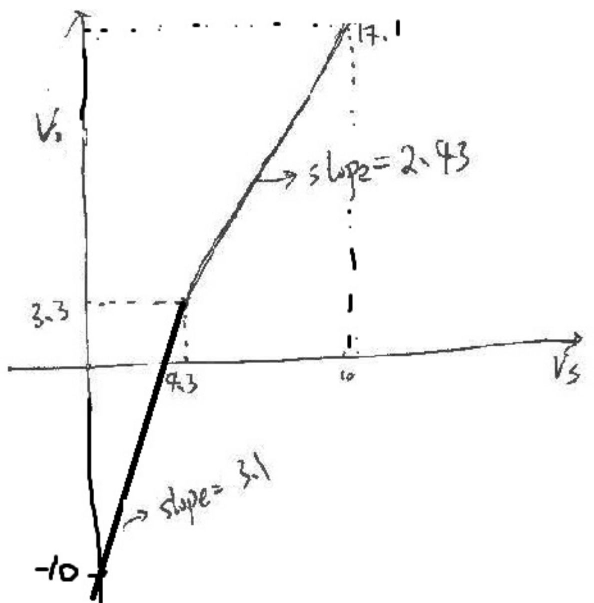


$$i_1 + i_3 = 0$$

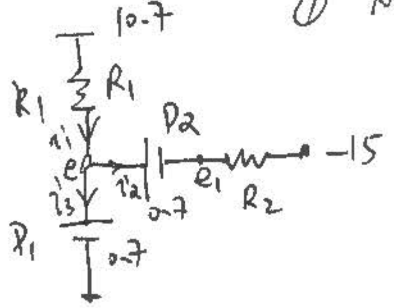
$$\frac{5 - V_s}{1.4k} + \frac{V_o - V_s}{2k} = 0 \Rightarrow$$

$$V_o = \frac{3.4V_s - 10}{1.4}$$

check. $V_D = 5 - i_D \cdot R_3 - V_s$
$< 5 - i_D \cdot R_3 - 4.3$
$= 0.7 - i_D \cdot R_3$
$\leq V_{th}$. "OFF"



Problem 2.



① Assume. D_1, D_2 Both "ON"

$e = 0.7$; $e_1 = 0$.

$$i_1' = \frac{10.7 - e}{R_1} = \frac{10.7 - 0.7}{10k} = 1\,mA$$

$$i_2' = \frac{e_1 - (-15)}{R_2} = \frac{0 + 15}{10k} = 1.5\,mA$$

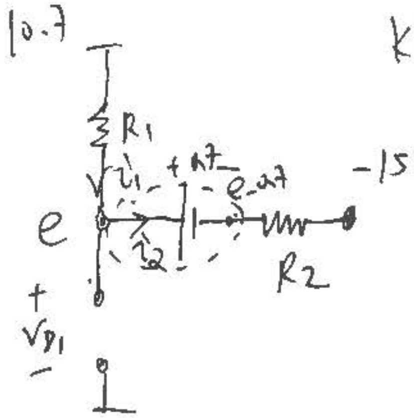
KCL:

$$i_1 - i_2 - i_3' = 0$$

$$i_3' = i_1' - i_2' = 1 - 1.5\,mA = -0.5\,mA.$$

So the Assumption of D_1 "ON" ✗

② Assumen D_1 "OFF" D_2 "ON"



KCL @ supernode:

$$i_1 - i_2 = 0$$

$$\frac{10.7 - e}{R_1} - \frac{e - 0.7 - (-15)}{R_2} = 0$$

$$\frac{10.7 - e}{10k} - \frac{e - 0.7 + 15}{10k} = 0$$

$$10.7 - e - e - 14.3 = 0$$

$$\Rightarrow e = -1.8\,(V)$$

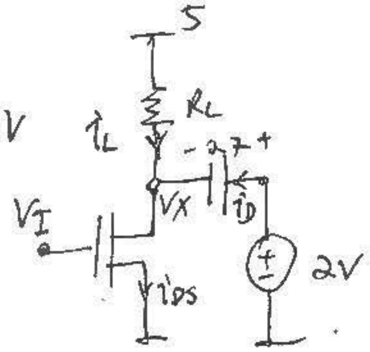
$$i_1' = \frac{10.7 - (-1.8)}{10k} = 1.25\,mA\ ;\quad i_2' = \frac{e - 0.7 - (-15)}{R_2} = \frac{-1.8 - 0.7 + 15}{10k} = 1.25\,mA$$

$$\underline{i_3' = 0}$$

Check: $V_{D_1} = e - 0 = -1.8 - 0 = -1.8 < V_{th}$ ✓.

Problem 3:

(a): ① Assume Diode "ON". Then $V_X = 2 - 0.7 = 1.3V$



Then NMOS:

$V_{GS} = V_{IN} - 0 = 2 - 0 = 2 > V_T \Rightarrow$ "ON"

$V_{DS} = V_X - 0 = 1.3 > V_{GS} - V_T = 2 - 1 = 1\ (V)$

$\Rightarrow$ Saturation:

$$So: i_{DS} = \frac{k}{2}(V_{GS} - V_T)^2 = \frac{5\times10^{-3}}{2}(2-1)^2 = 2.5mA.$$

KCL @ V_X: $i_D + i_L - i_{DS} = 0$

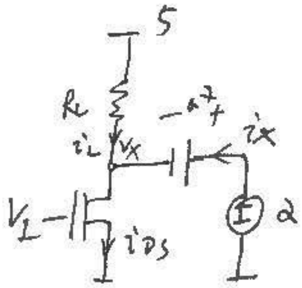
$$\Rightarrow i_D = i_{DS} - i_L$$

$$= 2.5mA - \frac{5-1.3}{10k}$$

$$= 2.5mA - 0.37mA > 0$$

Assumption "ON" ✓

(b). ① Assume Diode "ON". $V_X = 2.0 - 0.7 = 1.3V.$



NMOS: $V_{GS} = V_{IN} - 0 = 3 - 0 = 3 > V_T$ "ON"

$V_{DS} = V_X - 0 = 1.3 < V_{GS} - V_T = 3 - 1 = 2$

$\Rightarrow$ "Triode"

$$R_{ON} = \frac{1}{k(V_{GS}-1)} = \frac{1}{5\times10^{-3}(3-1)} = 0.1k.$$

$$i_{DS} = \frac{V_X - 0}{R_{ON}} = \frac{1.3V}{0.1k} = 13mA, \quad i_L = \frac{5 - V_X}{R_L} = \frac{5 - 1.3V}{10k}$$

$$= 0.37mA$$

KCL @ V_X:

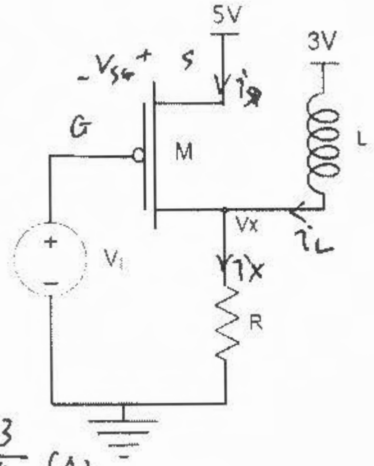
$$i_X + i_L - i_{DS} = 0$$

$$\Rightarrow i_X = i_{DS} - i_L = 13 - 0.37\ mA = 12.63\ (mA)$$

Check $i_D = i_X = 12.63mA > 0$. Diode "ON" ✓

Problem 4 (5 pt)

In the circuit below, $R = 3\ \Omega$, L = 1 H. V_I has been at 5 V for a long time. At = 0, V_I changes from 5 V to 0 V. Find $V_x(t)$ for $t > 0$. $K_p = \frac{1}{12}\ A/V^2$, $V_{TP} = -1$ V. (You need to justify the region of M in SCS model).



When $t > 0$. $V_{SG} = 5 - 0 > |V_{TP}|$. "ON"

finding $V_{x(0^+)}$ in order to determine Transistor region at the very Begining

① at $t = 0^+$, Assume "saturation"

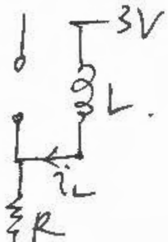
$$i_{SD} = \frac{k}{2}(V_{GS} - |V_{TP}|)^2 = \frac{1}{2} \times \frac{1}{12} \times (5-1)^2 = \frac{3}{4}\ (A)$$

KCL @ Vx .

$$i_{SD}(0^+) + i_L(0^+) - i_x(0^+) = 0$$

$$\frac{3}{4}A + \underbrace{i_L(0^+)}_{= i_L(0^-)} - \frac{V_{x(0^+)}}{3} = 0$$

find $i_L(0^-)$: when $t < 0$: $V_{SG} = 0 - 0 = 0 < |V_{TP}|$ ⇒ "OFF"



At steady state:

$$i_L = \frac{3}{R} = 1\ (A) = i_L(0^-)$$

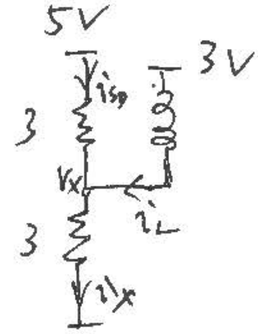
So: $V_{x(0^+)} = 3 \times \left(\frac{3}{4} + i_L(0^-)\right) = 3 \times \left(\frac{3}{4} + 1\right) = 3 \times 1.75 = 5.25\ (V)$

$V_{SD}(0^+) = 5 - V_x = 5 - 5.25 = -0.25 < V_{SG} - |V_{TP}|$

Assumption of "saturation" ✗

So Assume: $t=0^+$, PMOS = "Triode"

$$R_{ON} = \frac{1}{K(V_{SG}-|V_{tp}|)} = \frac{1}{\frac{1}{12}\times(5-1)} = 3\,(\Omega)$$



$$i_{SD}(0^+) + i_L(0^+) - i_X(0^+) = 0$$

$$\frac{5-V_x}{3} + \underbrace{i_L(0^+)}_{=\,i_L(0^-)\,=\,1A} - \frac{V_x}{3} = 0$$

$\hookrightarrow = i_L(0^-) = 1A$ (Based on previous Analysis)

$$\Rightarrow \frac{5-V_x}{3} + 1 - \frac{V_x}{3} = 0 \quad \Rightarrow V_{x(0^+)} = 4V$$

check: $V_{SD(0^+)} = 5-4 = 1 < V_{SG} - |V_{tp}| \Rightarrow$ "Triode" ✓

For $t>0$, Continue to Assume PMOS "Triode"

$$i_{SD} + i_L - i_X = 0$$

$$\frac{5-V_x}{3} + i_{L(0^+)} + \frac{1}{1}\int(3-V_x)\,dt - \frac{V_x}{3} = 0$$

Diff: $-\frac{1}{3}\frac{dV_x}{dt} + (3-V_x) - \frac{1}{3}\frac{dV_x}{dt} = 0$

$$-\frac{2}{3}\frac{dV_x}{dt} - V_x = -3 \quad \Rightarrow \frac{dV_x}{dt} + \frac{3}{2}V_x = \frac{9}{2}$$

$$V_x = 3 + k\cdot e^{-\frac{3}{2}\cdot t}$$

We know $V_x(0^+) = 4\,(V)$

So: $k=1$ So: $V_x = 3 + e^{-\frac{3}{2}\cdot t}$

check: when $t>0$:

$$V_{SD} = 5 - V_x = 5-(3+e^{-\frac{3}{2}t}) = 2-e^{-\frac{3}{2}t} < 2 < V_{SG}-|V_{tp}| = 4$$

"Triode" ✓