# ECE124: Discussion

Discussion #7

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# Minimization via K-Maps

To obtain minimized SOP expression for a Boolean function F…

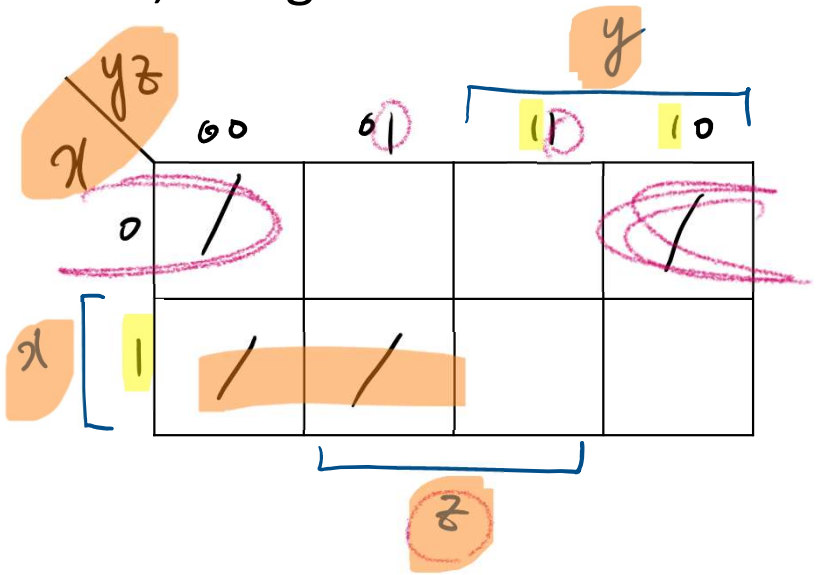
- Step1: Translate truth table to K-map
- Step2: Cover all 1's in the smallest number of largest blocks
- Step3: Extract one product term for each block
- Step4: Sum of extracted product terms=minimized SOP expression!

3.1 Simplify the following Boolean functions, using three-variable K-maps:

(a) F (x, y, z) = ∑ (0, 2, 4, 5)

F = ▭ + ▭

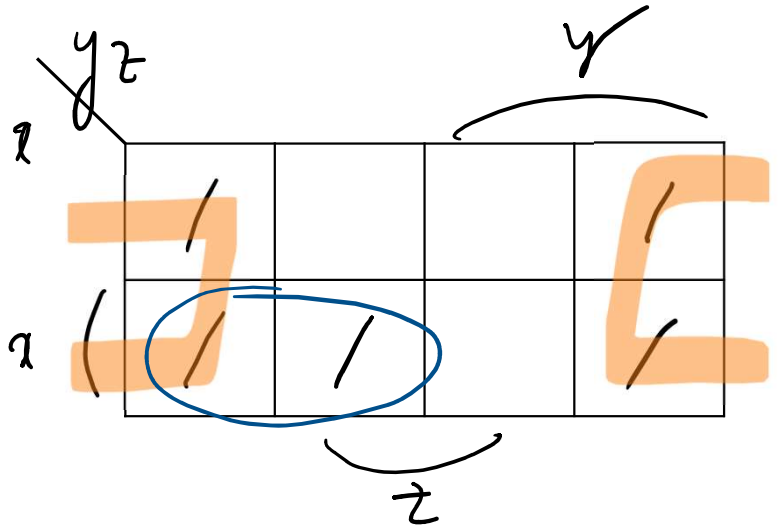
= x·y′ + x′z′



3.1 Simplify the following Boolean functions, using three-variable K-maps:

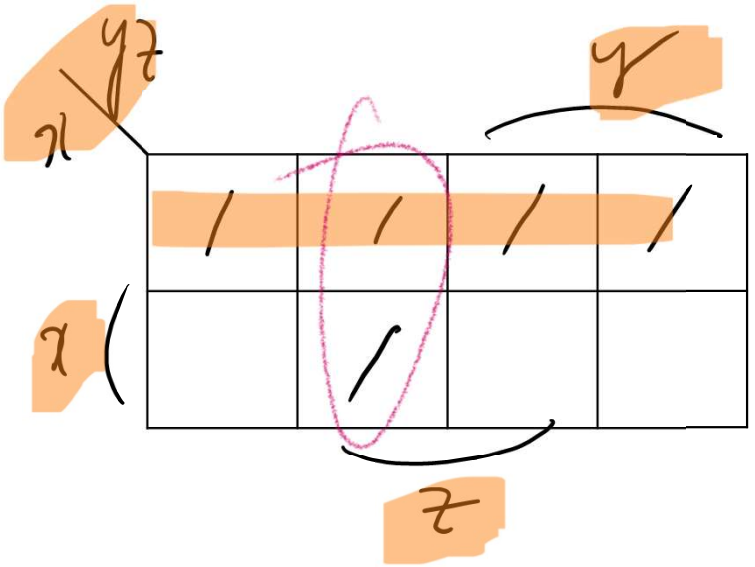
(b) F (x, y, z) = ∑ (0, 2, 4, 5, 6)

$F = z' + xy'$



3.1 Simplify the following Boolean functions, using three-variable K-maps:

(c) F (x, y, z) = ∑ (0, 1, 2, 3, 5)



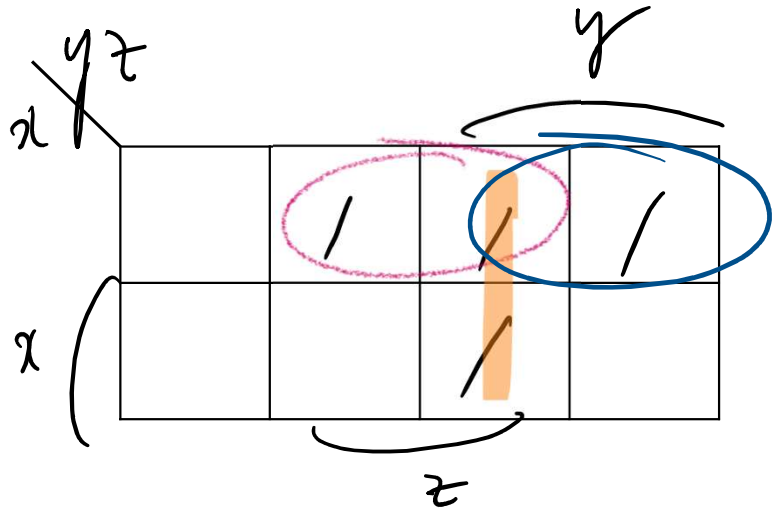

+ 0

$F = x' + y'z$

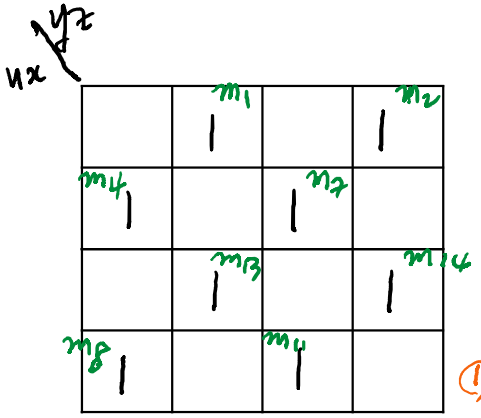
3.1 Simplify the following Boolean functions, using three-variable K-maps:

(d) F (x, y, z) = ∑ (1, 2, 3, 7)

$F = x'z + yz + x'y$



- 1(b). Boolean expression for F.




✱ All minterms are exclusive.

$F = m_1 + m_2 + m_4 + m_7 + m_8 + m_{11} + m_{13} + m_{14}$

$= w'x'y'z + w'x'yz' + w'xy'z' + w'xyz + wx'y'z' + wx'yz + wxy'z + wxyz'$

$0001_{(2)} = 1_{(10)}$ $0010_{(2)} = 2_{(10)}$ $0100_{(2)} = 4_{(10)}$ $0111_{(2)} = 7_{(10)}$ $1000_{(2)} = 8_{(10)}$ $1011_{(2)} = 11_{(10)}$ $1101_{(2)} = 13_{(10)}$ $1110_{(2)} = 14_{(10)}$

① $= w'x'(y'z + yz') + w'x(y'z' + yz) + wx'(y'z' + yz) + wx(y'z + yz')$

② $= (w'x' + wx)(y'z + yz') + (w'x + wx')(y'z' + yz)$

③ $= (w \oplus x)'(y \oplus z) + (w \oplus x)(y \oplus z)'$

($(w \oplus x)' = A'$, $(y \oplus z) = B$, $(w \oplus x) = A$, $(y \oplus z)' = B'$)

$= (w \oplus x) \oplus (y \oplus z)$

$= w \oplus x \oplus y \oplus z$

✱ $A \oplus B = A'B + AB'$

$(A \oplus B)' = A'B' + AB$

$= A \odot B$

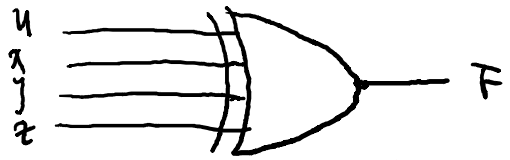
✱ $A'B' + AB \neq 1$

if, $A = B \rightarrow A'B' + AB = 1$

- 1(c). Draw a logic diagram for F.

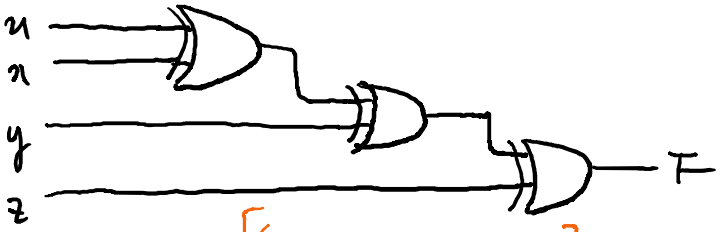
$F = u \oplus x \oplus y \oplus z$

i)



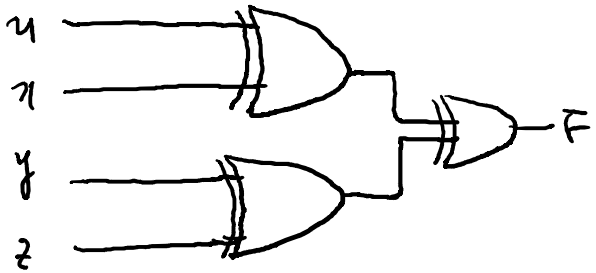

$F = u \oplus x \oplus y \oplus z$

ii)



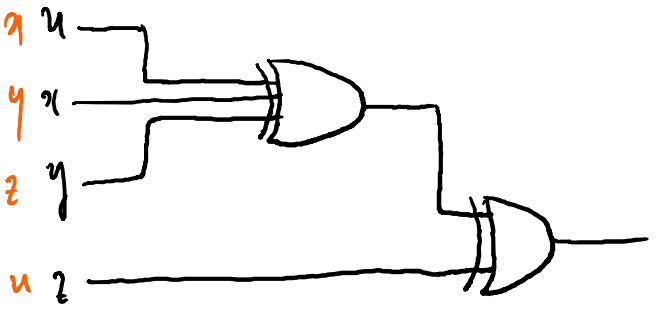

$F = [(z \oplus (y \oplus (u \oplus x))]$

iii)




$F = (u \oplus x) \oplus (y \oplus z)$

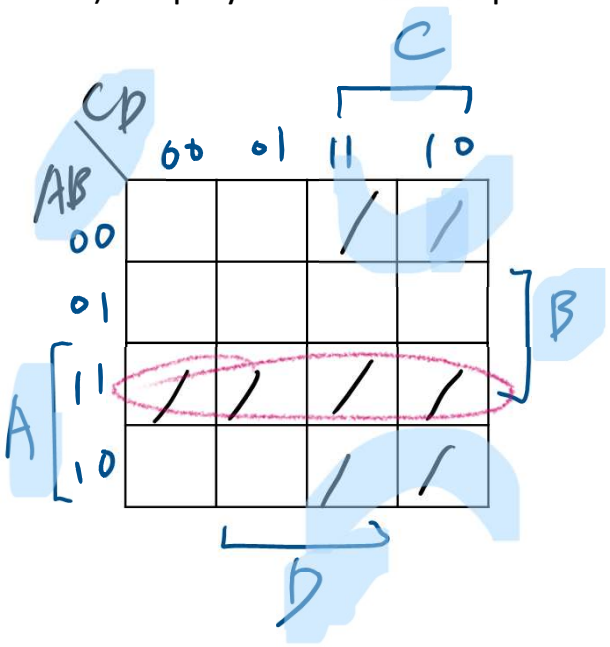
iv)




$F = (u \oplus x \oplus y) \oplus z$

$= u \oplus (x \oplus y \oplus z)$

- 1) Consider the Boolean function: F (A, B, C, D) = AB + B'C + ACD + ABD' + ACD'
- 2) Extract a minimized Boolean expression for F using the Karnaugh map of 1).
- 3) Simplify the Boolean expression F and Compare it with the minimized Boolean expression of 2)




$F = AB + B'C$

$F = AB + B'C + ACD + ABD' + ACD'$

$= AB(1 + D') + ACD + ACD' + B'C$

$= AB + AC + B'C$

$ABCD \quad ABCD \quad AB'CD$

$= AB + (B + B')AC + B'C$

$= AB + ABC + AB'C + B'C$

$= AB(1 + C) + B'C(A + 1)$

* Consensus Th.

$AB + AC + B'C$

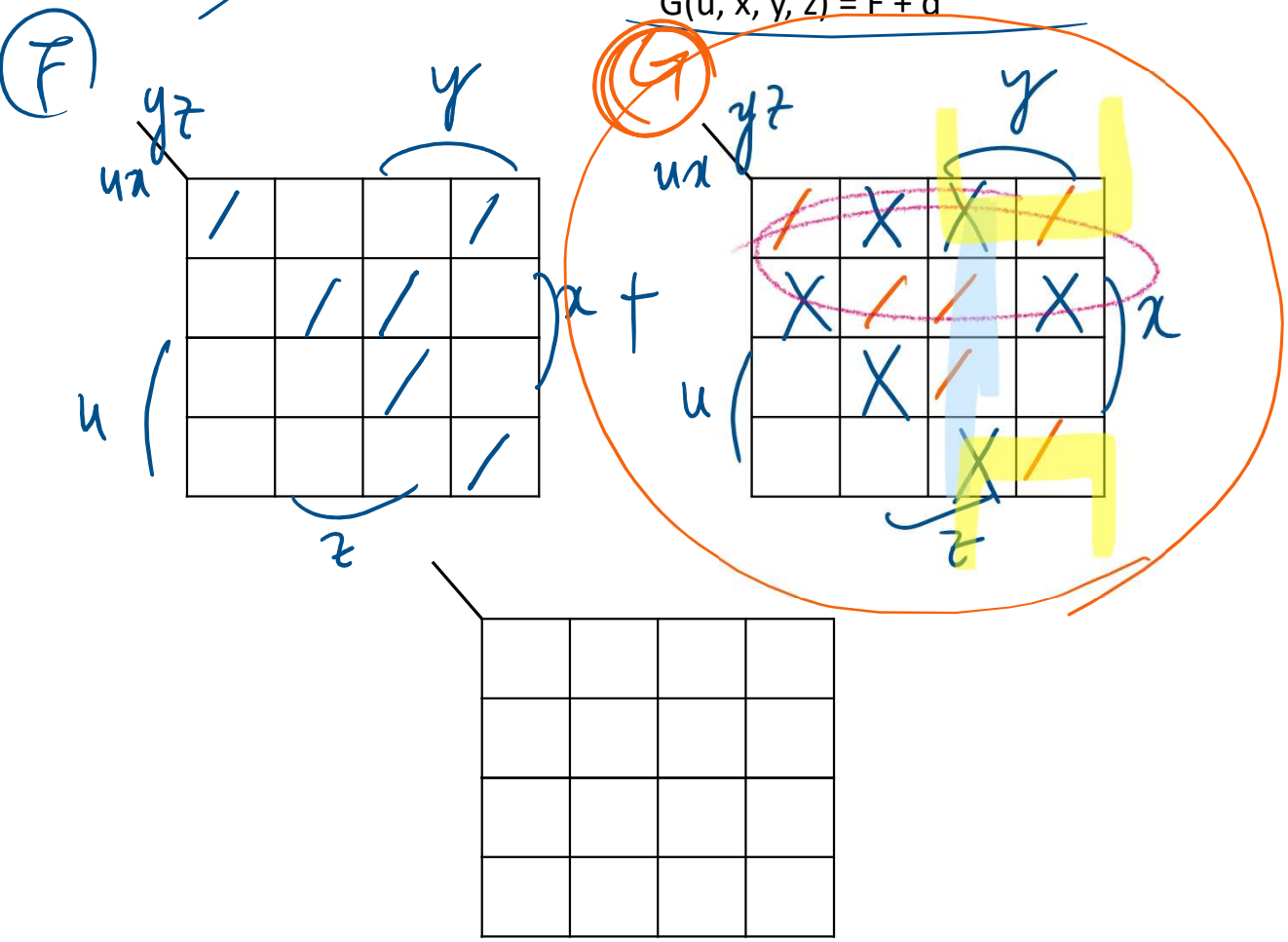
$(A + B) \cdot (A + C) \cdot (B' + C)$

$x'y + yz + xz$

- 2(a). Consider the Karnaugh map for F together with the don't care conditions, d. Show the Karnaugh map for G.

F(u, x, y, z) = ∑(0, 2, 5, 7, 10, 15)  d(u, x, y, z) = ∑(1, 3, 4, 6, 11, 13)

G(u, x, y, z) = F + d

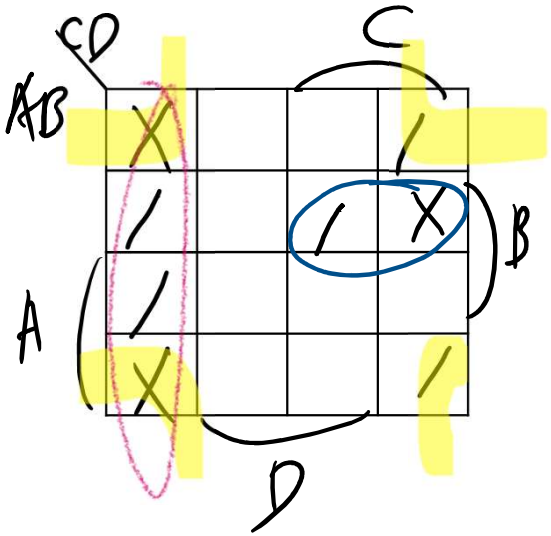



$F = u' + yz + x'y$

3.15 Simplify the following Boolean function F, together with the don't care conditions d, and then express the simplified function in sum-of-minterms form:

(d) F(A, B, C, D) = Σ(4, 12, 7, 2, 10)

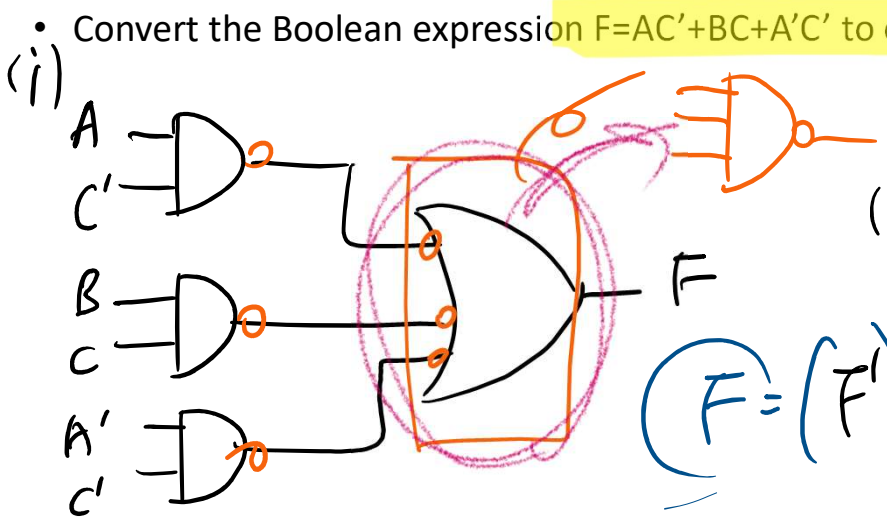
*d* (A, B, C, D) = Σ(0, 6, 8)

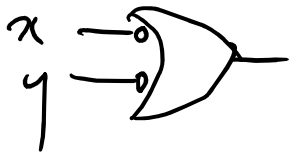



+ 0 + 0

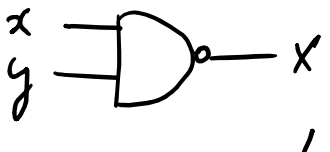
$F = B'D' + C'D' + A'BC$

- Convert the Boolean expression F=AC'+BC+A'C' to only NAND gates.




$x' + y'$

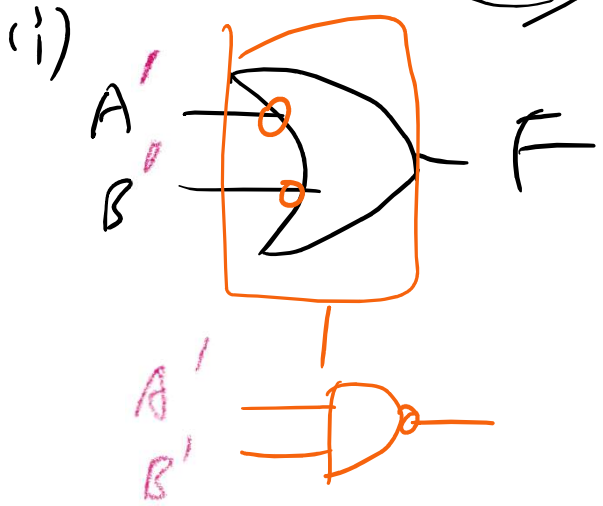



$(x \cdot y)'$

(ii) $F = AC' + BC + A'C'$

$$F = (F')' = \{(AC' + BC + A'C')\}' = \{(A \cdot C')' \cdot (B \cdot C)' \cdot (A' \cdot C')\}'$$

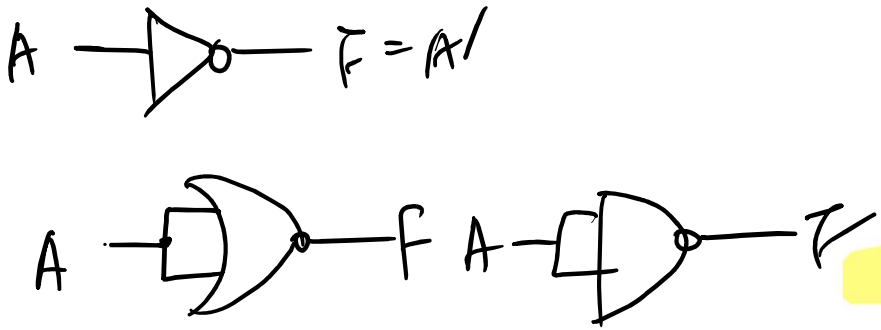
- Draw a logic diagram for F=A+B using only NAND gates.




(ii) $F' = (A + B)'$

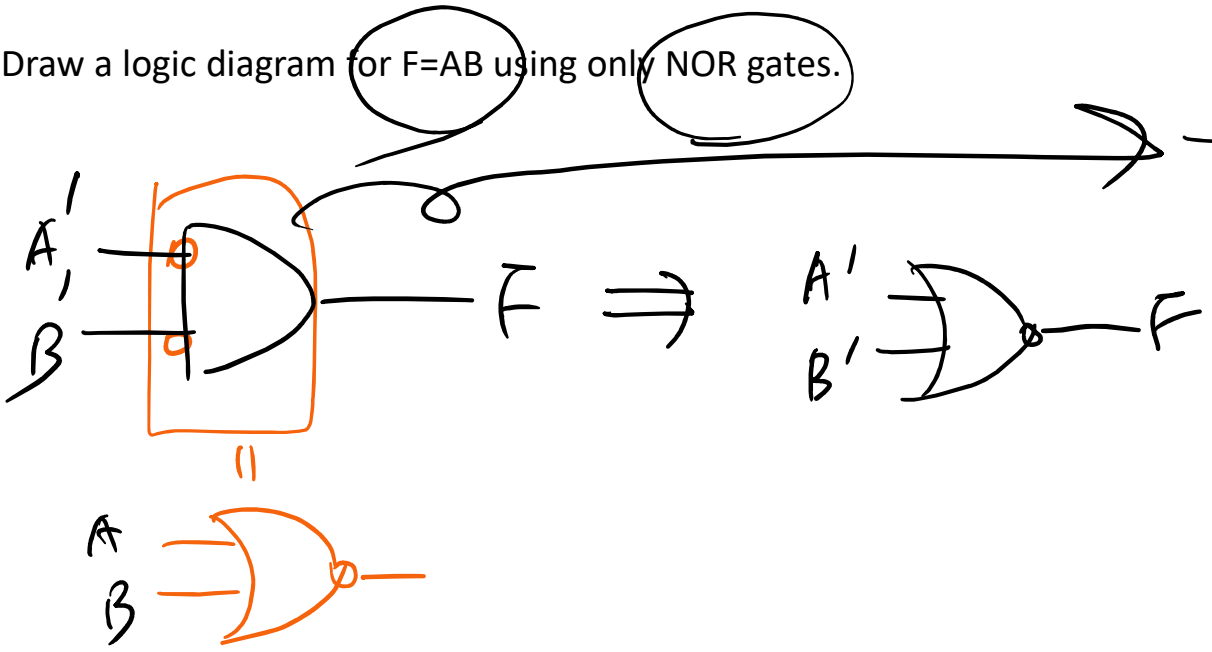
$$F = (A' \cdot B')'$$

- Draw a logic diagram for F=A' using only NOR gates.




| x | y = x | NOR | NAND |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |

- Draw a logic diagram for F=AB using only NOR gates.




$$\frac{A' \cdot B'}{(ii)} = (A+B)'$$

$$F = AB$$

$$F' = (AB)' = A' + B'$$

$$F = (A' + B')'$$