ECE124: Discussion

Discussion #5

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2.15 Simplify the following Boolean functions T_1 and T_2 to minimum number of literals:

A	B	C	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

$T_2(A,B,C) = M_0 \cdot M_1 \cdot M_2 = (A+B+C)\cdot(A+B+C')\cdot(A+B'+C)$

$(m_0)' = M_0$

$(A'B'C')' = A+B+C$

$T_1(A,B,C) = m_0 + m_1 + m_2$

$= A'B'C' + A'B'C + A'BC'$

$= A'B'(C'+C) + A'BC'$

$= A'B' + A'BC'$

$= A'\cdot(B' + BC')$

$B' + (B\cdot C')$

$= A'\{(B'+B)(B'+C')\} = A'(B'+C')$

SOP $\leftarrow$ $A'B' + A'C'$

$T_2 = \Sigma(3,4,5,6,7)$

$f = m_3 + m_4 + \ldots + m_7$

$= A + BC$

- For the Boolean function: F (a, b, c, d) = (c' + d)(b + c')

(a) Express the function in sum of minterms and product of maxterms forms.

(b) Obtain the truth table of F.

POS

$F = c' + bd$ SOP

a	b	c'	d	
0	0	0	0	(m_0)
0	0	0	1	(m_1)
0	1	0	0	(m_4)
0	1	0	1	(5)
1	0	0	0	(8)
1	0	0	1	(9)
1	1	0	0	(12)
1	1	0	1	(13)

a	b	c	d	
0	1	0	1	(m_5)
0	1	1	1	(7)
1	1	0	1	(13)
1	1	1	1	(15)

a	b	c'	d	
0	0	1	0	(M_2)
0	1	1	0	(M_6)
1	0	1	0	(M_{10})
1	1	1	0	(M_{14})

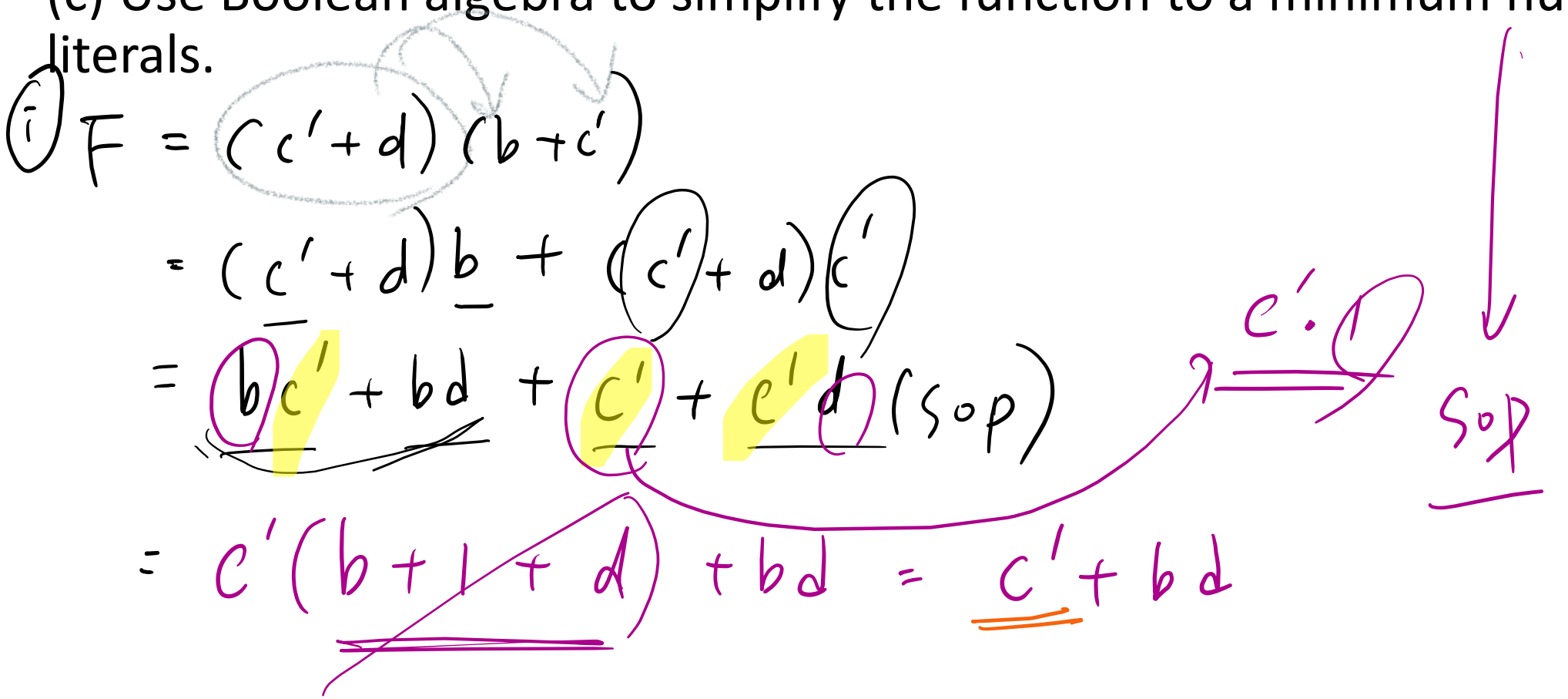
a	b	c'	d	
0	0	1	0	(2)
0	0	1	1	(3)
1	0	1	0	(10)
1	0	1	1	(11)

$F = \Pi(2, 3, 6, 10, 11, 14)$

$F = \Sigma(0, 1, 4, 5, 7, 8, 9, 12, 13, 15)$

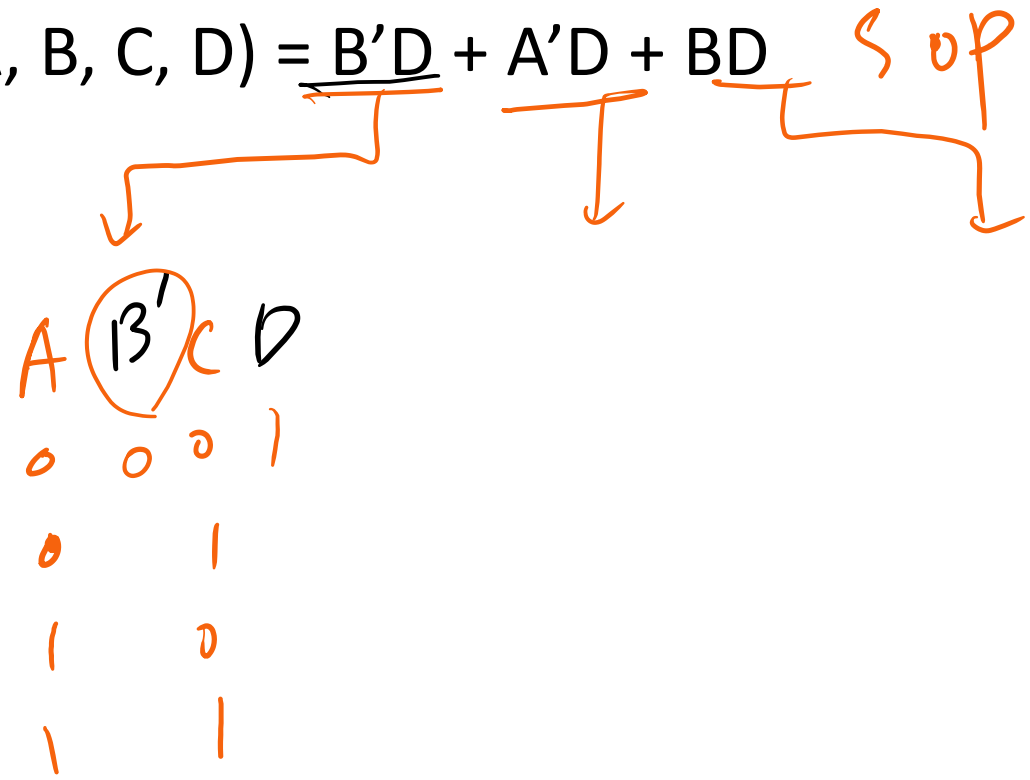
- For the Boolean function: F (a, b, c, d) = (c’ + d)(b + c’)

(c) Use Boolean algebra to simplify the function to a minimum number of literals.



2.19 Express the following function as a sum of minterms and as a product of maxterms:

F (A, B, C, D) = B'D + A'D + BD



$$F = \sum(1, 3, 5, 7, 11, 13, 15)$$

$$= \prod(0, 2, 4, 6, 8, 10, 12, 14)$$

2.29 Determine whether the following Boolean equation is true or false.

$$x'y' + x'z + x'z' \stackrel{?}{=} x'z' + y'z' + x'z$$

$F_1(x, y, z)$

$F_2(x, y, z)$

F_1 $\qquad$ F_2

$F_1 = x'y' + x'z + x'z'$

$= \sum(0, 1, 2, 3)$

$F_2 = x'yz' + xy'z' + x'yz$

0	0	0 (m_0)	0	0	0 (0)	0	0	1 (1)
0	1	0 (m_2)	1	0	0 (4)	0	1	1 (3)

$F_2 = \sum(0, 1, 2, 3, 4)$

2.30 Write the following Boolean expressions in sum of products form and in sum of minterms form.

SOP

$F(a, b, c, d) = (b + d)(a' + b' + c)$ → POS

$F = (b+d)(a'+b'+c)$

$= (b+d)a' + (b+d)b' + (b+d)c$

$= a'b + a'd + b'd + bc + cd$

	b				$a'+b'+c+d$
	$a+b+c+d$				
	0	0	0	0 (M_0)	1 1 0 0 (12)
	0	0	1	0 (M_2)	1 1 0 1 (13)
	1	0	0	0 (M_8)	
	1	0	1	0 (M_{10})	

$F = \Pi(0, 2, 8, 10, 12, 13)$

$= \Sigma($ _____ $)$

* Simplify function F (x, y) = x'y' + x'y + xy' using Boolean algebra

$$F(x,y) = x'y' + x'y + xy' \quad (\text{SOP})$$

$$= x'(y' + y) + xy'$$

$$= x' + xy'$$

$$= (x' + x)(x' + y')$$

$$= x' + y'$$

$x' + (x \cdot y')$

$y' + y = 1$

$y' \cdot y = 0$

* Simplify function X (A, B, C) = AB + ABC + AB'C' + AC' using Boolean algebra

$X(A,B,C) = AB(1+C) + AC'(B'+1)$

$= AB + AC'$ (SOP)

$= A(B+C')$ (POS)

$(A+0)\cdot(B+C')$

* Simplify function F= A'BC + AB'C + ABC' + ABC using Boolean algebra

$$F(A,B,C) = (A'B + AB')C + AB(C' + C)$$

$$= (A'B + AB') \cdot C + AB$$

$$(A' \cdot B) + A = (A'B + AB' + AB) \cdot (C + AB)$$

$$= \{A'B + A(B' + B)\} \cdot (C + AB)$$

$$= (A' + A)(B + A)$$

$$= (B + A)(C + AB)$$

$$= (B + A)C + (B + A)AB$$

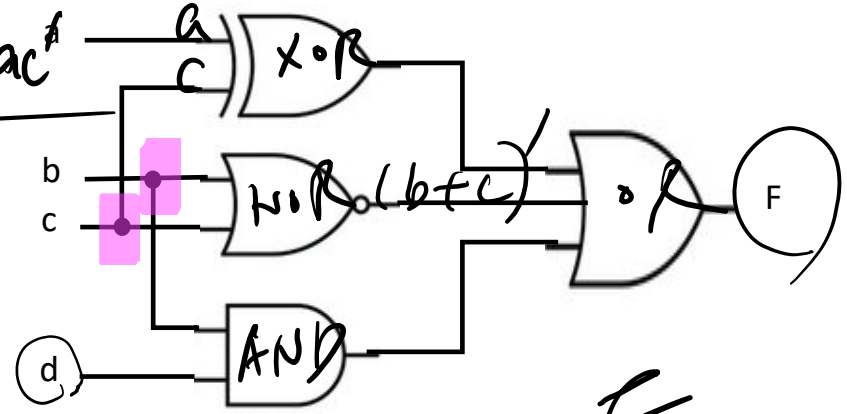
$$= BC + AC + AB$$

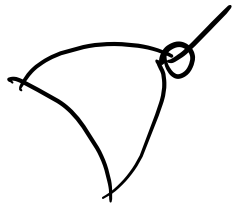
* From the logic diagram,

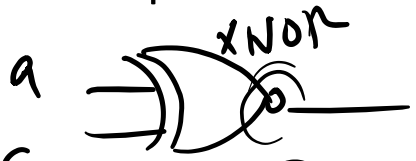
(a) Express the function F in Boolean expression.

(b) Construct the truth table describing the outputs

$a\,(XOR)\,c = a \oplus c = a'c + ac'$







$a \odot c = a'c' + ac$

$=$

$F = \underbrace{a \oplus c}_{a'c + ac'} + \underbrace{(b+c)'}_{} + \underbrace{bd}_{}$

$= a'c + ac' + b'c' + bd$

$=$

$F = \sum (0, 1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$

a	b	c	d	F
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1