ENGIN 112: Homework 12

Due 5pm, October 25, 2022

Please submit your answers on letter size paper. If you use multiple pages, please staple them together (no paper clips, please, since they fall off easily) and write your name on each page.

Question 1

(This question asks you to draw the block diagram of a feedback control system described in class.)

In class, we described the operation of a few control systems. Pick one and draw a block diagram of the associated feedback loop including the plant, sensor, controller, actuator and plant input and plant output. Identify each of these systems and signals.

Question 2

(This question asks you view and describe some of the content of a video that discusses the development of feedback control and its applications in systems such as drones.)

Watch the video *Feedback Control and the Coming Machine Revolution* at the following URL:

https://www.youtube.com/watch?v=C4IJXAVXgIo

(a) The video mentions three particular technological advances that have enabled improvements in feedback control systems, which in turn greatly enhance the capabilities of machines. Name and briefly describe the impact of the three technological advances described in the video.

(b) The video also describes the importance of mathematical modeling for feedback control system design. Summarize the arguments made in the video for why mathematical modeling is so useful.

Question 3

Consider the hand-stick system modeled in lecture and having D.E.

$$\ddot{\theta} - \frac{g}{\ell}\theta = \frac{1}{m\ell}f$$

where θ is the stick angle, f is the force on the hand, g is gravitational acceleration, and ℓ and m are the stick length and mass respectively.

(a) Suppose the force is zero; i.e., $f(t) = 0$ so that stick motion satisfies

$$\ddot{\theta} - \frac{g}{\ell}\theta = 0;\ \theta(0) \neq 0 \tag{1}$$

where $\theta(0)$ is a non-zero initial stick angle. Prove to yourself that

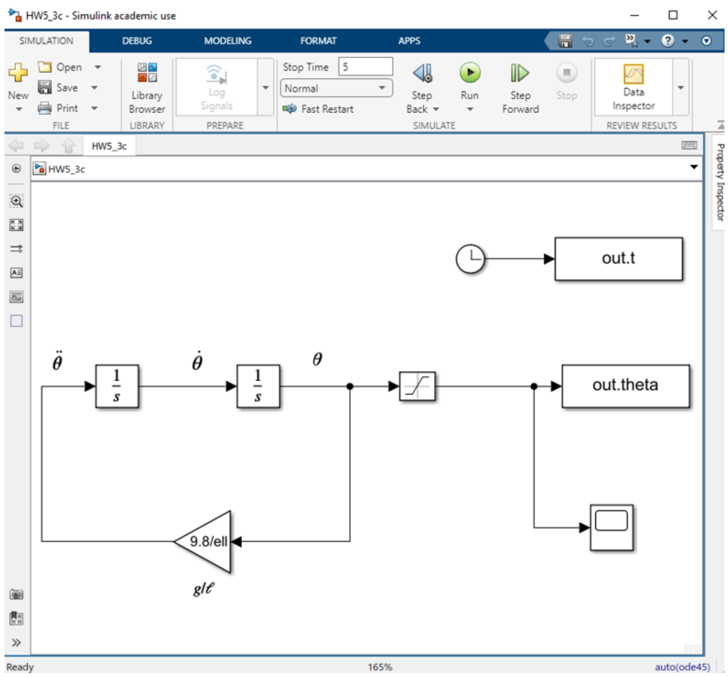
$$\theta(t) = \frac{\theta(0)}{2}(e^{-\sqrt{g/\ell}t} + e^{\sqrt{g/\ell}t}) \tag{2}$$

is a solution by substituting this $\theta(t)$ into (1).

 ©2017-2022 Patrick Kelly and C.V. Hollot

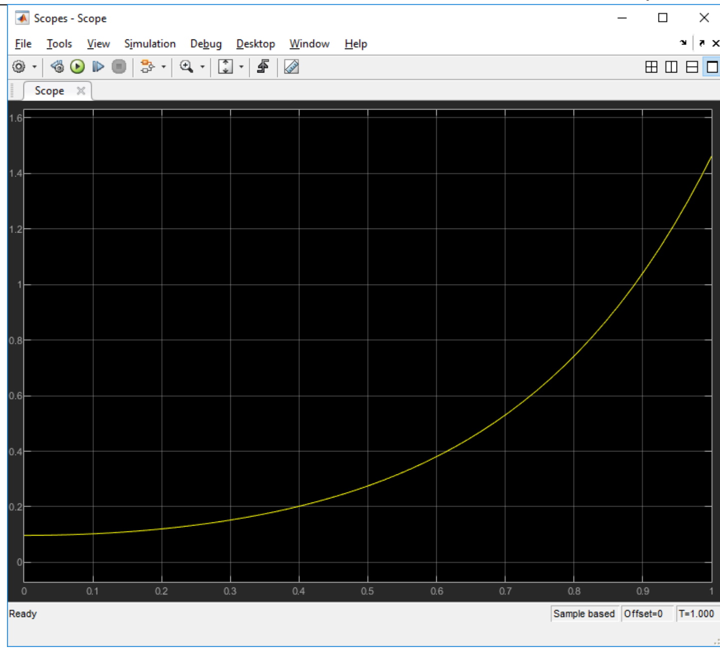
(b) For $g = 9.8m/s^2$, $\ell = 1m$ and $\theta(0) = 0.1rad$, use Matlab to plot $\theta(t)$ (using (2)) vs. time t for 1sec. Now, change the stick length to $\ell = 0.5m$ and replot $\theta(t)$. Use the *hold* command to overlay theses two plots. From the plots, argue that the shorter length stick falls faster.

(c) Now, we will repeat part (b) simulating the solution to (1) rather than leaning on the analytical solution (2). Open the Matlab Simulink file `HW5_3c.slx` resulting in the screen shot as shown below.



This is the simulation that we introduced in lecture, where the values for ℓ and $\theta(0)$ can be set in the Matlab command window; for example, by typing *ell=0.5* and *theta_0=0.1*.The simulation can be started and stopped with the buttons on the top ribbon. Clicking on the "Scope" box will open up a graph that plots θ as the simulation runs (see below). The blocks "out.t" and "out.theta" write t and θ to the workspace where they can be accessed via Matlab and plotted such as in `plot(out.t, out.theta)`. Now, repeat what was done in part (b) via Simulink simulations. Provide a single overlayed plot of θ vs. t for $\ell = 0.5, 1$m.

 ©2017-2022 Patrick Kelly and C.V. Hollot



Question 4

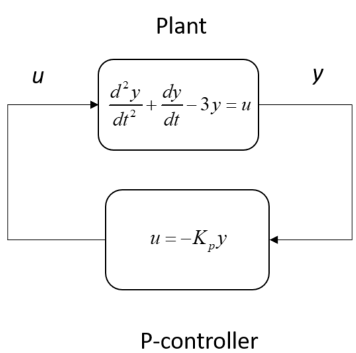
(This question asks you how to choose a feedback control parameter for a plant having a second-order differential equation model.)

Suppose that we have a plant modeled by the differential equation having plant input u and plant output y:

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 3y = u. \tag{3}$$

(a) Is this plant stable? Why?

(b) Consider the feedback control loop with this plant and P-controller with gain K_p as shown below.



Find a value of K_p that makes the system stable ($y \to 0$ as $t \to \infty$) .

Question 5

(This question asks you to use MATLAB Simulink to simulate the operation of a system having a second-order differential equation model, with and without the use of a feedback control loop.)

In this problem we will use a D-controller (derivative control) to add “”damping” to a system. (*Note:* The instructions in this problem refer to MATLAB version R2017a. If you have another version of MATLAB there may be some slight differences in how you access various Simulink functions.)

Suppose that we have a plant described by the differential equation

$$\frac{d^2y}{dt^2} + 0.3\frac{dy}{dt} + y = u. \tag{4}$$

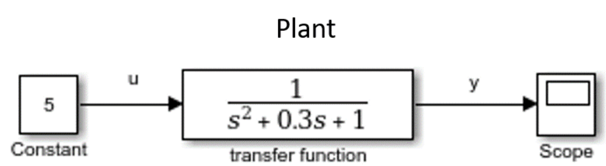
(a) Is the plant stable? Why?

(b) Suppose that the plant’s input signal is the constant $u = 5$. We’ll use Simulink to plot the output y over the period $0 \leq t \leq 50$ sec. Open MATLAB. Under the Home tab above the workspace, click on the item “Simulink” - a screen called the Simulink Start Page will open (this may take a few seconds). You’ll see a set of boxes - click on the“Blank Model” box, and then the “Create Model” button. A new screen will open, having the label “untitled.” This is where we will build the simulation model. Click on “View” and then “Library Browser.” You will see a list of items on the left - click on the item “Continuous.” On the right you will see several blocks. We’re going to use the block labeled “Transfer Fcn.” Drag the Transfer Fcn block into the “untitled” screen. The Transfer Fcn block implements the plant’s differential equation. To set the differential equation, double click on the block. A screen labeled “Function Block Parameters: Transfer Fcn” will open. In the box on the screen labeled “denominator coefficients”, enter “$[1\ 0.3\ 1]$”

(these are the coefficients in the plant’s differential equation). Then click on “Apply” and “OK.”

We need to add an input. From the list in the lefthand panel of the Simulink Library Browser, click on the item “Sources” (these are different types of input signals). We want a constant input $u = 5$. To set this, drag the block labeled “Constant” into the simulation box, and put it to the left of the Transfer Fcn block. You may see a light line connecting the blocks - click on the line to form an arrow from the Constant block to the Transfer Fcn block (if you don’t see the line, just drag your pointer from the Constant block output to the Transfer Fcn input.) Double click on the Constant block - in the screen that opens, set “Constant Value” to 5, then click on “Apply” and “OK.”

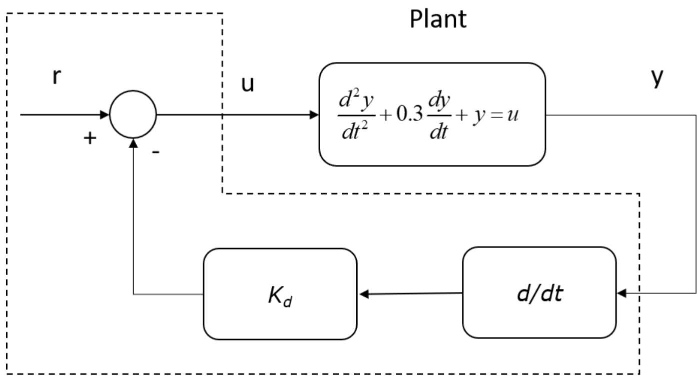
The last block we need to add is a way to see the output. In the list on the lefthand side of the Simulink Library Browser, click on “Sinks” (these are output devices), and drag the block labeled “Scope” into the simulation box - put it to the right of the Transfer Fcn block. The Scope block simulates an oscilloscope. Connect an arrow from the Transfer Fcn to the Scope. Your connection should now look like the diagram shown below. (*Note: the blocks may appear slightly different in your version of Simulink.*)



Double click on Scope to open a screen (the scope display) - you can drag on a corner of the screen to expand the screen if it seems too small. Also, the default screen has a black background, which uses a lot of ink or toner when printed. To change the screen colors click “View” and then “Style” on the Scope screen. You can adjust the background and grid colors using the “Axes colors” buttons, and the color and thickness of the output signal trace using the “Line” buttons.

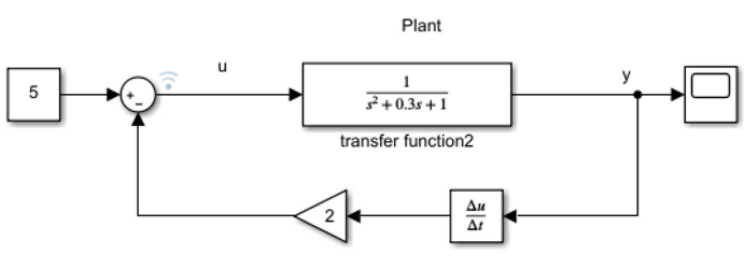
Now we’re set to run the simulation. In the simulation box, you’ll see a white box at the top with a number in it (usually set at 10.0). Change that to 50.0 (this sets how long our simulation will run). Then click on the icon that shows a green circle with a triangle inside - that starts the simulation. When the simulation is finished, you will see y traced out on the Scope screen. (To get the right axis scaling to see y, you might need to click on the screen icon labeled "Autoscale" - that's the icon at the top of the Scope screen that looks like a rectangle with four arrows inside.) Print the Scope screen (using the print icon at the upper left of the screen), and attach the print to your homework.

(c) You should see that, while the output settles around the value 5, it has large oscillations - these are often undesirable. To reduce the oscillations we will add a feedback control loop as shown in the figure below. (Note that the control signal u in this case is proportional to the derivative of the output y and is an example of *derivative (D)* control.)



D (derivative) controller w/ reference signal *r*

To form this block diagram, drag two new blocks into the simulation box: the "Sum" block from Math Operations, and the "Derivative" block from Continuous. Set up the connection shown in the diagram below. (*Note:* To connect the Transfer Fcn output to both the Derivative and Scope blocks: first drag your pointer to draw an arrow that connects Transfer Fcn to Derivative; then drag your pointer from the scope input back to the arrow from Transfer Fcn to Derivative.) Double click on the Sum block; under "List of signs" set the last sign to $-$. Also, double click on the Gain block, and set "Gain" to 2 (this multiplies the output of the derivative block (which is $\dot{y}$) by 2, as we want). (Be sure to click "Apply" and "OK" after each change.) Now run the simulation, and look at the output y on the Scope screen (again, you may need to use Autoscale to see the full trace). Did our control system succeed in reducing the oscillations? Print the Scope screen and attach it to your homework.



D controller with reference

Question 6

For each of the following, answer ('Yes' or 'No') on whether the differential equation (D.E.) has a stable solution:

 ©2017-2022 Patrick Kelly and C.V. Hollot

(a)

$$\ddot{\theta} + \theta = 0;\ \theta(0) \neq 0$$

(b)

$$\ddot{\theta} + \dot{\theta} = 0;\ \theta(0) \neq 0$$

(c)

$$\ddot{\theta} - 2\dot{\theta} + 3\theta = 0;\ \theta(0) \neq 0$$

(d)

$$\ddot{\theta} + 2\dot{\theta} - 3\theta = 0;\ \theta(0) \neq 0$$

 ©2017-2022 Patrick Kelly and C.V. Hollot