Example 2.3: Test if (a) integrator, (b) 1/2-wave rectifier, (c) modulator have the time invariance property.

Solution:
(i) Integrator [Eq. (1) in notes]
Consider a delayed input $x_1(t) = x(t-T)$. The output $y_1(t)$ is

$$\begin{aligned} y_1(t) &= \int_{-\infty}^{t} x_1(\tau)d\tau = \int_{-\infty}^{t} x(\tau - T)d\tau \\ &= \int_{-\infty}^{t-T} x(\tau')d\tau' = y(t-T) \end{aligned} \tag{E1}$$

with a change of variable $\tau' = \tau - T$. Hence, the integrator is time invariant.

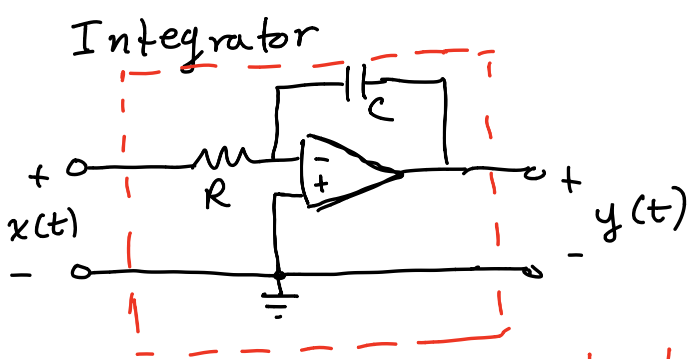

Figure 1: An integrator

Note that the system does not change with time.

(ii) 1/2-wave rectifier [Eq. (2)]
For the delayed input $x_1(t) = x(t-T)$, the output $y_1(t)$ is

$$y_1(t) = \begin{cases} x_1(t), & x_1(t) \geq 0 \\ 0, & x_1(t) < 0 \end{cases} \\ = \begin{cases} x(t-T), & x(t-T) \geq 0 \\ 0, & x(t-T) < 0 \end{cases}. \tag{E2}$$

From (2), the delayed output is

$$y(t-T) = \begin{cases} x(t-T), & x(t-T) \geq 0 \\ 0, & x(t-T) < 0 \end{cases}, \tag{E3}$$

which is the same as (E2). Hence, the rectifier is time-invariant.

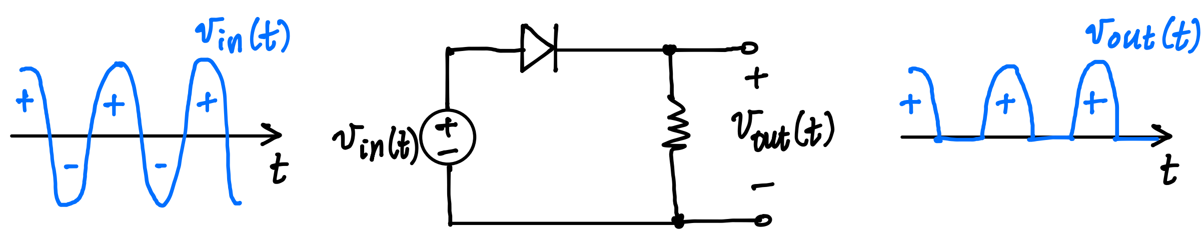

Figure 2: A 1/2-wave rectifier circuit.

Note that the rectifier circuit does not have a time-varying component.

(iii) Modulator [Eq. (3)]
Consider a delayed input $x_1(t) = x(t-T)$. Its output

$y_1(t)$ is

$$y_1(t) = x_1(t)\cos\omega_0 t = x(t-T)\cos\omega_0 t, \tag{E4}$$

From (3), the delayed output $y(t-T)$ is

$$y(t-T) = x(t-T)\cos\omega_0(t-T) \neq y_1(t), \tag{E5}$$

unless $\omega_0 T = 2n\pi$ (n = integer). For a system to be time-invariant, $y_1(t) = y(t-T)$ should hold for any T. So, the modulator is *not* time invariant.

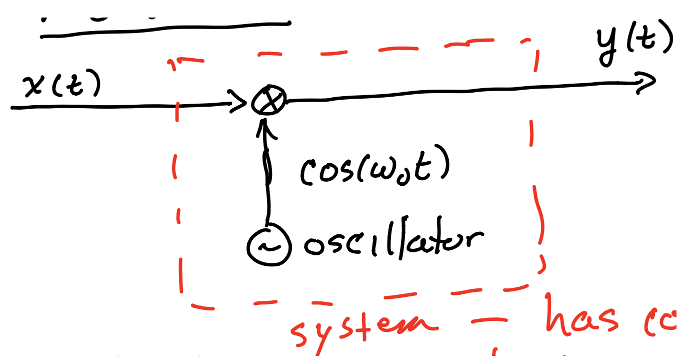

Figure 3: A modulator block diagram.

The system has a component that changes with time.