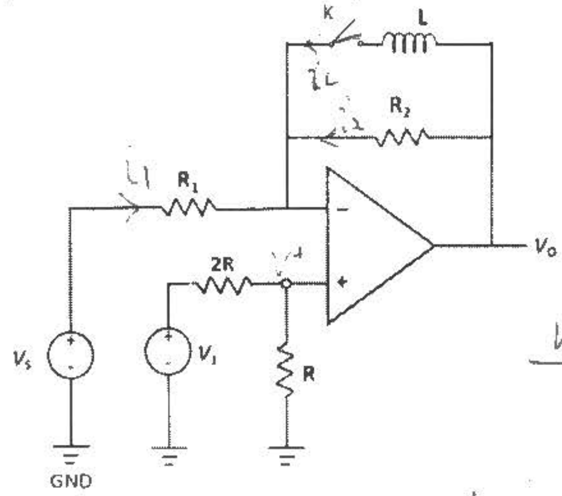
Homework 8

Problem 1 (4 pt)

Find the differential equation for v_o in the opamp circuit below for t>0 when the switch K is closed (You don't need to solve the equation). Assume the opamp is ideal with infinite gain.



① For V^+.

Voltage Divider: $V^+ = \frac{R}{R+2R} \times V_1 = \frac{1}{3} V_1 = V^-$

② KCL @ V^-:

$$i_1 + i_2 + i_L = 0$$

$$\frac{V_s - V^+}{R_1} + \frac{V_o - V^+}{R_2} + i_L(0^+) + \frac{1}{L}\int_0^t (V_o - V^+)\,dt = 0$$

$$\frac{1}{R_2}\frac{dV_o}{dt} + \frac{1}{L}(V_o - V^+) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\frac{dV^+}{dt} - \frac{1}{R_1}\frac{dV_s}{dt}$$

$$\frac{dV_o}{dt} + \frac{R_2}{L}\cdot V_o = \left(\frac{R_1+R_2}{R_1}\right)\cdot\frac{dV^+}{dt} - \frac{R_2}{R_1}\frac{dV_s}{dt} + \frac{R_2}{L}\cdot V^+$$

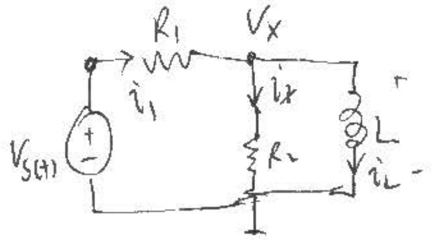
Suppose. V_s, V_1 are time independent.

$$\frac{dV_o}{dt} + \frac{R_2}{L}\cdot V_o = \frac{R_2}{L}\cdot V^+$$

$$\frac{dV_o}{dt} + \frac{R_2}{L}\cdot V_o = \frac{R_2}{L}\cdot\frac{1}{3}V_1$$

$$\frac{dV_o}{dt} + \frac{R_2}{L}\cdot V_o = \frac{1}{3}\cdot\frac{R_2\cdot V_1}{L}$$

Problem 2:



KCL @ V_x:

$$i_1 - i_L - i_x = 0$$

$$\frac{V_s - V_x}{R_1} - \left(i_L(0^+) + \frac{1}{L}\int_0^t (V_x - 0)\,dt\right) - \frac{V_x - 0}{R_2} = 0$$

Differential:

$$\frac{1}{R_1}\frac{d(V_s - V_x)}{dt} - \frac{1}{L}V_x - \frac{1}{R_2}\cdot\frac{dV_x}{dt} = 0$$

$$-\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\cdot\frac{dV_x}{dt} + \frac{1}{R_1}\frac{dV_s}{dt} - \frac{1}{L}V_x = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\cdot\frac{dV_x}{dt} - \frac{1}{R_1}\frac{dV_s}{dt} + \frac{1}{L}V_x = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\cdot\frac{dV_x}{dt} + \frac{1}{L}\cdot V_x = \frac{1}{R_1}\frac{dV_s}{dt}$$

$$\frac{dV_x}{dt} + \frac{R_1R_2}{L(R_1+R_2)}\cdot V_x = \frac{R_2}{R_1+R_2}\cdot\frac{dV_s}{dt}$$

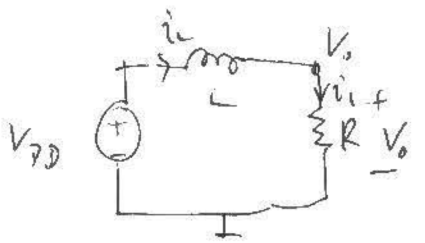
Since: $i_x = \dfrac{V_x}{R_2} \Rightarrow \underline{V_x = i_x \cdot R_2}$

$$\frac{d(i_x \cdot R_2)}{dt} + \frac{R_1R_2}{L(R_1+R_2)}\cdot i_x \cdot R_2 = \frac{R_2}{R_1+R_2}\cdot\frac{dV_s}{dt}$$

$$\Rightarrow \quad \frac{di_x}{dt} + \frac{R_1R_2}{L(R_1+R_2)}\cdot i_x = \frac{1}{R_1+R_2}\cdot\frac{dV_s}{dt}$$

Probem 3

$V_{(t)}$ when $t>0$



KCL: $i_L - i_1 = 0$

$$i_L(0^+) + \frac{1}{L}\int_0^t (V_{DD} - V_o)\,dt - \frac{V_o - 0}{R} = 0$$

Differential:

$$\frac{1}{L}(V_{DD} - V_o) - \frac{dV_o}{dt}\cdot\frac{1}{R} = 0$$

$$\Rightarrow \frac{dV_o}{dt} + \frac{R}{L}\cdot V_o = \frac{R}{L}\cdot V_{DD}$$

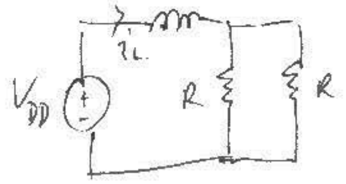
$$V_o(t) = V_{DD} + k\cdot e^{-\frac{R}{L}\cdot t}$$

find initial condition $V_o(0^+)$: (relate $V_o(0^+)$ to $i_L(0^+)$)

$$i_1(0^+) = i_L(0^+)$$

$$\frac{V_o(0^+)}{R} = i_L(0^+) \Rightarrow V_o(0^+) = i_L(0^+)\cdot R = i_L(0^-)\cdot R$$

find $i_L(0^-)$: (when $t<0$;)



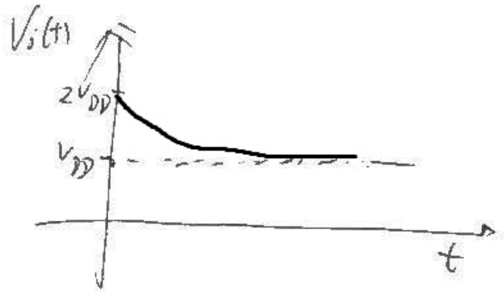
For steady state: inductor acts like short-circuit

$$i_L(0^-) = \frac{V_{DD}}{\frac{R}{2}} = \frac{2V_{DD}}{R}.$$

So: $V_o(0^+) = \frac{2V_{DD}}{R}\times R = 2V_{DD}$

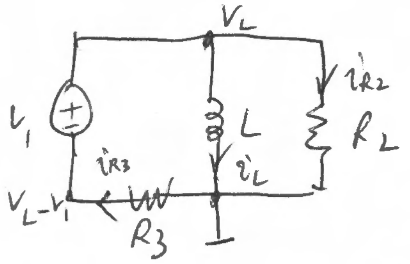
$V_o(0^+) = 2V_{DD} = V_{DD} + k \Rightarrow k = V_{DD}$

So: $V_o(t) = V_{DD} + V_{DD}\cdot e^{-\frac{R}{L}\cdot t}$



Problem 4

At $t > 0$.



KCL:

$$-i_L - i_{R2} + i_{R3} = 0 \quad \text{--- (1)}$$

$$-i_L(0^+) - \frac{1}{L}\int_0^t V_L\,dt - \frac{V_L}{R_2} + \frac{0-(V_L-V_1)}{R_3} = 0$$

diff: $-\frac{1}{L}V_L - \frac{1}{R_2}\frac{dV_L}{dt} - \frac{1}{R_3}\frac{dV_L}{dt} = 0$

$$\left(\frac{1}{R_2}+\frac{1}{R_3}\right)\frac{dV_L}{dt} + \frac{1}{L}V_L = 0$$

$$\left(\frac{1}{4k}+\frac{1}{4k}\right)\frac{dV_L}{dt} + \frac{1}{2mH}\cdot V_L = 0$$

$$\frac{1}{2k}\cdot\frac{dV_L}{dt} + \frac{1}{2mH}\cdot V_L = 0 \Rightarrow \frac{dV_L}{dt} + 10^6 V_L = 0$$

$$V_L(t) = k\cdot e^{-10^6\cdot t}$$

find: $V_L(0^+)$:

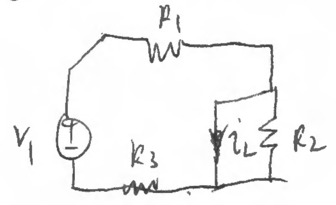
from (1): we have: $i_L(0^+) = i_{R3}(0^+) - i_{R2}(0^+) = \frac{0-(V_L(0^+)-V_1)}{R_3} - \frac{V_L(0^+)}{R_2}$

$$= -\left(\frac{1}{R_3}+\frac{1}{R_2}\right)\cdot V_L(0^+) + \frac{V_1}{R_3}$$

$$= -\frac{1}{2K}\cdot V_L(0^+) + \frac{3}{4k}$$

$i_L(0^+) = i_L(0^-)$.

@ $t = 0^-$. (steady state)



$$i_L(0^-) = \frac{V_1}{R_1+R_3} = \frac{3}{6k\Omega} = 0.5\,mA$$

So: $0.5\,mA = -\frac{1}{2k}\cdot V_L(0^+) + \frac{3}{4k} \Rightarrow 2 = -2V_L(0^+) + 3$

$$\Rightarrow V_L(0^+) = \frac{1}{2}\ (V)$$

So: $V_L(0^+) = k\cdot = \frac{1}{2}$

So $V_L(t) = \frac{1}{2}\cdot e^{-10^6\cdot t}$