Delta
Rasson
BILLIARDS

Announcements, Goals, and Reading

Announcements:

- HW10 due Tuesday 11/29
- Tomorrow is officially a “Friday Schedule” but there will not be a lecture.
- ***If needed***, an extra lecture may be recorded between now & end of semester
  - …but we seem to be on track.

Goals for Today:

- Conservation of Energy
- Conservative & Non Conservative Forces
- Momentum (time permitting)

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 10: Interactions and Potential Energy
- Chapter 11: Impulse and Momentum

Chapter 9 – Work and Kinetic Energy
Chapter 10: Interactions and Potential Energy

Energy measures the ability of a system to do **work** on another system. "**Work**" in this context refers to the transfer of energy via **forces**.

We will focus on **Kinetic Energy** and **Potential Energy, c**ollectively known as "**Mechanical Energy**"

Units: Joules ($1\,Joule = 1 Newton * 1\,meter = \frac{1kgm^2}{s^2}$).

Note: Historical unit is the Calorie; 1 kCal = 4184J. Dietary calories in food are actually measured in kCal.

Kinetic energy K: Energy of motion. For an object with velocity v…

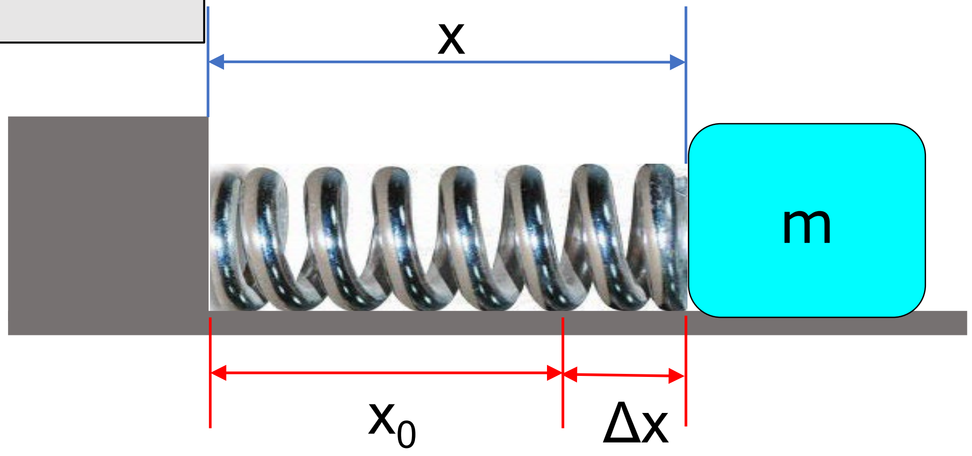
$$K = \frac{1}{2}mv^2$$

Potential energy U: Energy "stored" in a system. Mathematical form depends on the nature of the forces involved.

Gravitational Potential Energy: Depends on height **h** above the ground:

$$U = mgh$$

Springs and conservation of energy



Potential energy stored in stretched or compressed spring

$$U_s = \frac{1}{2}k(x - x_0)^2$$

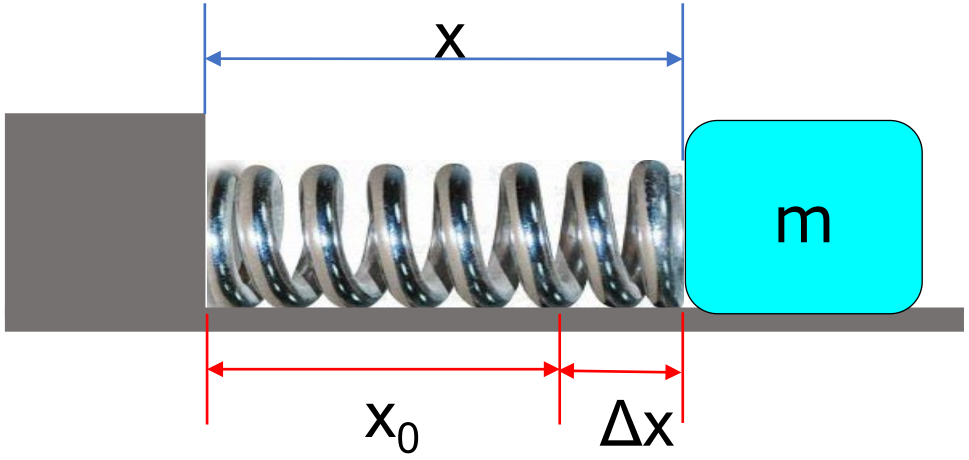
Mechanical energy is conserved: sum of *kinetic energy of block* and *potential energy of spring*

$$E = K + U_s = \frac{1}{2}mv^2 + \frac{1}{2}k(x - x_0)^2$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta x)^2$$

$$\Delta x = x - x_0$$

amount by which spring is stretched or compressed

$$U_s = \frac{1}{2}k(x - x_0)^2$$



We can prove that this form of *U(x)* leads to conservation law (same procedure as before for gravity):

velocity $v = \dot{x}$ acceleration $a = \ddot{x}$

$$\frac{dE}{dt} = \frac{d}{dt}\left[\frac{mv^2}{2} + \kappa\frac{(x-x_0)^2}{2}\right] = mv\dot{v} + \kappa(x - x_0)\dot{x} = vma + v\kappa(x - x_0)$$

Newton's second law ➡ $ma = F = -\kappa(x - x_0)$ for springs

$$\frac{dE}{dt} = vF + v\kappa(x - x_0) = 0 \checkmark$$

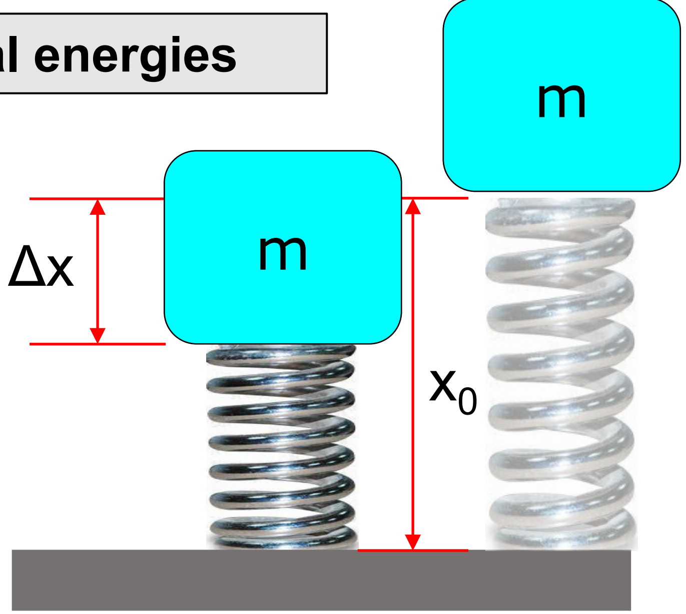
Mechanical energy is constant

Problems with spring and gravitational potential energies

A block of mass m=0.5 kg sits atop a spring with spring constant k= 100 N/m

The spring is compressed by Δx =0.3m and then released, launching the block into the air

How high up above its initial height does the block go?



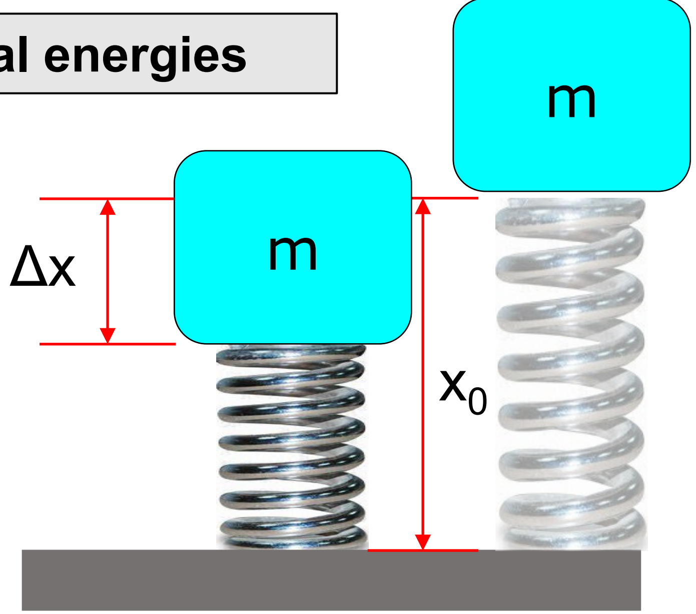
What are the relevant forms of energy? What is the total initial energy?*

Problems with spring and gravitational potential energies

A block of mass m=0.5 kg sits atop a spring with spring constant k= 100 N/m

The spring is compressed by Δx =0.3m and then released, launching the block into the air

How high up above its initial height does the block go?



Mechanical energy

height measured from initial elevation of block

$$E = K + U_g + U_s$$
$$= \frac{1}{2}mv^2 + mgh + \frac{1}{2}k(\Delta x)^2$$

Initially ➡ only spring potential energy

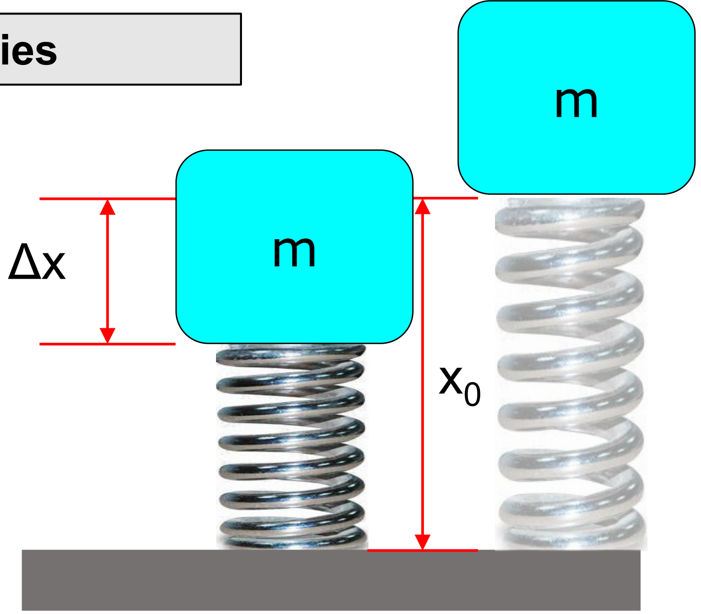
$$E_i = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}(100N/m)(0.3m)^2 = 4.5J$$

Problems with spring and gravitational potential energies

A block with mass m=0.2 kg sits atop a spring with spring constant k= 100N/m

The spring is compressed by Δx =0.3m and then released, launching the block into the air

How high up above its initial height does the block go?



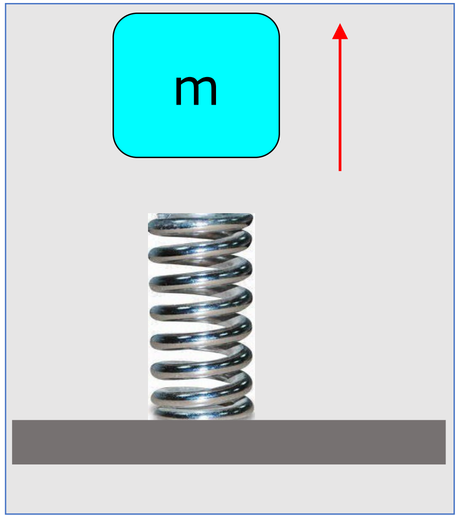
Mechanical energy $$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}k(\Delta x)^2$$

Initially $$E_i = 4.5J$$

At top of cheese's trajectory ➡ only gravitational potential energy

$$E_f = mgh$$

height at top, measured from starting height



Conservation of energy ➡ $$h = \frac{4.5J}{mg} = \frac{4.5J}{(0.2kg)(9.8m/s^2)}$$
$$= 2.3m$$

Potential energy diagram for mass on spring

Draw diagram for spring potential energy

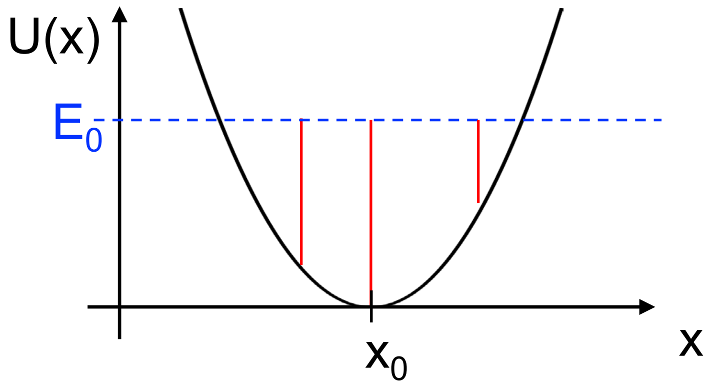
$$U = \frac{1}{2}k(x - x_0)^2$$

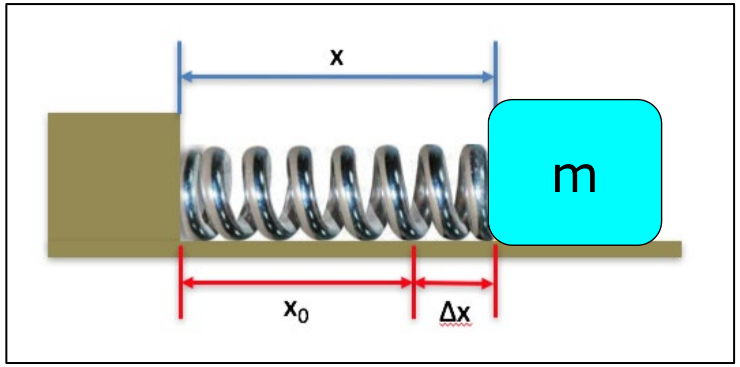
Conserved energy

$$E = K + U$$

Assume system has energy E_0 and add that to diagram

Difference between E_0 and potential energy curve is kinetic energy





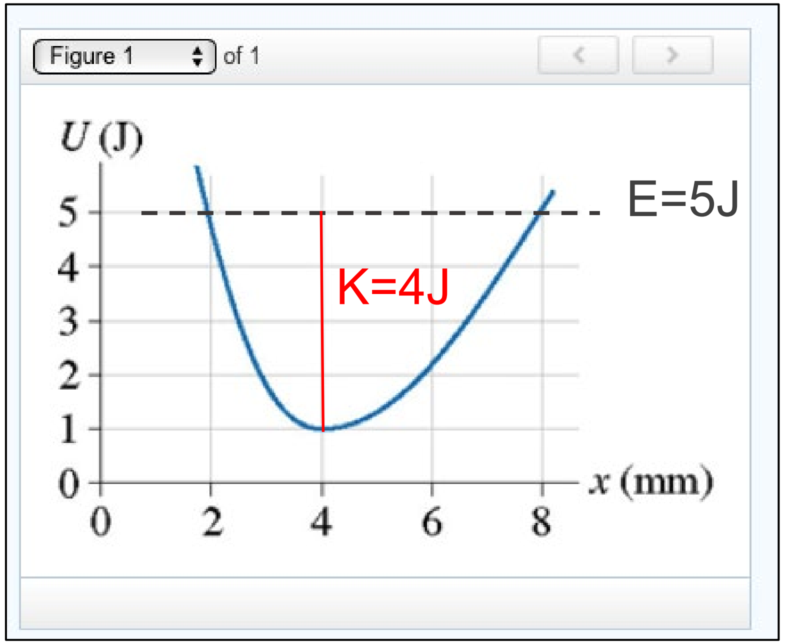
Maximum kinetic energy when $x=x_0$

Kinetic energy goes to zero when $U(x)=E_0$

➡ called "turning points" of motion

➡ Mass oscillates back and forth between turning points

Analysis of potential energy diagrams



$$E = K + U$$

$$\boxed{U_s = \frac{1}{2}k(x - x_0)^2}$$

Graphically determine that K=4 J is maximum kinetic energy

$$\frac{1}{2}mv^2 = 4J$$

$$v = \sqrt{\frac{2(4J)}{(0.0029kg)}} = 52m/s$$

What is the maximum speed of a 2.90 g particle that oscillates between $x = 2.0\ \mathrm{mm}$ and $x = 8.0\ \mathrm{mm}$ in the figure? (Figure 1)

turning points

Tells us where to draw in the horizontal energy line

More on conservative and non-conservative Forces

For **conservative** forces in 1D, the force is related to derivative of the potential energy! (Recall our checks for gravitational and spring forces).
(conservative force means work done is path-independent)

$$F(x) = -\frac{dU(x)}{dx}; \qquad U(x) = U_0 - \int_{x_0}^{x} F(s)\,ds$$

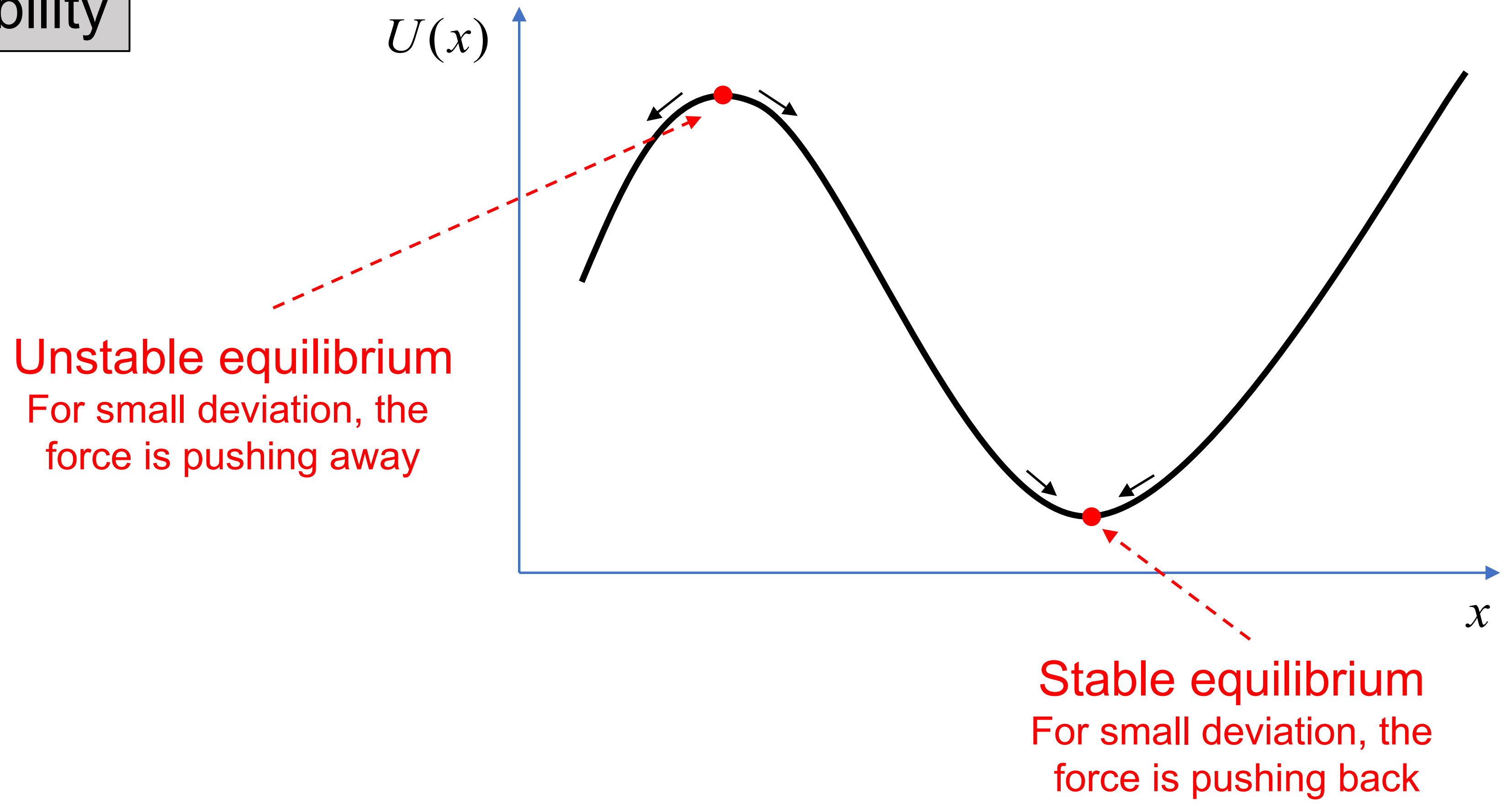
$$\Delta K = W = \int_{x(0)}^{x(t)} F(s)\,ds \quad \Rightarrow \quad \Delta K = W = -\Delta U$$

For conservative force only

$$\Delta K + \Delta U = 0 \Rightarrow E = K + U(x) \Rightarrow \Delta E = 0$$

(conservation of mechanical energy)

Stability



$$F(x) = -\frac{d}{dx}U(x)$$

Points where force is zero are call **equilibrium points**. If at rest at t=0, you will stay at equilibrium point forever (classically).

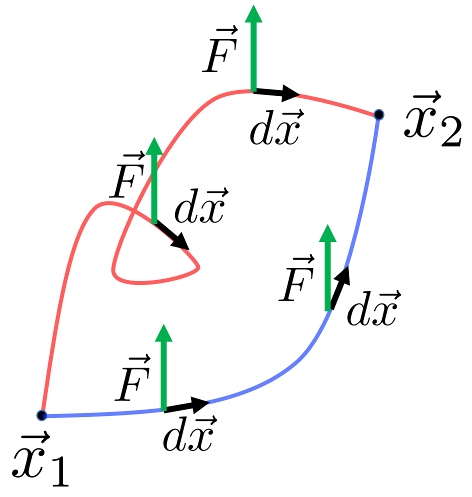
They are minima and maxima of *U(x)*

Conservative Forces

When can a force be associated with a derivative of potential energy?

Recall definition of work done by a force F as an object moves between positions $\vec{x}_1$ and $\vec{x}_2$

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} \cdot d\vec{x}$$



Work may be different along different paths between same two points

If work same for all paths connecting any two points, then force is "conservative"

Can be associated with potential energy function $U(\vec{x})$

If this is the only force acting, then mechanical energy will be conserved

$$E = K + U(\vec{x})$$

e.g. true for gravity

For **conservative** forces (any dimension), the force is related to the gradient of the potential energy!

$$U(x) = U_0 - \int_{\vec{x}_0}^{\vec{x}} \vec{F} d\vec{s} \qquad = U_0 - \int_{x_0}^{x} F_x(\vec{s}) dx - \int_{y_0}^{y} F_y(\vec{s}) dy - \int_{z_0}^{z} F_z(\vec{s}) dy$$

$$\boxed{F_x(\vec{x}) = -\frac{d}{dx} U(\vec{x});\ F_y(\vec{x}) = -\frac{d}{dy} U(\vec{x});\ F_z(\vec{x}) = -\frac{d}{dz} U(\vec{x});}$$

For the sake of compact notations introduce: $\boxed{\vec{\nabla} = \hat{x}\frac{d}{dx} + \hat{y}\frac{d}{dy} + \hat{z}\frac{d}{dz}}$

Gradient

$$\boxed{\vec{F}(\vec{x}) = -\vec{\nabla} U(\vec{x});}$$

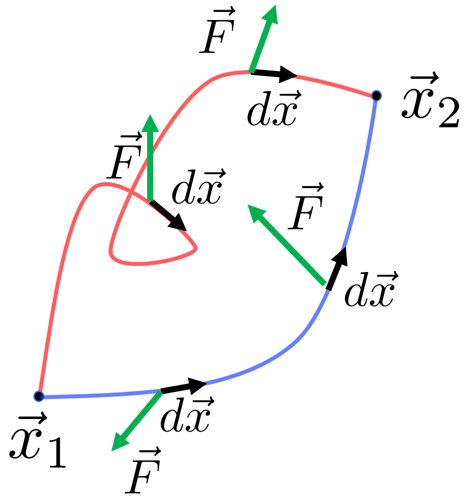
Minima and maxima of $U(\vec{x})$ are equilibrium points
Minima are stable and maxima are unstable, as in 1D

Conservative Forces in 3D

(conservative force means work done is path-independent)

Work may be different along different paths between same two points, but

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} d\vec{s}$$

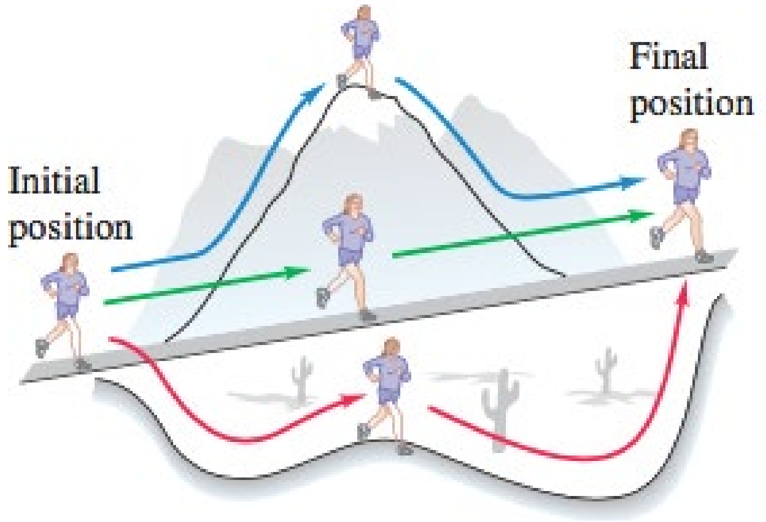


If work is same for **all** paths connecting any two points, then force is **"conservative."**

This is true for any $\vec{F} = -\vec{\nabla} U(\vec{x})$:

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} d\vec{s} = U(\vec{x}_1) - U(\vec{x}_2)$$

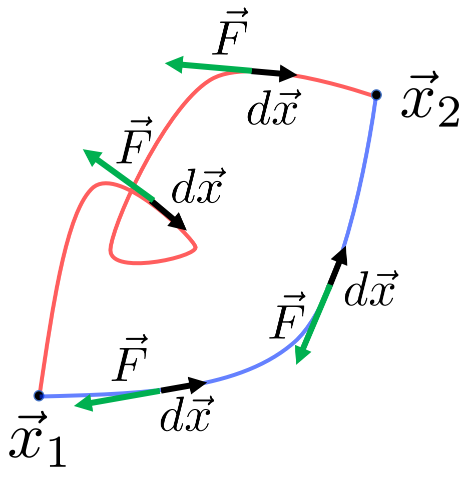
Because the gravitational force is conservative, the work it does is the same for all three paths.



Non-Conservative Forces

When is a force ***NOT*** associated with a form of potential energy?

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} \cdot d\vec{x}$$



Work may be different along different paths between same two points

If work depends on path, then force is “non-conservative”

Key example ➡ kinetic friction

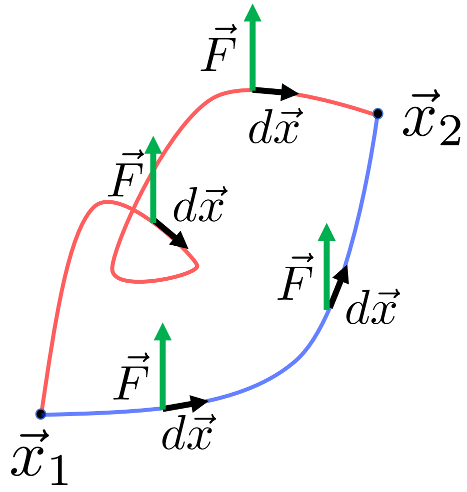
Points backwards along direction of motion

Work proportional to length of path

Can’t be described in terms of potential energy function

Alternative formulation

Conservative force ⟺ Work along blue path = Work along red path

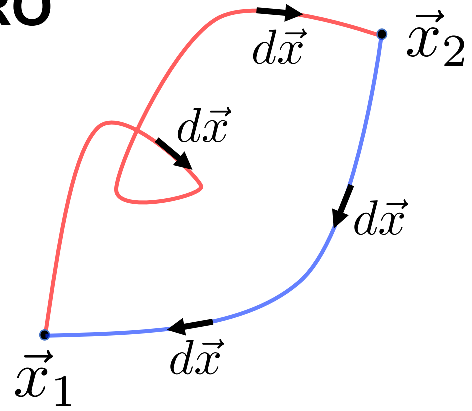


Reversing blue path makes a closed loop

Reversing path changes overall sign of work

Work along reversed blue path = - (Work along red path)

Total work done by force around closed loop is **ZERO**



⇒ For a conservative force, the work from any point back to itself along a closed path is **zero**

$$\oint \vec{F} \cdot d\vec{x} = 0$$

Symbol for line integral around a closed path

Summary: Conservative and Non-Conservative Forces

• **<u>Conservative force</u>**: work done is independent of path.
Equivalently: force is conservative if work done over any closed path is zero

• **<u>Non-conservative force</u>**: work done depends on the path.

• The component of a conservative force, in a particular direction, equals the negative of the derivative of the potential energy for that force, with respect to a displacement in that direction.

$$F_x = -\frac{\Delta U(x)}{\Delta x} = -\frac{dU(x)}{dx}$$

New Topic: Linear Momentum and Collisions

Start with **momentum**…

$$\vec{p} = m\vec{v}$$

Momentum of an object is the product of its mass and velocity

Momentum is a vector

It has both magnitude and direction

Momentum is a conserved quantity

Follows from Newton's 3^{rd} Law

Total momentum of an isolated system is constant

We'll come back to this..

A large object moving slowly can have the same momentum as a small object moving fast

$\vec{p} = m\vec{v}$ Momentum

A 100g chipmunk wants to run fast enough to have as much momentum as a 6000kg elephant moving at 25 mph.

How fast does the chipmunk need to run?

Only dealing with the magnitude of momentum here

$$m_c v_c = m_e v_e \Rightarrow v_c = \frac{m_e}{m_c} v_e = \left(\frac{6000kg}{0.1kg}\right) 25mph$$

$$= 1.5 \times 10^6 \, mph$$

This was just a dream …

Linear Momentum and Impulse

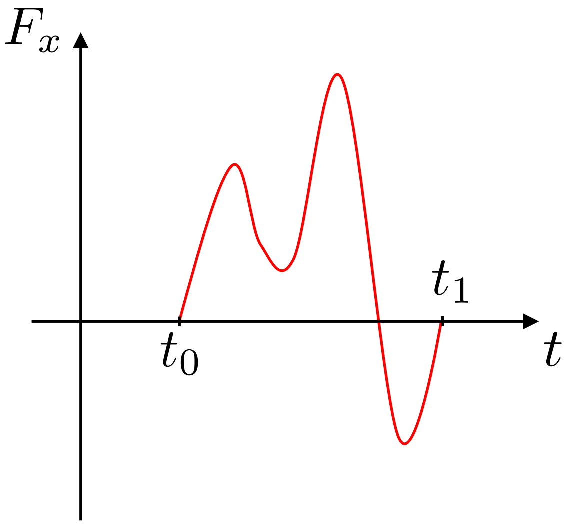
What happens if variable external force acts on object for some time?

If we have a force that is applied on an object for a certain amount of time

$$\vec{F}(t) = F_x(t)\hat{i} + F_y(t)\hat{j} + F_z(t)\hat{k}$$

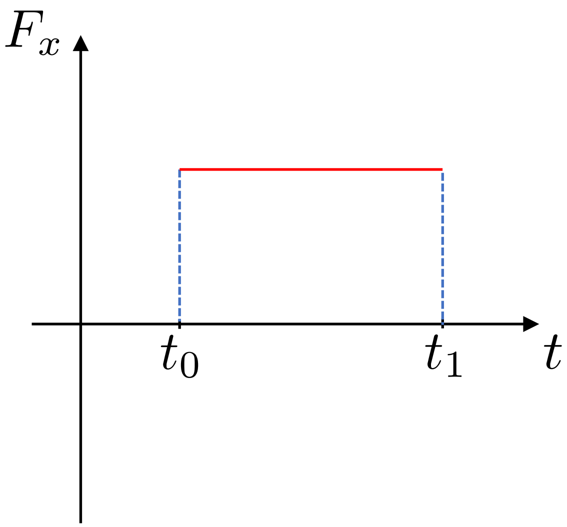
Assume force vanishes before time t_0 and after time t_1

So that graphs of components of force may look for example like…



Complicated time dependent force

Or perhaps like…



Constant force is turned on, then turned off

<u>Impulse vector</u> determines how much momentum of object changes under action of time dependent force.

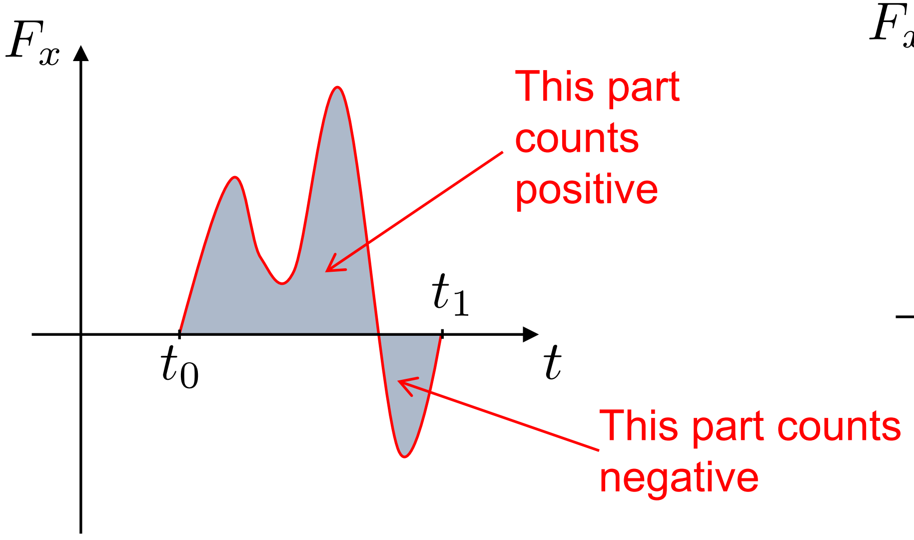
Adds up impact of force on object from t_0 to t_1

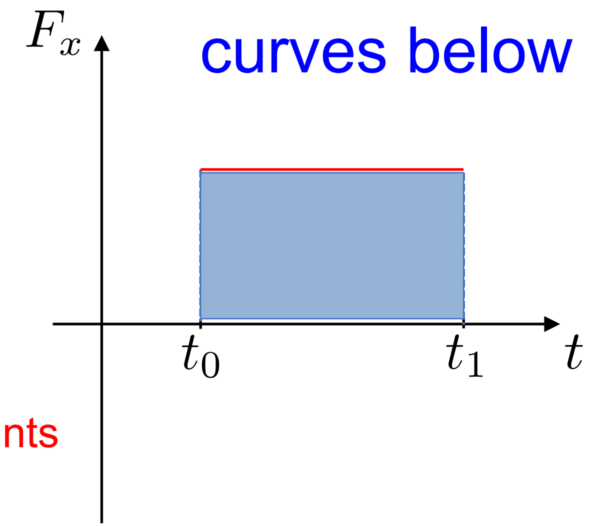
Define **Impulse vector** $\vec{J} = J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$

In words -

Components of impulse vector are the areas under graphs of components of force vector

In our examples the J_x would be given by the areas under the curves below

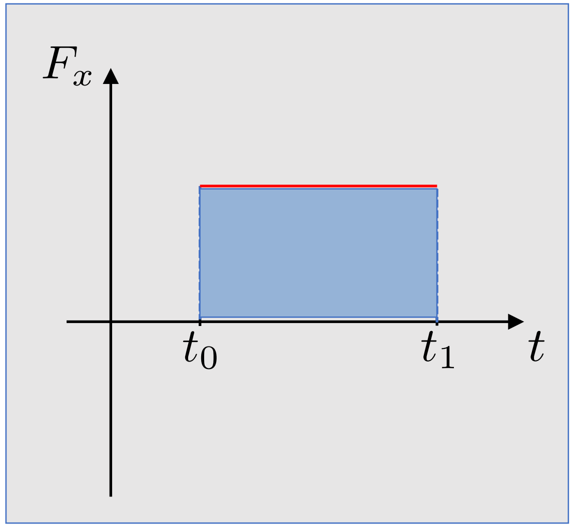




(Force is pointing in negative x-direction)

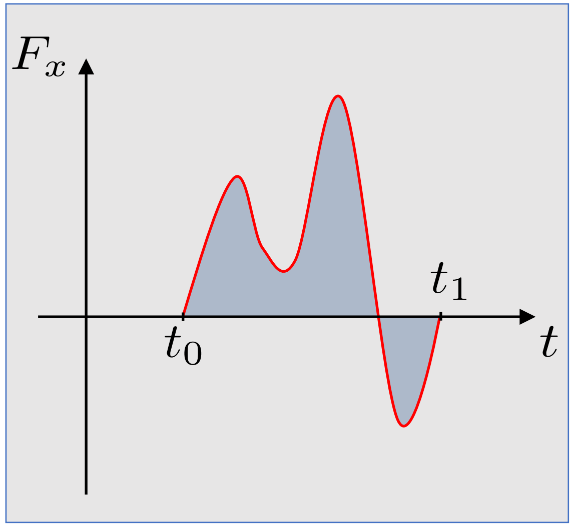
<u>Impulse vecto</u>r determines how much momentum of object changes under action of time dependent force.

Define **Impulse vector** $\vec{J} = J_x\hat{i} + J_y\hat{j} + J_z\hat{k}$



In words -

Components of impulse vector are the areas under graphs of components of force vector



In math formula –

Impulse is the integral of force over time

$$\vec{J} = \int_{t_0}^{t_1} dt\, \vec{F}(t)$$

$$J_x = \int_{t_0}^{t_1} dt\, F_x(t)$$

$$J_y = \int_{t_0}^{t_1} dt\, F_y(t)$$

$$J_z = \int_{t_0}^{t_1} dt\, F_z(t)$$

This vector expression means that to get the components of impulse we integrate the components of force

Impulse has units of Newton-seconds

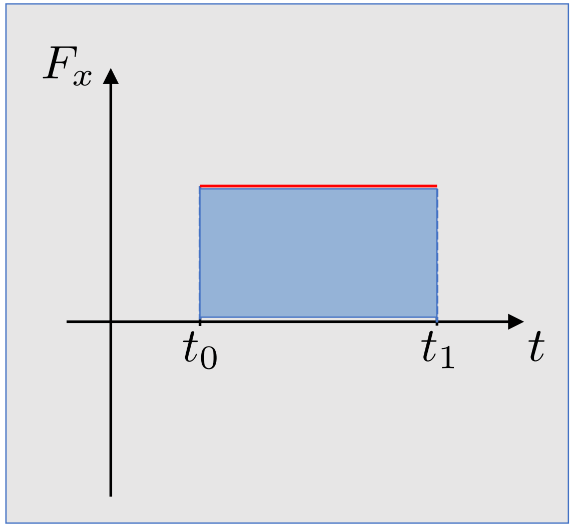
Impulse determines how much momentum of object changes under action of time dependent force.

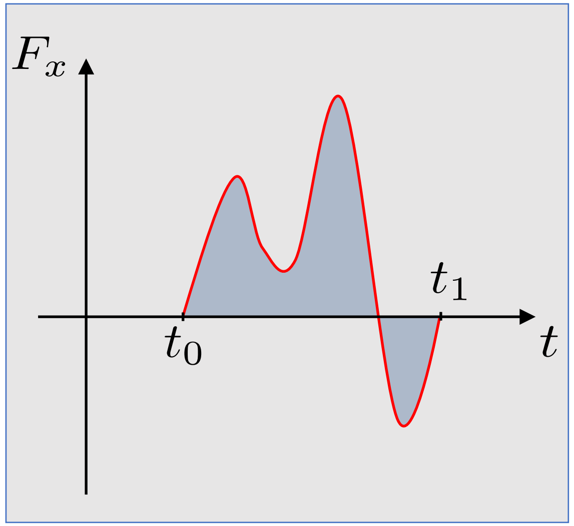
$$\Delta\vec{p} = \vec{p}(t_1) - \vec{p}(t_0)$$

Change in momentum between t_0 and t_1

$$\Delta\vec{p} = \vec{J}$$

Change in momentum equals impulse!





This does not involve any new principles of dynamics beyond Newton's laws

It comes from adding up the impact of Newton's 2nd law on an object's velocity over time

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

$$\Rightarrow \quad \Delta\vec{p} = \int_{t_0}^{t_1} dt\,\vec{F} = \vec{J}$$

Using fundamental theorem of calculus

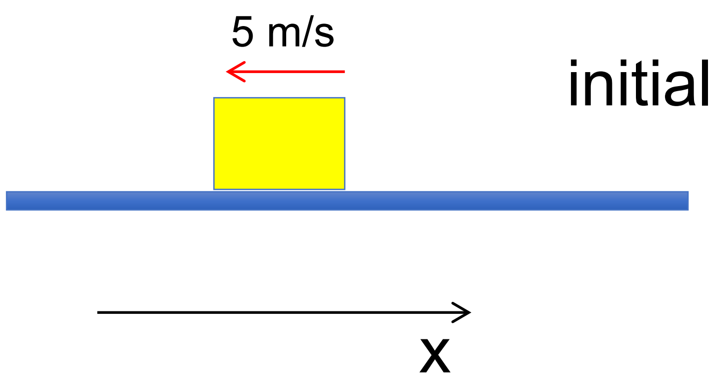
A 2 kg block starts sliding to the left along the floor with speed 5 m/s

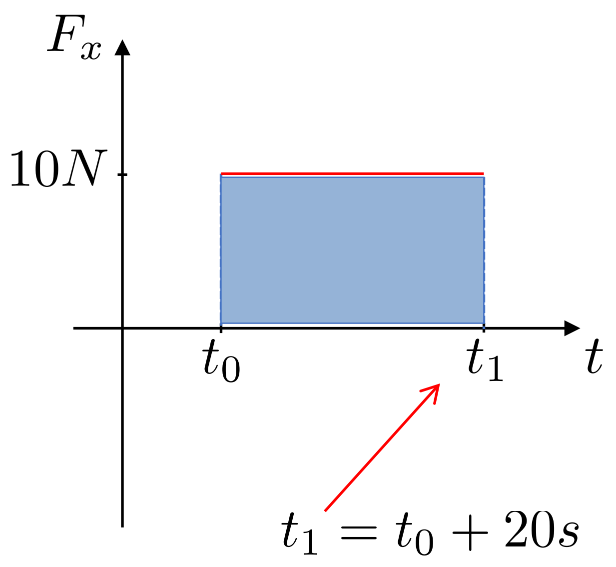
A force of 10 N directed to the right acts on the block for 20 s

What are the initial and final momenta of the block?

What is the impulse delivered by the force?

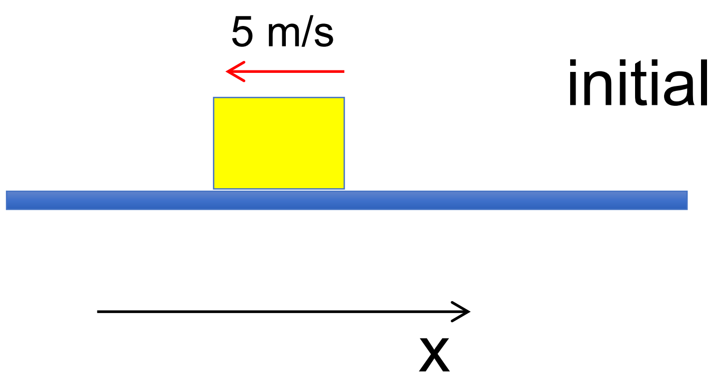
What is its final velocity?





A 2 kg block starts sliding to the left along the floor with speed 5 m/s

A force of 10 N directed to the right acts on the block for 20 s



What are the initial and final momenta of the block?

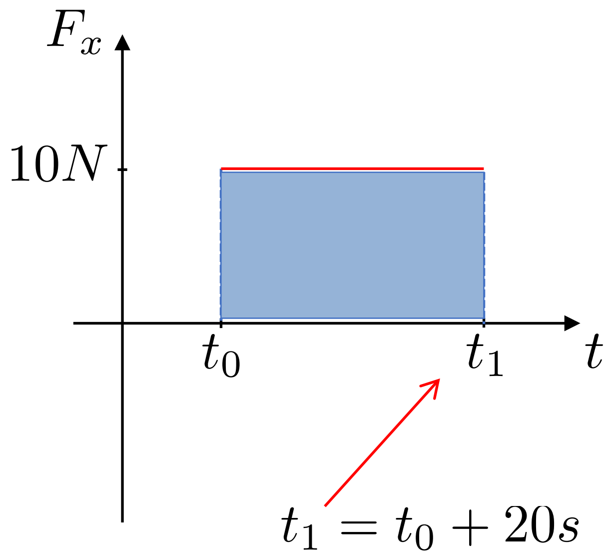
What is the impulse delivered by the force?

What is its final velocity?

$$p_i = -10N\ s$$

Find impulse

Rectangular area under force curve



$$J = (10N)(20s) = 200Ns$$

Final momentum

$$\Delta p = p_f - p_i = J$$

$$p_f = p_i + J$$

$$= -10Ns + 200Ns = +190Ns$$

Final velocity $v_f = \dfrac{p_f}{m} = +95m/s$

To the right

Can also figure out average force from change in momentum

0.5 kg hammer is moving at 10m/s when it hits a nail

Hammer comes to rest in 8 milliseconds

What impulse does the hammer deliver to the nail?

What is the average force exerted?

Change in hammers momentum gives the force acting on it.

By Newton's 3rd law, this is the same as the force the hammer exerts on the nail.

Impulse

Can also figure out average force from change in momentum

0.5 kg hammer is moving at 10m/s when it hits a nail

Hammer comes to rest in 8 milliseconds

What impulse does the hammer deliver to the nail?

What is the average force exerted?

Change in hammers momentum gives the force acting on it.

By Newton's 3rd law, this is the same as the force the hammer exerts on the nail.

$$J = \Delta p = (0.5kg)(10m/s) = 5Ns$$ Impulse

$$J = F_{avg}\Delta t \Rightarrow F_{avg} = \frac{J}{\Delta t} = \frac{5Ns}{0.008s} = 625N$$

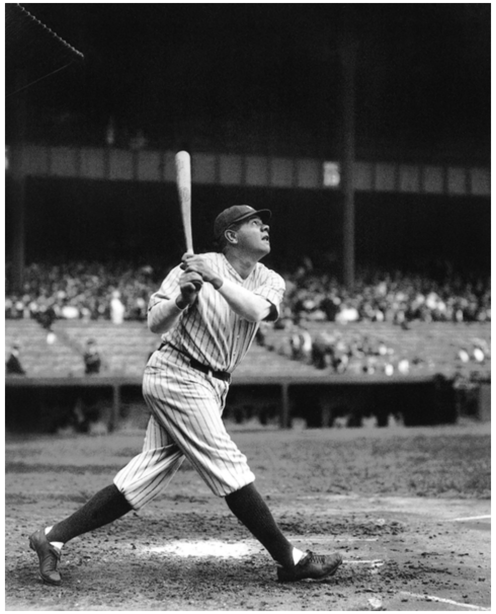
Average force is simply impulse divided by the time over which the force is exerted

Can be quite large, if time duration is short

0.13 kg baseball is pitched at 33 m/s and hit directly back towards the pitcher at 50 m/s

Contact time between bat and ball is 5.4 milliseconds

What is the average force of the bat on the ball?

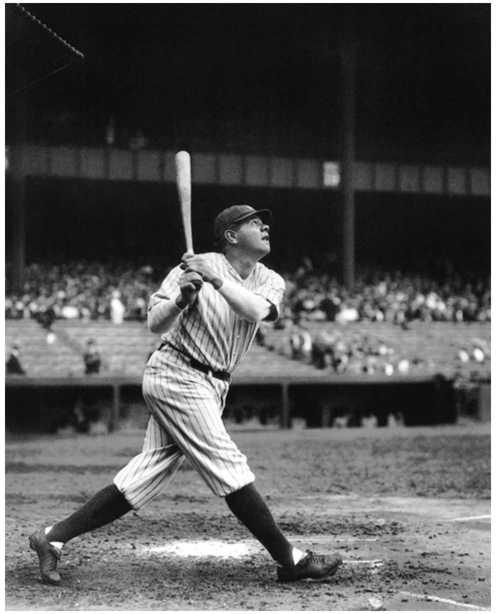


$$v_i = -33m/s$$

$$v_f = +50m/s$$

x

0.13 kg baseball is pitched at 33 m/s and hit directly back towards the pitcher at 50 m/s



Contact time between bat and ball is 5.4 milliseconds

What is the average force of the bat on the ball?

$$v_i = -33m/s$$

$$v_f = +50m/s$$

x

Impulse

$$J = \Delta p = p_f - p_i = mv_f - mv_i$$

$$= (0.13kg)(50m/s - (-33m/s)) = 10.8Ns$$

$$F_{avg} = \frac{J}{\Delta t} = \frac{10.8Ns}{0.0054s} = 2000N$$

Practical question…

You need to close a heavy door that is partway open, but you are on the couch across the room watching your favorite movie, The Matrix (1999).

You have two different things you could throw at the door to try to close it all the way

A bouncy Superball or a ball of clay

Which one should you throw?

Because superball bounces back, its momentum change is double that of clay ball.

Door has to deliver twice the impulse to it.

By Newton's 3rd law, the Superball must also deliver twice the impulse to the door