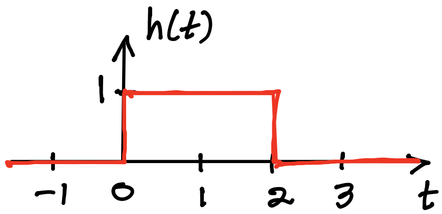
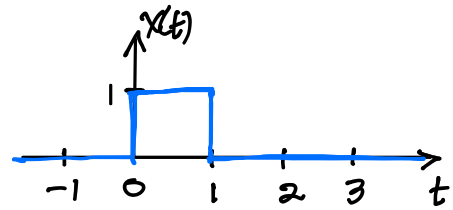
Example 2.5: Consider functions $h(t)$ and $x(t)$ shown below.





Use the graphical convolution method to find $y(t) = h(t) * x(t)$.

<u>Solution</u>: By definition of convolution,

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau. \tag{E1}$$

As a function of τ, $x(t-\tau) = x(-[\tau - t])$ is a time-reversed version of $x(\tau)$ followed by a time shift by t.

When $t < 0$ or $t \geq 3$, there is no overlap between non-zero portions of $h(\tau)$ and $x(t-\tau)$, leading to

$$y(t) = 0,\ t < 0 \text{ or } t \geq 3. \tag{E2}$$

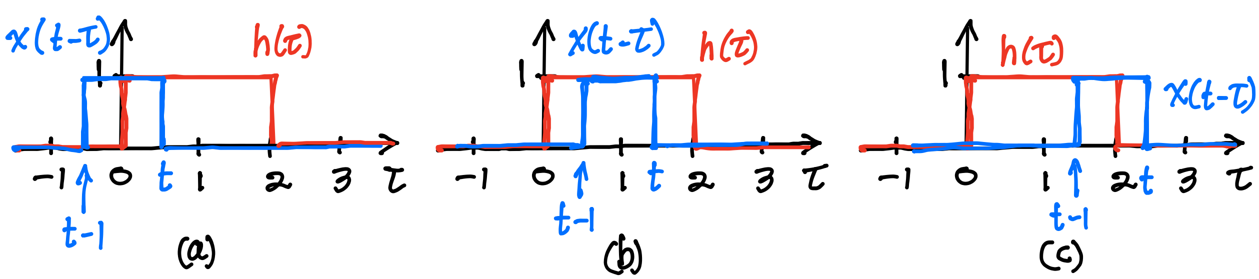


Figure 1: $x(t-\tau)$ and $h(\tau)$. (a) $0 \leq t < 1$. (b) $1 \leq t < 2$. (c) $2 \leq t < 3$.

For three different ranges of t, $h(\tau)$ and $x(t-\tau)$ are overlaid in Fig. 1. When $0 \le t < 1$, Fig. 1(a) gives[1]

$$y(t) = \int_0^t 1 \times 1 d\tau = t,\ 0 \le t < 1. \tag{E3}$$

When $1 \le t < 2$, Fig. 1(b) gives

$$y(t) = \int_{t-1}^t 1 \times 1 d\tau = t - (t-1) = 1,\ 1 \le t < 2. \tag{E4}$$

When $2 \le t < 3$, Fig. 1(b) gives

$$y(t) = \int_{t-1}^2 1 \times 1 d\tau = 2 - (t-1) = 3 - t. \tag{E5}$$

From (E2)–(E5),

$$y(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \le t < 1 \\ 1, & 1 \le t < 2 \\ 3-t, & 2 \le t < 3 \\ 0, & t \ge 3 \end{cases}. \tag{E6}$$

The result is shown in Fig. 2.

[1] Over the overlap range of τ, the problem definition happens to have $h(\tau) = 1$ and $x(t-\tau) = 1$.

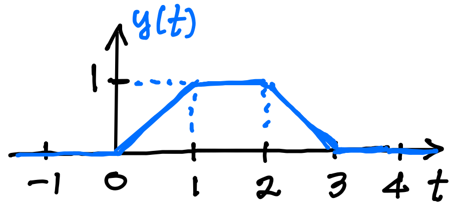


Figure 2: $y(t)$ from graphical convolution.