Lab 5 - Ideal Gas

Overview

In this lab you will learn about the properties of an ideal gas. *Real* gas behaves like ideal gas when the density of the gas is low and the temperature is much higher than the gas's boiling. The properties of an ideal gas are described in terms of the gas's pressure P, volume V, and temperature T.

If the pressure and number of molecules of the gas is constant, then the volume V is proportional to to the absolute temperature T:

$$V \propto T \tag{1}$$

Increase the temperature of a gas and the gas will expand to a bigger volume. This is known as Charle's Law.

If the temperature and number of molecules of the gas is constant, then the pressure of the gas is inversely proportional to the gas's volume:

$$V \propto \frac{1}{P} \tag{2}$$

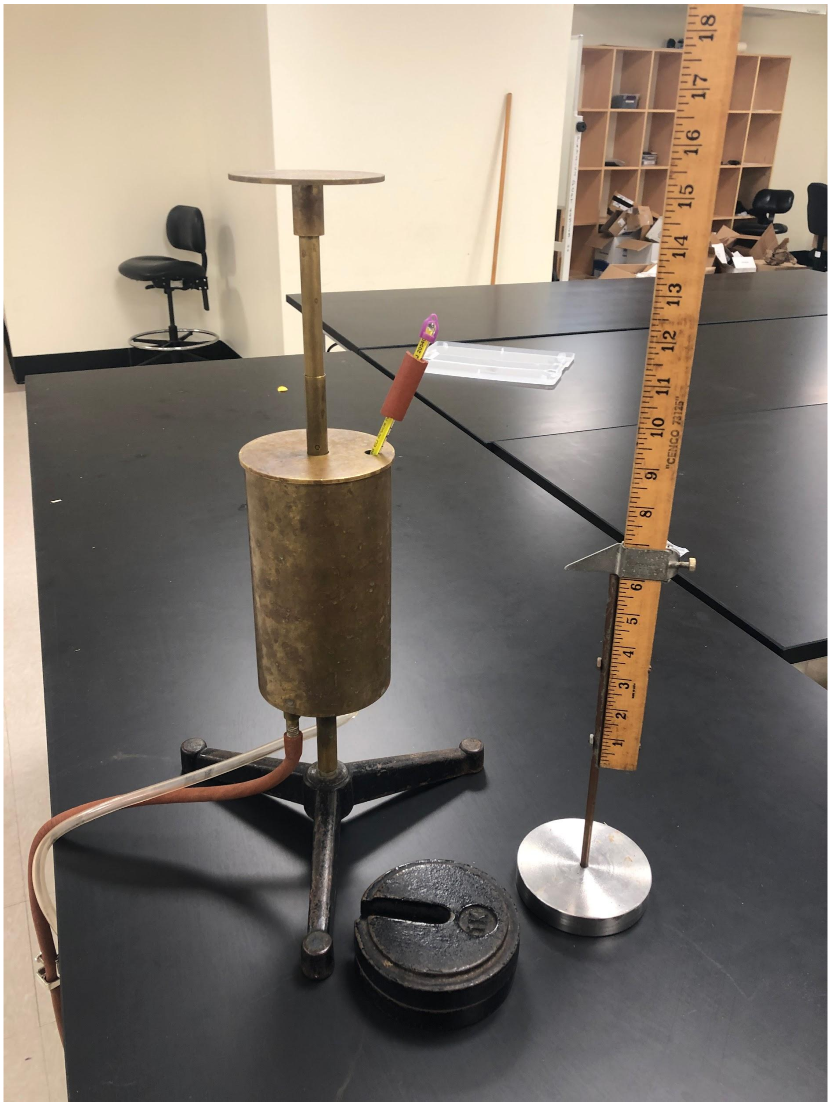
Exert a higher pressure on a gas and it will be squeezed into a smaller volume. This relation is known as Boyle's Law.

If we combine Charle's Law and Boyle's Law, we get the Ideal Gas Law.

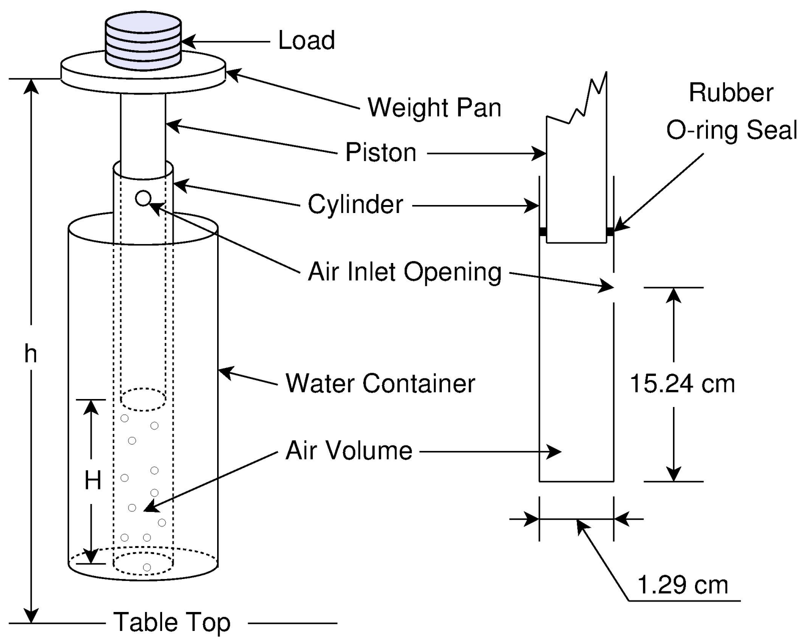
$$PV = Nk_BT \tag{3}$$

Here N is the number of molecules of gas and k_B is the Boltzmann constant. What is amazing about the Ideal Gas Law is it is independent of the chemical composition of the gas. Any gas (Oxygen, Nitrogen, Argon, Helium, etc.) that is low density and high temperature (higher than gas's boiling point) will behave like an ideal gas!

The Experiment



In this experiment you are going to measure the pressure, volume, and temperature of an ideal gas. Air, a mixture of Nitrogen and Oxygen, behaves like an ideal gas at typical temperatures we are accustomed to. The experimental apparatus pictured above is a double cylinder. The inner cylinder contains the gas held inside by a piston. The outer cylinder contains water at different temperatures to control the temperature of the gas inside the inter cylinder. Below is a diagram that shows the double cylinder in detail.



Measuring Pressure

There is a small air inlet opening on the side of the inner air cylinder. When the bottom of the piston is above the height of the air inlet, air can move freely in and out of the inner cylinder. When the piston is inserted into the cylinder past the air inlet, the air inside the cylinder will be trapped.

IMPORTANT: DO NOT RAISE THE PISTON ABOVE THE HEIGHT OF THE AIR INLET ONCE THE EXPERIMENT BEGINS. THE AIR INSIDE THE CYLINDER MUST STAY CONSTANT THROUGHOUT THE EXPERIMENT. IF THE PISTON RISES ABOVE AIR INLET, THE AMOUNT OF AIR INSIDE WILL CHANGE AND YOU WILL HAVE TO REPEAT THE WHOLE EXPERIMENT.

Now the air inside the cylinder is held there by the *pressure* exerted by the piston. What contributes to the pressure exerted by the piston? Let's think about it. Pressure is the force F exerted over a surface area A:

$$P = \frac{F}{A} \tag{4}$$

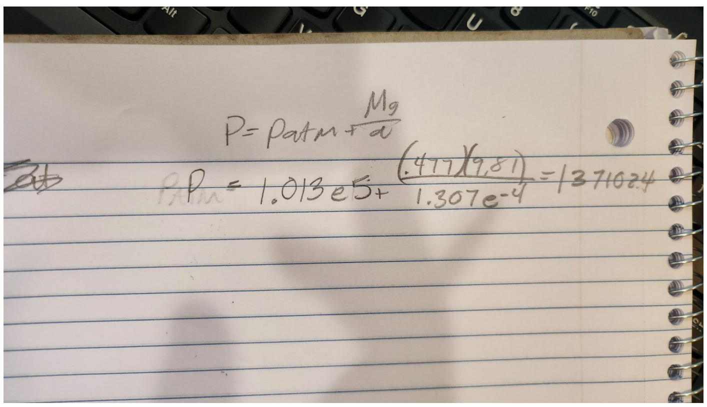
So pressure in the cylinder is ALL the forces pushing down on the cross sectional area A of the piston. What are the forces? There is the weight of the piston, the weight of the pan, AND the weight of the atmosphere in the room too! You can even add more masses (Load) on top of the pan to increase the pressure of the gas inside the cylinder.

$$P = P_{atm} + \frac{Mg}{A} \tag{5}$$

Here:

- $P_{atm} = 1.013 \times 10^5 \frac{N}{m^2}$
- Mass of piston and pan is $M = 0.477kg$
- The cross sectional area of the piston is $A = 1.307 \times 10^{-4} m^2$

1. What is the pressure of the gas inside the cylinder using the atmospheric pressure P_{atm}, mass of piston and pan M, and cross sectional area of piston A? Show your work.



Measuring Volume

If you push down on the piston, the volume of the gas inside will decrease. The more you push, the smaller the volume. Changing the amount of pressure *changes* the volume of the gas. The Volume of the cylinder is the cross sectional area A and the height h of the piston above the bottom of the cylinder:

$$V = Ah \tag{6}$$

So the change in the volume IS the change in the height of the piston!

Measuring Temperature

The outer cylinder can be filled with water at different temperatures. The temperature water is the temperature of the air inside the inner cylinder. You can use a thermometer inserted into the water to measure the temperature in Celsius. BUT, the Ideal Gas Law relates the thermodynamic properties of pressure, volume to the *absolute* temperature scale in Kelvins:

$$T_K = T_C + 273.15 \tag{7}$$

Hot Water Measurement

You are now going to perform an experiment to measure the change in the volume of the gas inside the cylinder when you increase the pressure on the gas. From the sink in the laboratory fill a large beaker with hot water. Let the hot run a minute or to make sure it is hot.

You will record your measurements in the Google Sheet Lab1 - Ideal Gas - Student Copy. When you open the spreadsheet click the menu File > Make a copy. A copy of the spreadsheet will be in your Google Drive that you can edit.

Lab 5 - Ideal Gas - Student Copy

- Make sure the value clamps on the hose of the apparatus are CLOSED tight and the ends of the hose are hanging in the waste bucket.
- Pour the hot water into the outer cylinder of the apparatus. Allow the apparatus a minute or two to reach thermal equilibrium. Insert the thermometer into the hole in the cap of the apparatus. Record the temperature of the hot water.
- Measure the height h of the pan using the vertical half meter stick and indicator as shown in picture below.
- Give the pan a *small* push down then release the pan. Measure the height of the pan after the push down.
- Give the pan a *small* push up then release the pan. Measure the height of the pan after the push up.

- Now make a series of measurements of the height h of the pan as you add more mass on top of the pan. There are 1 kg and 2 kg masses at each station. Record the height h for additional mass 1kg to 4 kg. Remember to measure the height of the pan after a small push down and push up each time.
- After making your series of height measurements, record the temperature again of the hot water.

2. When you add masses to the pan, is the pressure of the gas increasing, decreasing or staying the same? Explain your answer. Hint: look at Eq.(5).

When you add masses to the pan the pressure increases because the amount force pushing on the cross sectional area increases.

3. Does the volume of the gas increase, decrease, or stay the same when you add mass to the pan? Explain your answer. Hint: Look at Boyle's Law Eq.(2).

The volume decreases because as the pressure increases, the volume decreases. They are inversely proportional.

Ice Water Measurement

Now you will repeat the measurements, but this time you will measure the height of the pan when the out cylinder is filled with ice water. Open the value clamp on the hose and allow the water in the cylinder to drain out in the waste bucket at your lab station. Your lab TA will have a bin of crushed ice. Use the plastic beaker at your station to get ice. Fill the outer cylinder with crushed ice, then add a little COLD water from the lab sink.

- Allow the inner cylinder a minute or two to reach thermal equilibrium. Insert the thermometer into the hole in the cap and record the temperature.
- Measure the height h of the pan with NO additional weight.
- Give the pan a *small* push down then release the pan. Measure the height of the pan after the push down.
- Give the pan a *small* push up then release the pan. Measure the height of the pan after the push up.

4. Is the height h of the pan higher, lower, or the same as the height when the apparatus was filled with hot water? Explain your answer. Hint: look at Charle's Law Eq.(1).
   The pan is lower because as a gas cools, the volume decreases, temperature is directly proportional to volume, and therefore also height.

- Now repeat the series of measurements of the height h of the pan as you add masses on top of the pan. Record the height h for additional masses 1kg to 4 kg. Remember to measure the height of the pan after a small push down and push up each time.
- After making your series of height measurements, record the temperature again of the hot water.

Data Analysis

Now let's analyze the data and see if air has the properties of an ideal gas. First, let's calculate the pressure of the gas. Spreadsheets are *calculators*! You can do the calculations right in the spreadsheet. Spreadsheet uses the address of the cell (columns are letters, rows are numbers) as variables. The value in the cell address is used in the equation. All equations start with an equal sign =. Here is an example. If you want to calculate the pressure of the gas according to Eq. (5):

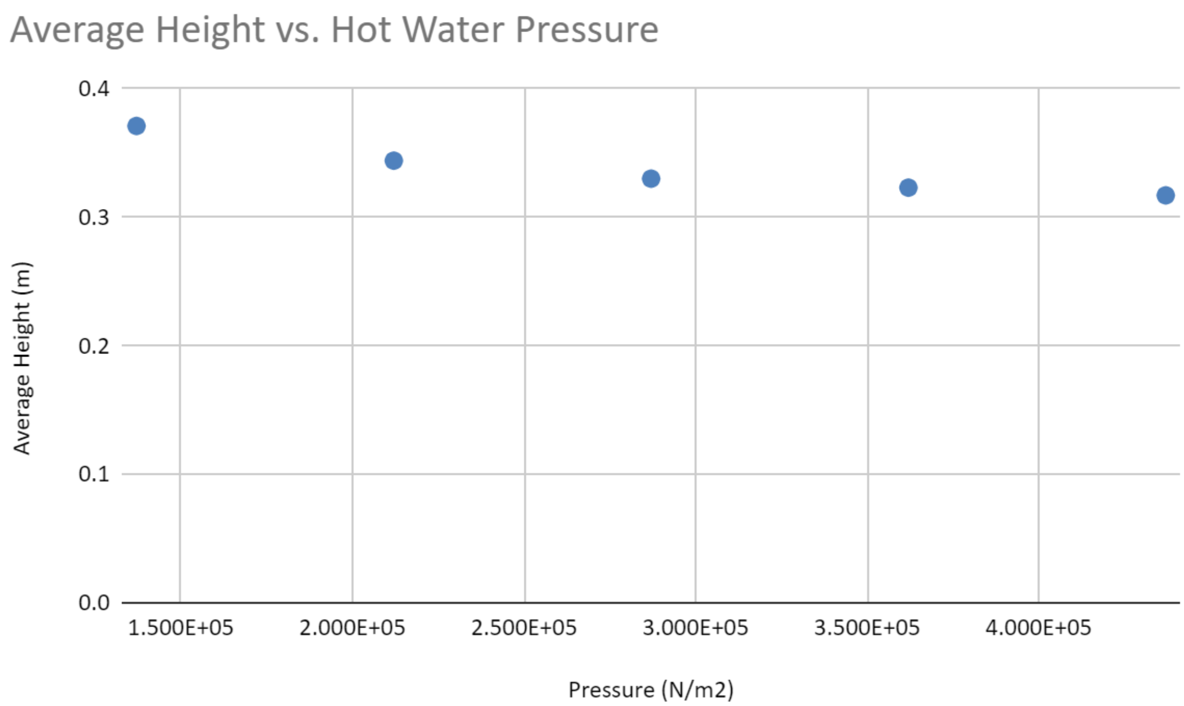
- In the cell D15, enter the equation =$B$5+(A15+$B$3)*9.8/$B$7
    - $B$5 is the value of atmospheric pressure.
    - A15 is the value of the additional mass added to the pan.
    - $B$3 is the value of the piston and pan mass.
    - $B$7 is the cross sectional area of the piston.
- Some of the cell addresses have dollar signs $. The dollar signs hold the values fixed because they are constants.

Now do the calculations for your Hot Water Measurement

5. Calculate the pressure of each additional mass added to the pan.

6. Calculate the average height of the pan using the height after a *small* push up and *small* push down.

7. Make a scatter plot of average h vs Pressure:
    a. Click on the header P (N/m2) and drag down the column to select the values of pressure.
    b. Press and hold the Ctrl key and click on the header h_avg (m) and drag down the column to select the values of average height.
    c. Release Ctrl key and open the Insert menu and select Chart.
    d. The Charter Editor will open. In Setup choose Chart Type > Scatter Chart,
    e. Make sure the X-axis indicates P (N/m2).
    f. Make sure the Series indicates h_avg (m). If P (N/m2) is in the Series, remove it.
    g. In Customize > Chart & axis titles add axes titles with units to your graph.
    h. Copy and paste your plot into this document below here:



Repeat the calculations for your Ice Water Measurements

8. Calculate the pressure of each additional mass added to the pan.

9. Calculate the average height of the pan using the height after a *small* push up and *small* push down.

10. Make a scatter plot of average h vs Pressure:
    a. Click on the header P (N/m2) and drag down the column to select the values of pressure.
    b. Press and hold the Ctrl key and click on the header h_avg (m) and drag down the column to select the values of average height.
    c. Release Ctrl key and open the Insert menu and select Chart.
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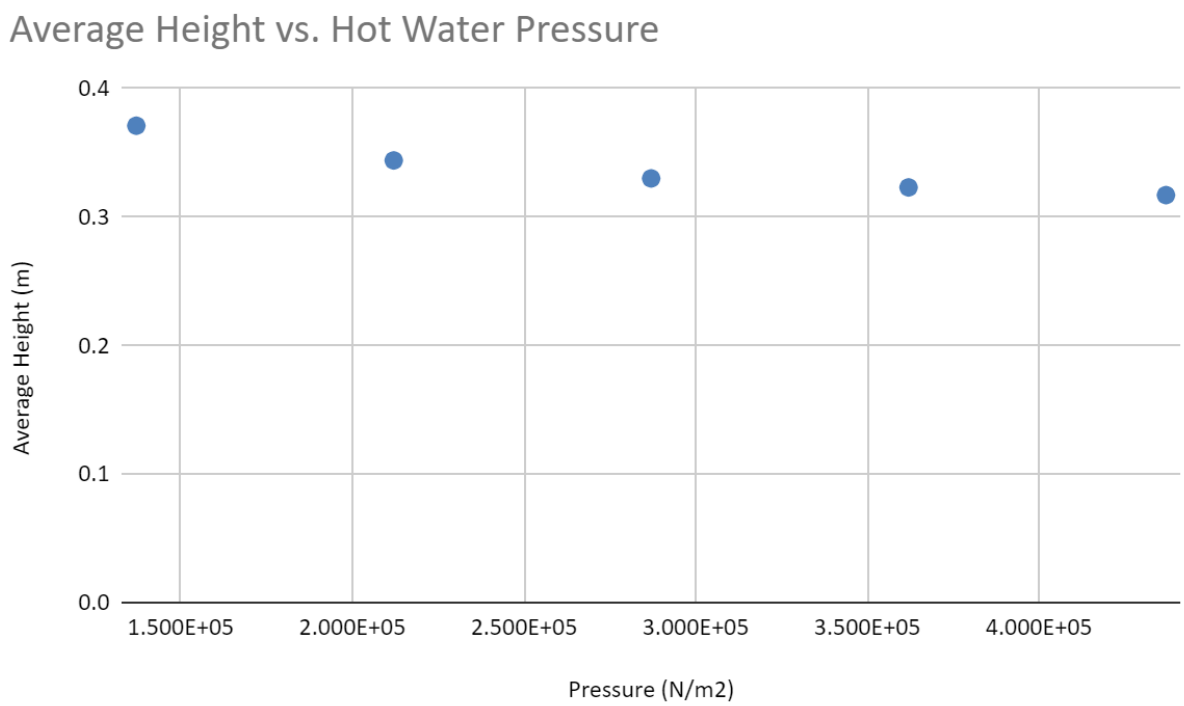


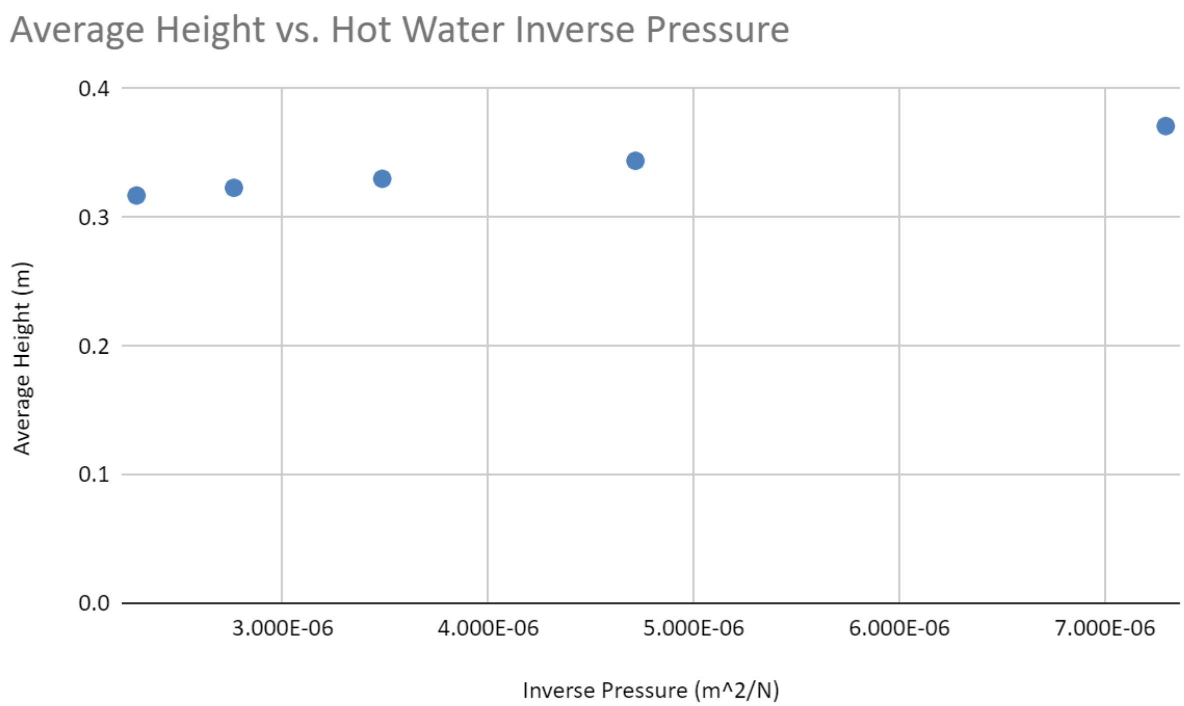
11. Look at your two plots of average h vs. Pressure. Describe the *mathematical* relationship of height vs. Pressure. Hint: Look at Boyle's Law Eq.(2)

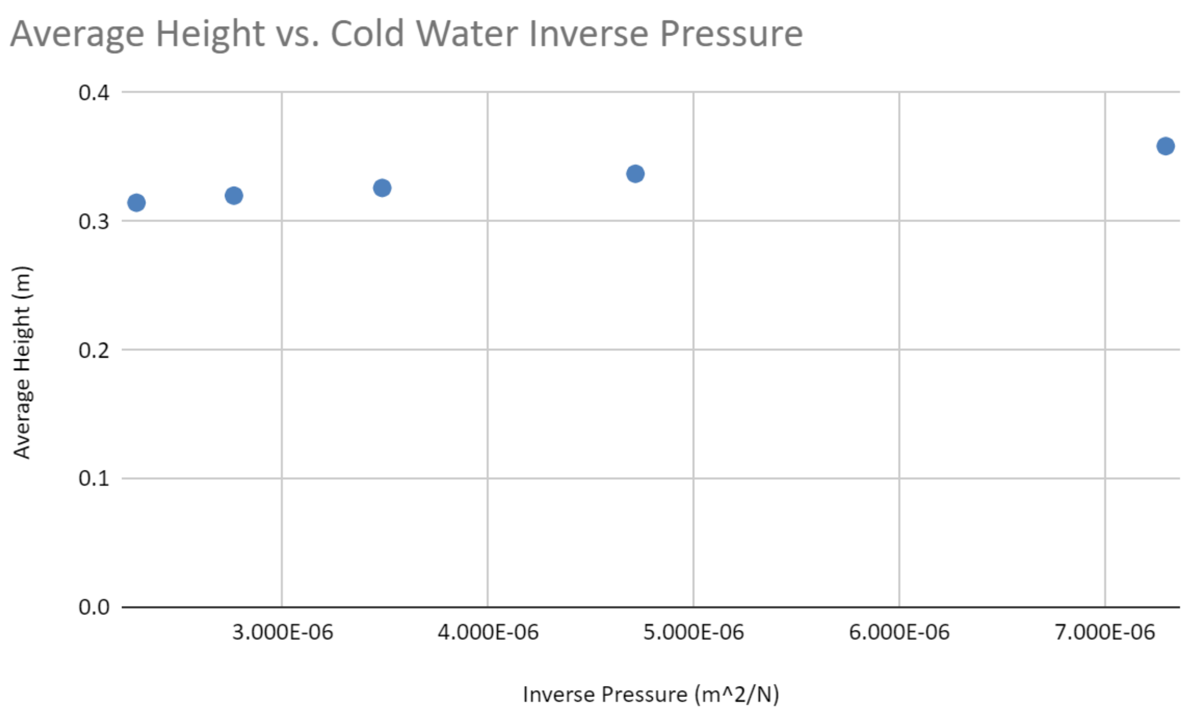
Height vs pressure is inversely proportional, this is because volume = 1/pressure, and volume is directly proportional to volume.

If the height of the piston is inversely proportional to the pressure, let's test it. Perform the calculations and make scatter plots for both the Hot Water and Ice Water measurements.

12. Calculate inverse pressure for each additional mass added to the pan.

13. Make a scatter plot of average h vs inverse Pressure:
    a. Click on the header 1/P (m2/N) and drag down the column to select the values of pressure.
    b. Press and hold the Ctrl key and click on the header h_avg (m) and drag down the column to select the values of average height.
    c. Release Ctrl key and open the Insert menu and select Chart.
    d. The Charter Editor will open. In Setup choose Chart Type > Scatter Chart,
    e. Make sure the X-axis indicates 1/P (m2/N).
    f. Make sure the Series indicates h_avg (m). If 1/P (m2/N) is in the Series, remove it.
    g. In Customize > Chart & axis titles add axes titles with units to your graph.
    h. Copy and paste your plot into this document below here:





14. Look at both scatter plots of average h vs. inverse pressure (one for Hot Water, one for Ice Water). Describe the *mathematical* relationship of height vs. inverse pressure. Is this what you expected?

The relationship between average height and inverse pressure is a positive linear relationship, this is expected because height is directly proportional to volume, and volume is proportional to the inverse of pressure. As height increases it is expected that the inverse of pressure will increase at the same rate.
Let's find the slope height vs. inverse pressure:

- Click on the a chart to open the Chart editor
- Click Customize > Series.
- Put a check mark next to Trendline.
- In the Type menu select Linear.
- In the Label menu select Use Equation.

15. Record the values of the slopes in the tables below:

Hot Water	10736
Ice Water	86680

16. Which slope is higher (Hot Water or Cold Water). Is this what you expected? Explain your answer. Hint: look at Charle's Law Eq.(1).
    The slope of hot water is higher, this is expected because temperature is proportional to volume, and volume is inversely proportional to pressure, so as temperature increases, pressure is also increased. So hot water would have a steeper slope than cold water.