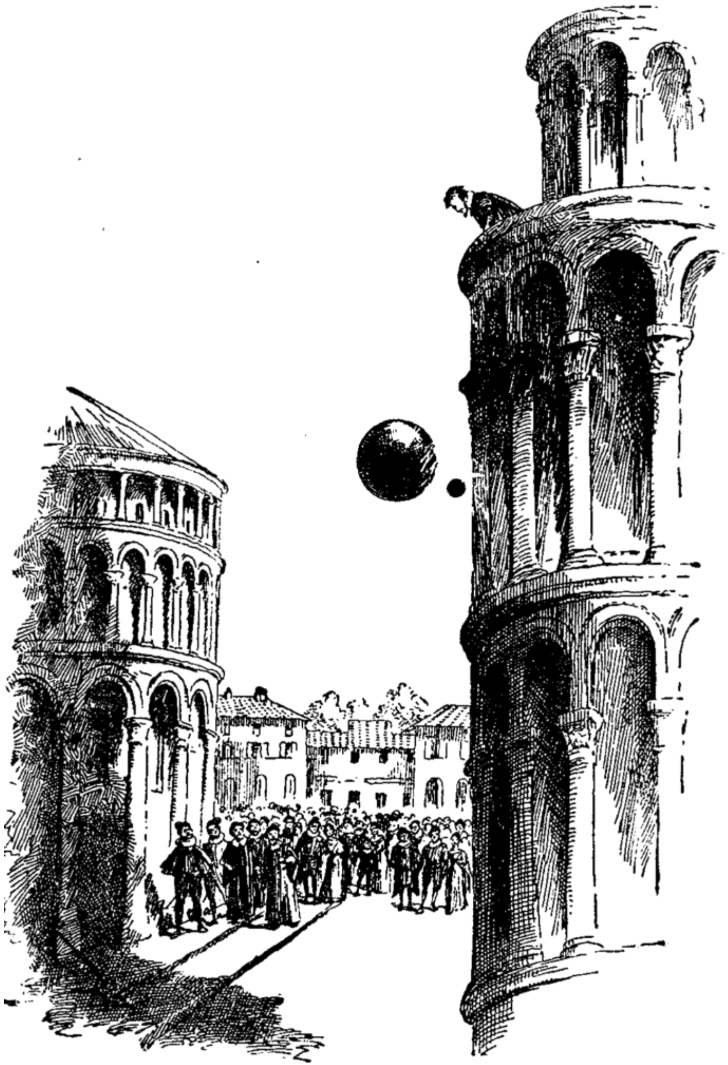


Announcements, Goals, and Reading

Announcements:

- HW02 due Tuesday Sep 27th, 11:59 pm on Mastering Physics
- HW01 grace period ends tonight
- **Help Resources: See moodle**

Goals for Today:

- Motion w/ constant acceleration **a**
- Free Fall
- Vectors

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 2: Kinematics in One Dimension
- Chapter 3: Vectors

Summary

Position x(t)	Velocity v(t)	Acceleration a(t)	Description
$x(t) = x_0$	$v(t) = 0$	$a(t) = 0$	Constant position
$x(t) = x_0+v_0\, t$	$v(t) = v_0$	$a(t) = 0$	Constant velocity
$x(t) = x_0 + v_0\, t + (1/2)\, a_0\, t^2$	$v(t) = v_0 + a_0\, t$	$a(t) = a_0$	Constant acceleration

$$a(t) = \frac{dv}{dt} \Longleftrightarrow v(t) = v_0 + \int_{t_0}^{t} a(t')\, dt'$$

$$v(t) = v_0 + a(t - t_0)$$

$$v(t) = \frac{dx}{dt} \Longleftrightarrow x(t) = x_0 + \int_{t_0}^{t} v(t')\, dt'$$

$$\boxed{x(t) = x_0 + v_0(t - t_0) + a(t - t_0)^2 / 2}$$

Motion with constant acceleration

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

Example…

- Alice is driving down the road at 30 m/s when she passes Bob, who is at rest in his car
- Bob accelerates at 0.1 m/s^2 in order to catch up and pass Alice
- How much time does it take Bob to catch up?
- How far has he travelled?
- How fast is he going when he catches up?

Write equations for positions

$$\boxed{x_A(t) = v_A t} \longleftarrow \boxed{\begin{array}{l} x_0 = a = 0 \\ v_A = 30\text{m/s} \end{array}}$$

Alice

$$\boxed{x_B(t) = at^2 / 2} \longleftarrow \boxed{\begin{array}{l} x_0 = v_0 = 0 \\ a = 0.1\text{m/s}^2 \end{array}}$$

Bob

- Alice is driving down the road at 30 m/s when she passes Bob, who is at rest in his car
- Bob accelerates at 0.1 m/s^2 in order to catch up and pass Alice
- How much time does it take Bob to catch up?
- How far has he travelled?
- How fast is he going when he catches up?

$$\boxed{x_A(t) = v_A t}$$

$$\boxed{x_B(t) = at^2/2}$$

Assume they meet at time T

$$\Rightarrow (30m/s)T = \frac{1}{2}(0.1m/s^2)T^2$$

$$\Rightarrow T = \frac{2(30m/s)}{(0.1m/s^2)} = 600s$$

Where they meet up

>Plug T into position formula

$$x_A(600s) = \left(30\frac{m}{s}\right)600s = 18,000m$$

How fast Bob is going

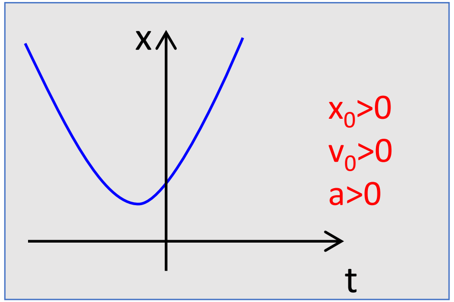
>Plug T into velocity formula

$$v_B(T) = aT$$

$$v_B(600s) = \left(0.1\frac{m}{s^2}\right)600s = 60\frac{m}{s}$$

Another type of problem…

- At **t=0** an object is at position $\mathbf{x_0}$, moving with velocity $\mathbf{v_0}$, and is accelerating at the constant rate **a.**
- **How far does it move before reaching velocity $\mathbf{v_1}$?**
- **Relevant for highway on-ramps, stopping distance, …**



Leads to a standard physics formula

Start with basic formulas for constant acceleration

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = v_0 + at$$

Solve first for the time T at which this happens

$$v_1 = v_0 + aT \quad \Rightarrow \quad T = \frac{1}{a}(v_1 - v_0)$$

Plug this time into x(t) to see how far the object has moved

How far does the object move before reaching velocity v_1?

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$\boxed{T = \frac{v_1 - v_0}{a}}$$

Let $\Delta x = x(T) - x_0$

Plugging in T gives…

$$\Delta x = v_0 \left(\frac{v_1 - v_0}{a} \right) + \frac{1}{2} a \left(\frac{v_1 - v_0}{a} \right)^2$$

$$= \frac{1}{a} \left(\cancel{v_0 v_1} - v_0^2 + \frac{1}{2} v_1^2 - \cancel{v_0 v_1} + \frac{1}{2} v_0^2 \right)$$

$$= \frac{1}{2a} (v_1^2 - v_0^2)$$

$$\boxed{\Delta x = \frac{v_1^2 - v_0^2}{2a}}$$

Commonly used by itself when not interested in how long process took. **Very useful formula!**

final velocity

Initial velocity

Example…

$$\Delta x = \frac{v_1^2 - v_0^2}{2a}$$

final velocity

Initial velocity

- A car can decelerate at a maximum rate of **$a = -5 m/s^2$**
- It is initially travelling at **$v_0 = 40 m/s$**
- How much distance is required for it to come to a stop?

Stop ➡ $v_1 = 0$ Final velocity is zero

$$\Delta x = \frac{0 - 40^2}{2(-5)} = \frac{-1600}{-10} = 160m$$

Similar example…

- A car can accelerate from rest to **50 m/s** in a distance of **100 m**.
- What is its acceleration, **a**?

$$\Delta x = \frac{v_1^2 - v_0^2}{2a}$$

final velocity

Initial velocity

Rearrange formula to solve for acceleration

$$a = \frac{v_1^2 - v_0^2}{2\Delta x}$$

Here

$$v_0 = 0$$
$$v_1 = 50m/s$$

Plug in to get… $a = \frac{1}{2(100m)}\left((50m/s)^2 - (0m/s)^2\right) = 12.5m/s^2$

- What car is it?

$$12.5\frac{m}{s^2}\frac{mile}{1609m}\frac{3600}{hour} \approx 28mph\frac{1}{s}$$

$$= 60mph\frac{1}{2.14s}$$

Fastest production cars by acceleration 0-60 MPH

2014 Nissan GT-R Track Edition: 2.7 s
2005 Bugatti Veyron 16.4: 2.7 s
2017 Audi R8 V10 Plus: 2.6 s
2018 Lamborghini Huracán Performante: 2.6 s
2015 Porsche 918 Spyder: 2.1 s
2020 Porsche 911 Turbo S: 2.2 s
…..

Motion with Constant Acceleration: Free Fall

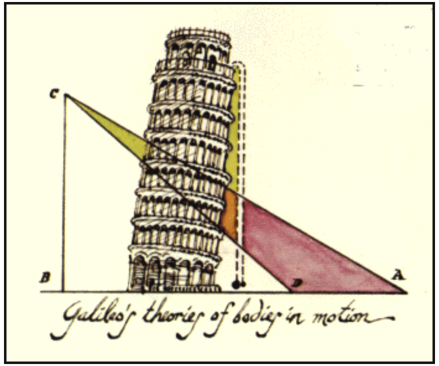
Acceleration due to gravity near Earth's surface happens at a very nearly constant rate.

…if we can ignore air resistance

$g = 9.8 m/s^2$ downwards

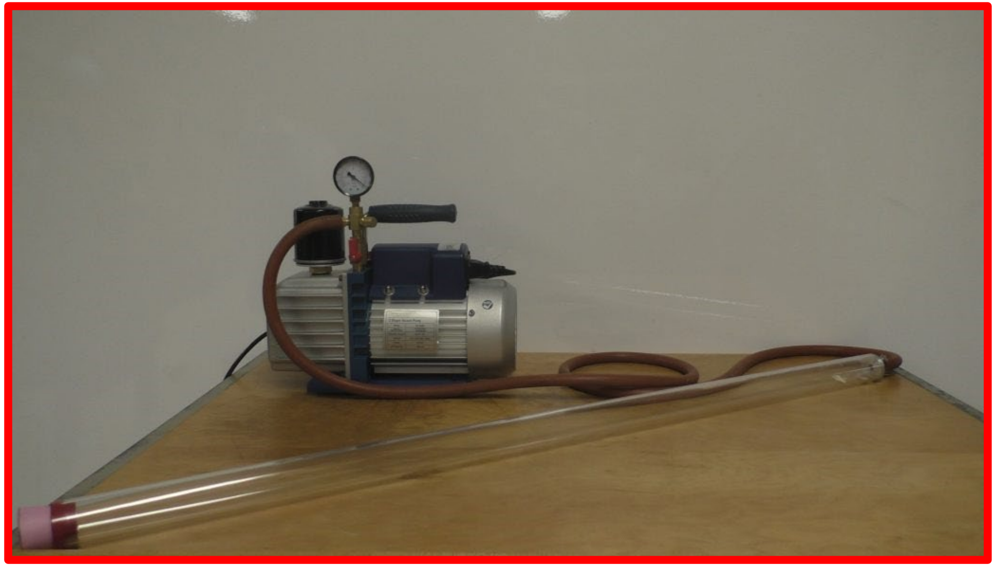
Everything falls at the same rate, regardless of its mass!

Demonstrated (according to lore) by Galileo dropping things off Leaning Tower of Pisa



For us…
Nice opportunity to use our constant acceleration formulas!

Motion with constant acceleration: Free Fall

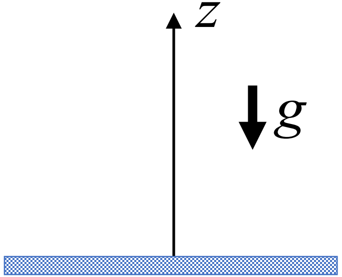


Free Fall: Penny versus the Feather

https://youtu.be/XydJkXnklsI


NASA

Free Fall



Let z-coordinate measure height above ground

Basic Equations…

Height

$$z(t) = z_0 + v_0 t - \frac{1}{2} g t^2$$

Initial height

Initial velocity in z-direction

Constant gravitational acceleration $g=9.8 m/s^2$ is a (nearly) constant positive quantity.

a=-g, acceleration acting along the -z direction

Velocity

$$v(t) = \frac{dz}{dt} = v_0 - gt$$

Can tailor these formulas to numerous physics problems!

Free Fall

- A diver jumps from a 100 m cliff
- How long does it take to hit the sea?
- How fast is he moving when he hits

$$z(t) = z_0 + v_0 t - \frac{1}{2}gt^2$$
$$v(t) = v_0 - gt$$

$g=9.8m/s^2$

Initial conditions

$$z_0 = 100m$$
$$v_0 = 0$$

$$\Rightarrow \quad z(t) = 100m - \frac{1}{2}gt^2$$

Sea level $\Rightarrow$

$$z(T) = 0$$
$$100m - \frac{1}{2}(9.8m/s^2)T^2 = 0$$
$$T = \sqrt{\frac{2(100m)}{(9.8m/s^2)}} = 4.5s$$

Velocity

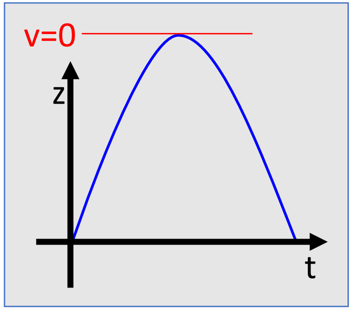
$$v(t) = -gt \quad \Rightarrow \quad v(4.5s) = -(9.8m/s^2)(4.5s) = -44m/s$$

Free Fall

- A ball is thrown straight upwards with speed 25m/s
- How long does it take to reach the top of its trajectory?
- How high does it go?

$$z(t) = z_0 + v_0 t - \frac{1}{2} g t^2$$
$$v(t) = v_0 - g t$$

$g=9.8m/s^2$



Initial conditions
$$z_0 = 0$$
$$v_0 = +25m/s$$

At top $\Rightarrow$ $v(T) = 0$ Velocity must pass through zero in changing from positive to negative

$$v_0 - gT = 0 \Rightarrow T = \frac{v_0}{g} = \frac{25m/s}{9.8m/s^2} = 2.6s$$

How high?

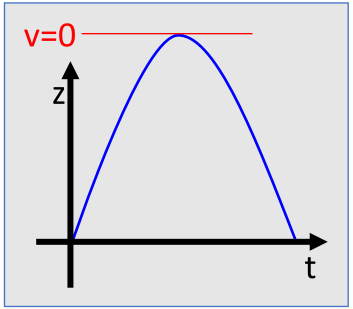
$$z(T) = (0m) + (25m/s)(2.6s) - \frac{1}{2}(9.8m/s^2)(2.6s)^2 = 65m - 33m = 32m$$

Free Fall

- A ball is thrown straight upwards with speed 25m/s
- ~~How long does it take to reach the top of its trajectory?~~
- How high does it go?

$$z(t) = z_0 + v_0 t - \frac{1}{2} g t^2$$
$$v(t) = v_0 - gt$$

g=9.8m/s^2



Can also get height from
Setting a=-g, and using..

$$D = \frac{1}{2a}(v_1^2 - v_0^2)$$

$v_0 = 25m/s$ Initial velocity

$v_1 = 0$ Final velocity at top

$$D = \frac{1}{2(-9.8m/s)}\left((0m/s)^2 - (25m/s)^2\right) = 32m \checkmark$$

As we found before

1D Motion: Determine velocity, acceleration from position vs time

A drone moves in 1 dimension. The x component of its position as a function of time is:

$$x(t) = At^3 + Bt^2 - Ct + D$$

What are the units of the constants?

Units of A:

Units of B:

Units of C:

Units of D:

1D Motion: Determine velocity, acceleration from position vs time

A drone moves in 1 dimension. The x component of its position as a function of time is:

$$x(t) = At^3 + Bt^2 - Ct + D$$

A = 5 m/s^3, B = 2 m/s^2, C = 10 m/s, D = 3 m

What are the velocity and acceleration of the drone at t=4 seconds?

$$x(t) = At^3 + Bt^2 - Ct + D$$

Remember "Calculus Move #1": $\frac{d}{dx}(cxn) = ncx^{n-1}$

$$v(t) = \frac{dx}{dt} = 3At^2 + 2Bt - C$$

$$a(t) = \frac{dv}{dt} = 6At + 2B$$

$$v(4) = 3 \times 5\ (m/s^3) \times (4s)^2 + 2 \times 2\ (m/s^2) \times 4s - 10\ m/s = 246m/s$$

$$a(4) = 6 \times 5 \times 4 + 2 \times 2 = \ 124m/s^2$$

Motion in 1 Dimension: Free Fall

Example: A ball is thrown vertically upward at 20 m/s from the edge of cliff 30 m high. Time to reach top=? Max height? Time to reach ground=? What is v at ground?

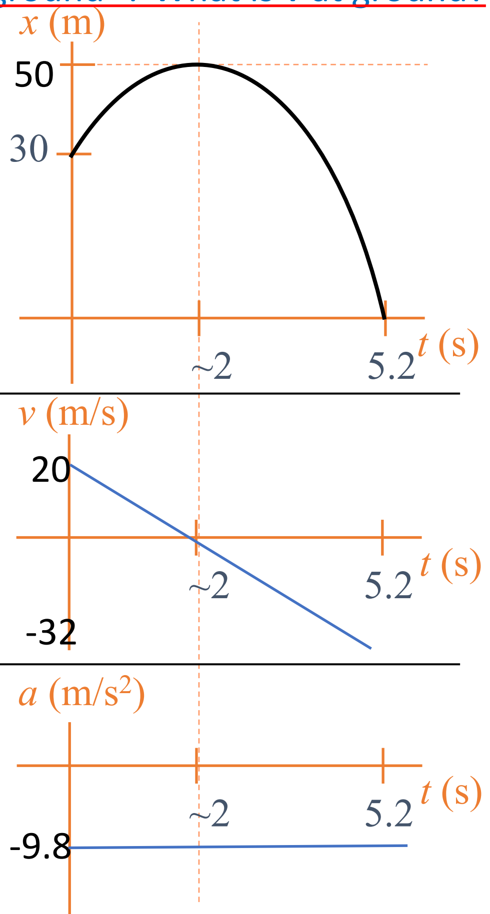
Position: $x(t) = 30 + 20t - 0.5gt^2$

T_{gd} is where x=0

Velocity = dx/dt= v(t) = 20 – 9.8t

Acceleration a(t) = dv/dt = -9.8 m/s^2

[Acceleration is constant in this case.]



Summary of kinematics along a line:

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$
$$v(t) = v_0 + at$$

$\forall a =$ for any a

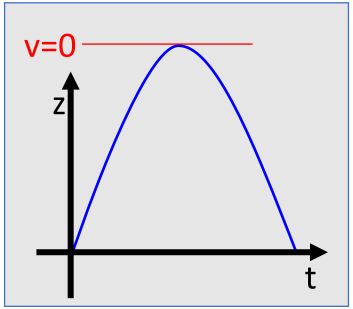
$$z(t) = z_0 + v_0 t - \frac{1}{2} g t^2$$
$$v(t) = v_0 - gt$$

Gravity on Earth case g=9.8m/s^2

$$D = \frac{1}{2a}(v_1^2 - v_0^2)$$

final velocity

Initial velocity



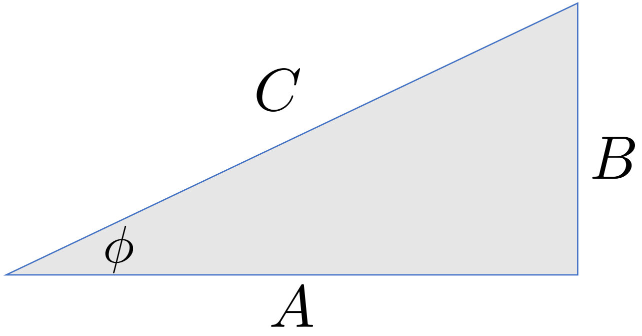
Trigonometry review

(this will be important soon)

A Length adjacent to angle

B Length opposite to angle

C Hypotenuse



$$\sin\phi = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{B}{C}$$

$$\cos\phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{C}$$

$$\tan\phi = \frac{\text{opposite}}{\text{adjacent}} = \frac{B}{A}$$

Length of $C = \sqrt{A^2 + B^2}$

Vectors and Coordinate Systems

Recall…

Scalars & Vectors

- A **scalar** is a quantity that only has a magnitude, such as…

  Mass, length, distance, speed, temperature

- A **vector** is a quantity that has both magnitude and direction, such as…

  Displacement, velocity, acceleration, force,
  momentum, angular momentum, wind

Vectors and Components

- 2D vector: Pair of numbers

(x,y) coordinates of tip, when tail is placed at origin

For a vector not drawn with tail at origin…

2 equivalent ways to find components

1. Translate tail over to origin

2. Measure x and y components of tip relative to tail of vector left in place

For this vector both methods give…

$$\vec{w} = (-2, 2)$$