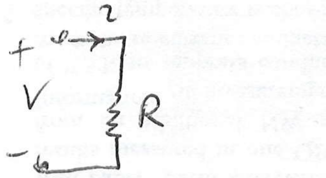


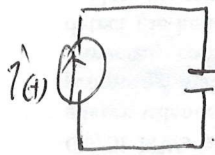

**(Always follow Associated Variable Convension)**

$i = \frac{1}{R} \cdot V \qquad i = C \cdot \frac{dV}{dt}$ $\qquad V \rightarrow$ Voltage drop across the Dev. or Branch Voltage

$\downarrow$ (G)

Current is proportional to $\begin{cases} \text{Voltage} \\ \text{Voltage change} \end{cases}$ for $\begin{cases} R \\ C \end{cases}$

Given Current. to find Voltage [Integral form)



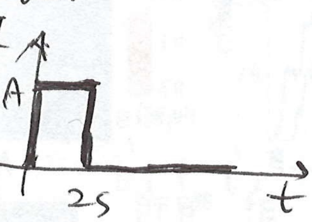

$i(t) = C\frac{dV}{dt}$

$$\int_0^t i(t)\,dt = C \cdot \int_0^t \frac{dV}{dt}\,dt = C\left(V_{(t)} - V_{(0)}\right)$$

$$\boxed{V_{(t)} = V_{(0)} + \frac{1}{C}\int_0^t i(t)\,dt}$$

$\boxed{\text{e.g. 1}}$ $C = 0.1\,F$, stores $q_{(0)} = 2\,C$.

Input a current:




What is $V_{(t)}$?

For $0 < t < 2$

$$V_{(t)} = V_0 + \frac{1}{C}\int_0^t 1A \cdot dt$$

$$= \frac{Q_{(0)}}{C} + \frac{1}{C}\int_0^t 1A\,dt$$

$$= \frac{Q_{(0)}}{C} + \frac{1}{C} \cdot t$$

$$= \frac{2}{0.1} + \frac{1}{0.1} \cdot t \quad = 20 + 10t.$$

For $t > 2\,s$

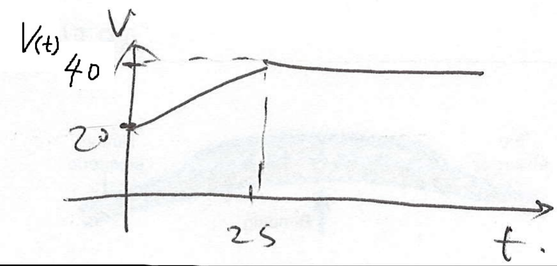
$$V_{(t)} = \frac{Q_{(0)}}{C} + \frac{1}{C}\int_0^2 1A\,dt + \underbrace{\frac{1}{C}\int_2^t 0A \cdot dt}_{\to 0}$$

$$= \frac{Q_{(0)}}{C} + \frac{1}{C}[2 - 0)$$

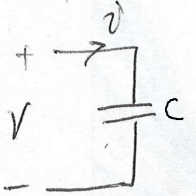
$$= 20V + 20V$$

$$= 40V$$

So:




## 2. Energy stored in a capacitor.




$$W = \int_0^t p_{(t)}\,dt = \int_0^t i \cdot v\,dt$$ **(Work done to any circuit)**

$$= \int_0^t C \cdot \frac{dv}{dt} \cdot v \cdot dt = C \int_0^t v \cdot \frac{dv}{dt} \cdot dt$$

$$= C \int_{V_{(0)}}^{V_{(t)}} v \cdot dv = \frac{C}{2}\left(V_{(t)}^2 - V_{(0)}^2\right)$$

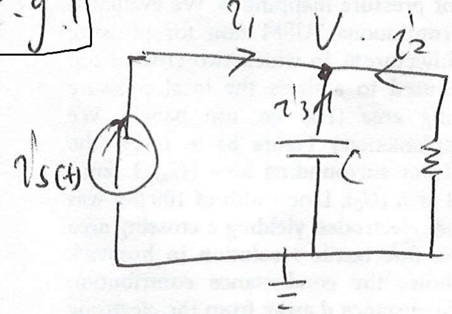
Assume Initially. $V_{(0)} = 0$

So: $E = C \cdot \frac{V^2}{2} = \frac{Q^2}{2C}$

e.g. $100\,\mu F$, $V = 200\,V$. $E = \frac{1}{2} \times (200V)^2 \times 10^{-4} F = 2J$

## 3. Differential equation for RC Circuit.

e.g.1

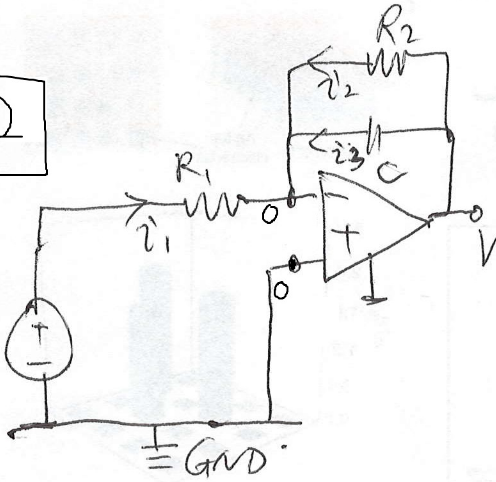



KCL: $i_1 + i_2 + i_3 = 0$

$$i_{s(t)} + \frac{0 - v}{R} + C \cdot \frac{d(0 - v)}{dt} = 0$$

$$i_{s(t)} - \frac{v}{R} - C \cdot \frac{dv}{dt} = 0$$

e.g.2




Ideal OpAmp

$$\boxed{V^+ \simeq V^- = 0 \; ; \quad i^+ = i^- = 0}$$

$$KCL: \; i_1 + i_2 + i_3 = 0$$

$$\frac{V_s - 0}{R_1} + \frac{V_0 - 0}{R_2} + C\frac{d(V_0 - 0)}{dt} = 0$$

$$\frac{dV_0}{dt} + \frac{V_0}{R_2 C} = -\frac{V_s}{R_1 C}$$

**A special case:**

$$I_f \; R_2 \to \infty$$

$$\frac{dV_0}{dt} = -\frac{V_s}{R_1 C} \Rightarrow \text{Integrator.}$$

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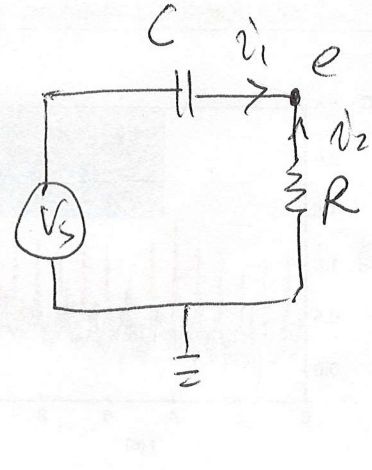
e.g.3




KCL:

$$i_1 + i_2 = 0$$

$$\frac{0 - e}{R} + C\cdot\frac{d(V_s - e)}{dt} = 0.$$