Announcements, Goals, and Reading

Announcements:

- HW09 due Friday 11/18
- Midterm 2 grades posted by end of week

Goals for Today:

- Work
- Power
- Conservation of Energy

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 9: Work and Kinetic Energy
- Chapter 10: Interactions and Potential Energy

"Work" done by force equals change in kinetic energy of object $W = \Delta K$

"Work" depends on relative orientation of force and direction of motion

In general...

$$W = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F} \cdot d\vec{x}$$

1D, constant force:

$$W = \int_{x_1}^{x_2} F_x \, dx \; = F_x(x_2 - x_1)$$

Multiple forces acting on object

$$i = 1, 2, \ldots, n$$

Work associated with each force $W_i = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F}_i \cdot d\vec{x}$

Change in kinetic energy equals total work done on object by all forces

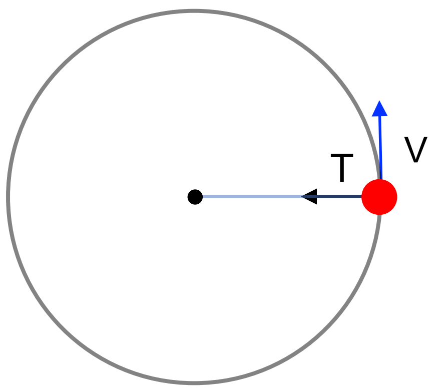
$$\Delta K = W_1 + \cdots + W_n = W_{tot}$$

A ball of mass m swings around a circle of radius R with constant angular velocity ω

A rope attached to the center provides the centripetal force

a) What is the tension in the rope?

b) How much work does the rope do as the ball swings through ¼ of a revolution?



Tension ➡ $$T = m\omega^2 R$$

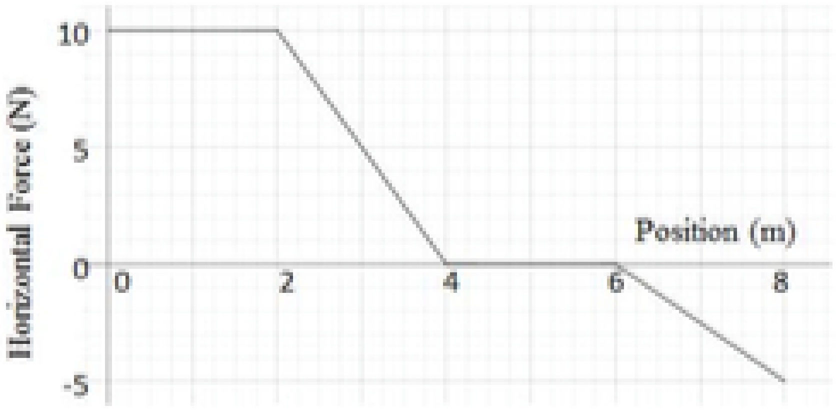
Work ➡ $$W = 0$$

Because tension force is always perpendicular to direction of motion

-KE is constant, no net work done

Non-constant force

A 5.0 kg toolbox is initially at rest on the floor in the back of a large truck as the truck starts driving straight along a flat road. The only horizontal force that the toolbox feels comes from static friction, due to static friction with the truck bed. That horizontal force (from the truck) is indicated on the graph for the first 8 m of a very short trip. How fast is the toolbox moving after 8 m?



Go back to expression for work in 1D

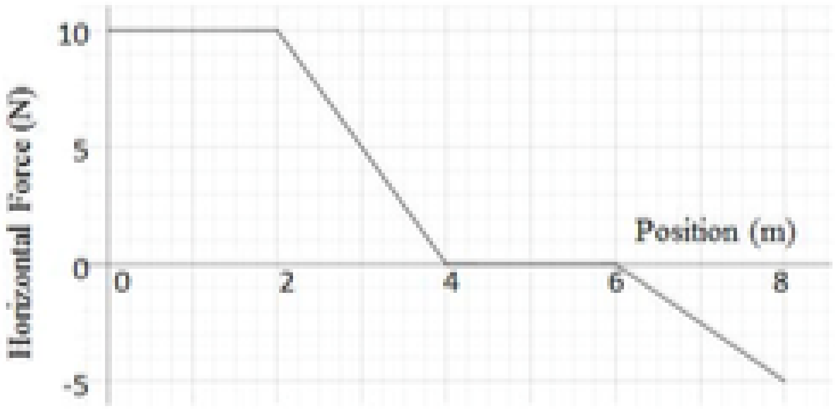
$$W = \int_{x_1}^{x_2} F_x \, dx$$

Work = area under force vs position curve

Need to add up areas for different parts of curve. Then determine speed from KE.

Non-constant force

A 5.0 kg toolbox is initially at rest on the floor in the back of a large truck as the truck starts driving straight along a flat road. The only horizontal force that the toolbox feels comes from static friction, due to static friction with the truck bed. That horizontal force (from the truck) is indicated on the graph for the first 8 m of a very short trip. How fast is the toolbox moving after 8 m?



Go back to expression for work in 1D

$$W = \int_{x_1}^{x_2} F_x \, dx$$

Work = area under force vs position curve

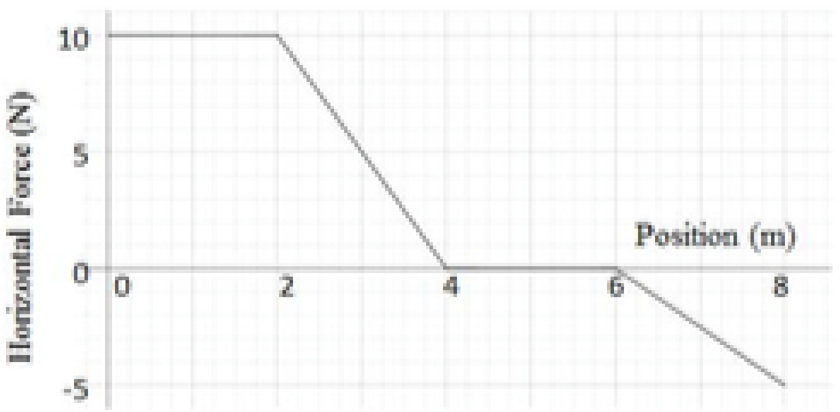
Need to add up areas for different parts of curve

$$W = (10N)(2m) + \frac{1}{2}(10N)(2m) + 0 + \frac{1}{2}(-5N)(2m)$$

$$= 25Nm = 25J$$

Non-constant force

A 5.0 kg toolbox is initially at rest on the floor in the back of a large truck as the truck starts driving straight along a flat road. The only horizontal force that the toolbox feels comes from static friction, due to static friction with the truck bed. That horizontal force (from the truck) is indicated on the graph for the first 8 m of a very short trip. How fast is the toolbox moving after 8 m?



$W = 25J$ Work gives change in kinetic energy

$$\Delta K = K_f - K_i = W \quad \Rightarrow \quad K_f = \frac{1}{2}mv_f^2 = 25J$$

Gives $v_f = \sqrt{\frac{2(25J)}{5kg}} = 3.2m/s$

Summary: Work in 3D, Dot Product, Relation to Kinetic Energy

Work:

Work done over small time Δt equals force times displacement in 1D

$$\boxed{W = F\Delta x}$$

In 3D add in y and z components

$$W = F_x \Delta x + F_y \Delta y + F_z \Delta z$$
$$= \vec{F} \cdot \Delta \vec{x}$$

Each component of force vector multiplies the corresponding component of displacement vector

Dot Product:

$$\boxed{\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z}$$

$$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}|\,\cos\theta}$$

Work from many forces i:

$$W_i = \int_{\vec{x}_1}^{\vec{x}_2} \vec{F}_i \cdot d\vec{x}$$

Change in Kinetic Energy from Work :

Change in kinetic energy equals total work done on object by all forces

$$\boxed{\Delta K = W_1 + \cdots + W_n = W_{tot}}$$

Power: Rate at which energy is used or transferred

Units of power are units of energy divided by time

$$1W = 1\,Watt = 1\frac{Joule}{second}$$

In the context of work ⇒ Power = rate at which work is done on an object, P = dW / dt

For a **constant** force $\vec{F}$ acting on an object moving with constant velocity $\vec{v}$

Work $W = \vec{F}\cdot\Delta\vec{x}$

Power $P = \vec{F}\cdot\frac{\Delta\vec{x}}{\Delta t}$ ⇒ $\boxed{P = \vec{F}\cdot\vec{v}}$

Example

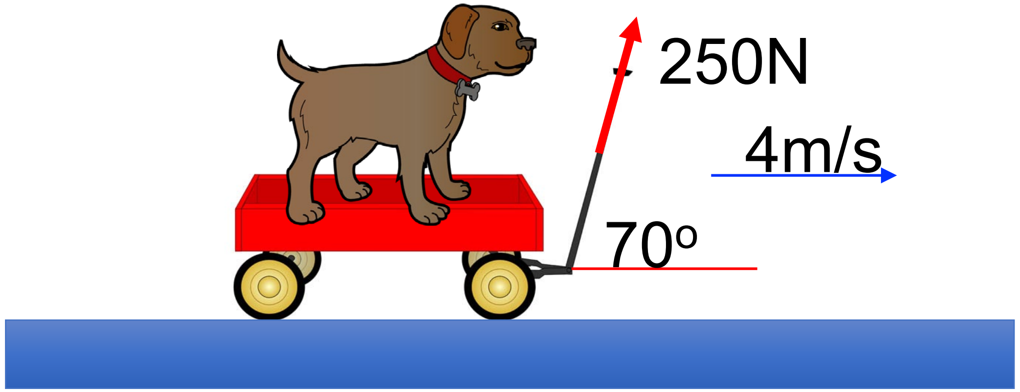
Dog on wagon is pulled along by 250N force applied at angle of 70°

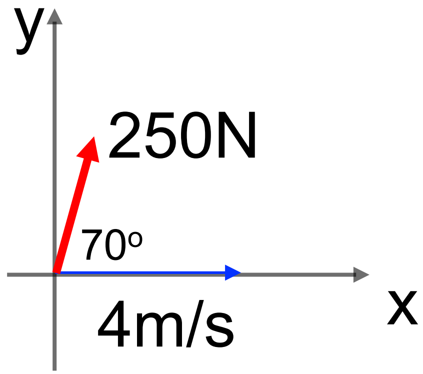
Moves at velocity 4m/s

How much power is being delivered by the force?

$$\vec{v} = (4m/s)\hat{x} \quad *$$

$$P = \vec{F} \cdot \vec{v}$$



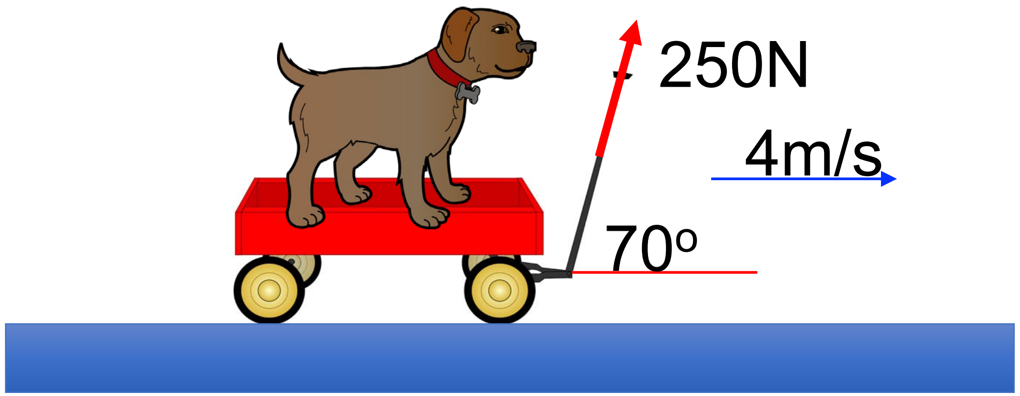


Example

Dog on wagon is pulled along by 250N force applied at angle of 70°

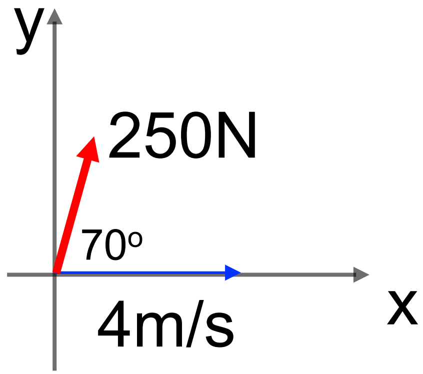
Moves at velocity 4m/s

How much power is being delivered by the force?



$$\vec{v} = (4m/s)\hat{x}$$

$$\vec{F} = (250N)\cos 70^o\hat{x} + (250N)\sin 70^o\hat{y}$$

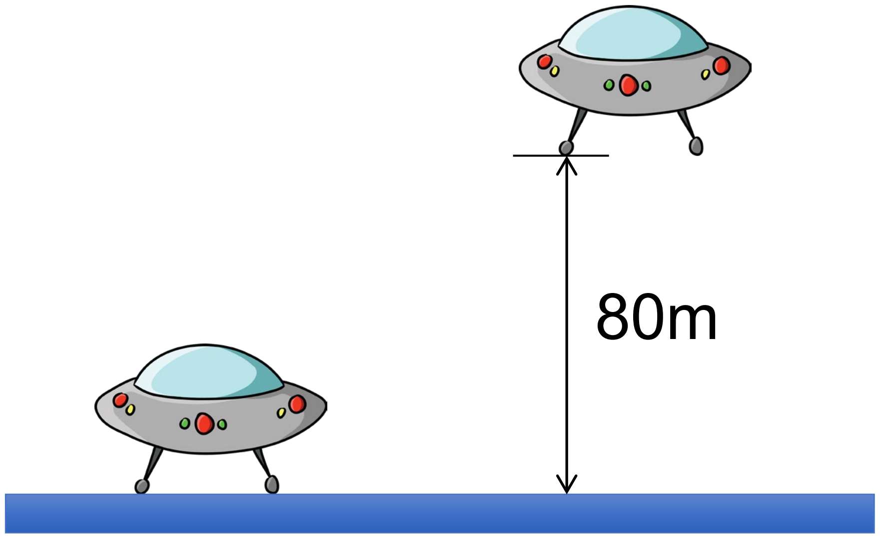


$$P = \vec{F}\cdot\vec{v} = (250N)(\cos 70^o)(4m/s)$$

$$= 342W$$

Power Example

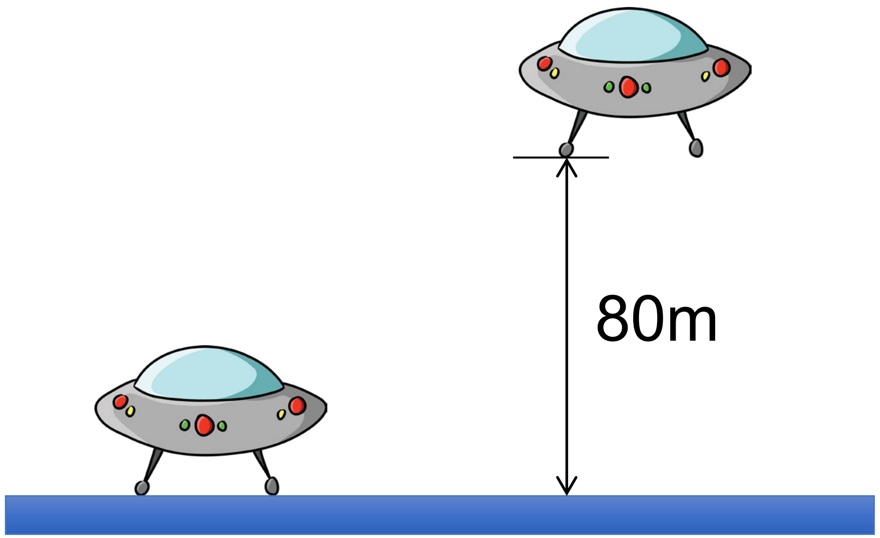
What average power is required to lift a 1500kg flying saucer by 80m in 20s?



$$P_{avg} = \frac{\Delta E}{\Delta t} \quad *$$

Power Example

What average power is required to lift a 1500kg flying saucer by 80m in 20s?



$$P_{avg} = \frac{\Delta E}{\Delta t}$$

$$= \frac{mg\Delta h}{\Delta t}$$

change in gravitational potential energy

$$= \frac{(1500kg)(9.8m/s^2)(80m)}{20s}$$

$$= 59,000W$$

Note P = F v where F=mg, and v is the average velocity v= Δh/Δt

Power Example

A 1000 kg car accelerates from 30 mph to 60 mph in 9.2 s

(13.4 m/s to 26.8 m/s)

What average power is supplied by the engine?

$$P_{avg} = \frac{\Delta E}{\Delta t} \quad *$$

Power Example

A 1000 kg car accelerates from 30 mph to 60 mph in 9.2 s

(13.4 m/s to 26.8 m/s)

What average power is supplied by the engine?

$$P_{avg} = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\Delta t}$$

$$= \frac{1}{2}(1000kg)\frac{(26.8m/s)^2 - (13.4m/s)^2}{9.2s}$$

$$= 29,000W$$

Note: **1 horsepower is about 746 Watts.** 29,000W = 39 HP

Humans at rest: 70-100W. Cyclists: 400+ Watts

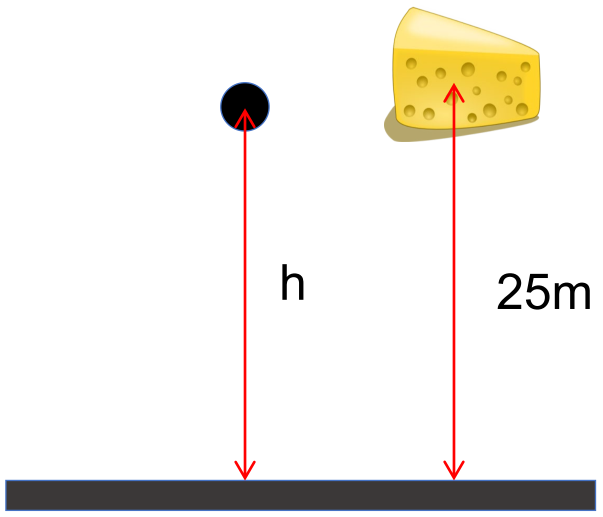
Olympic weight lifters: 2100 W, 3 HP (215 kg, 1 m, 1 sec)

Chapter 10: Interactions and Potential Energy

Potential energy U

There are different types of potential energy associated with different forces

Start with gravitational potential energy



Object at height h above ground has gravitational potential energy

$$U = mgh$$

Example

100kg block of cheese is 25m above ground

What is its gravitational potential energy?

$$U = (100kg)(9.8m/s^2)(25m) = 24,500\frac{kg\,m^2}{s^2} = 24,500J$$

Can measure h from any starting point.

Nothing special about ground; often just convenience

Only <u>changes</u> in height will matter

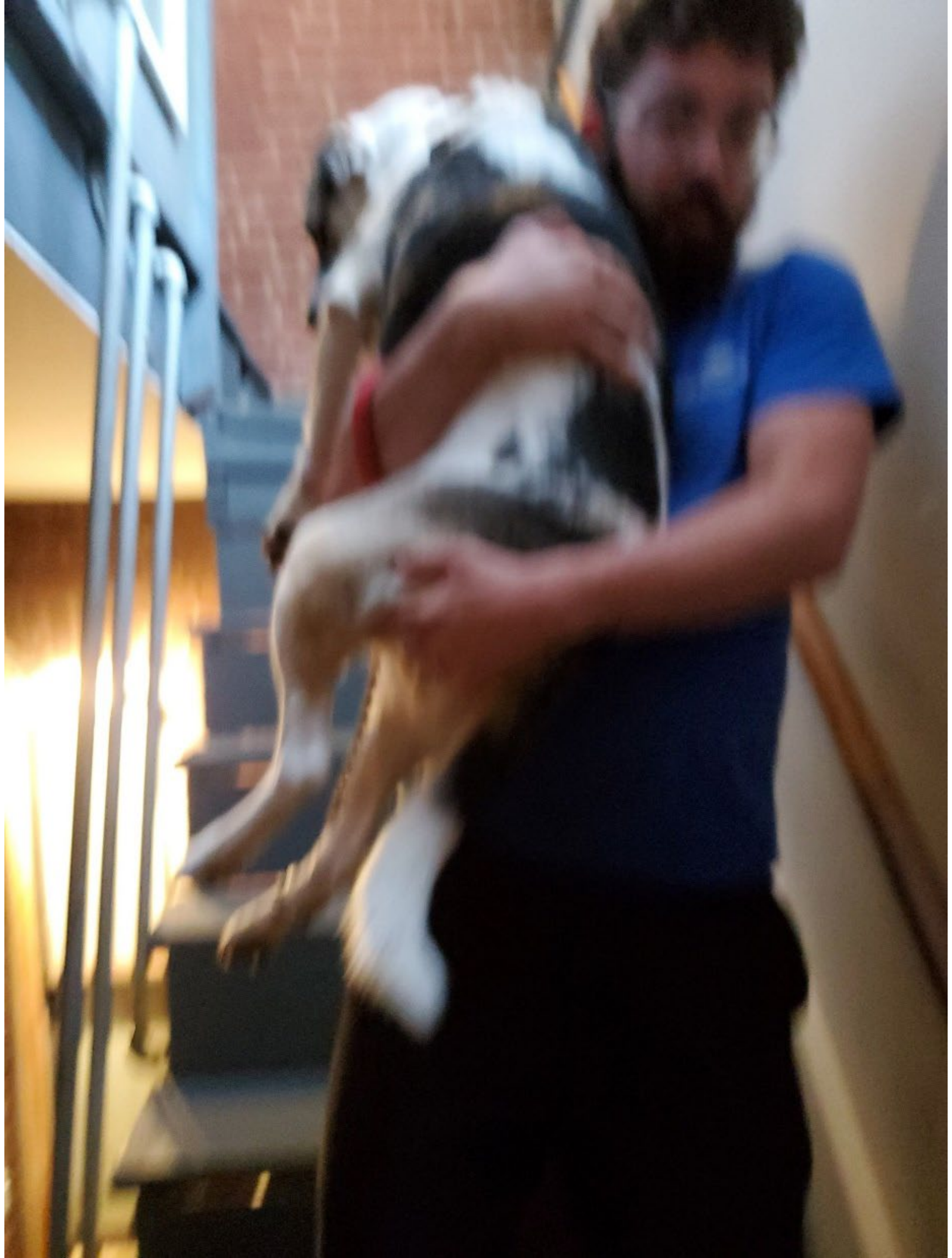
Professor Hamilton stays in a 4th story loft while visiting St. Louis, roughly 20m above ground level. Unfortunately, his 60lb (27kg) dog is afraid of stairs. Roughly how much energy is required to carry the dog up the stairs?

$U_{ground}=mgh_{ground}=0$

$U_{4th\ floor}=mgh_{4th\ floor}$

$= (27kg)(9.8m/s^2)(20m)$

$E_{dog} = \Delta U = 530J$



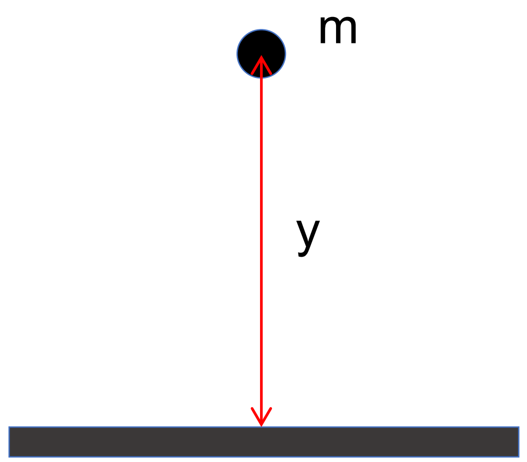
<u>Mechanical energy E</u> of object acted on by gravity is sum of its kinetic energy and gravitational potential energy

$$E = K + U$$
$$= \frac{1}{2}mv^2 + mgh$$

If gravity is only force acting on object, then
<u>mechanical energy is conserved</u>

Like conservation of momentum, this is nothing new

Follows from Newton's 2nd law



Easy to show in 1 dimension

Consider only vertical motion

y(t) = height above ground

velocity $v = \frac{dy}{dt}$ acceleration $a = \frac{dv}{dt}$

$$\frac{dE}{dt} = \frac{d}{dt}(\frac{1}{2}mv^2 + mgy) = mv\frac{dv}{dt} + mg\frac{dy}{dt} = mv(a+g) = 0$$

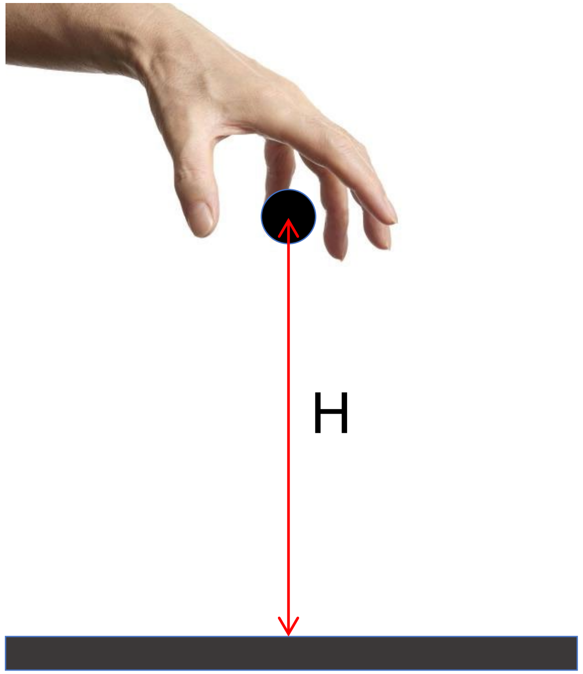
Newton's second law $\Rightarrow$ $a = -g$ Free fall

Mechanical energy is constant

Application of conservation of mechanical energy E=K+U

Ball of mass m dropped from height H above the ground

Find speed v_f when it hits the ground



Can solve via kinematics, but use conservation of energy instead

Top

Ball is not moving
Has potential energy but no kinetic energy

Initial energy $$E_i = mgH$$

Just before it hits the ground

Bottom

Ball is moving, but is at ground level
Has kinetic energy buy no potential energy

Final energy $$E_f = \frac{1}{2}mv_f^2$$

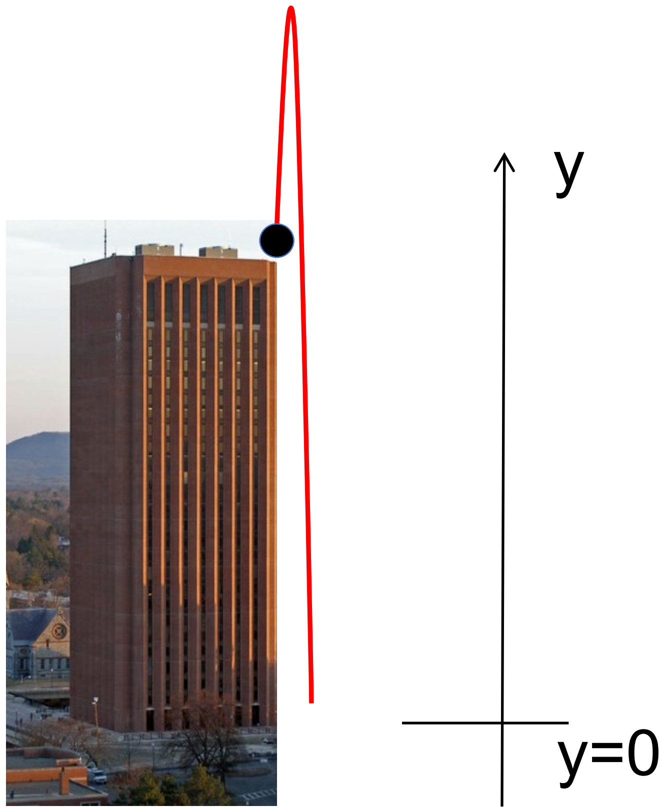
Conservation of Energy $$E_f = E_i \Rightarrow \frac{1}{2}mv_f^2 = mgH \Rightarrow v_f = \sqrt{2gH}$$

2 kg rock is thrown straight upwards from the edge of a 25 m tall building with speed of 15 m/s

What is the rock's mechanical energy?

How high does the rock go?

What is its speed when it hits the ground?

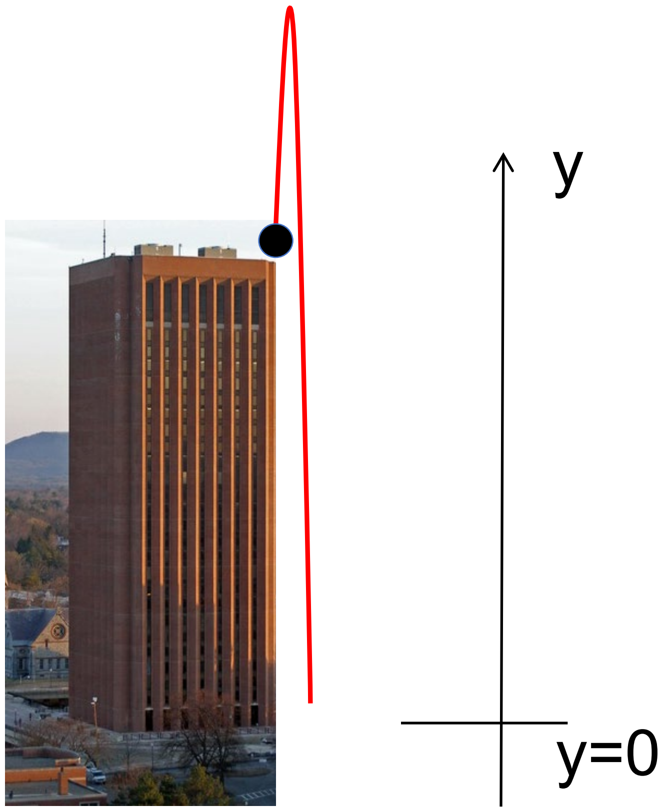


2 kg rock is thrown straight upwards from the edge of a 25 m tall building with speed of 15 m/s

What is the rock's mechanical energy?

How high does the rock go?

What is its speed when it hits the ground?



Mechanical energy

Can evaluate it anywhere along the path.
It will be the same

Top of building

$$E = \frac{1}{2}mv^2 + mgy$$

v=15m/s
y=25m

$$= \frac{1}{2}(2kg)(15m/s)^2 + (2kg)(9.8m/s^2)(25m)$$

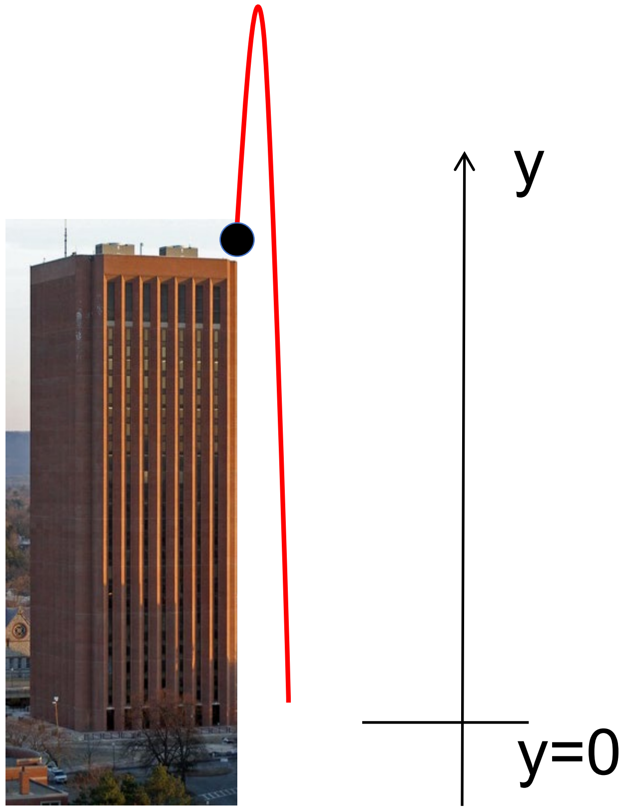
$$= 715J$$

2 kg rock is thrown straight upwards from the edge of a 25 m tall building with speed of 15 m/s

What is the rock's mechanical energy?

How high does the rock go?

What is its speed when it hits ground?



Mechanical energy $E = 715J$

Top of trajectory ➡ $v = 0$ ➡ $E = \frac{1}{2}\cancel{mv^2} + mgy$

$$= (2kg)(9.8m/s^2)y = 715J$$

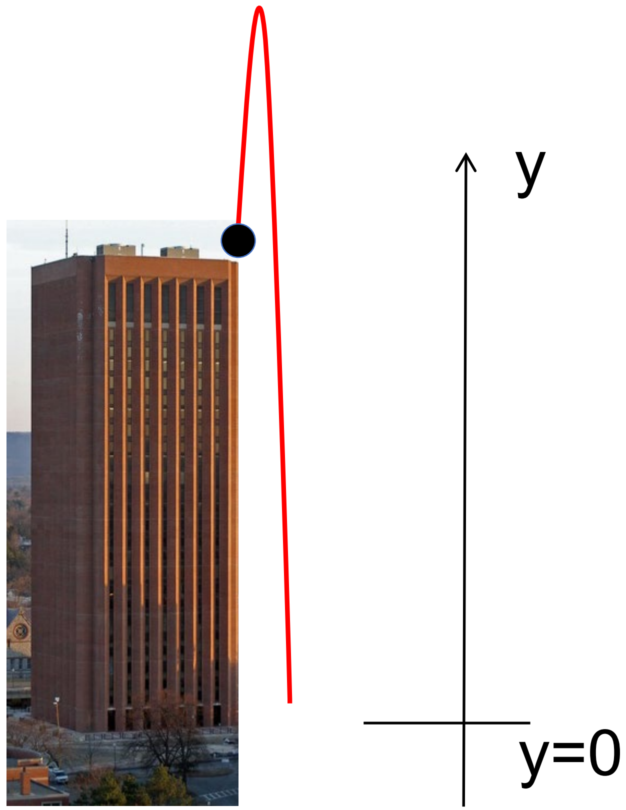
➡ $$y = \frac{715J}{(2kg)(9.8m/s^2)} = 36.5m$$

2 kg rock is thrown straight upwards from the edge of a 25 m tall building with speed of 15 m/s

What is the rock's mechanical energy?

How high does the rock go?

What is its speed when it hits ground?



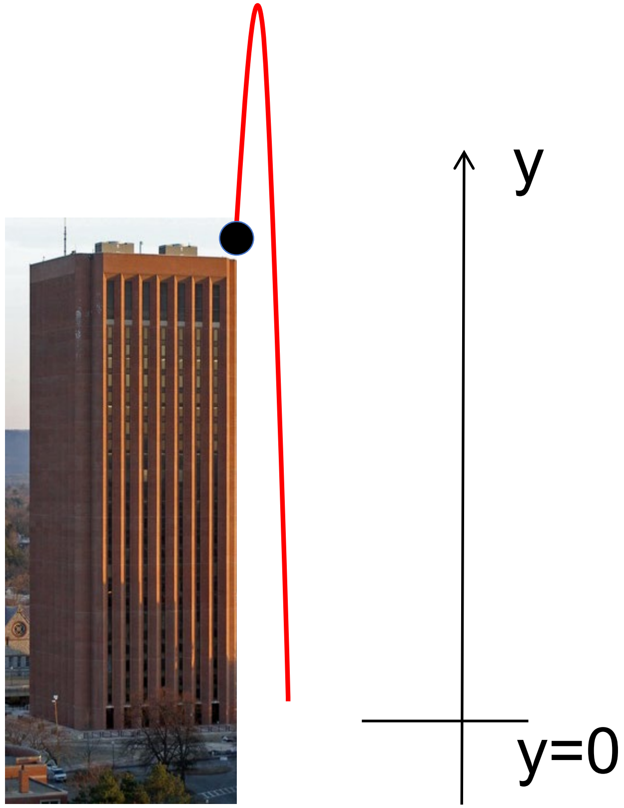
Mechanical energy $E = 715J$

Top of trajectory $y = 36.5m$

Ground ➡ $y = 0$ ➡ $E = \frac{1}{2}mv^2 + \cancel{mgy} = 715J$

➡ $v = \sqrt{\frac{2(715J)}{2kg}} = 26.7m/s$

2 kg rock thrown up from 25 m tall building with speed of 15 m/s:
Rock's mechanical energy: 715 J
How high does the rock go: 36.5 m
Speed when it hits ground: 26.7 m/s



Is this consistent with results from kinematics part of the course?

How high does the rock go?

$$v = v_0 - gt. \quad \text{At peak } v = 0 \quad \Rightarrow \quad t_{peak} = \frac{v_0}{g} = 1.53 \text{ s}$$

$$\begin{aligned} y &= y_0 + v_0 t - \frac{1}{2} g t^2 \\ &= 25 + 15\frac{v_0}{g} - \frac{1}{2} g \left[\frac{v_0}{g}\right]^2 \\ &= 36.5\ m \end{aligned}$$

Speed when it hits the ground?

$$D = v^2/2g \Rightarrow v = \sqrt{2gD} = \sqrt{2 \times 36.5 \times 9.8} = 26.7 m/s$$

Former President Obama shoots a basketball with mass 0.62 kg at 10 m/s at an angle of 55^{o} with respect to the horizontal

Ball is released at height 2 m

Hoop is at height 3 m

How fast is ball moving when it goes through hoop?

Former President Obama shoots a basketball with mass 0.62 kg at 10 m/s at an angle of 55° with respect to the horizontal

Ball is released at height 2 m

Hoop is at height 3 m

How fast is ball moving when it goes through hoop?

Use conservation of mechanical energy

Let… v_0 = magnitude of initial velocity h_0 = initial height

v_1 = magnitude of final velocity h_1 = final height

Conservation of energy

$$E_0 = \frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_1^2 + mgh_1 = E_1$$

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_0^2 + mg(h_0 - h_1)$$

Former President Obama shoots a basketball with mass 0.62 kg at 10 m/s at an angle of 55^o with respect to the horizontal

Ball is released at height 2 m

Hoop is at height 3 m

How fast is ball moving when it goes through hoop?

Conservation of energy

$$E_0 = \frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_1^2 + mgh_1 = E_1$$

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_0^2 + mg(h_0 - h_1)$$

$$v_1^2 = v_0^2 + 2g(h_0 - h_1)$$

Only change in height matters!
Vector properties of velocity do not.

$$v_1 = \sqrt{(10m/s)^2 + 2(9.8m/s^2)(2m - 3m)} = 9.0m/s$$

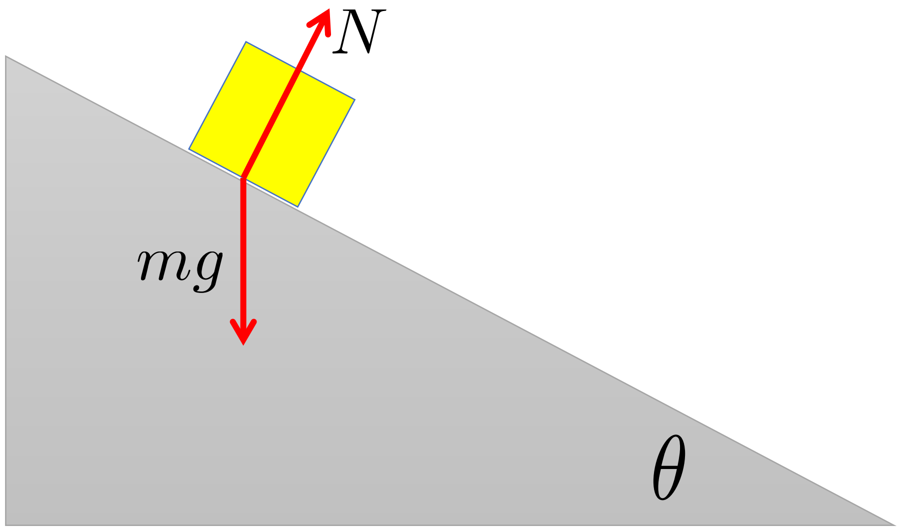
Mechanical energy and inclined plane

Block of mass m slides up frictionless inclined plane at angle θ

In addition to gravity, now have normal force too

Show later that conservation of block's mechanical energy still holds when normal force acts

Turns out that normal force does not change block's energy (more when we discuss "work" later)



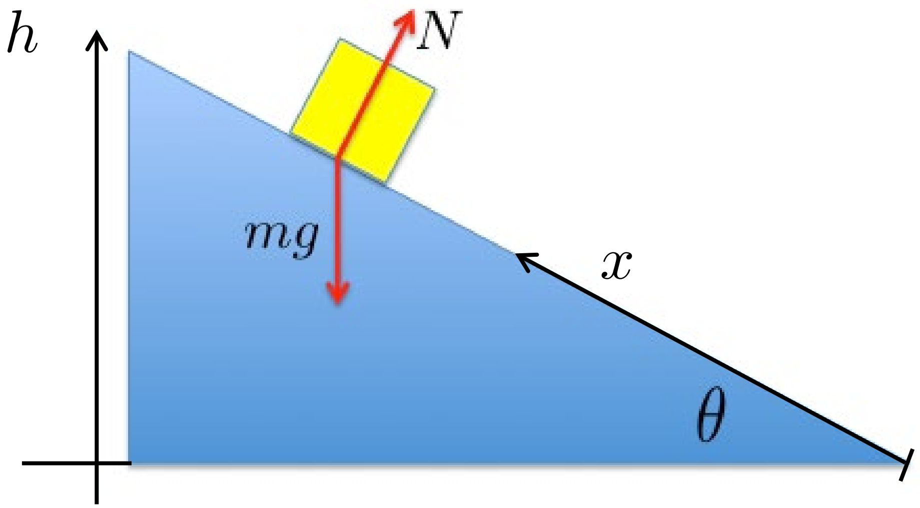
First, it is exactly compensated by the gravity component normal to the surface.
Second, there is no friction.
Third, normal force is acting perpendicular to the direction of motion

Mechanical energy and inclined plane

Block of mass m slides up frictionless inclined plane at angle θ

Block starts at bottom of plane with velocity v_0 up the plane.

How far up the plane does it go?



Let x measure distance up the plane and v be velocity along plane

Height above base $h = x \sin\theta$

Conservation of energy $E_1 = E_0$

$$E = \frac{1}{2}mv^2 + mgx\sin\theta$$

$$\boxed{x_1 = \frac{v_0^2}{2g\sin\theta}}$$

Initially at bottom $x = 0$ $\Rightarrow$ $E_0 = \frac{1}{2}mv_0^2$

Top of trajectory $v = 0$ $\Rightarrow$ $E_1 = mgx_1\sin\theta$