Lab 3 - RC Circuits

Your Name (First Last):

Overview

In this lab you will construct electrical circuits that contain capacitors. A capacitor is an electrical element that can store electrical charge. The amount of charge a capacitor can store depends upon its capacitance. If a voltage V is applied across a capacitor, and amount of charge Q is stored on the capacitor, then its capacitance is:

$$C = \frac{Q}{V} \quad (1)$$

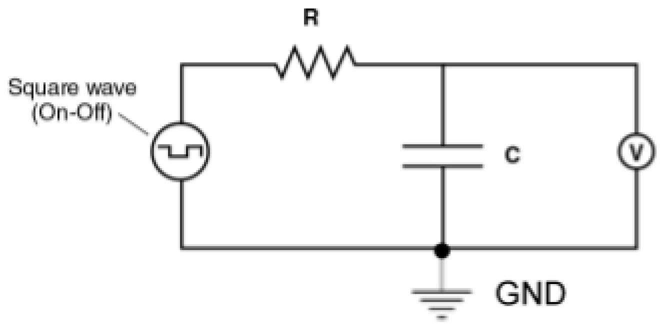
Now you are going to construct circuits using capacitors. In Lab 2 you constructed circuits with resistors. When you apply a voltage across a resistor, a current flows through the resistor according to Ohm's Law. The more voltage applied, the more current will flow. Likewise, when you apply a voltage across a capacitor, charge will be stored in the capacitor. What do you get when you combine a resistor and a capacitor together in a circuit? You get an RC circuit! An important property of an RC circuit is its charging/discharging time constant τ:

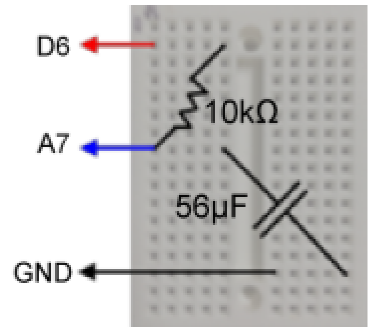
$$\tau = RC \quad (2)$$

In this lab you will construct different RC circuits and measure the time constant τ for each one.

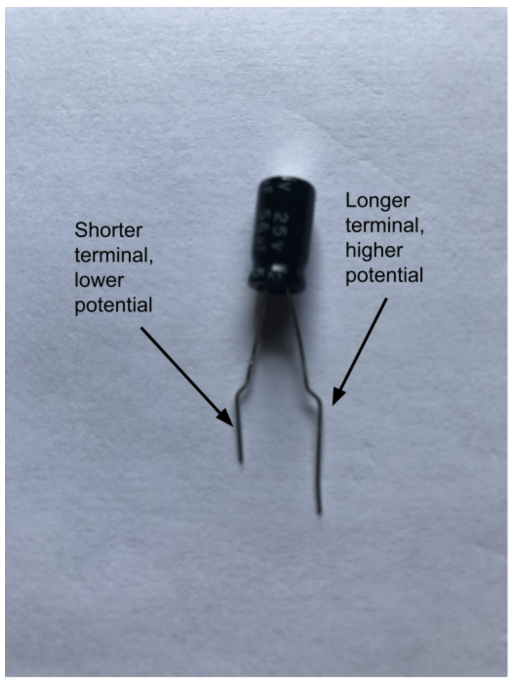
Part 1 - Short Time Constant

From Eq. 2 up above, it is easy to see the duration of the time constant τ depends on the size of the capacitance of the capacitor in the circuit. A large capacitor can hold more charge, so it takes longer to discharge the capacitor. A smaller capacitor holds less charge so the time to discharge the capacitor is shorter. Construct the following circuit using a 10kΩ resistor in series with a 56μFarad capacitor. Remember, 1μ Farad = 1$\times 10^{-6}$Farad.

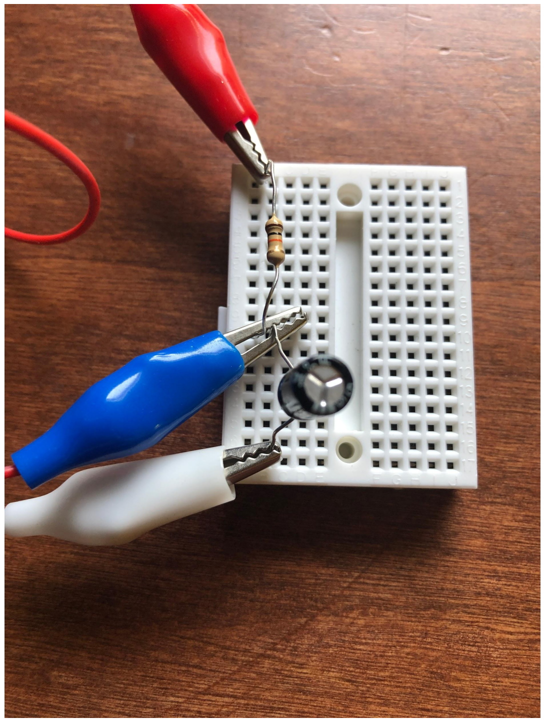




You may notice that the terminals of the capacitor are different lengths. When you connect the capacitor to the circuit, the longer terminal should be connected to the higher potential and the short terminal should connect to the lower potential.



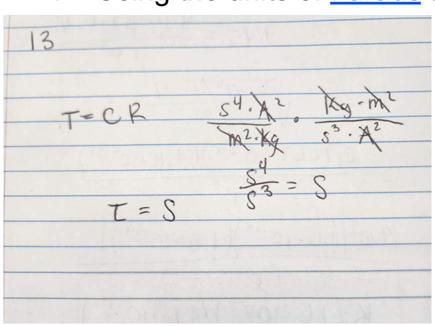
Once your circuit is connected, it should look like this:



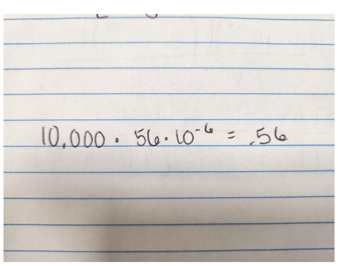
- Red alligator clip connected to D6.
- Blue alligator clip connected to A7.
- White alligator clip connected to GND.

The time constant τ is measured in seconds. In Eq. 2, you can see τ is the product of C measured in Farads and R measured in Ohms.

1. Using the units of Farads and Ohms, and show τ is measured in units seconds.



2. Calculate the value of the time constant for your circuit with R = 10kΩ and C = 56μF.



.56 seconds

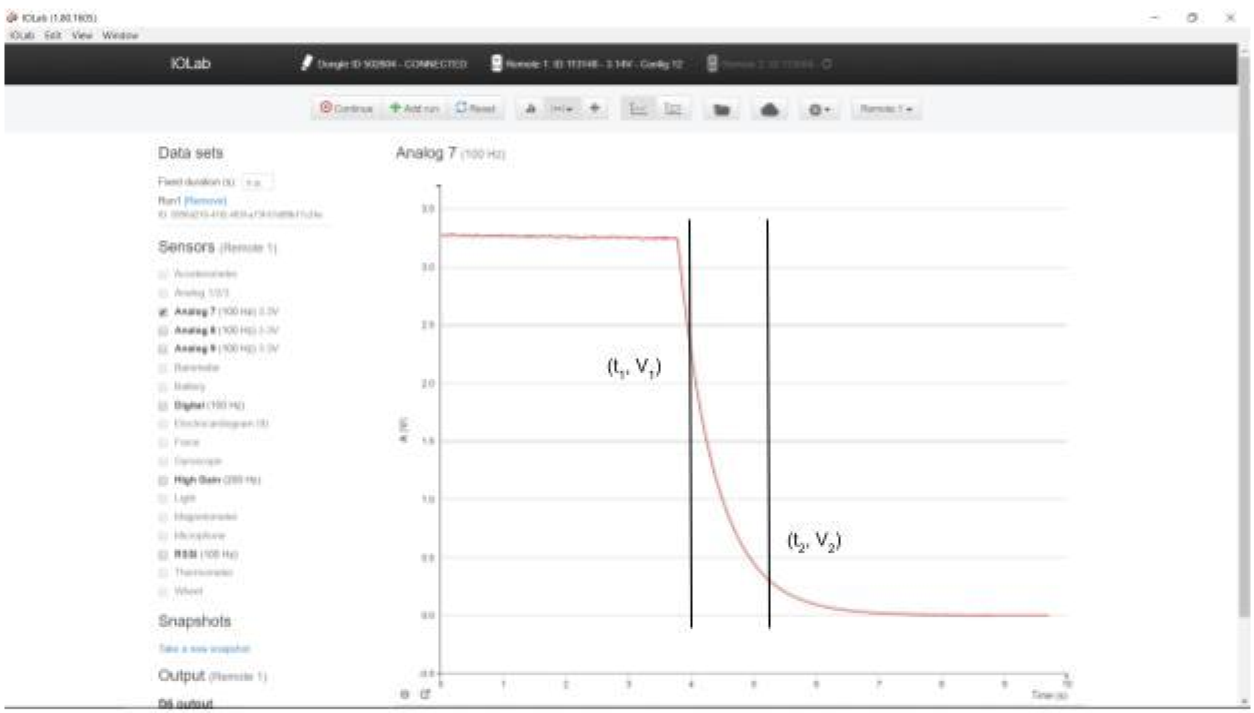
Setup the iOLab and application:

- Launch the iOlab application on your computer.
- Make sure the dongle is connected to the USB port of your computer and the iOLab is turned on.
- Check the A7 sensor.
- On the toolbar click the settings button (cog) > Expert Mode > Output configuration.
- At the bottom of the iOLab app window the D6 output should appear.
- For the D6 Output select 3.3V and then click On. Allow the capacitor to charge for about 10 seconds.

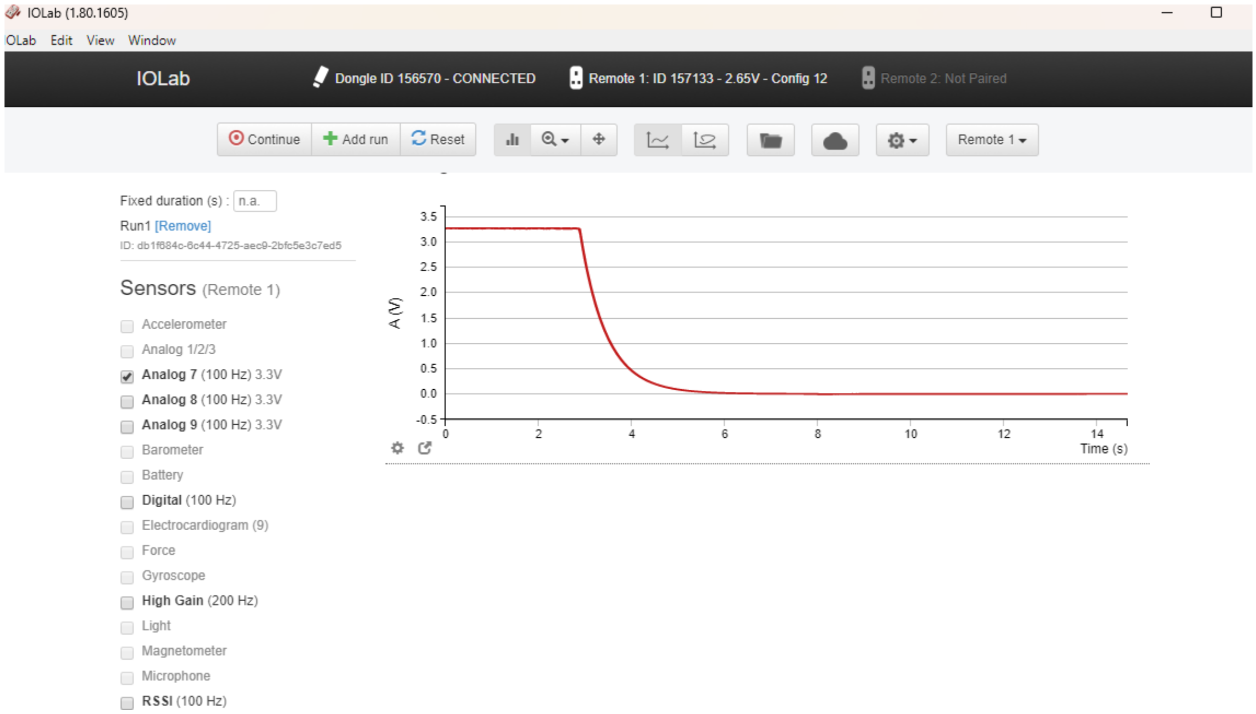
Now it’s time to take some data:

- Click the Record button and record the A7 voltage for about 5 seconds.
- While still recording, change the D6 output to Off.
- Continue recording for about 10 - 15 seconds, then click Stop.

Your data should look something like this:



3. Take a screenshot of your iOLab application with the data and paste here:



When the voltage supplied by the iOLab, the voltage across the capacitor does NOT instantaneously drop to zero. The voltage does decrease over time because the charge stored decreases, BUT not instantaneously. If you look closely at your data, you may notice the decrease in voltage follows a curve. This is an *exponential* curve. The voltage across the capacitor V_C decreases with time:

$$V_C(t) = V_0 e^{-t/\tau} \tag{3}$$

Now let's record some data. In the application when you move the mouse arrow, a vertical line will track with the arrow. In the upper left corner of the graph, you should see the value of time t and voltage V for that position of the vertical line.

4. Record two sets of time and voltage across the discharging capacitor: One set at a high point along the curve, just after turning off D6, and the other set at a low point on the curve. Do not measure the voltage along the flat part of the curve. Record your value in the table below.

	Time (sec)	Voltage (V)
first point (t_1, V_1)	2.99	2.724
second point (t_2, V_2)	4.35	.261

In Eq. (3) above we see how the voltage across the discharging capacitor is related to the time. At the first point the voltage is:

$$V_1 = V_0 e^{-t_1/\tau} \tag{4}$$

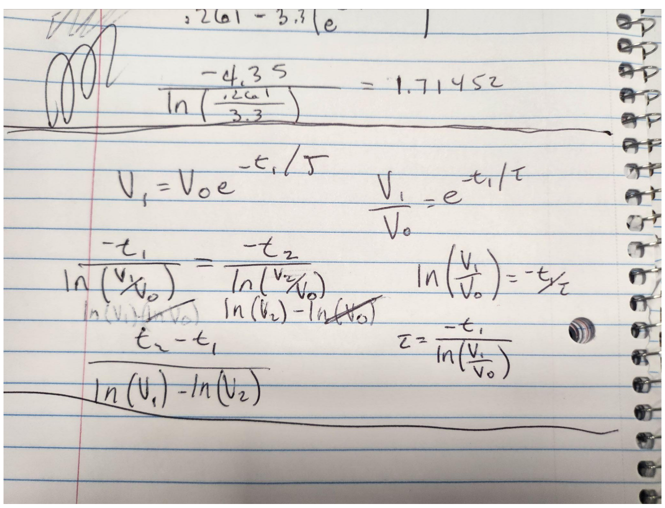
At the second point the voltage is:

$$V_2 = V_0 e^{-t_2/\tau} \tag{5}$$

You can combine this two equations and show the time constant τ is:

$$\tau = \frac{t_2 - t_1}{ln(V_1) - ln(V_2)} \tag{6}$$

5. Combine Eqs. (4) and (5) and show τ is equal to Eq. (6). Here is the procedure:
   a. Divide Eq. (4) by Eq.(5)
   b. Take the natural log of both sides of the equation.
   c. Use the logarithmic identity $ln(A/B) = ln(A) - ln(B)$
   d. Show your work here:



6. Use your values of (t_1, V_1) and (t_2, V_2), and calculate the value of τ you measured in your DC circuit. Record your value of τ in table below:

τ for 56μF Capacitor (sec)
.579874

7. How does your measure value compare the value of τ calculated in question 2?

It is very close, only .02 seconds off. Our predicted time value of .56 is only about 4% off

Part 2 - Long Time Constant

Now replace the 56μF capacitor with a 220μF capacitor. Remember, the capacitor has a long and a short terminal. The long terminal should be on the higher potential and the short terminal should be on the lower potential.

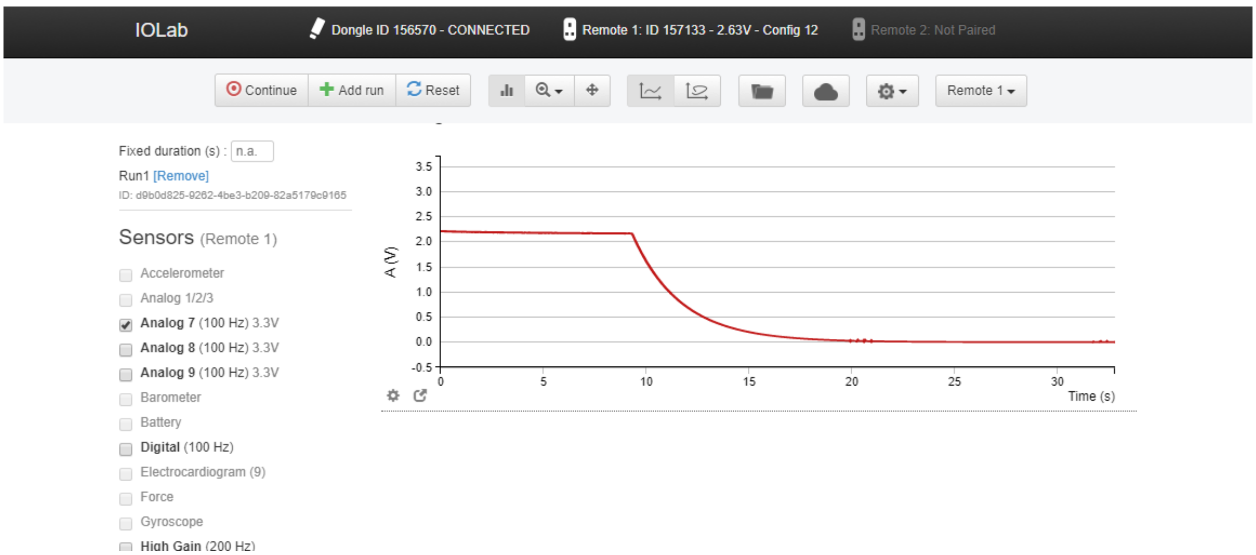
8. Calculate the value of the time constant for your circuit with R = 10kΩ and C = 220μF.

2.2 seconds 10000 * 220E-6 = 2.2 seconds

Repeat the same procedure of recording data:

- For the D6 Output select 3.3V and then click On. Allow the capacitor to charge for about 10 seconds.
- Click the Record button and record the A7 voltage for about 5 seconds.
- While still recording, change the D6 output to Off.
- Continue recording for about 20 - 30 seconds, then click Stop.

9. Take a screenshot of your iOLab application with the data and paste here:



10. Record two sets of time and voltage across the discharging capacitor: One set at a high point along the curve, just after turning off D6, and the other set at a low point on the curve. Do not measure the voltage along the flat part of the curve. Record your value in the table below.

	Time (sec)	Voltage (V)
first point (t_1, V_1)	10	1.631
second point (t_2, V_2)	15	.205

11. Use your values of (t_1, V_1), (t_2, V_2), and Eq. (6), and calculate the value of τ you measured in your RC circuit. Record your value of τ in table below:

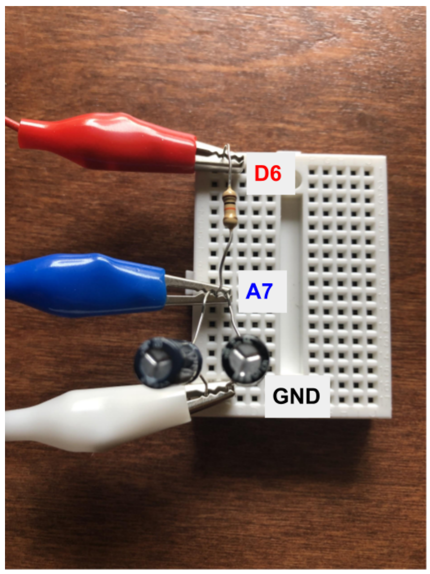
τ for 220μF Capacitor (sec)
2.41

12. How does your measure value compare the value of τ calculated in question 8?

Our predicted value of 2.2 seconds is very close to the actual value of 2.41, the percent error is only about 10 %

Part 3 - Parallel Capacitors

Just like you did in Lab 2 with resistors, you can combine capacitors in different ways to make a new capacitor. First combine the 56μF capacitor with the 220μF capacitors in *parallel* with each other. Your circuit should look like this:



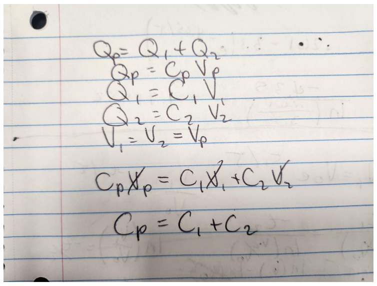
- Red alligator clip connected to D6.
- Blue alligator clip connected to A7.
- White alligator clip connected to GND.

When capacitors are combined in parallel, the equivalent capacitance is the sum of the individual capacitors:

$$C_p = C_1 + C_2 + \cdots \tag{7}$$

13. If two capacitors are connected in parallel, what must be true about the *voltage* across each capacitor? Explain your answer.
    The voltage across each capacitor is the same, this is because the voltage in parallel is split evenly

14. Using your answer to question 13, conservation of electric charge (Hint: Kirchhoff's Junction Rule), and the equation $Q = CV$, derive the equation for equivalent capacitance for capacitors in parallel. Show your work.



15. Will the time constant τ be longer or shorter when capacitors are combined in parallel? Explain your answer.

The time constant will be longer when the capacitors are in parallel because the capacitance is increased

16. Calculate the equivalent capacitance of the 56μF and 220μF capacitors combined in parallel.

276 microfarad

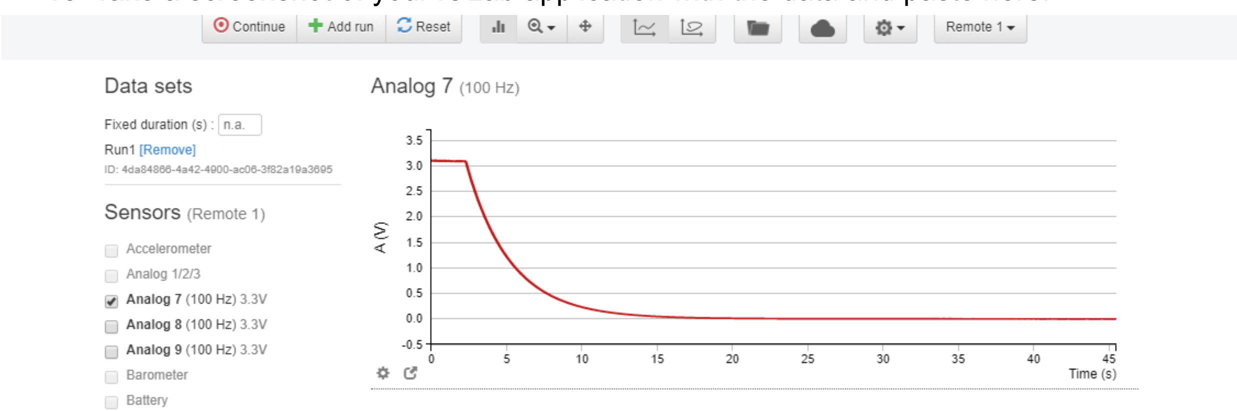
17. Calculate the value of the time constant τ for your circuit with R = 10kΩ and the equivalent parallel capacitor.

2.76 seconds 276E-6 * 10,000 = 2.76 seconds

Repeat the same procedure of recording data:

- For the D6 Output select 3.3V and then click On. Allow the capacitor to charge for about 10 seconds.
- Click the Record button and record the A7 voltage for about 5 seconds.
- While still recording, change the D6 output to Off.
- Continue recording for about 40 - 50 seconds, then click Stop.

18. Take a screenshot of your iOLab application with the data and paste here:



19. Record two sets of time and voltage across the discharging capacitor: One set at a high point along the curve, just after turning off D6, and the other set at a low point on the curve. Do not measure the voltage along the flat part of the curve. Record your value in the table below.

	Time (sec)	Voltage (V)
first point (t_1, V_1)	2.99	2.446
second point (t_2, V_2)	8.98	.326

20. Use your values of (t_1, V_1), (t_2, V_2), and Eq. (6), and calculate the value of τ you measured in your RC circuit. Record your value of τ in table below:

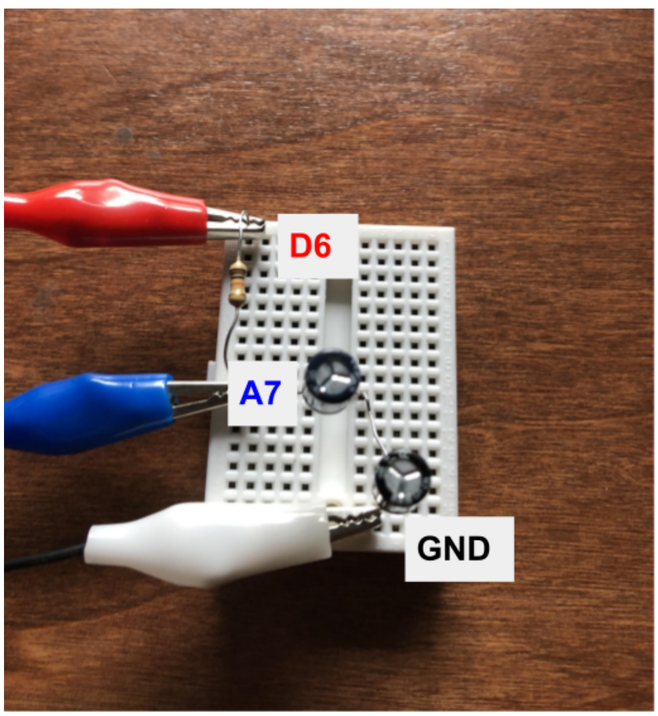
τ for Parallel Capacitor (sec)
2.97 seconds

21. How does your measure value compare the value of τ calculated in question 15?

Our predicted value of 2.76 seconds is very close to the actual value of 2.97 seconds
This is a percent error of about 8%

Part 4 - Series Capacitors

Finally you can combine the capacitors in *series*. combine the 56μF and 220μF capacitors in *series* with each other. Your circuit should look like this:

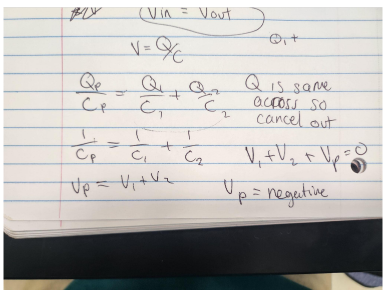


- Red alligator clip connected to D6.
- Blue alligator clip connected to A7.
- White alligator clip connected to GND.

When capacitors are connected in series, the equivalent capacitance is:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \tag{8}$$

22. If two capacitors are connected in *series*, what must be true about the *charge* stored in each capacitor? Explain your answer.
The charge stored in each capacitor would need to be equal, this is because the voltage coming in for both capacitors must be equal to the original voltage, and the stored must be equal to the whole system
23. Using your answer to question 22, conservation of energy (Hint: Kirchhoff Loop Rule), and the equation $V = Q/C$, derive the equation for equivalent capacitance for capacitors in series. Show your work.



$V_{in} = V_{out}$

$V = Q/C$

$\frac{Q_p}{C_p} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$ Q is same across so cancel out

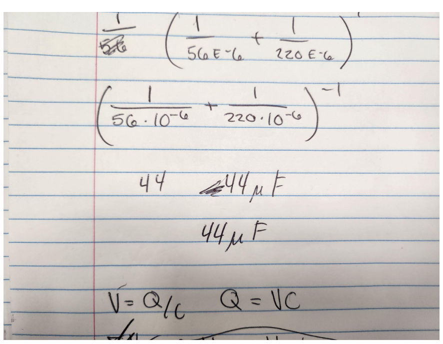
$\frac{1}{C_p} = \frac{1}{C_1} + \frac{1}{C_2}$ $V_1 + V_2 + V_p = 0$

$V_p = V_1 + V_2$ V_p = negative

24. Will the time constant τ be longer or shorter when capacitors are combined in series? Explain your answer.

The time constant is shorter than when they are in parallel, this is because the overall capacitance lowers

25. Calculate the equivalent capacitance of the 56μF and 220μF capacitors combined in series.



$\left(\frac{1}{56E-6} + \frac{1}{220E-6}\right)$

$\left(\frac{1}{56 \cdot 10^{-6}} + \frac{1}{220 \cdot 10^{-6}}\right)^{-1}$

44 44μF

44μF

$V = Q/C$ $Q = VC$

26. Calculate the value of the time constant τ for your circuit with R = 10kΩ and the equivalent series capacitor.

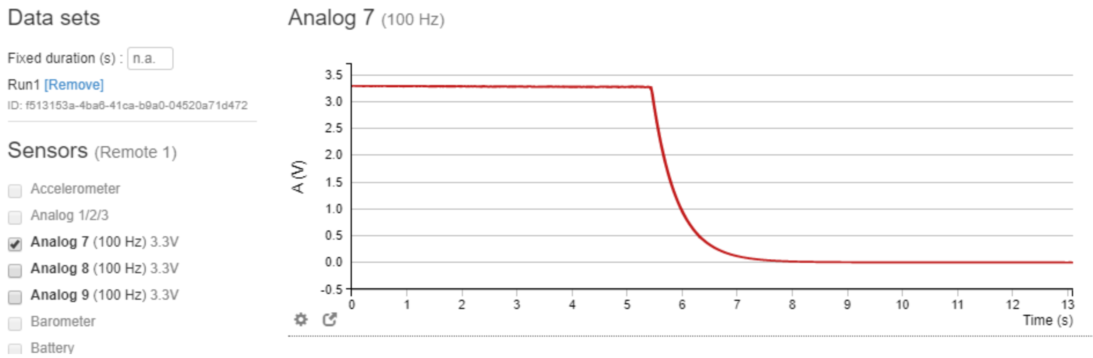
44E-6 * 10,000 = .44 seconds

Repeat the same procedure of recording data:

- For the D6 Output select 3.3V and then click On. Allow the capacitor to charge for about 10 seconds.
- Click the Record button and record the A7 voltage for about 5 seconds.

- While still recording, change the D6 output to Off.
- Continue recording for about 10 - 20 seconds, then click Stop.

27. Take a screenshot of your iOLab application with the data and paste here:



28. Record two sets of time and voltage across the discharging capacitor: One set at a high point along the curve, just after turning off D6, and the other set at a low point on the curve. Do not measure the voltage along the flat part of the curve. Record your value in the table below.

	Time (sec)	Voltage (V)
first point (t_1, V_1)	5.49	2.994
second point (t_2, V_2)	5.99	1

29. Use your values of (t_1, V_1), (t_2, V_2), and Eq. (6), and calculate the value of τ you measured in your RC circuit. Record your value of τ in table below:

τ for Series Capacitors (sec)
.455 seconds

30. How does your measure value compare the value of τ calculated in question 22?
The predicted value of .44 is very close to the actual value of .455 this is a percent error of about 3%