RC/RL

① 1^{st} order diff. eq., $t>0$

$$\boxed{\frac{d}{dt}V(t) + A \cdot V(t) = B}$$

② Write Solution to, $t>0$

$$\boxed{V(t) = \frac{B}{A} + Ke^{-At}}$$

③ Find K using initial Conditions

- need $V(0^+)$
- Use $i_L(0^-) = i_L(0^+)$

  $V_c(0^-) = V_c(0^+)$

RLC

① 2^{nd} order diff. eq., $t>0$

$$\boxed{\frac{d^2}{dt^2}V(t) + A\frac{d}{dt}V(t) + BV(t) = C}$$

② Determine damping

$2\alpha = A$, $\omega_0^2 = B$

$\alpha^2 > \omega_0^2 \rightarrow$ overdamped $\rightarrow V(t) = \frac{C}{B} + A_1e^{S_1t} + A_2e^{S_2t}$, $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$\alpha^2 < \omega_0^2 \rightarrow$ underdamped $\rightarrow V(t) = \frac{C}{B} + e^{-\alpha t}[A_1\cos(\omega_d t) + A_2\sin(\omega_d t)]$

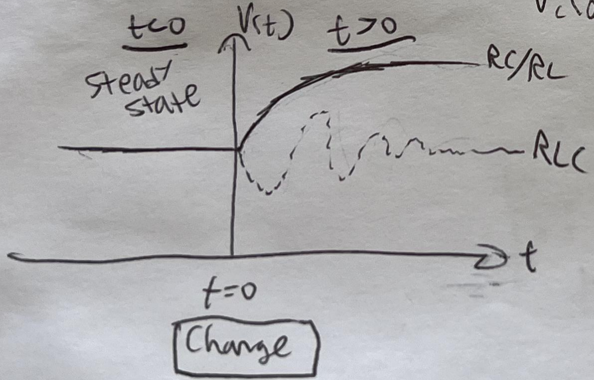
$\alpha^2 = \omega_0^2 \rightarrow$ critically damped $\rightarrow V(t) = \frac{B}{C} + A_1e^{-\alpha t} + A_2te^{-\alpha t}$

③ Write Solution to, $t>0$

③ Find A_1, A_2 Using initial conditions

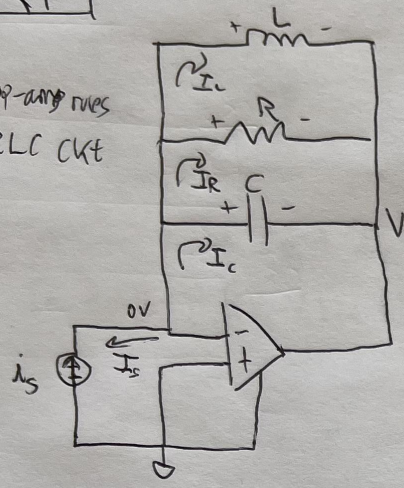
- need $V(0^+)$, $\frac{d}{dt}V(0^+)$
- Use $i_L(0^-) = i_L(0^+)$

  $V_c(0^-) = V_c(0^+)$



HW9 Q1

① apply op-amp rules
② solve RLC ckt



a) Find diff. eq.

Goal: $\boxed{\frac{d^2}{dt^2}V_0 + A\frac{d}{dt}V_0 + BV_0 = 0}$

a ① $V^+ = 0V = V^-$

$i^- = 0A$

$V_C = V_R = V_L = 0 - V_0 = \underline{-V_0}$

$I_s + I_C + I_R + I_L = 0$

② $-i_s + C\frac{d}{dt}V_C + \frac{V_R}{R} + i_L(0) + \frac{1}{L}\int V_L\,dt = 0$

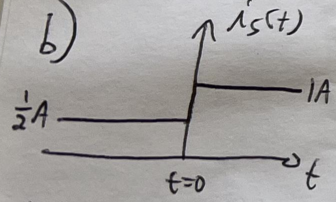
$\frac{d}{dt}\left[-i_s - C\frac{d}{dt}V_0 - \frac{V_0}{R} + i_L(0) - \frac{1}{L}\int V_0\,dt = 0\right]$

$-\frac{d}{dt}i_s - C\frac{d^2}{dt^2}V_0 - \frac{1}{R}\frac{d}{dt}V_0 + 0 - \frac{1}{L}V_0 = 0$

rearrange

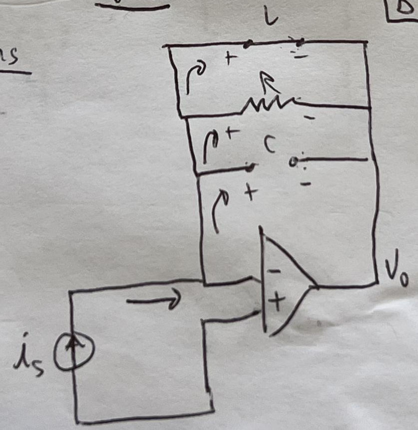
$$\boxed{\frac{d^2}{dt^2}V_0 + \underbrace{\frac{1}{RC}}_{A}\frac{d}{dt}V_0 + \underbrace{\frac{1}{LC}}_{B}V_0 = \underbrace{-\frac{1}{C}\frac{d}{dt}i_s}_{C}}$$

b)



Find initial conditions
- $V_0(0^+)$
- $\frac{d}{dt}V_0(0^+)$

t<0



b ① $V_C = -V_0$

$\underline{V_C(0^-) = V_C(0^+)}$

$V_C(0^-) = 0V = V_C(0^-) = V_C(0^+) = \boxed{V_0(0^+) = 0V}$

@ 0^+

$i_s = i_C + i_R + i_L$

$i_C = i_s - i_R - i_L$

② $\frac{d}{dt}V_0(0^+) = \frac{d}{dt}V_C(0^+) = \frac{i_C(0^+)}{C} = \boxed{\frac{i_s(0^+) - i_R(0^+) - i_L(0^+)}{C}}$

given

find

$i_L(0^-)$

$\frac{d}{dt}V_0(0^+)$

c) damping?

α vs w_0

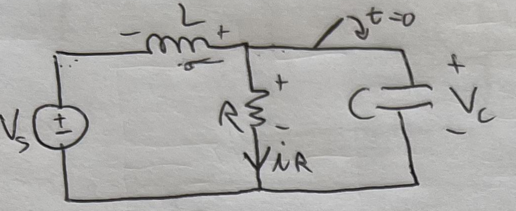
Q?

$Q = \frac{w_0}{2\alpha} = \#$ cycles

$Q \cdot T = \text{answer}$

sine wave period

HW 9 P2



Goal: equation for $i_R(t)$, $t>0$

$V_c(t) =$ ~~~

$i_R(t)\cdot R =$ ~~~

div by R   div by R

$i_R(t) = \frac{V_c(t)}{R}$

①

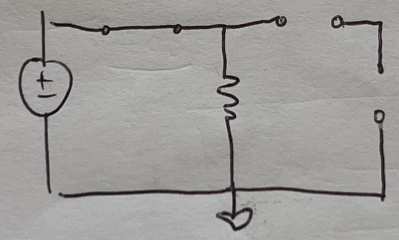
$$I_L + I_R + I_C = 0$$

$$\frac{d}{dt}\left[i_L(0) + \frac{1}{L}\int (V_c - V_s)\,dt + \frac{V_c}{R} + C\frac{d}{dt}V_c = 0 \right]$$

- rearrange into std. form
- determine damping
- write solution

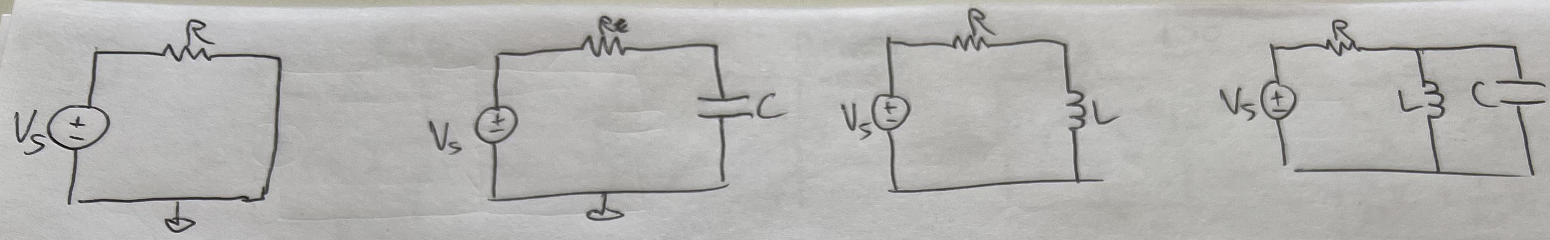
② initial conditions

$t<0$



- $V_c(0^+) = V_c(0^-) = 1V$
- $\frac{d}{dt}V_c(0^+) = 0.4V$

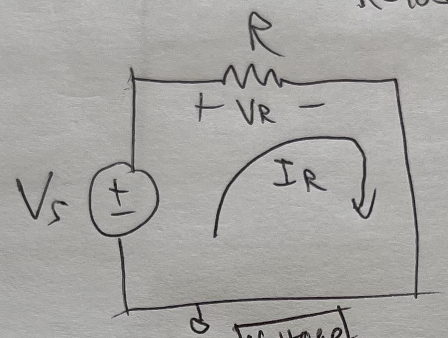
$$\frac{d}{dt}V_c(0^+) = \frac{i_c(0^+)}{C}$$



$\rightarrow V_s = 1V \ (t<5_s)$
$\rightarrow V_s = 5V \ (t>5_s)$

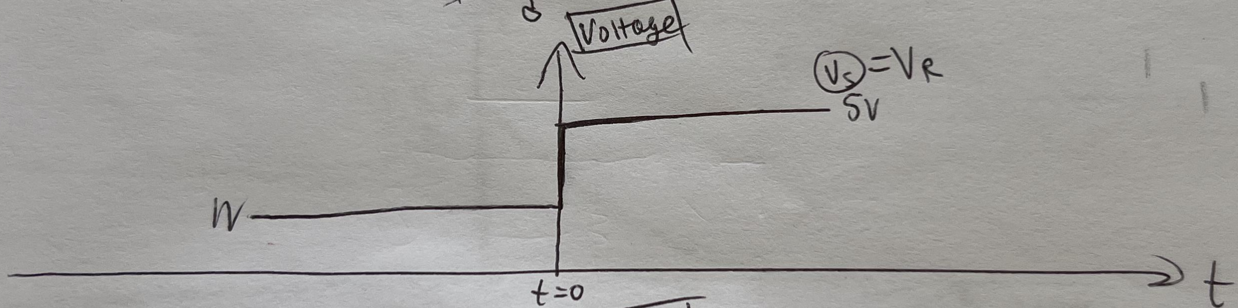
$\boxed{R}$

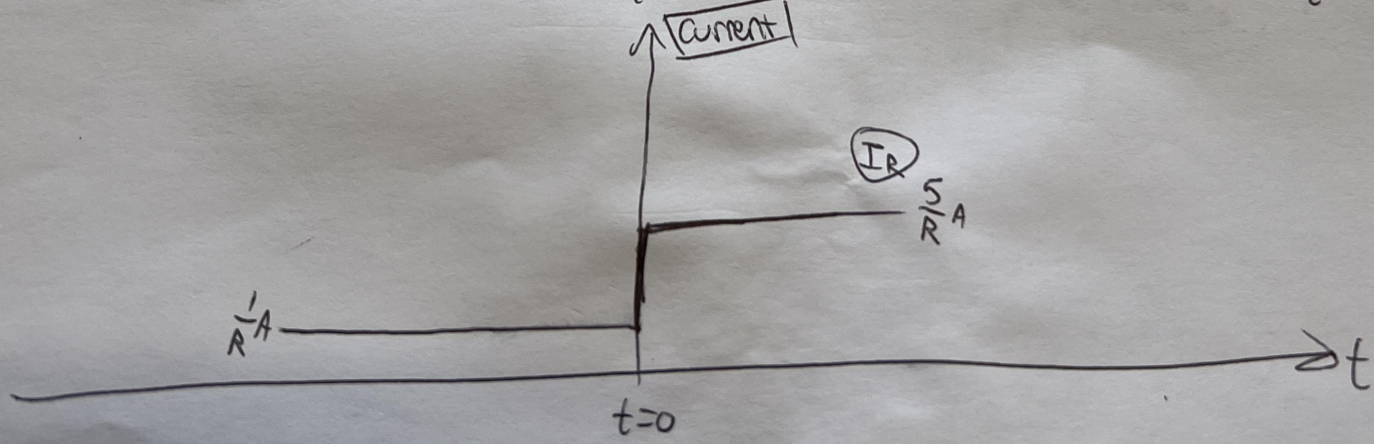
$R = 10\Omega$



$V_R = V_s$

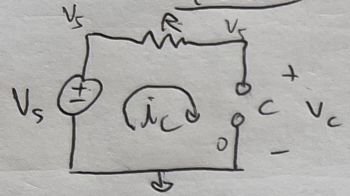
$I_R = \frac{V_s}{R}$



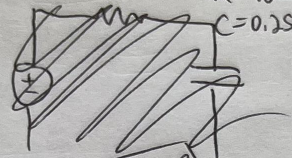


RC

t < 0

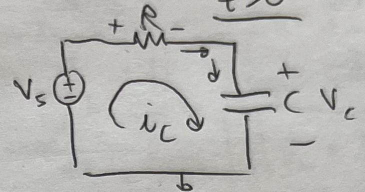


$V_c(0^-) = V_s(0^-) = 1V$



$R = 10\Omega$
$C = 0.25F$

t > 0



$I_R = i_c$

$\frac{V_s - V_c}{R} = C\frac{d}{dt}V_c$

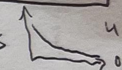
$\boxed{\frac{d}{dt}V_c + \frac{1}{RC}V_c = \frac{V_s}{RC}}$

$\boxed{V_c(t) = V_s + ke^{-t/RC}}$

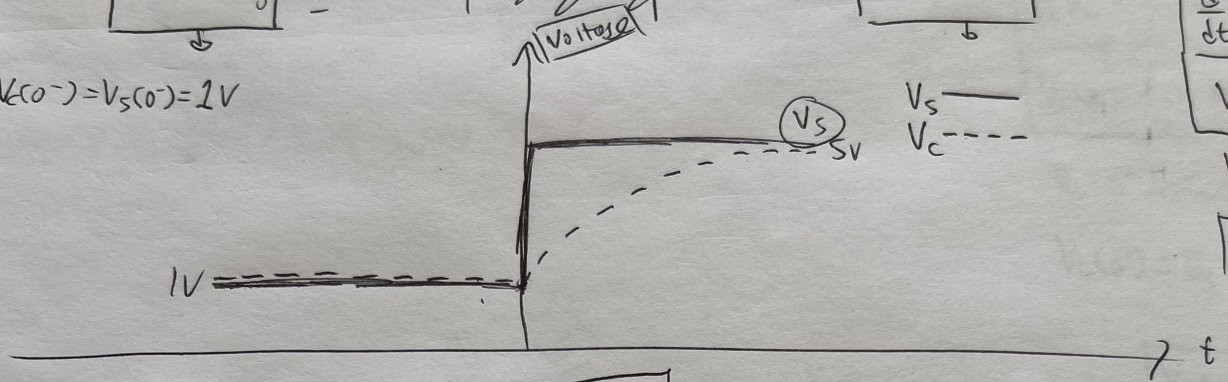
$V_c(0^+) = V_c(0^-) = V_s(0^-) = 1V$
$\rightarrow k = -4$

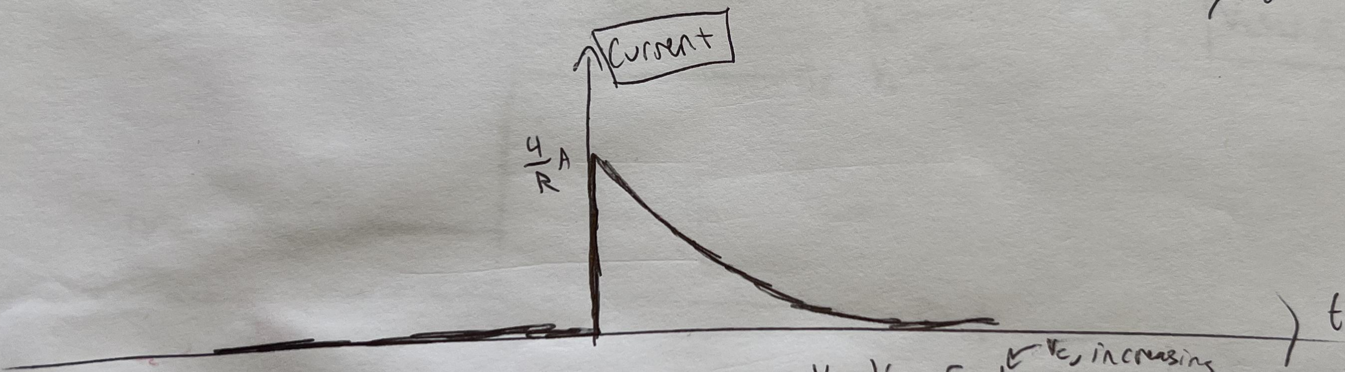
$\boxed{V_c(t) = 5 - 4e^{-t/2.5}}$

5V minus



@t=0, 1V
@t=∞, 5V

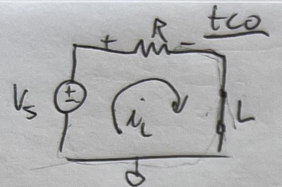




V_c increasing

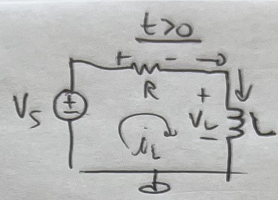
@ $t = 0^+$, $i_c = \frac{V_s - V_c}{R} = \frac{5 - 1}{R} = \frac{4}{R}$

RL



$R = 10\Omega$

$L = 10H$



$I_R = I_L$

$\frac{V_S - V_L}{R} = i_L(0) + \frac{1}{L}\int V_L\,dt$

$\boxed{\frac{d}{dt}V_L + \frac{R}{L}V_L = 0}$

$\boxed{V_L(t) = 0 + Ke^{-t\frac{R}{L}}}$

$V_L(0^+)$ ← use $i_L(0^+)$ to get

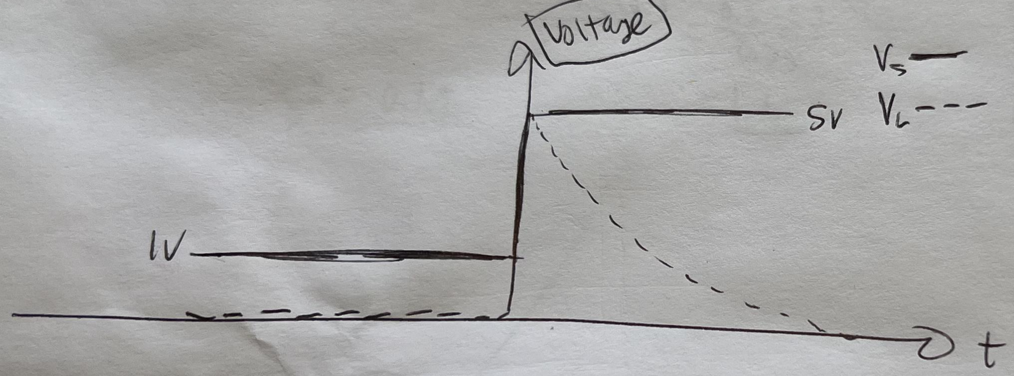
$V_L(0^+) = V_S - i_L(0^+)\cdot R = 5 - 0.1\cdot 10 = \underline{4V}$

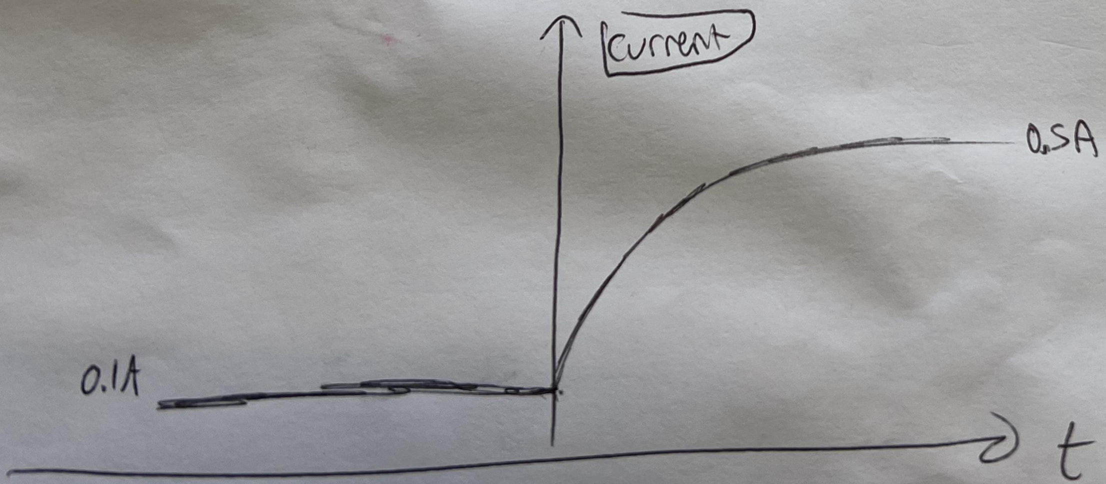
$K = 4$

$\boxed{V_L(t) = 4e^{-t}}$

$i_L(0^-) = i_L(0^+)$

$i_L(0^-) = \frac{\overset{1V}{V_S(0^-)}}{R} = 0.1A = i_L(0^+)$

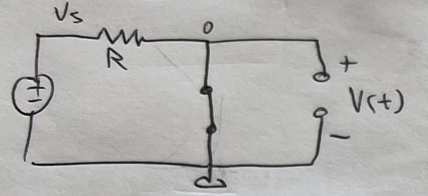




$i_L = \frac{V_S - V_L}{R} = \frac{5 - \ }{R}$

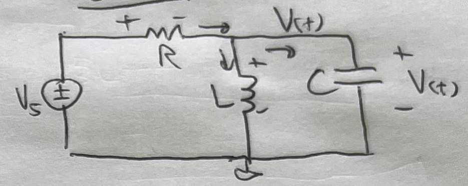
RLC

$t<0$



$R = 10\Omega$
$L = 10H$
$C = 0.25F$

$t>0$

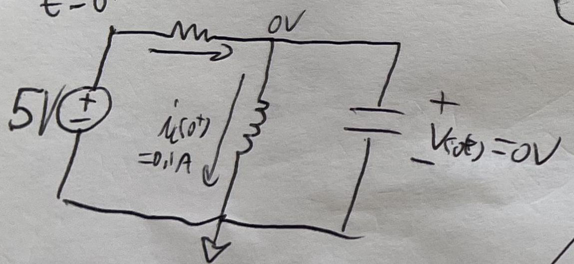


① $V_c(0^-) = \underline{0V}$ Inductor $= V_c(0^+) = \underline{V(0^+)}$ ✓

② $\frac{d}{dt}V(0^+) = \frac{d}{dt}V_c(0^+) = \frac{i_c(0^+)}{C} = \frac{i_R(0^+) - i_L(0^+)}{C} = \frac{0.5-0.1}{0.25} = \underline{1.6}$

$i_L(0^+) = i_L(0^-) = \frac{V_s - 0}{R} = \underline{0.1A}$

$t=0^+$



$i_R(0^+) = \frac{5-0}{R} = \underline{0.5A}$

$I_R = I_L + I_C$

$\frac{d}{dt}\left(\frac{V_s - V(t)}{R} \pm i_L(0) + \frac{1}{L}\int V(t)\,dt + C\frac{d}{dt}V(t) = 0\right)$

rearrange

$$\boxed{\frac{d^2}{dt^2}V(t) + \underbrace{\frac{1}{RC}}_{A}\frac{d}{dt}V(t) + \underbrace{\frac{1}{LC}}_{B}V(t) = \underbrace{0}_{C}}$$

$2\alpha = A$, $\omega_0^2 = B$

$\alpha = 0.1$, $\omega_0 = 0.632$

$\alpha < \omega_0 \rightarrow$ underdamped

$$\boxed{V(t) = \cancel{\frac{C}{B}}^{0} + e^{-\alpha t}\left[A_1\cos(\omega_d t) + A_2 \sin(\omega_d t)\right]}$$

$A_1 = ?$ $A_2 = ?$

- $V(0^+) = \underline{0V}$
- $\frac{d}{dt}V(0^+) = \underline{1.6}$

① $\cancel{V(0^+)}^{0} = \cancel{e^{-\alpha(0)}}^{1}\left[A_1 \cancel{\cos(0)}^{1} + A_2 \cancel{\sin(0)}^{0}\right]$

$\boxed{0 = A_1}$

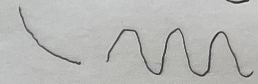
② $\cancel{\frac{d}{dt}V(0^+)}^{1.6} = 0.1A_2\cancel{e^{-0.1(0)}}^{1}\left[2\cancel{\cos(0)}^{1} - \cancel{\sin(0)}^{0}\right]$

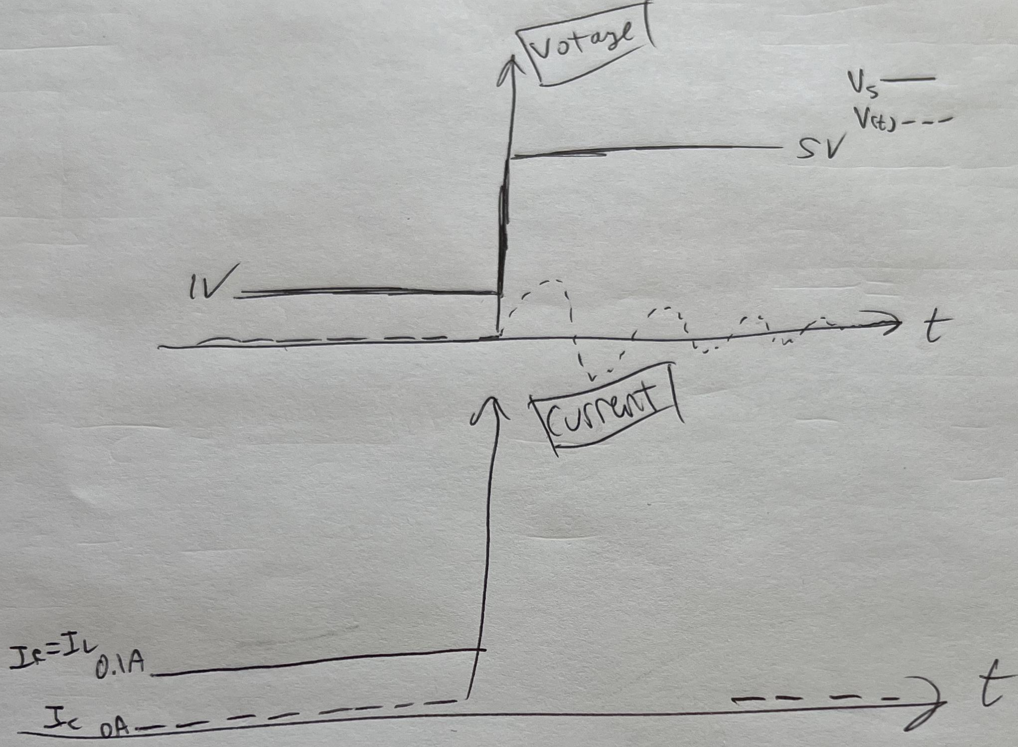
$\boxed{A_2 = 8}$

$$\boxed{V(t) = e^{-0.1t}\left[8\sin(\omega_d t)\right]}$$

$\boxed{\text{RLC}}$

$$V(t) = e^{-0.1t}\left[8\sin(\omega_d t)\right]$$





Final

$$I_C(\infty) = 0\,A$$

$$I_L = I_R(\infty) = \frac{5V}{10\Omega} = 0.5\,A$$