#### \*\*You may detach this page from the test booklet\*\*

#### PLEASE READ THESE INSTRUCTIONS CAREFULLY

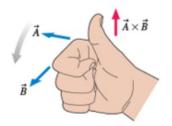
- Allowed: Calculator and an 8.5"×11" reference sheet with handwritten notes on both sides
- **Not Allowed:** phones, laptops, tablets, headphones, music players, cameras, anything with internet connectivity. Put these away while the exam is in progress.

#### • PRINT YOUR NAME AND SPIRE ID ON THE EXAM BOOKLET AND ANSWER SHEET

## • >>> USE #2 PENCIL TO FILL IN THE CIRCLES ON ANSWER SHEET WITH YOUR NAME (last name first) and SPIRE ID. <<<

- Please go to the restroom before the midterm starts.
- Unless friction or air resistance are mentioned, you can assume that they are negligible.
- Use #2 Pencil to fill the circles with your answers in spaces 1 through 27. Each question is worth 1 point. Only bubble in one circle per answer, or you may not receive credit. Erase pencil marks cleanly.
- When done, hand in ANSWER SHEET, EXAM BOOKLET, and show your UMass ID.
- There are 27 questions but the exam will be graded out of 25. This means you can get one question incorrect and still get a perfect score.

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center	L	$\frac{1}{12}ML^2$	Cylinder or disk, about center	R	$\frac{1}{2}MR^2$
Thin rod, about end	$L \longrightarrow$	$\frac{1}{3}ML^2$	Cylindrical hoop, about center	R	$MR^2$
Plane or slab, about center	a b	$\frac{1}{12}Ma^2$	Solid sphere, about diameter	R	$\frac{2}{5}MR^2$
Plane or slab, about edge	b b	$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$



#### Math

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$ax^2 + bx + c = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}} = u_x v_x + u_y v_y + u_z v_z = \vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}} = |\vec{\boldsymbol{u}}| |\vec{\boldsymbol{v}}| cos(\theta)$$

$$\vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}} = (u_y v_z - u_z v_y) \hat{\boldsymbol{i}}$$

$$+ (u_z v_x - u_x v_z) \hat{\boldsymbol{j}}$$

$$+ (u_x v_y - u_y v_x) \hat{\boldsymbol{k}}$$

$$|\vec{\boldsymbol{u}} \times \vec{\boldsymbol{v}}| = |\vec{\boldsymbol{u}}| |\vec{\boldsymbol{v}}| sin(\theta)$$

## Angular Motion

$$a = v^{2}/r = \omega^{2}r$$

$$v = \omega r$$

$$\omega = 2\pi/T$$

$$L = r\theta; v = r\omega; a = r\alpha$$

$$\theta(t) = \theta_{0} + \omega_{0}t + \frac{1}{2}\alpha t^{2}$$

$$\omega(t) = \omega_{0} + \alpha t$$

$$\omega^{2} = \omega_{0}^{2} + 2\alpha\Delta\theta$$

#### 1D Kinematics

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = v_0 + a t$$

$$\Delta x = \frac{v_1^2 - v_0^2}{2a}$$

## Projectile Motion

Range: 
$$D = \frac{v_0^2 \sin(2\theta)}{g}$$
  
[Same initial/final height only]

## **Dynamics**

$$\sum_{\vec{F}_{ext}} \vec{F}_{ext} = m\vec{a}$$

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

$$\vec{F}_{c} = \frac{mv^{2}}{r} (toward - \hat{r})$$

$$\vec{F}_{spring} = -k\Delta \vec{x}$$

## Rotational Dynamics

$$\vec{x}_{CM} = \sum m_i \vec{r_i} / \sum m_i$$

$$I = \sum m_i r_i^2$$

$$I_{parallel} = I_{CM} + Md^2$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}| |\vec{F}| \sin \phi$$

$$\sum \vec{\tau} = I\alpha$$

$$v_{CM} = R\omega \text{ (rolling without slipping)}$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = I\omega \text{ (fixed axis)}$$

#### Friction

$$\begin{aligned} f_k &= \mu_k N \\ f_s &\leq \mu_S N \end{aligned}$$

#### Momentum

$$\vec{p} = m\vec{v}$$

$$\vec{p}_i = \vec{p}_f \ if \ \sum_i \vec{F}_{ext} = 0$$

### Elastic collision in 1D:

$$v_{1f} = \frac{(m_1 - m_2)}{(m_1 + m_2)} v_{1i} + \frac{2m_2}{(m_1 + m_2)} v_{2i}$$

$$v_{2f} = \frac{2m_1}{(m_1 + m_2)} v_{1i} + \frac{(m_2 - m_1)}{(m_1 + m_2)} v_{2i}$$

## *Impulse*

$$\vec{J} = \int \vec{F} \, dt$$
$$\Delta \vec{p} = \vec{F}_{avg} \Delta t$$

## Energy

$$KE_{trans} = \frac{1}{2}mv^{2}$$

$$KE_{rot} = \frac{1}{2}I\omega^{2}$$

$$U_{grav} = mgh$$

$$U_{spring} = \frac{1}{2}kx^{2}$$

$$W = \int \vec{F} \cdot d\vec{l}$$

$$W = \vec{F} \cdot \Delta \vec{x} \text{ (if F constant)}$$

$$W_{net} = \Delta KE$$

$$P = \frac{dW}{dt}$$

$$P = \vec{F} \cdot \vec{v} \text{ (if F constant)}$$

# **Conversion Factors and Constants**

1 minute = 60s 1 hour = 3600s 1 mile = 1.60934 km 1 mile = 5280 feet 1 foot = 0.3048 meters 1 foot = 12 inches 1 inch = 2.54 cm  $g = 9.8 m/s^2$ 

$$\begin{array}{c} \text{Hypotenuse} \\ \theta \\ \text{Adjacent} \end{array} \quad \begin{array}{c} \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \\ \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \\ \tan \theta = \frac{\text{opposite}}{\text{adjacent}} \end{array}$$

1. A simple watermelon launcher is designed as a spring with a light platform for the watermelon. When an 8.00 kg watermelon is put on the launcher, the launcher spring compresses by 10.0 cm. The watermelon is then pushed down by an additional 30.0 cm and it's ready to go. Just before the launch, how much energy is stored in the spring?

- A) 35 J
- B) 63 J
- C) 120 J
- D) 140 J
- E) 160 J



2. A 2 kg block sliding along a rough table hits a spring and comes to a stop after the spring is compressed 0.2 m. If the spring constant is 20 N/m and the coefficient of friction between the block and the table is 0.15, what was the speed of the block right before it hit the spring?

- A) 0.2 m/s
- B) 0.4 m/s
- C) 0.6 m/s
- D) 0.8 m/s
- E) 1.0 m/s

3. A 80kg silverback gorilla is standing atop a spring in an elevator as it accelerates upwards at 3m/s<sup>2</sup>. The spring constant is 2500N/m. By how much is the spring compressed?

- A) 0.21 m/s
- B) 0.41 m/s
- C) 0.61 m/s
- D) 0.81 m/s
- E) 1.01 m/s

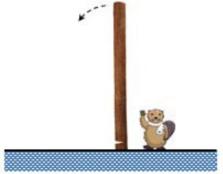
4. Starting from rest, a 28 kg child goes down a slide 3.0 m high and 4.0 m wide (see diagram). If her speed at the bottom is 2.5 m/s, how much energy has been lost to friction?



- A) 630 J
- B) 680 J
- C) 700 J
- D) 740 J
- E) 790 J

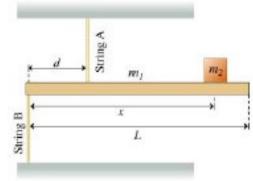
5. An industrious beaver manages to chew through the base of a wooden tree trunk 18 m tall, causing it to fall over. If the base of the trunk does not slide while it falls over, how fast is the other end of the trunk moving right before it hits the ground? [The thin rod moments of inertia are useful here.]

- A) 17 m/s
- B) 20 m/s
- C) 23 m/s
- D) 26 m/s
- E) 29 m/s



6. The diagram shows a uniform horizontal bar of mass m1 = 3 kg supporting a box of mass m2 = 10 kg. The bar is held up from the ceiling by String A and the bar is held down by String B tied to the floor. Both strings are pulling vertically. Use d = 20 cm, x = 60 cm, and L = 70 cm. What is the tension in String B when the system is in equilibrium?

- A) 220 N
- B) 240 N
- C) 260 N
- D) 270 N
- E) 280 N



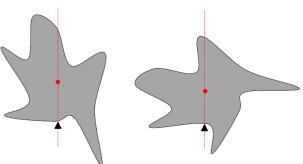
- 7. A circular pulley of mass M and radius R has a light cord wrapped around it and initially supports an anvil of mass m. Unfortunately the pulley gets loose and starts rotating (in place) freely, allowing the cord to unravel. What is the total kinetic energy of the system when the anvil reaches the speed v? [Use moment of inertia for pulley  $I=MR^2/2$ ]
- A) m  $v^2/2$
- B)  $M v^2/2$
- C)  $(m+M/2) v^2/2$
- D)  $(m+M) v^2/2$
- E)  $(m/2+M) v^2/2$
- 8. A potter's wheel (a solid, uniform disk) of mass 6.5 kg and radius 0.50 m spins about its central axis. A 2.1 kg lump of clay is then dropped onto the wheel at a distance 0.40 m from the axis. What is the moment of inertia for the combined system about the axis? (Use moment of inertia for the disk)
- A)  $0.81 \text{ kg m}^2$
- B)  $1.15 \text{ kg m}^2$
- C)  $1.31 \text{ kg m}^2$
- D)  $3.25 \text{ kg m}^2$
- E)  $0.71 \text{ kg m}^2$

- 9. Prof. Swarzenegger picks up his 45 kg brief case on the 17<sup>th</sup> floor of his office building (85 m above the ground) and prepares to go to class. He walks 24 meters straight to the elevator at 1.5 m/s, takes the elevator to the ground floor at a steady 3 m/s, and walks 175 m in a straight line at 2 m/s south to the classroom, where he deposits his briefcase on the floor. About how much work has he done on the briefcase over the entire trip?
- A) 40kJ
- B) 4kJ
- C) zero
- D) -40kJ
- E) -4kJ
- 10. A thin rod of mass m=3kg and L=5m length is rotating about an axis located at a distance D=0.28m from the edge. What is its moment of inertia?
- A) 27 m
- B) 25 m
- C) 18 m
- D) 16 m
- E) 21 m





- 11. One can easily find the center of mass for a complex body by locating balancing points and drawing vertical lines (see figure). The center of mass is at the intersection of these lines. Where should one place the axle with predetermined direction so that the moment of inertia for rotation about this axle is the smallest possible?
- A) As far from the center of mass as possible
- B) Anywhere on the edge
- C) Through the center of mass
- D) In the middle of the balancing line
- E) There is no easy way to figure it out

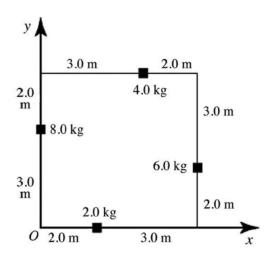


12. A monkey sits in a tree 60 m off the ground. He throws a 0.5 kg banana up and through the branches at 45 m/s (101 mph). (This monkey pitches for the New York Yankees during baseball season.) The banana lands on the ground 75 m away. Which of the following is closest to the banana's speed right before it hits the ground? (You may neglect air drag.)

- A) 80 m/s
- B) 60 m/s
- C) 40 m/s
- D) 30 m/s
- E) not enough information

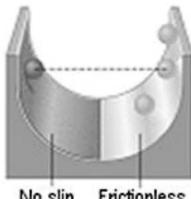
13. In the figure, four point masses are placed as shown. The x and y coordinates of the center of mass are closest to

- A) (2.2 m, 2.6m)
- B) (2.2 m, 2.7m)
- C) (2.3 m, 2.6m)
- D) (2.3 m, 2.7m)
- E) (2.3 m, 2.8m)



14. A ball is released from rest on a no-slip surface, as shown in the figure. After reaching the lowest point, the ball begins to rise again, this time on a frictionless surface as shown in the figure. When the ball reaches its maximum height on the frictionless surface it is

- A) at a greater height than when it was released
- B) at a lesser height than when it was released
- C) at the same height as when it was released
- D) impossible to answer without knowing its mass
- E) impossible to answer without knowing its radius



- 15. A hollow sphere (moment of inertia  $I=(2/3)MR^2$ ) is rolling along a horizontal floor at 5.0 m/s when it comes to a  $30^0$  incline. How far up the incline does it roll before reversing direction?
- A) 3.35 m
- B) 3.50 m
- C) 3.85 m
- D) 4.25 m
- E) 4.05 m
- 16. A father and his daughter are out having a running race. The father is running with half the kinetic energy of his daughter, but she has half the mass of her father. If the daughter is running atspeed *v*, then how fast is the father running?
  - *A*. 0.25*v*
  - *B*. 0.5*v*
  - C. v
  - D. 2*v*
  - E. 4*v*
- 17. A 1000-kg coal-powered train engine rolls by a coal bin at 11 mph. As the engine passes, a 250-kg load of coal is suddenly dropped straight down onto the engine. Howfast is the engine moving right after the coal has landed?
  - **A)** 4 mph
  - **B)** 6 mph
  - **C**) 9 mph
  - **D)** 11 mph
  - **E)** 44 mph

18. Typical torque for tightening a lug nut on the wheels of a Mustang is 95 ft-lbs (129 Nm). How hard would a mechanic need to push on the end of a 0.5 m torque wrench to achieve this?

- A) 258 N
- B) 228 N
- C) 278 N
- D) 298 N
- E) 238 N



19. The Sun's mass is  $2.0 \times 10^{30}$  kg, its radius is  $7.0 \times 10^{5}$  km, and it has a rotational period of approximately 28 days. If the Sun should collapse into a neutron star of radius 15 km, what would its period be if no mass were ejected (not realistic but let's assume) and a sphere of uniform density can model both the Sun and the neutron star?

- A) 100 s
- B) 10 s
- C) 1 s
- D) 0.01 s
- E) 0.001 s

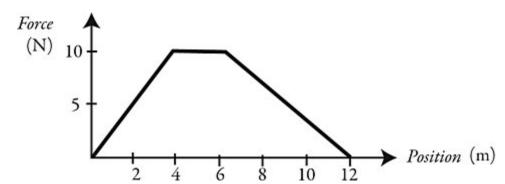
20. Suppose the polar ice sheets broke free and quickly floated toward Earth's equator without melting. What would happen to the duration of the day on Earth?

- A) It will remain the same
- B) Days will become longer
- C) Days will become shorter

21. What's the dot product of  $(2i + 2j) \cdot (4i + 2k)$ ?

- A) 8
- B) 12
- C) 24
- D) 48
- E) 0

- 22. Safety standards call for a 1900-kg car colliding at 12 m/s with a concrete wall to experience an average force (during the collision) not greater than 50,000 N. What is the minimum permissible time for the car to come to a stop during such a collision?
  - **A)** 0.12 s
  - **B)** 0.18 s
  - **C)** 0.22 s
  - **D)** 0.38 s
  - **E)** 0.46 s
- 23. Kendall pulls on a suitcase strap at an angle 36° above the horizontal. If 650J of work are done by the strap while moving the suitcase a horizontal distance of 15 m, what is the tension in the strap?
- A) 54N
- B) 61N
- C) 44N
- D) 66N
- 24. An object is acted upon by a force that represented by the force vs. position graph in the figure. What is the work done as the object moves from 4m to 6m?



- A) 0J
- B) 10J
- C)20J
- D)30J
- E)40J

- 25. A 7-kg bowling ball moving at +21 m/s collides with a bowling pin of mass 7 kg in ahead-on elastic collision. What is the velocity of the ball immediately after the collision?
- A. 10.5 m/s
  - B. -10.5 m/s
  - C. 3 m/s
  - D. -3 m/s
  - E. 0 m/s
  - 26. A Ford Escort of mass 800 kg collides with a Mack Truck of mass 12,000 kg at rest. The Escort's incoming speed is 30 m/s and after the collision it bounces straight back with a speed of 15 m/s. If the collision lasts 2.0 seconds, what is the average force of the truck on the car during this time?
  - A. 6 kN
  - B. 18 kN
  - C. 27 kN
  - D. 36 kN
  - E. 270 kN
  - 27. Jocko the clown (who has a mass of 60 kg) is standing still at the center of the ice rink when his rival Bozo throws a 7 kg bowling ball in his direction at 10 m/s. Assuming Jockocatches the ball, how much energy is lost in the collision?
  - A. 207 J
  - B. 412 J
  - C. 313 J
  - d. 540 J
  - e. 820 J

## Answers

1	В
	E
3	В
4	D
5	C
6	A
7	C
8	В
9	D
10	E
11	С
12	В
13	E
14	В
15	D
16	В
17	С
18	A
19	Е
20	В
21	A
22	Е
23	A
24	С
25	Е
26	В
27	С