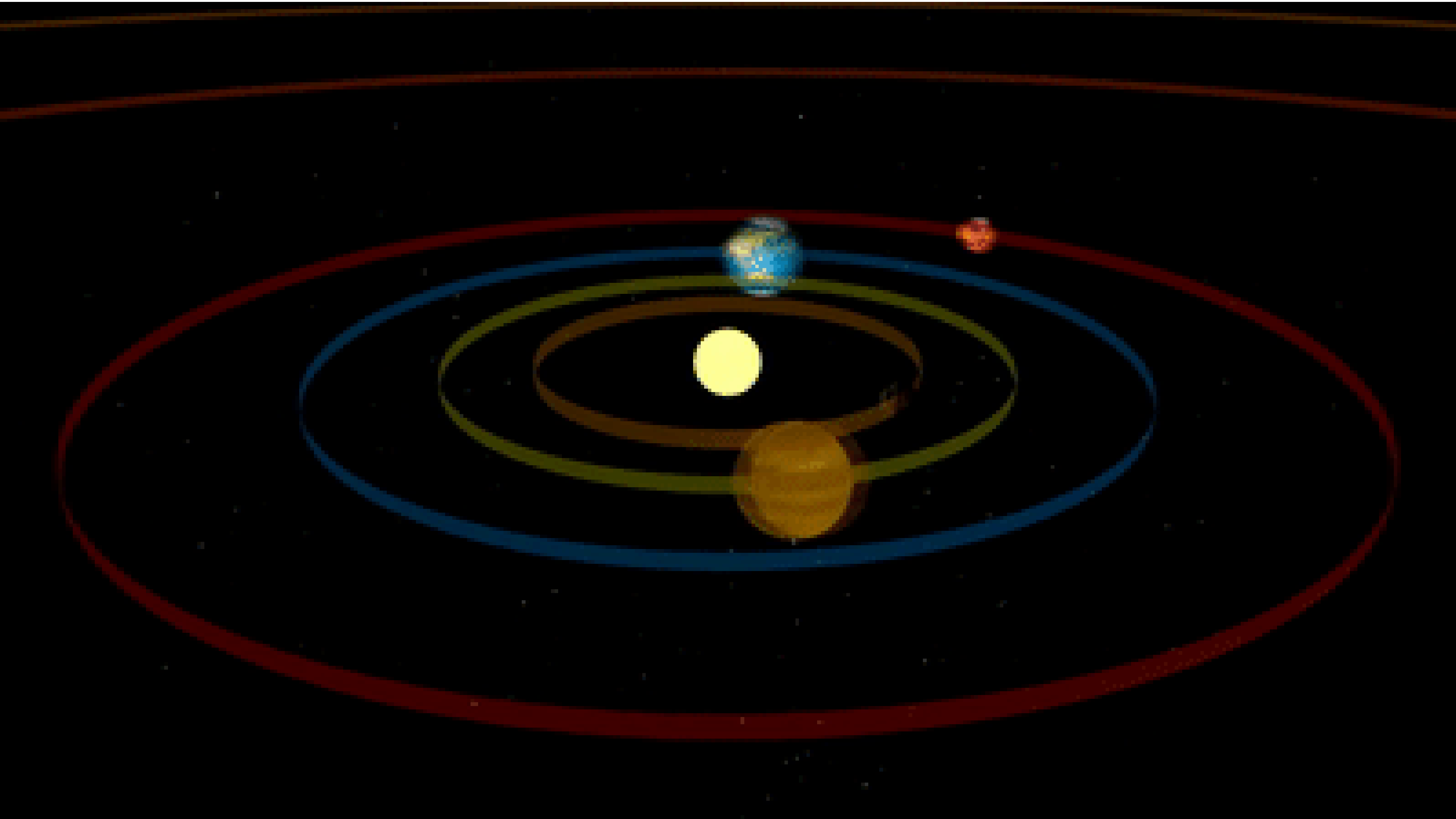


Announcements, Goals, and Reading

Announcements:

- HW04 due Tuesday October 11th, 11:59pm on Mastering Physics
- HW03 is past due. Grace period ends Friday.
- Midterm 1: Thursday 10/20, 7-9PM

Goals for Today:

- Circular Motion

Reading (Physics for Scientists and Engineers 4/e by Knight)

- Chapter 4: Kinematics in 2D

Midterm 1

October 20th, 7-9PM

- **Covers Chapters 1-5* from Knight textbook, Homework 1-5***
- **Key topics: Motion, Acceleration, Position, Velocity, Kinematics, Projectile Motion, Circular Motion, and Forces*.** *No questions about sig. figs or relative motion.*
- Location depends on 1st letter of your last name:
  - **HAS20 – Last Name A-F**
  - **HAS124 - Last Name G-H**
  - **ISB135 - Last Name I-M**
  - **ILCN151 - Last Name N-T**
  - **HAS126 - Last Name U-Z**
  - **HAS138 - Reduced distraction / Extra time accommodation**
  - **Online-only students (UWW/FLEX sections): You will be contacted about details this week.**
  - ***If you have extra time accommodations, please take the exam in HAS 138. I will come at the end to proctor the extra time. You can also take the exam with Disability Services. If you need other disability accommodations, please contact me.***

ISB: Integrated Sciences Building
ILC: Integrative Learning Center

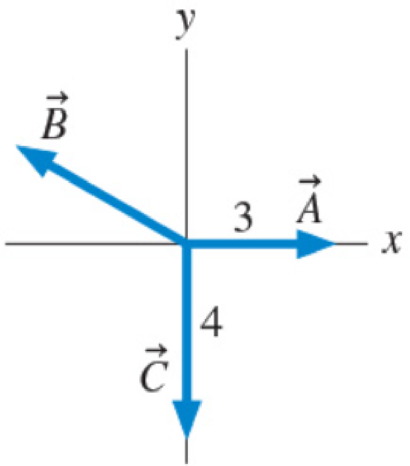
- **Allowed a Calculator and an 8.5”×11” reference sheet with handwritten notes on both sides.** Be sure to bring a calculator as there are never enough spares.
- ~25 Multiple Choice Questions; **Bring a #2 pencil**
- Practice problems will be posted on Moodle and Mastering Physics
- SI/TA exam review sessions will be held on exam week.
- Makeup Exams: If you have a conflict with another exam, please let us know as soon as possible. Friday 10/14 will be the last day to request a makeup exam without penalty. E-mail our TA, Joanna Wuko (jwuko@physics.umass.edu) and CC me.

**Questions about Force will be limited in number, scope and complexity.*

Frequently Asked Question: How to enter unit vectors in Mastering?

For the three vectors shown in (Figure 1), $\vec{A}+\vec{B}+\vec{C}=1\hat{j}$. What is vector $\vec{B}$?

Figure 1 of 1



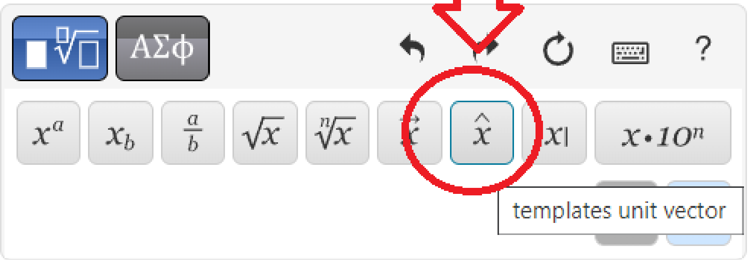
Part A

Write $\vec{B}$ in component form.

Express your answer in terms of the unit vectors $\hat{i}$ and $\hat{j}$. Use the 'unit vector' button to denote unit vectors in your answer.

Unit vector button is here

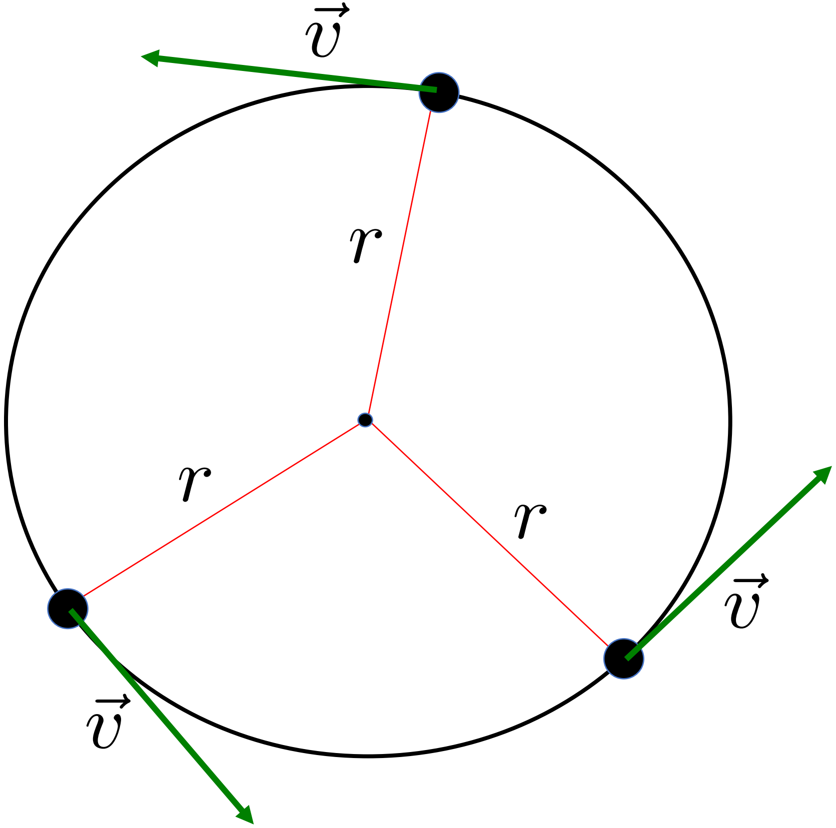
View Available Hint(s)



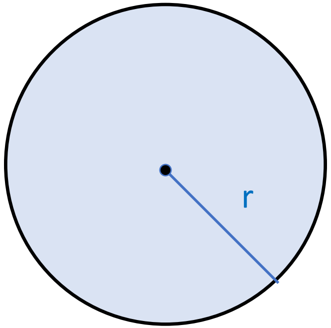
$\vec{B}$ =

Submit

Uniform Circular Motion

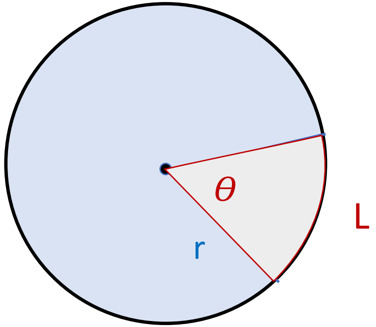


Circles – Essential Info



Circumference $C = 2\pi r$ (circle perimeter length)

Arc length $L = \theta r \Longleftrightarrow \theta = \dfrac{L}{r}$



length of portion of circle subtended by angle θ

<u>Potentially confusing point!</u>

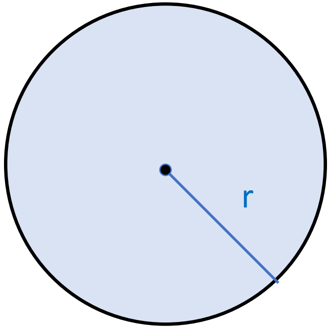
What are the dimensions of angles?

Infer from arc length that angles are dimensionless

Nonetheless, apart from **radians** (just a label attached to a number to signal that it is an angle) we also use **degrees** and **revolutions** to measure angles

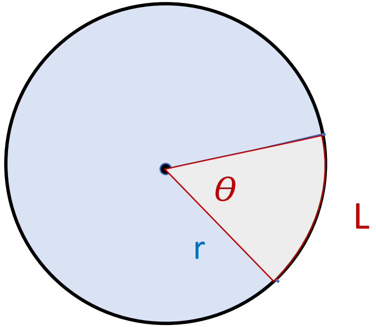
Circles – Essential Info

Circumference $C = 2\pi r$ (circle perimeter length)



Arc length $L = \theta r \Leftrightarrow \theta = \frac{L}{r}$

length of portion of circle subtended by angle θ



2π radians gives a full circle.

Hence circumference is arc length of full circle

Taking θ=2π in arc length reproduces circumference

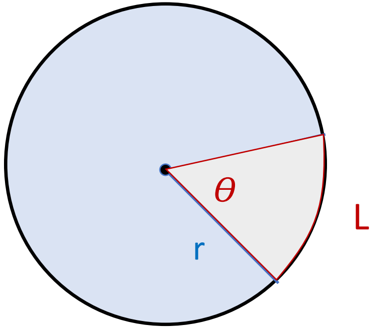
For fixed angle θ ⇨ Increasing radius increases arc length, and vice-versa

Important to keep this in mind

Circles – Essential Info

3 Units for angles

Radians, Degrees, and Revolutions



360 degrees = 2π radians = 1 revolution

Example: A turntable spins at 33.3 revolutions per minute (rpm).

How many radians per second is this?

1 revolution is going one full circle around the center

$$\left(33.3\frac{rev}{min}\right)\left(\frac{2\pi rad}{1rev}\right)\left(\frac{1min}{60s}\right)$$

$$= 3.49rad/s$$

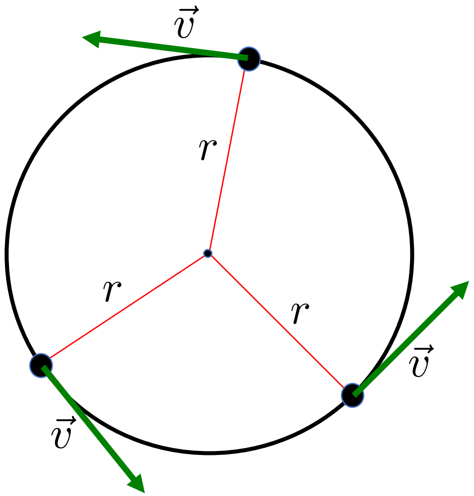
Uniform Circular Motion

Come back to this

Particle moving in a circular path with constant speed

Magnitude of velocity stays constant, but direction changes

Particle is accelerating!



Characterizing uniform circular motion…

Radius ➡ r

Period ➡ T time required for one full revolution $= \dfrac{Total\ Time}{Total\ Number\ of\ Revolutions}$

Speed ➡ $v = \dfrac{\text{circumference}}{\text{period}} = \dfrac{2\pi r}{T}$

$|\vec{v}| = v$ Wherever particle is on circle

Example

Rotating cylinder

Diameter 4.0 cm
Turns at 2400 rpm = 2400 turns in 60 seconds
Find the speed of a point on its surface?

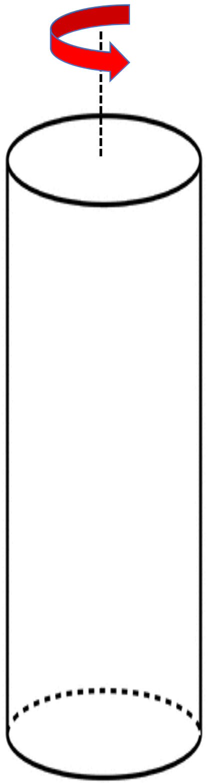
Revolutions per Minute

$$v = \frac{\text{circumference}}{\text{period}} = \frac{2\pi r}{T}$$

Radius r = .02m

Period $$T = \frac{60s}{2400} = 0.025s$$

Speed $$v = \frac{2\pi(.02m)}{(.025s)} = 5.0m/s$$



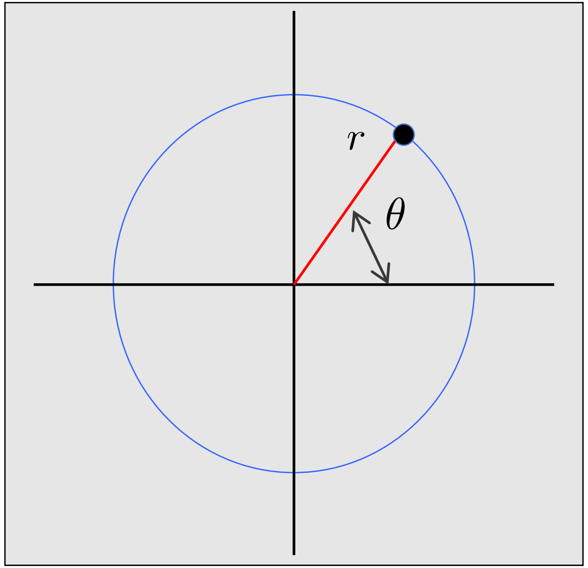
Particle on a circular path

Also talk about more general motion for particle constrained to move on a circle

Not necessarily at a constant rate

Define angle θ with respect to positive x-axis

Particle path can be described by angle as a function of time: $\theta(t)$



Position of particle described by a single function of time

Very similar to 1D motion described by x(t)

Introduce angular analogues of velocity and acceleration

How fast is particle going around circle at any point in time?
Is it speeding up or slowing down?

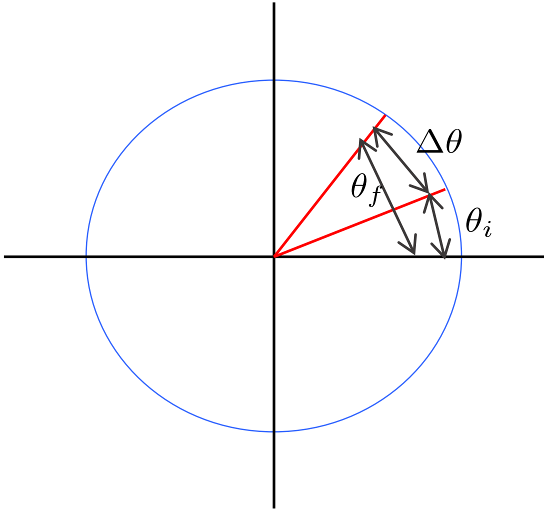
Pose similar problems as in 1D case

Call these “angular velocity” and “angular acceleration”

Particle on a circular path

Particle path can be described by angle as a function of time $\theta(t)$

Develop "angular velocity" as rotational analogue of linear velocity



Between times t_i and t_f
angle of particle changes from θ_i to θ_f

Angular displacement $\Delta\theta = \theta_f - \theta_i$

Change in time $\Delta t = t_f - t_i$

Analogous to definition of average linear velocity as displacement over change in time

Average angular velocity:

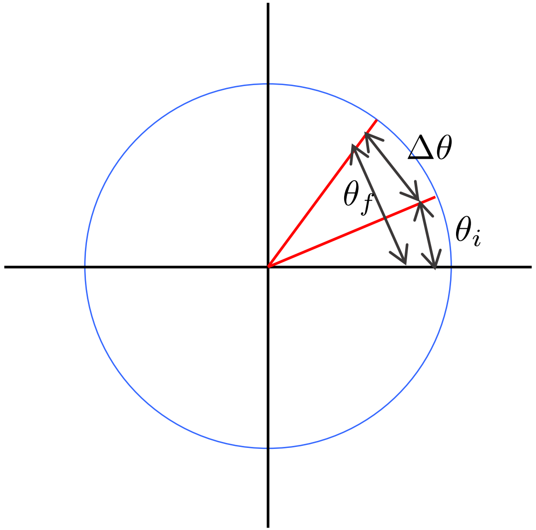
$$\omega_{avg} = \frac{\text{angular displacement}}{\text{change in time}} = \frac{\Delta\theta}{\Delta t}$$

Particle on a circular path

$$\theta(t)$$

Between times t_i and t_f

Angle of particle changes from θ_i to θ_f



Angular displacement $\Delta\theta = \theta_f - \theta_i$

Change in time $\Delta t = t_f - t_i$

Average angular velocity

$$\omega_{avg} = \frac{\text{angular displacement}}{\text{change in time}} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous angular velocity

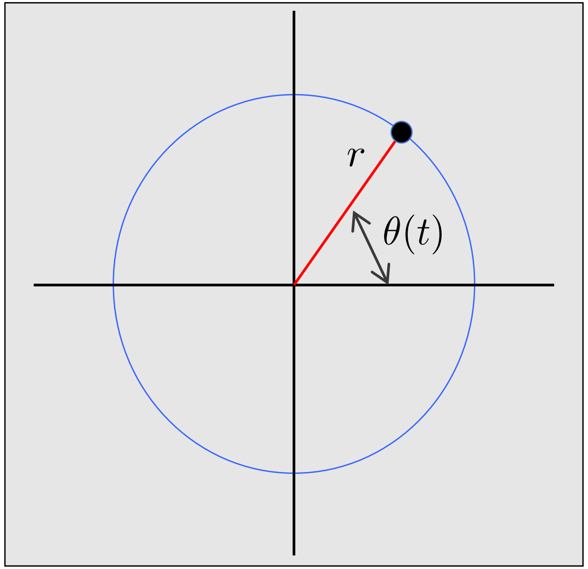
$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Analogous to limiting process that defined instantaneous linear velocity

Think of circular motion as like 1D motion, but with position on a circle instead of a line

Particle on a circular path

$\theta(t)$



Instantaneous angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

What are units of angular velocity?

Angles are actually dimensionless

Not length, mass or time!

Change in arc length for a certain change in angle is proportional to radius of circle

Take "units" of angle = radians

Units of angular velocity = radians/second

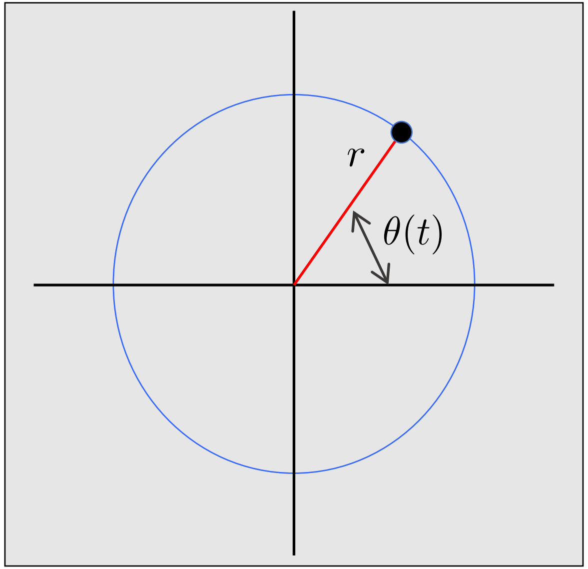
1 revolution of full circle = 2π radians
1 radian ~ 57.3 degrees

Example…

Particle goes around circle 1 time per each second

$$\omega = 2\pi\,\mathrm{rad}/s$$

Particle on a circular path $\theta(t)$



Often angular velocities are given in revolutions/second or revolutions/minute (rpm)

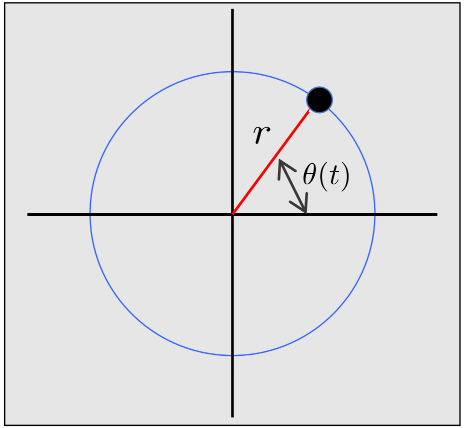
Convert these to radians/second

$$17\, rev/s = (17\, rev/s)\left(\frac{2\pi\, rad}{1\, rev}\right)$$

$$= 34\pi\, rad/s = 110\, rad/s$$

$$1.00\, RPM = \left(1\frac{rev}{min}\right)\left(\frac{2\pi\, rad}{1\, rev}\right)\left(\frac{1\, min}{60\, s}\right) = 0.105\, rad/s$$

Particle on a circular path $\theta(t)$



Define **angular acceleration**

Rate of change of angular velocity

Analogous to ordinary acceleration in 1D

ω_i Initial angular velocity at time t_i

ω_f Final angular velocity at time t_f

$$\Delta\omega = \omega_f - \omega_i$$ Change in angular velocity

$$\Delta t = t_f - t_i$$ Change in time

Average angular *acceleration*

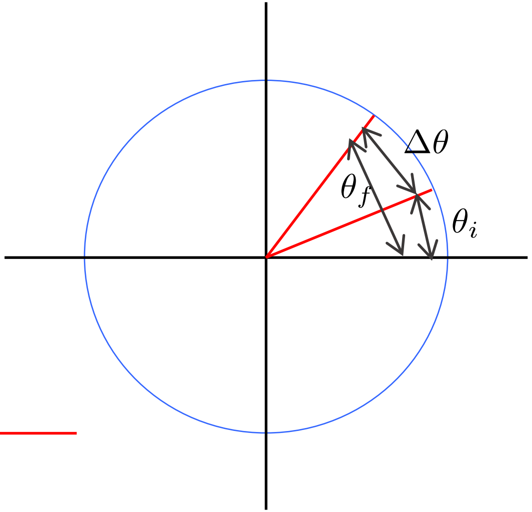
$$\alpha_{avg} = \frac{\text{change in angular velocity}}{\text{change in time}} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular *acceleration*

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Particle on a circular path

$\theta(t)$



Between times t_i and t_f
angle of particle changes from θ_i to θ_f

Angular displacement $\Delta\theta = \theta_f - \theta_i$

Change in time $\Delta t = t_f - t_i$

Average angular velocity:

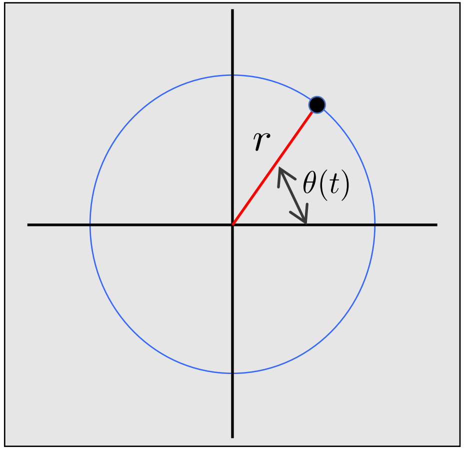
$$\omega_{avg} = \frac{\text{angular displacement}}{\text{change in time}} = \frac{\Delta\theta}{\Delta t}$$

Particle goes around circle once per second:
ω = 2π rad/s
ω = 360 degrees/s
ω = 1 rev/s

Instantaneous angular velocity:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Particle on a circular path $\theta(t)$



Define **angular acceleration**

Rate of change of angular velocity

Analogous to ordinary acceleration in 1D

$t_i \longrightarrow \omega_i$ Initial angular velocity

$t_f \longrightarrow \omega_f$ Final angular velocity

$$\Delta\omega = \omega_f - \omega_i$$ Change in angular velocity

$$\Delta t = t_f - t_i$$ Change in time

Average angular *acceleration*

$$\alpha_{avg} = \frac{\text{change in angular velocity}}{\text{change in time}} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous angular *acceleration*

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

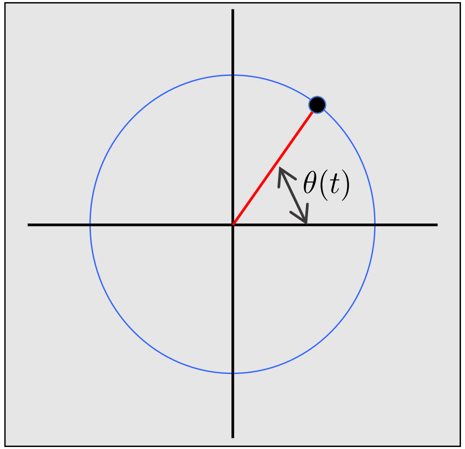
Very important analogy between linear motion and circular motion

Circular motion with constant angular acceleration

Analogous to linear motion with constant acceleration

$$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v(t) = v_0 + at$$



$$\theta(t) = \theta_0 + \omega_0\, t + \frac{1}{2}\alpha\, t^2$$

$$\omega(t) = \frac{d\theta}{dt} = \omega_0 + \alpha\, t$$

$$\alpha = \frac{d\omega}{dt} = \alpha \quad \checkmark$$

θ_0	Angular position at t=0
ω_0	Angular velocity at t=0
α	Constant angular acceleration

Example Problem

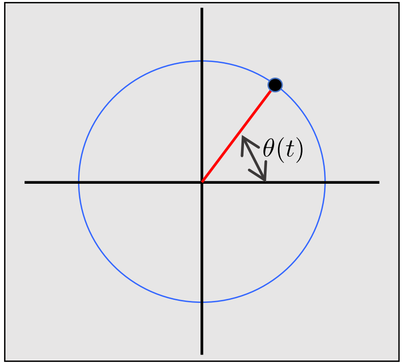
counterclockwise

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

What is the drill's angular acceleration?

Assume constant angular acceleration

$$\omega(t) = \frac{d\theta}{dt} = \omega_0 + \alpha\, t$$



Counterclockwise rotation → Θ is increasing with time

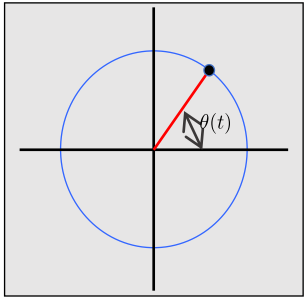
ω_0 is positive

Slows to rest → α is negative

Clockwise rotation has angular velocity ω<0

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

What is the drill's angular acceleration?



$$\omega(t) = \omega_0 + \alpha\, t$$

Find ω_0 in rad/sec

$$\omega_0 = (2400\frac{rev}{min})(\frac{2\pi\ rad}{1\ rev})(\frac{1\ min}{60\ s}) = 251.3\, rad/s$$

Find angular acceleration

$$\omega(3s) = \omega_0 + \alpha(3\, s) = 0$$

Solve for α in rad/s^2

Correct

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

$$\omega(t) = \omega_0 + \alpha\, t$$

Easiest to redo part A → Express angular acceleration in rev/s^2 rather than rad/s^2

$$\omega_0 = (2400\frac{rev}{min})(\frac{1\,min}{60\,s}) = 40.0\,rev/s$$

$$\omega(T) = 0 \;\Rightarrow\; \alpha = -\frac{\omega_0}{T} = -\frac{40.0\,rev/s}{3.00\,s} = -13.3\,rev/s^2$$

$$T = 3.00\,s$$

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

Now look at angular position at T=3.00s

$$\omega_0 = 40.0\, rev/s$$

$$\theta(t) = \theta_0 + \omega_0\, t + \frac{1}{2}\alpha\, t^2$$

$$\alpha = -13.3\, rev/s^2$$

$$\theta_0 = 0$$

Don't really care about angle at t=0

Plug in to get…

$$\theta(3s) = (40.0\, rev/s)(3\, s) + \frac{1}{2}(-13.3\, rev/s^2)(3\, s)^2 = 60\, rev$$

If we had found θ(3s) in radians…

→ Divide by 2π to convert to revolutions

A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

Plot results…

Probably should have started by drawing a graph!

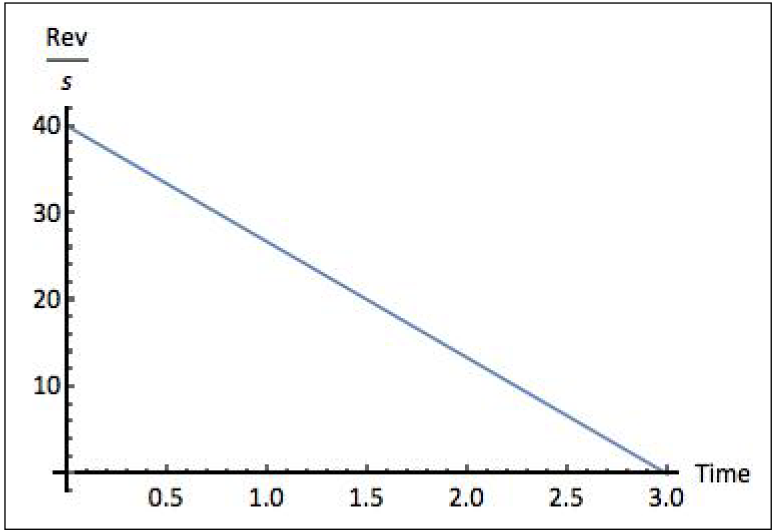
Plot angular velocity as a function of time first

Straight line corresponding to constant angular acceleration

Can also find number of revolutions before stopping from area under angular velocity curve

(40 rev/s)(3 s)/2 = 60 rev ✓

$$\omega(t) = 40.0\,rev/s + (-13.3\,rev/s^2)t$$



A high-speed drill rotating ccw at 2400 rpm comes to a halt in 3.00 s .

Part B

How many revolutions does it make as it stops?

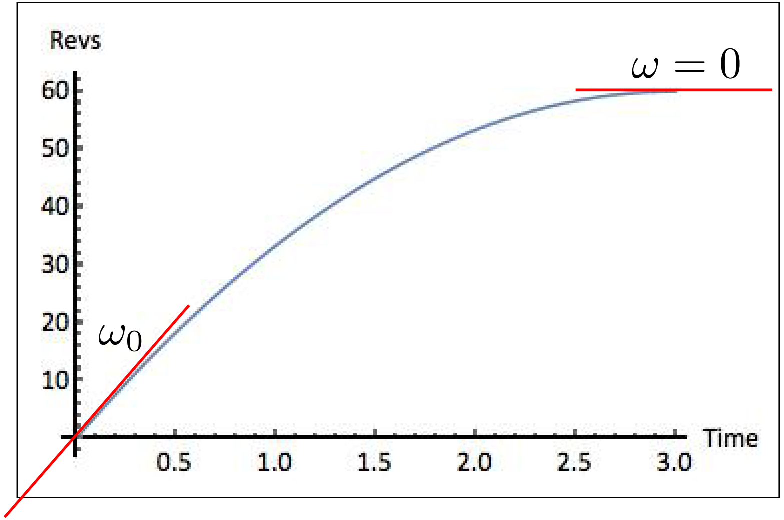
Plot results…

Now plot angular position as function of time

Angular velocity is slope of angular position graph

Slope gradually decreases to zero as drill slows down

$$\theta(t) = (40.0\, rev/s)t + \frac{1}{2}(-13.3\, rev/s^2)t^2$$



Uniform Circular Motion: relate angular variables with 1D motion variables

Constant angular velocity $\omega = \dfrac{2\pi}{T}$

Constant linear speed $v = \dfrac{2\pi r}{T}$

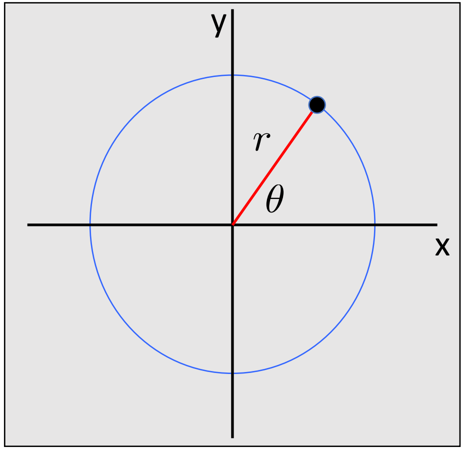
By inspection these are related according to…

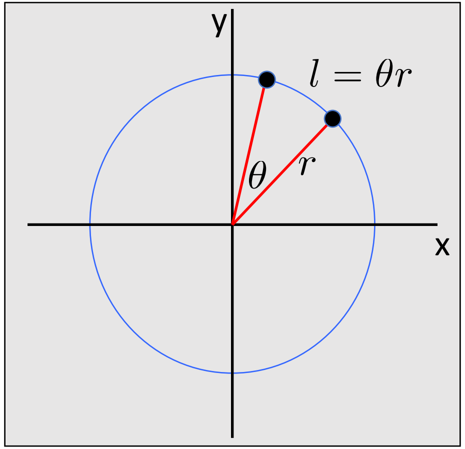
$$\boxed{v = \omega\, r}$$

Makes sense in terms of "arc length" for a portion of a circle

Arc of angular extent θ of circle of radius r has length $l = \theta\, r$

As particle sweeps out angle with angular velocity ω, it sweeps through arc length with velocity v = ωr





Two objects with **same angular velocities** can have very **different velocities**

$$\boxed{v = \omega r}$$

Bob walks around a circular track of radius 5 km, once per day. What are his **angular velocity** and **velocity**?

$$\omega = \frac{2\pi rad}{1 day} = \frac{2\pi rad}{24 \cdot 60 \cdot 60 s} = 7.3 \times 10^{-5} rad/s$$

$$v = \omega r = (7.3 \times 10^{-5} rad/s)(5000m) = 0.36m/s$$

The Earth spins around its axis once per day. What is the velocity of someone standing at the equator?

Same angular velocity! Earth radius = 6400km

$$v = \omega r = (7.3 \times 10^{-5} rad/s)(6.4 \times 10^{6} m) = 470m/s$$

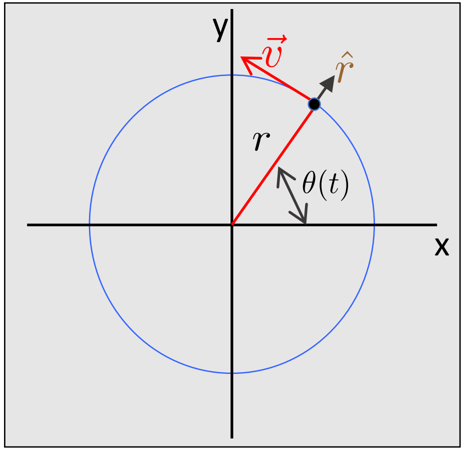
Uniform circular motion

speed $v = \omega\, r$

What are **velocity** and **acceleration**?

Magnitude of velocity stays fixed, but direction is constantly changing

Velocity always in direction tangent to circle



Can show that acceleration is always directed towards center of circle

Called “centripetal” acceleration

Latin for “center seeking”

Magnitude of centripetal acceleration stays constant

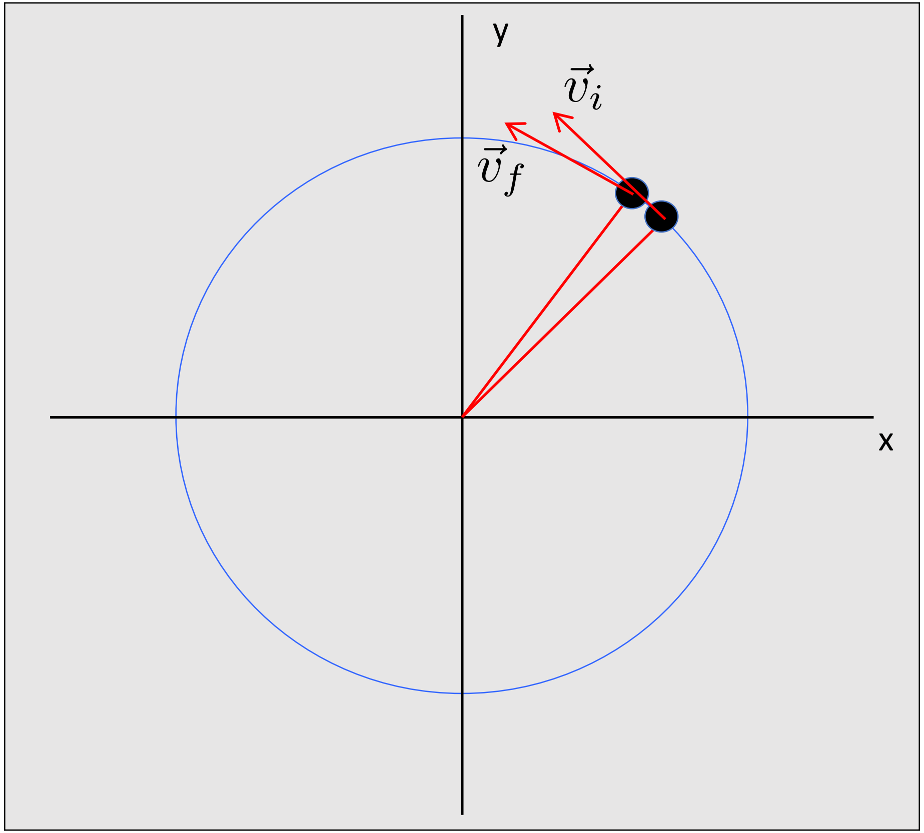
$$a = |\vec{a}| = \frac{v^2}{r} = \omega^2\, r$$

Unit vector in radial direction

$$\vec{a} = -\frac{v^2}{r}\hat{r}$$

Centripetal acceleration

Find direction of acceleration by looking at change in velocity vector between two nearby times t_i and t_f

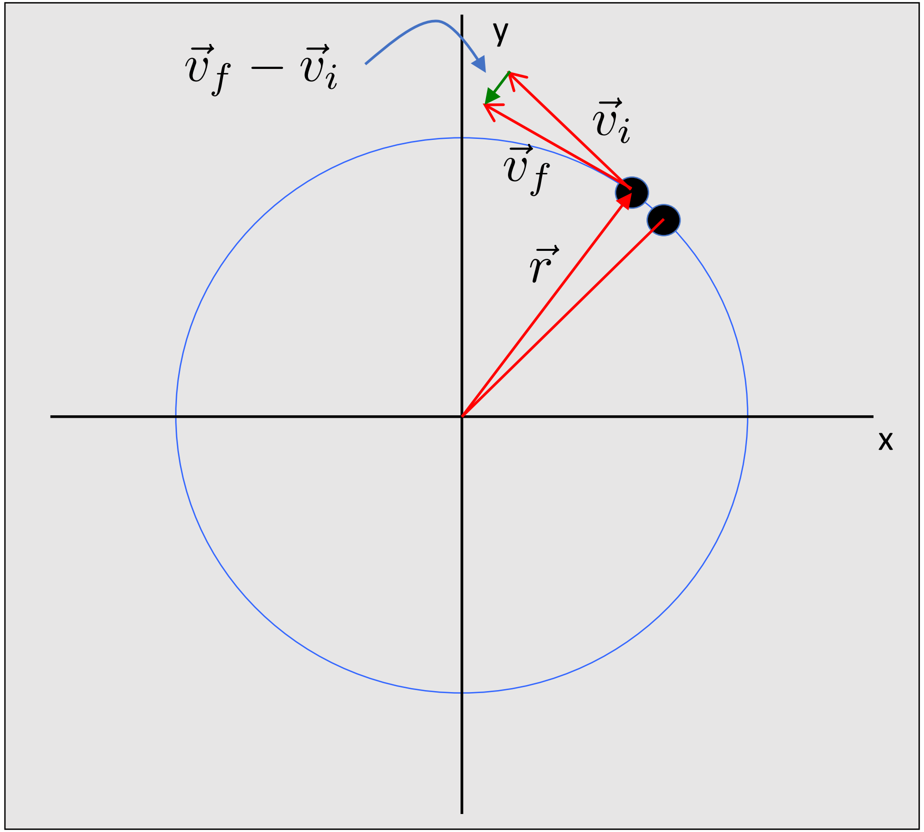


Centripetal acceleration

The vector $\vec{v}_f - \vec{v}_i$

is proportional to the acceleration vector and points in the opposite direction as the radial vector $\vec{r}$

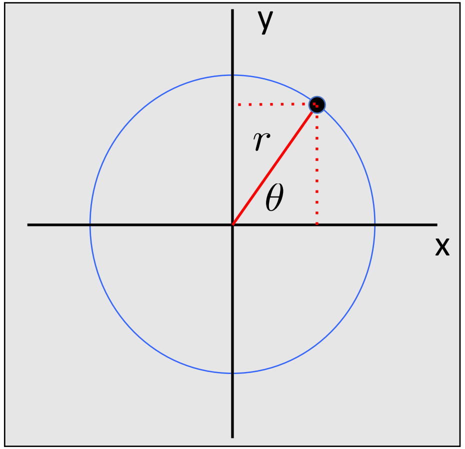
Can demonstrate this more precisely by computing the acceleration



Uniform circular motion

Speed $v = \omega r$

Centripetal acceleratio n $\boxed{\vec{a} = -\frac{v^2}{r}\hat{r}}$



Compute formula for centripetal acceleration

$$x(t) = r\cos\theta(t) = r\cos(\omega\, t)$$

$$y(t) = r\sin\theta(t) = r\sin(\omega\, t)$$

Position in plane as function of time

x & y oscillate in time

Take time derivative to find x & y components of velocity

Derivatives of trig functions

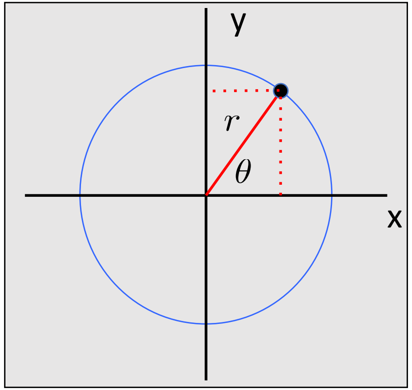
Really need...

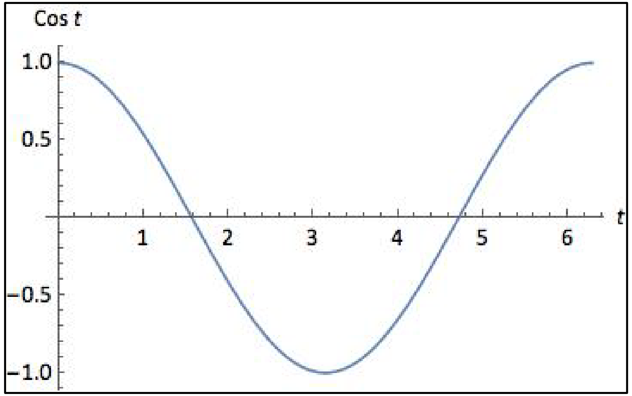
$$\frac{d}{dt}\sin t = \cos t$$

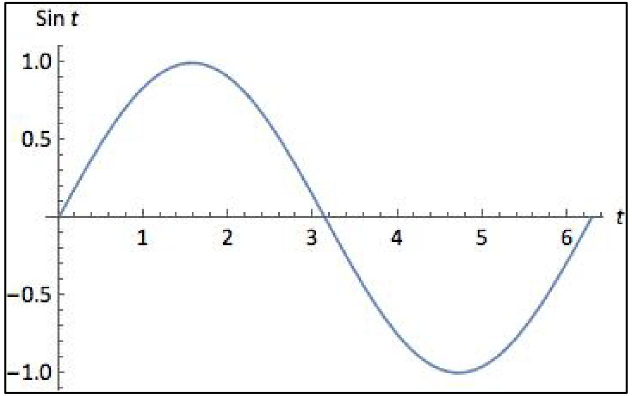
$$\frac{d}{dt}\cos t = -\sin t$$

$$\frac{d}{dt}\sin \omega t = \omega \cos \omega t$$

$$\frac{d}{dt}\cos \omega t = -\omega \sin \omega t$$







Uniform circular motion

Compute formula for centripetal acceleration

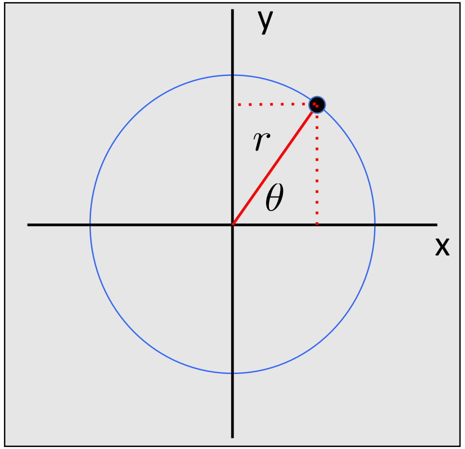
$$x(t) = r\cos\theta(t) = r\cos(\omega\, t)$$
$$y(t) = r\sin\theta(t) = r\sin(\omega\, t)$$

Take time derivative to find x & y components of **velocity**

$$v_x(t) = \frac{dx}{dt} = -\omega r\sin(\omega t)$$
$$v_y(t) = \frac{dy}{dt} = \omega r\cos(\omega t)$$

Take another time derivative to find x & y components of **acceleration**

$$a_x(t) = \frac{dv_x}{dt} = -\omega^2 r\cos(\omega t) = -\omega^2\, x(t)$$
$$a_y(t) = \frac{dv_y}{dt} = -\omega^2 r\sin(\omega t) = -\omega^2\, y(t)$$

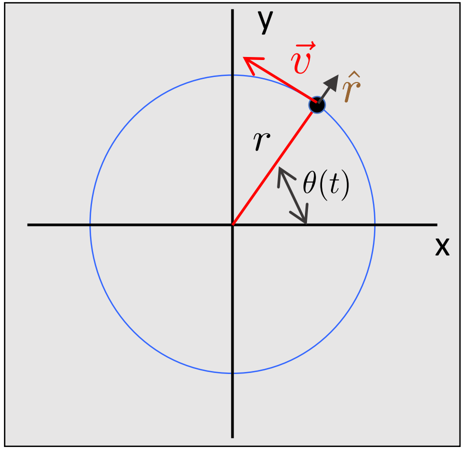


$$\frac{d}{dt}\sin\omega t = \omega\cos\omega t$$
$$\frac{d}{dt}\cos\omega t = -\omega\sin\omega t$$

Compute formula for centripetal acceleration

$$x(t) = r\cos\theta(t) = r\cos(\omega\, t)$$
$$y(t) = r\sin\theta(t) = r\sin(\omega\, t)$$

$$a_x(t) = -\omega^2\, x(t)$$
$$a_y(t) = -\omega^2\, y(t)$$



Write explicitly as vectors

$$\vec{r} = r\cos(\omega\, t)\hat{i} + r\sin(\omega\, t)\hat{j}$$

Position vector

$$\vec{a} = -\omega^2\vec{r} = -(\omega^2\, r)\hat{r}$$

Acceleration vector

Radial unit vector

$$\hat{r} = \cos(\omega\, t)\hat{i} + \sin(\omega\, t)\hat{j}$$

Formula for centripetal acceleration

$$\boxed{\vec{a} = -\omega^2\hat{r} = -\frac{v^2}{r}\hat{r}}$$

Uniform circular motion

$$v = \omega\, r$$

Speed

$$\vec{a} = -\frac{v^2}{r}\hat{r}$$

$$a = |\vec{a}| = \frac{v^2}{r} = \omega^2\, r$$

Centripetal acceleration

Example…

A Ferris wheel has radius 9m and rotates 4 times per minute.
Find its angular velocity.
Find the speed and acceleration experienced by the riders.

Try not to be disturbed by "unit" of radians going away

Angular velocity $\omega = \frac{2\pi\, rad}{15\, s} = 0.42\, rad/s$

Speed $v = \omega\, r = (0.42\, rad/s)(9\, m) = 3.8\, m/s$

Acceleration $a = \omega^2\, r = (0.42\, rad/s)^2(9\, m) = 1.6\, m/s^2$

Summary: Angular motion with constant angular acceleration

$$\theta(t) = \theta_0 + \omega_0\, t + \frac{1}{2}\alpha\, t^2$$

$$\omega(t) = \frac{d\theta}{dt} = \omega_0 + \alpha\, t$$

$$\alpha = \frac{d\omega}{dt} = \alpha \quad \checkmark$$

θ_0	**Angular position at t=0**
ω_0	**Angular velocity at t=0**
α	**Constant angular acceleration**

Speed $\boxed{v = \omega\, r}$

Centripetal acceleration $\boxed{a = |\vec{a}| = \frac{v^2}{r} = \omega^2\, r}$

Analogous to linear motion with constant acceleration

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v(t) = \frac{dx}{dt} = v_0 + at$$

$$a(t) = \frac{dv}{dt} = a$$