DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

Math 331 **Midterm Exam** **Spring 2020**

Name: ______________________________ **Student ID Number:** ________________________

Instructor name: ____________________________ **Your section number** _________________

In this exam there are 5 sheets, including this one, and there are 6 problems.
Instructions:

- Calculators and outside notes are **not allowed** to be used during the exam.
- You must explain how you arrived at your answers, and show your algebraic calculations.
- You can leave fractions and square roots in your answers – no need to give decimal expansions.

Question	Points	Score
1	15	
2	15	
3	20	
4	15	
5	20	
6	15	
Total:	100	

1. For the questions below, consider the differential equation:

$$2xy + 1 + \left(x^2 + 1\right)\frac{dy}{dx} = 0$$

   (a) (5 points) Test the exactness of the differential equation

   (b) (10 points) Find the general solution of the differential equation.

2. (15 points) Find the *explicit* solution to the initial value problem:

$$\frac{dy}{dx} = -6xy^2 + 2y^2 \qquad\qquad y(0) = 1$$

3. (20 points) Find the solution to the Initial Value Problem:

$$y'' + 3y' - 10y = 0 \qquad\qquad \text{with } y(0) = 3 \text{ and } y'(0) = 5$$

4. (15 points) Find the solutions, $y(t)$, to the differential equation:

$$y'' + \frac{y'}{t} = 3 + t$$

using the substitution $v(t) = y'(t)$
Hint: Your solutions, y(t), will include two unknown constants.

5. A population of a colony of duck billed platypi in Australia can be modelled by the logistic population model:

$$\frac{dP}{dt} = 4P(1 - \frac{P}{2})$$

Here, P is given in hundreds of duck billed platypi and t is measured in years.

(a) (10 points) Draw a phase line for this autonomous differential equation, and classify all equilibria as stable or unstable.

(b) (10 points) Sketch the graphs of the equilibrium solutions as well as the two solutions given the starting populations: $P_1(0) = 1, P_2(0) = 5$;

Each graph should clearly show the solution's initial condition and long-term behavior as $t \to \infty$.

6. A 5 gallon vat is full of pure water. At time $t = 0$ salt water is added to the vat through a pipe carrying water at a rate of 2 gallons per minute and a concentration of $\frac{1}{4}$ a pound per gallon. Water drains out of the vat at a rate of 2 gallon per minute, so that the level of the vat is always 5 gallons. Assume that the salt is always evenly mixed throughout the vat. Let $S(t)$ denote the amount of salt in the vat at time t, and let t be measured in minutes.

   (a) (10 points) Set up the differential equation and initial condition for $\frac{dS}{dt}$ for the situation above.

   (b) (5 points) Find $\lim\limits_{t\to\infty} S(t)$. Justify your answer by classifying the equilibrium point.

This page is intentionally left blank for work. If you want any work done on this page to be looked at by the grader, please make note in the problem to check this page.