DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

Math 331 **Midterm Exam** **Fall 2021**

Name: _____ **Student ID Number:** _____

Instructor name: _____ **Your section number** _____

In this exam there are 5 sheets, including this one, and there are 6 problems.
Instructions:

- Calculators and outside notes are **not allowed** to be used during the exam.
- You must explain how you arrived at your answers, and show your algebraic calculations.
- You can leave fractions and square roots in your answers – no need to give decimal expansions.
- Be sure that your work on each problem stays inside of the boxed area.
- If you need to use the blank page on the back of the exam to finish your work on a problem, be sure to make a note on the problem that additional work can be found on the blank page and, also, label any/all additional work on the back page by its problem number(s).

Question	Points
1	15
2	15
3	20
4	15
5	15
6	20
Total:	100

1. For the questions below, consider the differential equation:

$$4xy - 2x + \left(ax^2 + 1\right)\frac{dy}{dx} = 0$$

(a) (5 points) For what value of a is the differential equation **exact**?

(b) (10 points) Using the value of a found above, find the general solution of the diff. eq.
Note 1: You can leave your solution in its implicit form.
Note 2: If you did not find a above, find the gen. sol. in terms of the parameter a.

2. (15 points) Find the *explicit* solution to the initial value problem:

$$\frac{dy}{dx} = -6e^{3x}y^2 \qquad\qquad y(0) = 3$$

3. Consider the differential equation:

$$y'' - 3y' - 10y = 0$$

(a) (10 points) Find two solutions to the diff. eq. that can be used to build the general solution (i.e. find a fundamental set).

(b) (5 points) Use the Wronskian Determinant to prove that the two solutions that you found can be used to build a general solution (form a fundamental set).

(c) (5 points) Find the general solution of the differential equation.

4. (15 points) Find the solution to the Initial Value Problem:

$$y'' + 6y' + 10y = 0 \qquad \text{with } y(0) = 2 \text{ and } y'(0) = -7$$

5. Suppose that the population of a colony of quokka can be modeled by the logistic population model:

$$\frac{dP}{dt} = 3P(1 - \frac{P}{3})(P - 1)$$

Here, P is given in hundreds of quokka and t is measured in years.

(a) (10 points) Draw a phase line for this autonomous differential equation, and classify all equilibria as stable or unstable.

(b) (5 points) If the initial population is given by $P(0) = P_o$, find all values P_o so: $\lim_{t\to\infty} \gtrsim 0$.

6. Consider a 10 gallon tank containing 5 gallons of pure water. At time $t = 0$ salt water is added to the tank through a pipe carrying water at a rate of 4 gallons per minute and a concentration of $\frac{1}{2}$ a pound per gallon. Water drains out of the tank at a rate of 3 gallon per minute, so that the amount of water in the tank increasing by 1 gallon per minute. Assume that the salt is always evenly mixed throughout the tank. Let $S(t)$ denote the amount of salt in the tank at time t, and let t be measured in minutes.

   (a) (10 points) Set up the differential equation and initial condition to model $\dfrac{dS}{dt}$.

   (b) (5 points) Solve the initial value problem that you set up in part (a) to find $S(t)$.

   (c) (5 points) How much salt is in the tank once it is full?

This page is intentionally left blank for work. If you want any work done on this page to be looked at by the grader, please make note in the problem to check this page.