Homework Assignment 2
Advanced robotics & Robotics Systems

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Abstract

This homework assignment is dedicated to the calibration of cylindrical robot, identifying compliance parameters of joints and implementing error compensation technique.

Estimating stiffness parameters of both links and joints from experimental data is a very challenging task as it requires identification of more than 250 parameters for 6 DoF industrial manipulator. For this reason, a reduced model that describes stiffness of links and joints with only six parameters are so appealing.

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Introduction

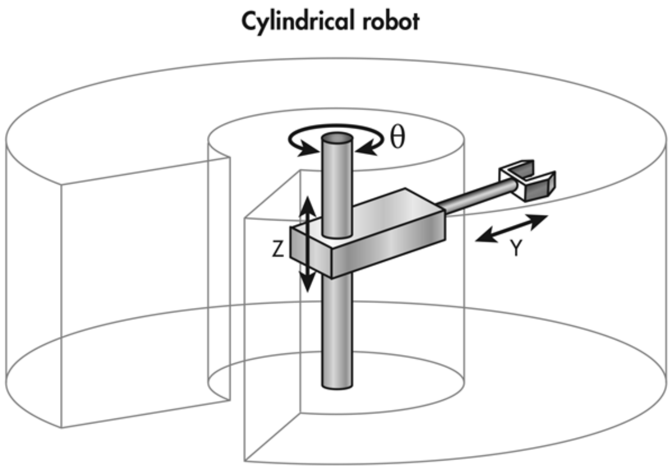

Figure 1: Cylindrical robot.

Task

The task of this HA is to:

1. Generate experimental data for 30 experiments;
2. Identify compliance parameters of joints for experimental data and compare results with the original one;
3. Implement error compensation technique [1], compare efficiency of calibrated and non-calibrated robots.

Also, the task has some requirements:

1. Tables with estimated and real parameters;
2. Figures with trajectories before and after calibration
3. Analysis of obtained results;
4. Link to the project on GitHub

Code

My code you can find here.

Inverse Kinematics

The kinematic scheme of the robot you can see in the fig. 2.

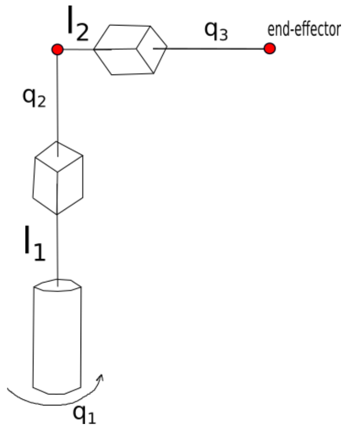

Figure 2: Kinematics scheme of cylindrical robot.

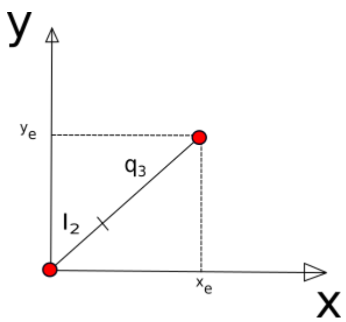

Figure 3: Top view of the robot

The solution for IK is pretty simple. According to the figures 2 and 3, the next equations are derived:

$$q_1 = atan2(y_e, x_e); \tag{1}$$
$$q_2 = z_e - l_1; \tag{2}$$
$$q_3 = \sqrt{x_e^2 + y_e^2} - l_2 \tag{3}$$

Forward Kinematics (VJM)

The VJM approach allows developing a more detailed and complete geometric model of the manipulator which provides a more accurate estimate of the end-effector position and orientation. It is assumed that the original model is complemented by virtual joints which describe the deformations of links. Besides, virtual springs are included in actuated joints, in order to take into consideration the stiffness of the transmission and control loop.

Let's see VJM model of our robot in the fig. 4.

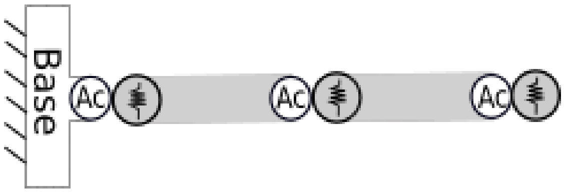

Figure 4: VJM model of the robot

According to the figure and [2], let's derive homogeneous transformation equation for the robot:

$$T = T_{base}R_z(q_1)R_z(\Theta_a^1)T_z(Link_1)Tz(q_2)Tz(\Theta_a^2)T_x(Link_2)T_x(q_3)T_x(\Theta_a^3) \quad (4)$$

where:

- q_i - the actuator displacement;
- $\theta(i)$ - virtual joints' coordinates for actuator and of the links.

Parameter	**Value**
Young's modulus, Pa	$7.0 \cdot 10^7$
Poisson's ratio	0.3
Density, $\frac{kg}{m^3}$	$2.7 \cdot 10^3$

Table 1: Material parameters

Joint	**Stiffness,** $10^6 \cdot \frac{N}{m}$	**Lower limit, rad**	**Upper limit, rad**
1	1	$-\pi$	π
2	2	0	1
3	0.5	0	1

Table 2: Joint stiffness values and upper and lower limits

Parameters identification

The main goal here is to find unknown stiffness parameters from torque and displacement measurements. The eq. 5 sets the relationship between the end-effector displacement and applied wrench through joint stiffness matrix and allows us to formulate parameter identification problem.

$$\boldsymbol{\Delta t} = \boldsymbol{J}_\theta \boldsymbol{K}_\theta^{-1} \boldsymbol{J}_\theta^T \boldsymbol{w} \tag{5}$$

where:

- $\boldsymbol{\Delta t}$ - the end-effector deflection;
- $\boldsymbol{J}_\theta$ - the Jacobian matrix with respect to $\boldsymbol{\theta}$
- $\boldsymbol{K}_\theta$ - the virtual stiffness matrix (diagonal)
- w - wrench vector

But firstly, it's more convenient to rewrite it in a form:

$$\boldsymbol{\Delta t} = \sum_{i=1}^{n} (\boldsymbol{J}_{\theta,i} \boldsymbol{k}_{\theta,i}^{-1} \boldsymbol{J}_{\theta,i}^T) \boldsymbol{w} \tag{6}$$

- n is a number of experiments
- $\boldsymbol{k}_{\theta,i}$ - the link or joint compliances to be identified
- $\boldsymbol{J}_{\theta,i}$ - sub-Jacobians ($\boldsymbol{J}_\theta = [\boldsymbol{J}_{\theta,1}, \boldsymbol{J}_{\theta,2} ...]$)

Further, it's possible to rewrite equation 6 in a form standard for identification (with respect to parameters to be identified):

$$\boldsymbol{\Delta t} = \boldsymbol{A}_k(\boldsymbol{q}, \boldsymbol{w})\boldsymbol{k} + \boldsymbol{\epsilon} \tag{7}$$

where:

- $\boldsymbol{A}_k = [\boldsymbol{J}_{\theta,1} \boldsymbol{J}_{\theta,1}^T \boldsymbol{w}, ..., \boldsymbol{J}_{\theta,n} \boldsymbol{J}_{\theta,n}^T \boldsymbol{w}]$ is so-called observation matrix;
- $\boldsymbol{k} = (\boldsymbol{k}_{\theta,1}, \boldsymbol{k}_{\theta,2}, ..., \boldsymbol{k}_{\theta,n})^T$ is a compliance vector;
- $\boldsymbol{e}$ - measurement noise.

So, to get $\boldsymbol{k}$, . For this reason, the least square approach is applied, which minimizes the sum of squared residuals:

$$\sum_{i=1}^{m} ||\Delta \boldsymbol{t}_i - \boldsymbol{A}_k(\boldsymbol{q}_i, \boldsymbol{w}_i)\boldsymbol{k}|| \rightarrow \min \boldsymbol{k}. \tag{8}$$

Index i defines the manipulator configuration number. Solution for eq. 8 has form

$$\widehat{\boldsymbol{k}} = (\sum_{i=1}^{m} \boldsymbol{A}_i^T \boldsymbol{A}_i)^{-1} (\sum_{i=1}^{m} \boldsymbol{A}_i^T \Delta \boldsymbol{t}_i) \tag{9}$$

Analysis

The random technique for stiffness estimation through 30 experiments was used.

So, I got the next results for stiffness coefficients:

Joint	**Stiffness,** $10^6 * \frac{N}{m}$
1	0.984
2	1.994
3	0.500

Table 3: New Joint stiffness values

The graphs for trajectories you can see in the fig. 5.

For more details, you can run my code with different parameters and trajectories.

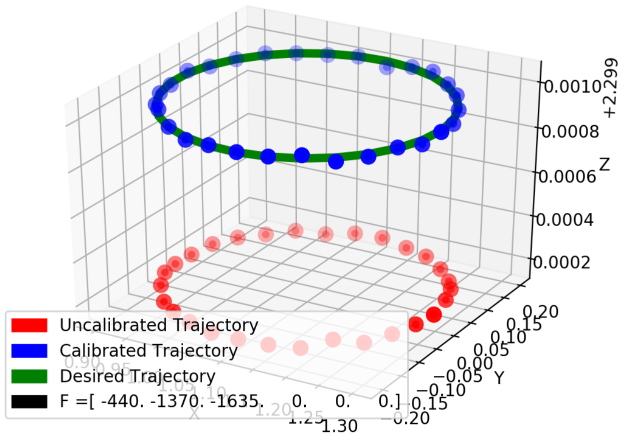

Figure 5: Compensated and uncompensated tool trajectories.

Bibliography

[1] Mamedov, Shamil and Popov, Dmitry and Mikhel, Stanislav and Klimchik, Alexandr. (2019). Increasing Machining Accuracy of Industrial Manipulators Using Reduced Elastostatic Model. 10.1007 978-3-030-31993-9-19.

[2] Alexandr Klimchik. Enhanced stiffness modeling of serial and parallel manipulators for robotic-based processing of high performance materials. Robotics [cs.RO]. Ecole Centrale de Nantes (ECN); Ecole des Mines de Nantes, 2011. English. fftel-00711978f