PHYSICS 4AL

EXPERIMENT 5: HARMONIC OSCILLATOR PART I: SPRING OSCILLATOR

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Date of Lab: May 16th, 2017
Lab Section: Tuesday, 5 P.M.
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Abstract: 165 words
Report: 2189 words

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INVESTIGATING RESONANT FREQUENCY IN DAMPED AND UNDAMPED OSCILLATIONS

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In Newtonian mechanics, a mass suspended on a spring will undergo harmonic motion if acted by an outside force. The objective of the experiment was to investigate the motion of damped and undamped oscillations. Specifically, we wanted to find whether resonant frequencies in both types of oscillations would be equivalent. A mass was hung from a spring attached to a vertical force sensor, which measured both voltage and time. Magnets provided a damping force when passed through a metal tube. Graphs of voltage vs time were plotted to find the relationship between these values. Based on the resonant frequency of the undamped oscillation found from our data, we calculated a value of the damped resonant frequency and comapred it to the measured value of damped resonant frequency. Through this, we confirmed the experimental resonant frequency matched calculated predicted values, which was 0.67 ± 0.02 Hz. We found that the resonant frequency for undamped oscillations and damped oscillations (both measured and calculated) are the exact same.

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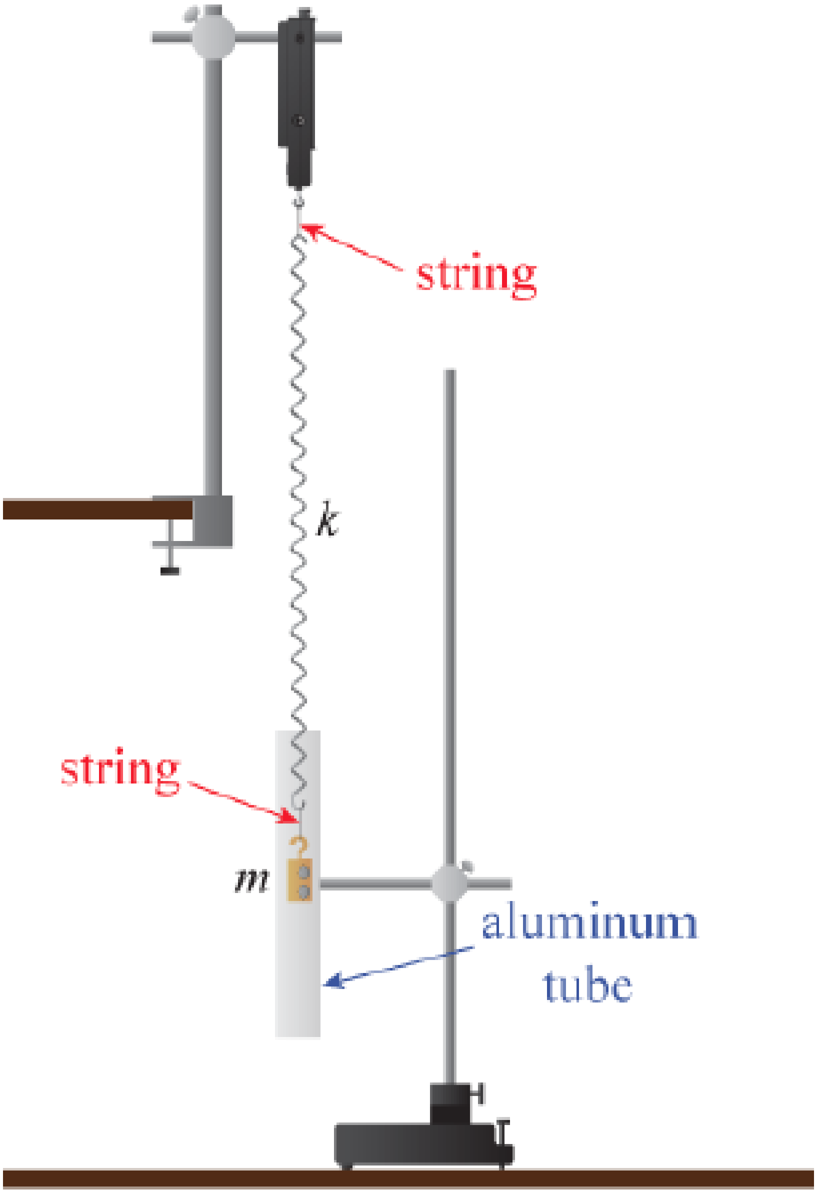

Figure 5.1 Setup for Oscillation Experiment. The sensor provides the readout of voltage during oscillation motion. The weight has magnets that can provide a damping force if passed through the aluminum tube. Be sure to decouple the spring from the mass and the force sensor to produce the most accurate data. Figure reproduced (with permission) from Fig. 5.1 by Campbell, W. C. et al.[1]

Introduction

Oscillations are caused by an acting force and a restoring force exerted by a spring. If an object is oscillating and there is no other force acting on it, it is in simple harmonic motion and has a constant amplitude and frequency. This frequency is also known as the resonant frequency, where it's amplitude will be greatest.

Damped oscialltions are oscillations that are affected by an outside force that causes the amplitudes of its oscillation to decrease exponentially. This is known as the damping term. Damping time is the time it takes for a damping force to decay the amplitude by a factor of $\frac{1}{e}$. Damped oscillations are affected by something called the Q-factor, which describes how energy is lost in a resonant element which in our experiment is the mass and spring. These terms can all be used to find the resonant frequency of a damped oscillation, which we will see later.

In our experiment, we sought to observe both undamped and damped oscillations, with the ultimate goal of finding the resonant frequency. By hanging a mass onto a force sensor and measuring voltage produced by the mass over time, we were able to measure the resonant frequency produced by undamped and damped oscillations, and compare these values.

Methods

The first step of the experiment is to determine the spring constant of the spring we will hang our masses from. A series of masses were hung from the spring and the distance from the mass to the floor was measured and plotted. The slope of the best fit line gives our spring constant k (**Figure 5.2**).

Figure 5.1 shows the layout of our experiment. With the force sensor (measuring voltage) attached to the computer, set a high sample rate for the sensor so the voltage data is accurate. Attach a small string to the bottom of the hook, then attach the mass to the string (this is done to avoid uncertainty that occurs when the spring rotates and turns as it stretches and compresses). The mass should have magnets on it's side so an aluminnum tube can be used for the damped oscillation trial and slow the mass. A small vertical force was given to the spring mass system and let to oscillate for 40 seconds, during which voltage and time are recorded. Do this twice, once without damping and once with damping through the aluminum tube.

Any data recorded should be transferred over to Excel for analysis. Other data important to collect would be an offset voltage(to account for the fact the voltage data may not be centered at zero) and mass of the weight used.

Analysis

Derivations

We mentioned terms such as damping term, resonant frequency, damping time, and Q-factor before, now we shall see how these terms are related.

The equation

$$m\ddot{x} = -kx - b\dot{x}$$

is the differential equation that represents damped motion. One solution to this differential equation is

$$x(t) = Ae^{iwt}$$

By substituting this into the first equation we get

$$-w^2 mAe^{iwt} = -kAe^{iwt} - iwbAe^{iwt}$$

which can simplify to

$$mw^2 = k + iwb$$

By solving this quadratic equation, we find that

$$w = \frac{ib}{2m} \pm \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

If we substitute what we just found into the solution to the differential equation stated previously, we find that

$$x(t) = Ae^{i\left(\frac{ib}{2m} + \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\right)t}$$

$$x(t) = Ae^{i\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}\,t} \times e^{-bt}2m$$

As stated previously, damping time τ is defined to be the time is takes for a damped oscillation to decay by a factor of $\frac{1}{e}$. Damping time is usually defined as

$$\tau = \frac{2m}{b}$$

The frequency of damped oscillations can be found by dividing the angular frequency by 2π, or

$$f_{damped} = \frac{w_{damped}}{2\pi} \equiv \frac{1}{2\pi}\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = f_0\sqrt{1 - \frac{b^2}{4m^2}}$$

By definition, we also know that

$$f_{damped} \equiv f_0\sqrt{1 - \frac{1}{4Q^2}}$$

By setting these two equations equal to one another, we find

$$\sqrt{1-\frac{1}{4Q^2}}=\sqrt{1-\frac{b^2}{4km}}$$

We solve and simpilfy for Q to get

$$Q=\frac{\sqrt{km}}{b}$$

Thus, Q can be found if we know the mass, the spring constant, and the damping term. We can find the damping term b by first knowing the damping time τ and dividing twice the mass by b.

Plots and Graphs

We first determined the spring constant of the spring by hanging masses of 0 g, 50 g, 100 g, 150 g, and 200 g, and plotting these masses against the distance remaining from the ground to the spring when placed onto the spring (the uncertainty of the masses was 0.0005 kg, half the lowest precision of our scale). This is illustrated in **Figure 5.2**. The spring constant is determined to be 3.2 ± 0.2 N/m.

The mass m on the spring we used for our experiment was 175 ± 0.5 g.

To predict our resonant frequency, we used the equation $f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where k is the spring constant found on **Figure 5.2** and m is the mass of the weight on spring. Using the propagation of uncertainty, $\frac{\delta f_0}{f_{0,best}} = \sqrt{\left(\frac{\delta k}{2k}\right)^2 + \left(\frac{\delta m}{2m}\right)^2}$, to find the uncertainty for f_0, we find that $f_{0,predict} = 0.67 \pm 0.02$ Hz.

In both **Figure 5.3** and **Figure 5.4**, we plotted our measured voltage values at each time. The data is presented with an offset to the voltage, which accounts for the fact the equilibrium voltage was not 0 V and to avoid drifting in our data for the damping oscillation in **Figure 5.4**. The offset value is 0.0327 V.

Figure 5.3 shows our plotted values of voltage V at time t for the undamped oscillation. We derived our experimental value of $f_{0,undamped}$ by taking the maxima at several different points, zooming in and finding the maximum value and time of each peak. The undamped resonant frequency f_0 is then equal to $\frac{n-1}{t_n-t_1}$, where n is the peak number and t is the time when that peak occured. We took several trials of this data to find an uncertainty range (standard deviation of the data divided by the number of trials), and the resulting frequency was $f_0 = 0.661 \pm 0.002$ Hz.

Similarly, we plotted the measured voltage values at each time for the damped oscillation in **Figure 5.4**. We derived our experimental value of f_{damped} by taking the maxima at several different points, zooming in and finding the maximum value and time of each peak. The damped resonant frequency is then equal to $\frac{n-1}{t_n-t_1}$, where n is the peak number and t is the time when that peak occured. We took several trials of this data to find an uncertainty range (standard deviation of the data divided by the number of trials), and the resulting frequency was $f_{damped} = 0.660 \pm 0.003$ Hz.

We can see that in both the damped and undamped oscillations, the resonant frequencies f_0 and f_{damped} calculated from our experiment were within the uncertainty range of our predicted resonant

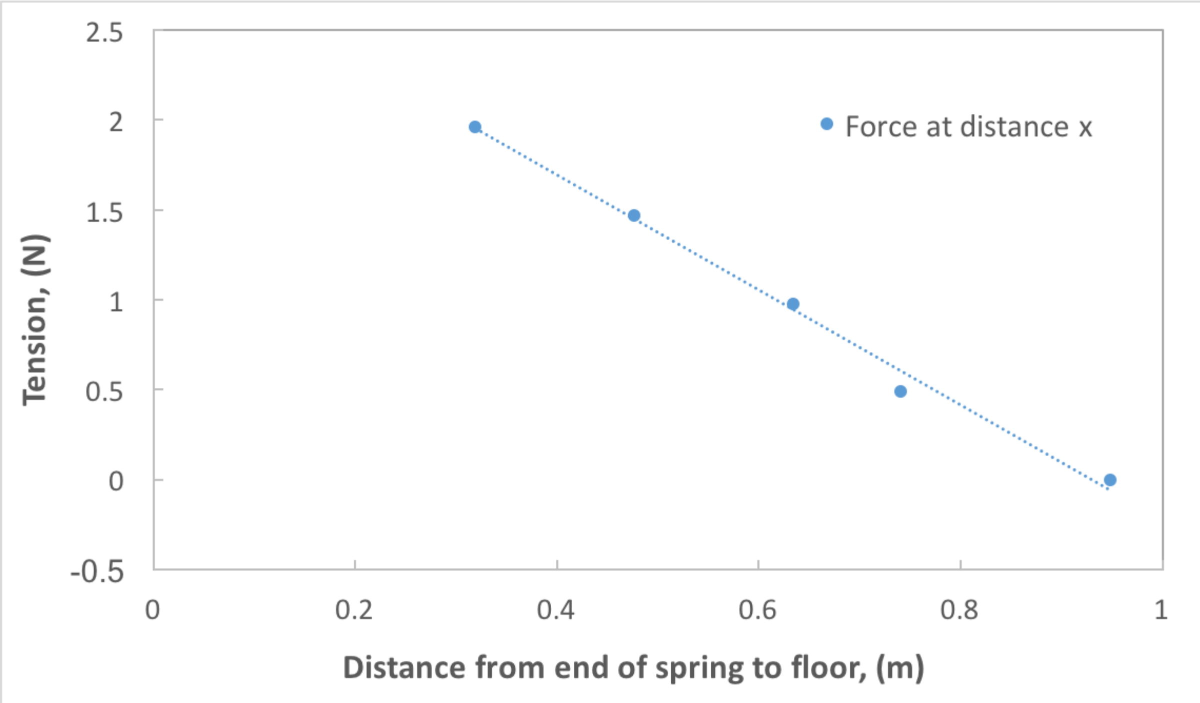

Figure 5.2 Measuring Spring Constant of a Spring. Masses used were 0 g, 50 g, 100 g, 150 g, and 200 g. The best fit line has the equation $F = (-3.2 \pm 0.2)x + (2.9 \pm 0.1)$ The spring constant was found to be 3.2 ± 0.2 N/m.

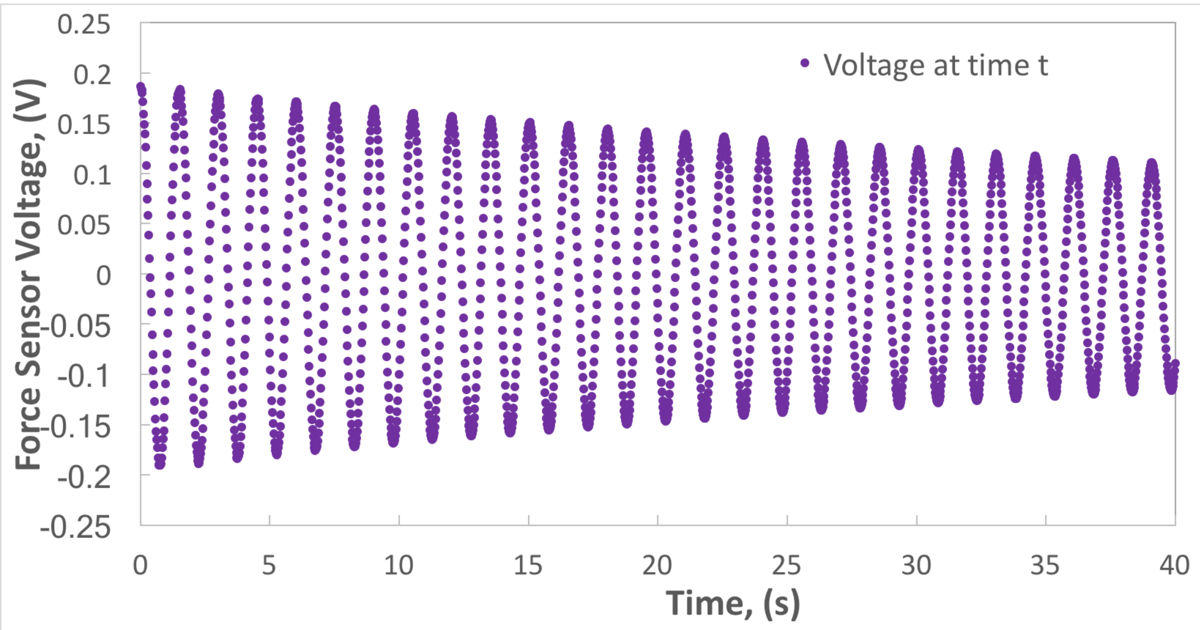

Figure 5.3 Simple Harmonic Motion of Spring. The x-axis shows time of reading and the y-axis shows the voltage measured by the force sensor at that time. An offset of 0.0327 V was added to each voltage to center our readings at V = 0. The f_0 found from this figure is 0.661 ± 0.002 Hz.

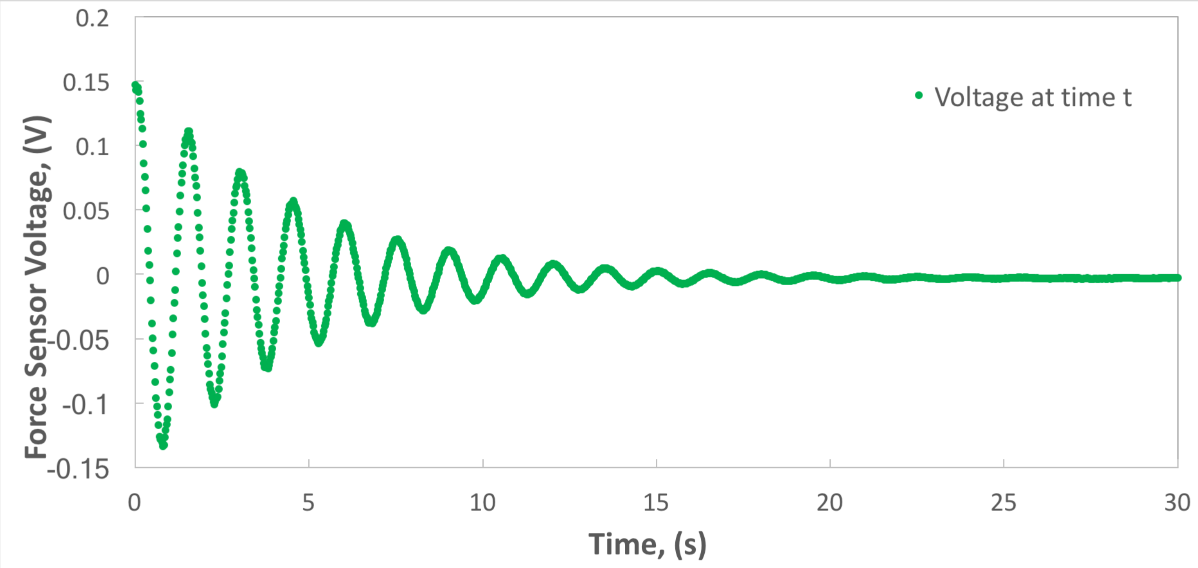

Figure 5.4 Damped Oscillation of a Spring. The x-axis shows the time of reading and the y-axis shows the voltage measured by the force sensor at that time. The offset was 0.0327 V, which was added to each voltage value to center our readings. The f_0 value derived from this graph is 0.660 ± 0.003 Hz.

frequency $f_{0,predict}$, which was 0.67 ± 0.02 Hz. Thus our calculated values are accurate relative to our predicted value and resonant frequencies in undamped and damped oscillations are the same.

We can also determine damping time from **Figure 5.4**. Damping time is the time for the damped osciallation's amplitude to decrease by $\frac{1}{e}$, which will only decay exponentially if damping is caused by a velocity-dependent force, and not by a friction force. After ensuring our voltage-time plot in **Figure 5.4** is centered around V = 0, we took the peak voltages for several extrenum, and calculated the ratio of peak voltages from one peak to the next. The ratio between amplitudes was plotted in **Figure 5.5**, which we can see ranges from 0.81 to 0.86. The ratio values shown in **Figure 5.5** do not indicate any upward or downward trend, which means we have chosen a correct value for the voltage offset. By setting a correct offset value, we have avoided a drift in our voltage data and now our future measurements not be affected by the fact that the equilibrium voltage was not centered at 0 V.

Each ratio measurement was converted to damping time τ with this equation: $\tau = -\left(\frac{T}{ln\left(\frac{V(t+T)}{V(t)}\right)}\right)$. We collected each damping time calculated for each ratio and averaged them to find the mean damping time, which was $\tau = 8.5 \pm 0.3$ s (uncertainties found by taking the standard deviation divided by number of ratios).

We can determine damping term b by dividing twice the mass by the mean damping time, or $b = \frac{2*m}{\tau}$. With a mass of 0.175 kg and using the previously found mean damping time, we find that $b = 0.04 \pm 0.03$ kg/s (uncertainties found by propagation of uncertainties).

Next, we can also determine something called the Q-factor from our damped oscillation, which indicates energy loss in a resonant element which in this experiment is the spring-mass system. The

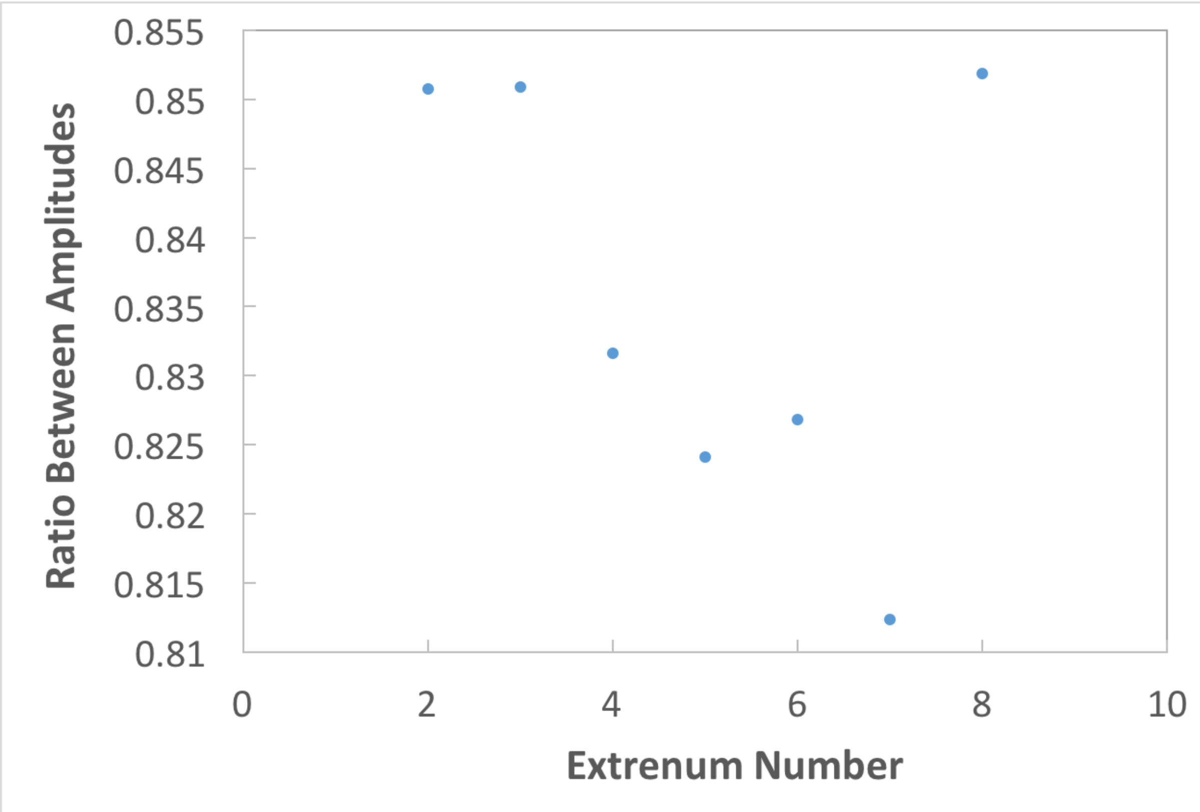

Figure 5.5 Ratios of Peak Amplitudes Graph of ratios of voltages from peaks of damped oscillations. Ratios range from 0.81 to 0.86.

equation for Q is $Q = \frac{\sqrt{k*m}}{b}$, so we can substitute our previously found value for b, as well as the spring constant k and mass m to find Q. By doing this, we calculate that $Q = 18.2 \pm 0.8$.

Lastly, we can use the Q factor to see if our calculated value for f_{damped} matches the one we measured by sampling extrenum of **Figure 5.4**. Using the equation $f_{damped} = f_0 * \sqrt{1 - \frac{1}{4Q^2}}$, where f_0 is the resonant frequency found by sampling peaks in **Figure 5.3** and Q is the Q-factor found previously. We calculate that $f_{damped} = 0.66 \pm 0.04$ Hz. We find the measuring and calculating the value of f_{damped} results in the exact same values, as our measured value of f_{damped} falls within the uncertainty range of the f_{damped} value we just calculated.

As a bonus, we tried to find the resonant frequency by doing a Fast Fourier Transformation(FFT) on our damped oscillation data. By taking the width of the peak at $\frac{1}{\sqrt{2}}$ of the height of the maximum value, we can determine the width of the change of resonance Δf. The FFT's peak was 0.025 Hz wide. Because Q is large, Q can be found as the ratio of resonance frequency to resonance width, or $Q = \frac{f_0}{\Delta f}$. The base of the FFT determines the resonant frequency of the oscillations, which was at 0.6468 Hz. Using the previously defined formula, $Q = 25.872$. The value of Q calculated from our FFT is higher by 7.7 than the value of Q derived from our calculations of the damped oscillation.

CONCLUSION

We performed our experiment to study simple harmonic motion and damped oscillations. We wanted to verify if resonant frequency in both damped and undamped oscialltions would be the same. Our predicted value of the resonant frequency was 0.67 ± 0.02 Hz. The resonant frequency from the undamped oscillation was 0.661 ± 0.002. Our measured value of f_{damped} from **Figure 5.4** was 0.660 ± 0.003 Hz. The calculated value of f_{damped} was 0.66 ± 0.04 Hz. All values of the resonant frequency from both undamped and damped oscillations fell inside the uncertainty range of the predicted f_0. We verified that the measured and caluclated value of f_{damped} were the same. Our experiment verifies that the resonant frequency was the same regardless of damping and that multiple ways of calculating the resonant frequency results in the same f_0.

We note that although the our values of f_0 fell within our predicted frequency's uncertainty range our experimental values were lower than the predicted value. As such, we can attribute some external factors such as friction to a loss of energy that makes our experimental values slightly lower than the predicted value, which could slow the oscillations and lower the frequency. Another source of error could stem from the uncertainty value of our spring constant was to the first decimal place, which gives a wide range of possible error. This wide range could have attributed to the slightly lesser experimental values of f_0 if the spring constant was actually lower than k_{best} for our experimental trials. This is a source of systematic error that could be avoided by measuring the spring constant to a higher degree of precision.

References

1. Campbell, W. C. et al. Physics 4AL: Mechanics Lab Manual (ver. April 3, 2017). (Univ. California Los Angeles, Los Angeles, California).