

## Biped Patrol

### Task 3.3: Think & Answer

Team Id	9
College	B.M.S. College Of Engineering
Team Leader Name	Akshay S Rao
e-mail	akshayrao.ec17@bmsce.ac.in
Date	January 15, 2020

Question No.	Max. Marks	Marks Scored
Q1	10	
Q2	20	
Q3	5	
Q4	5	
Q5	5	
Q6	10	
Q7	15	
Q8	8	
Q9	4	
Q10	8	
Q11	10	
Total	100	

## Biped Patrol

### Task 3.3: Think & Answer

#### Instructions:

- There are no negative marks.
- Unnecessary explanation will lead to less marks even if answer is correct.
- If required, draw the image in a paper with proper explanation and add the snapshot in your corresponding answer.

**Q 1.** Describe hardware design for the Medbot, your team is constructing. Describe various parts with well labeled image. Give reasons for selection of design. [10]

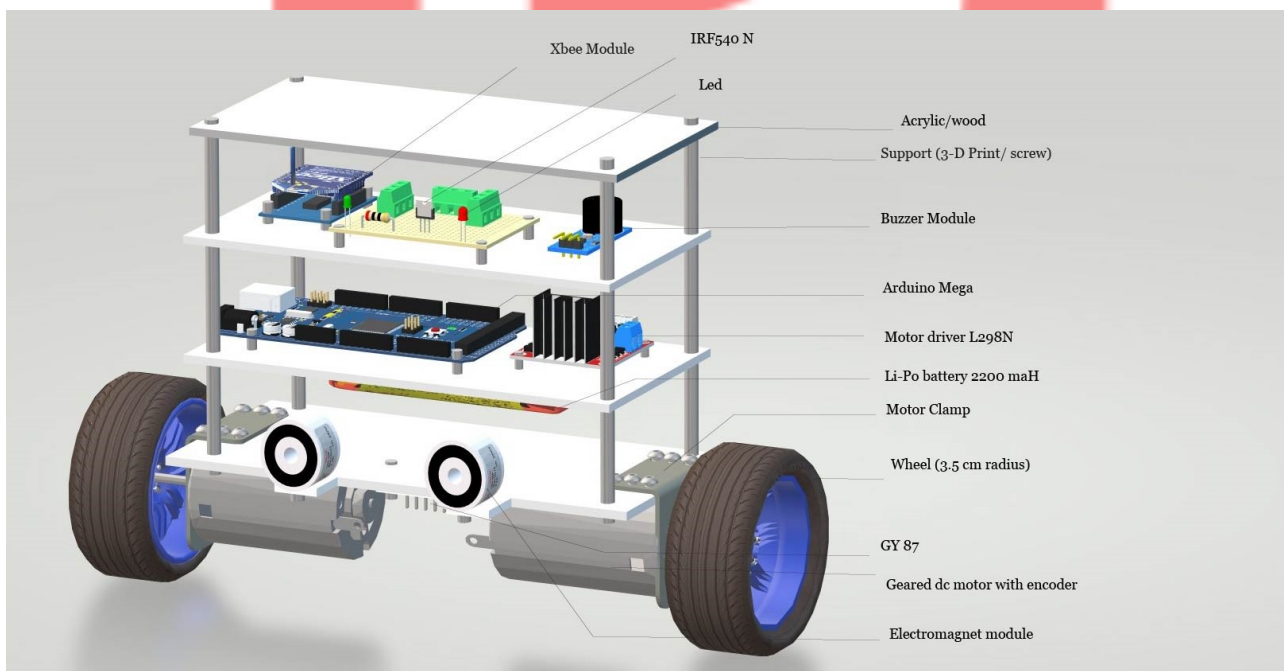
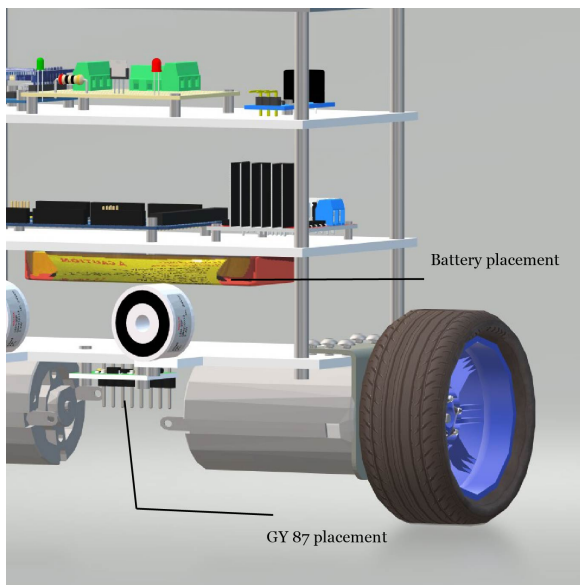
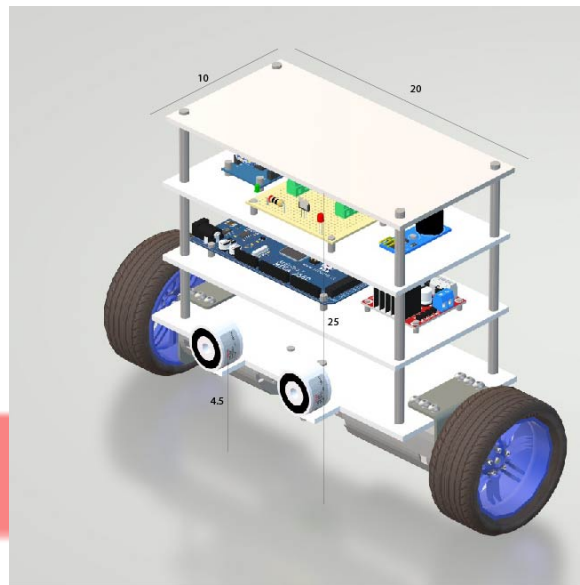


Figure 1: 3D Model of Medbot



(a) Gyro and Battery Placement



(b) Dimensions

Figure 2: MedBot from different angles

**A 1. Body:** Stacked type of design is employed. This helps in varying the position of centre of mass and also helps in programming point of view since the bot has to be tuned many times. To reduce the weight of the body for torque requirement acrylic material is used. The support material is a 3-D printed material. Stack is of size 20x10, The total height of the bot is around 25cm from the bottom.

**Gyroscope Placement:** GY-87 sensor is placed at the bottom stack along the axis of centre of mass and parallel to the wheel axis. The values measured by the gyroscope is not affected by the placement of the sensor, as it will vary by the same amount at all points on the robot. As the distance from the ground increases, linear acceleration and vibrations, measured at higher points, increases. This could lead to false readings on the accelerometer. Therefore, the sensor is placed at the bottom.

**Centre of mass:** Mass is distributed in such a way that the position of centre of mass is as low as possible. If the position is too high its stable to disturbances but torque requirement will be very high and lower recovery angle range. By lowering the position we get higher recovery angle and has lesser chance of toppling. From the calculation, it is at a height of around 14 cm

**Electromagnet placement:** Electromagnet is placed at the bottom at a height of 4.5cm from the bottom. The centres of the supply iems are at a height of 5cm and 4cm from the arena surface. The average of these height is considered for electromagnet position.

**Q 2.** In Task 1.2, you were asked to model different systems such as Simple Pulley, Complex Pulley, Inverted Pendulum with and without input and stabilizing the unstable equilibrium point using Pole Placement and LQR control techniques. There you had to choose the states; Derive the equations (usually non-linear), find equilibrium points and then linearize around the equilibrium points. You were asked to find out the linear system represented in the form

$$\dot{X}(t) = AX(t) + BU(t) \quad (1)$$

Where  $X(t)$  is a vector of all the state, i.e.,  $X(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$ , and  $U(t)$  is the vector of input to the system, i.e.  $U(t) = [u_1(t), u_2(t), \dots, u_m(t)]^T$ .  $A$  is the State Matrix &  $B$  is the Input Matrix.

In this question, you have to choose the states for the Medbot you are going to design. Model the system by finding out the equations governing the dynamics of the system using Euler-Lagrange Mechanics. Linearize the system via Jacobians around the equilibrium points representing your physical model in the form given in equation 1.

**Note:** You may choose symbolic representation such as  $M_w$  for Mass of wheel, etc. [20]

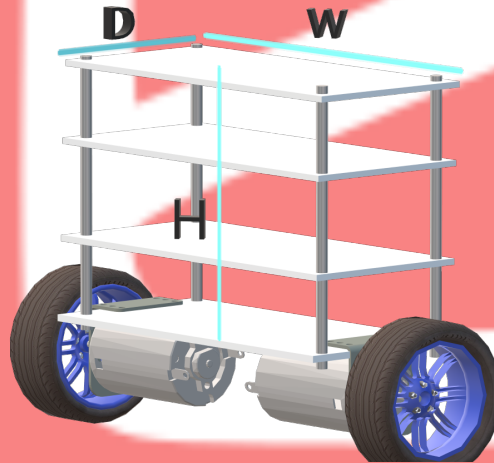


Figure 3: Medbot



Figure 4: MedBot from different angles

**A 2.**  $\psi$  : body pitch angle;  $\theta_{l,r}$  : wheel angle( $l, r$  indicates left and right)

Physical parameters of the model are the following.

Parameter	Units	Description
$g = 9.81$	[m / sec <sup>2</sup> ]	Acceleration due to gravity
$m$	[kg]	Wheel mass
$R$	[m]	Wheel radius
$J_W$	[kg m <sup>2</sup> ]	Wheel inertia moment
$M$	[kg]	Body weight
$W$	[m]	Body width
$D$	[m]	Body depth
$H$	[m]	Body height
$L$	[m]	Distance of the center of mass from the wheel axle
$J_\psi$	[kgm <sup>2</sup> ]	Body pitch inertia moment
$J_\phi$	[kgm <sup>2</sup> ]	Body yaw inertia moment
$J_m$	[kgm <sup>2</sup> ]	DC motor inertia moment
$R_m$	[ $\Omega$ ]	DC motor resistance
$K_b$	[V sec / rad]	DC motor back EMF constant
$K_t$	[Nm / A]	DC motor torque constant
$n$		Gear ratio
$f_m$		Friction coefficient between body and DC motor
$f_\pi$		Friction coefficient between wheel and floor

### Equations of Motion Of Two-Wheeled Inverted Pendulum(Medbot).

We can derive equations of motion of two-wheeled inverted pendulum by the Lagrangian method based on the coordinate system in Figure 4. If the direction of two-wheeled inverted pendulum is x-axis positive direction at  $t = 0$ , each coordinates are given as the following.

$$(\theta, \phi) = \left( \frac{1}{2} (\theta_l + \theta_r) \frac{R}{W} (\theta_r - \theta_l) \right)$$

$$(x_m, y_m, z_m) = \left( \int \dot{x}_m dt, \int \dot{y}_m dt, R \right), \quad (\dot{x}_m, \dot{y}_m) = (R\dot{\theta} \cos \phi, R\dot{\theta} \sin \phi)$$

$$(x_l, y_l, z_l) = \left( x_m - \frac{W}{2} \sin \phi, y_m + \frac{W}{2} \cos \phi, z_m \right)$$

$$(x_r, y_r, z_r) = \left( x_m + \frac{W}{2} \sin \phi, y_m - \frac{W}{2} \cos \phi, z_m \right)$$

$$(x_b, y_b, z_b) = (x_m + L \sin \psi \cos \phi, y_m + L \sin \psi \sin \phi, z_m + L \cos \psi)$$

The translational kinetic energy  $T_1$ , the rotational kinetic energy  $T_2$ , the potential energy  $U$  are

$$T_1 = \frac{1}{2} m (\dot{x}_l^2 + \dot{y}_l^2 + \dot{z}_l^2) + \frac{1}{2} m (\dot{x}_r^2 + \dot{y}_r^2 + \dot{z}_r^2) + \frac{1}{2} M (\dot{x}_b^2 + \dot{y}_b^2 + \dot{z}_b^2)$$

$$T_2 = \frac{1}{2} J_w \dot{\theta}_l^2 + \frac{1}{2} J_w \dot{\theta}_r^2 + \frac{1}{2} J_\psi \dot{\psi}^2 + \frac{1}{2} J_\phi \dot{\phi}^2 + \frac{1}{2} n^2 J_m (\dot{\theta}_l - \dot{\psi})^2 + \frac{1}{2} n^2 J_m (\dot{\theta}_r - \dot{\psi})^2$$

$$U = M g z_b$$

The fifth and sixth term in  $T_2$  are rotation kinetic energy of an armature in left and right DC motor. The Lagrangian  $L$  has the following expression.

$$L = T_1 + T_2 - U$$

We use the following variables as the generalized coordinates.

$\theta$  : Average angle of left and right wheel

$\psi$  : Body pitch angle

$\phi$  : Body yaw angle

The state variables are

$$[x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6] = [\theta \ \psi \ \phi \ \dot{\theta} \ \dot{\psi} \ \dot{\phi}]$$

Lagrange equations are the following.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = F_\theta$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\psi}} \right) - \frac{\partial L}{\partial \psi} = F_\psi$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) - \frac{\partial L}{\partial \phi} = F_\phi$$

The following equations are found by evaluating the above expressions.

$$\begin{aligned} [(2m + M)R^2 + 2J_w + 2n^2 J_m] \ddot{\theta} + (MLR \cos \psi - 2n^2 J_m) \ddot{\psi} - MLR \dot{\psi}^2 \sin \psi &= F_\theta \\ (MLR \cos \psi - 2n^2 J_m) \ddot{\theta} + (ML^2 + J_\psi + 2n^2 J_m) \ddot{\psi} - MgL \sin \psi - ML^2 \dot{\phi}^2 \sin \psi \cos \psi &= F_\psi \\ \left[ \frac{1}{2} m W^2 + J_\phi + \frac{W^2}{2R^2} (J_w + n^2 J_m) + ML^2 \sin^2 \psi \right] \ddot{\phi} + 2ML^2 \dot{\psi} \dot{\phi} \sin \psi \cos \psi &= F_\phi \end{aligned}$$

In consideration of DC motor torque and viscous friction, the generalized forces are given as the following

$$(F_\theta, F_\psi, F_\phi) = \left( F_l + F_r, F_\psi, \frac{W}{2R} (F_r - F_l) \right)$$

$$F_l = nK_t i_l + f_m (\dot{\psi} - \dot{\theta}_l) - f_w \dot{\theta}_l$$

$$F_r = nK_t i_r + f_m (\dot{\psi} - \dot{\theta}_r) - f_w \dot{\theta}_r$$

$$F_\psi = -nK_t i_l - nK_t i_r - f_m (\dot{\psi} - \dot{\theta}_l) - f_m (\dot{\psi} - \dot{\theta}_r)$$

where  $i_{l,r}$  is the DC motor current. We cannot use the DC motor current directly in order to control it because it is based on PWM (voltage) control. Therefore, we evaluate the relation between current  $i_{l,r}$  and voltage  $v_{l,r}$  using DC motor equation. If the friction inside the motor is negligible, the DC motor equation is generally as follows

$$L_m i_{l,r} = v_{l,r} + K_b (\dot{\psi} - \dot{\theta}_{l,r}) - R_m i_{l,r}$$

Here we consider that the motor inductance is negligible and is approximated as zero. Therefore the current is

$$i_{l,r} = \frac{v_{l,r} + K_b (\dot{\psi} - \dot{\theta}_{l,r})}{R_m}$$

The generalized force can be expressed using the motor voltage.

$$F_\theta = \alpha (v_l + v_r) - 2(\beta + f_w) \dot{\theta} + 2\beta \dot{\psi}$$

$$F_v = -\alpha (v_l + v_r) + 2\beta \dot{\theta} - 2\beta \dot{\psi}$$

$$F_\phi = \frac{W}{2R} \alpha (v_r - v_l) - \frac{W^2}{2R^2} (\beta + f_w) \dot{\phi}$$

$$\alpha = \frac{nK_t}{R_m}, \quad \beta = \frac{nK_t K_b}{R_m} + f_m$$

To find the expressions for  $\ddot{\theta}$ ,  $\ddot{\psi}$ ,  $\ddot{\phi}$  the following notations are used.

$$A = (2m + M)R^2 + 2J_w + 2n^2 J_m + 2(\beta + f_w)$$

$$B = MLR \cos \psi - 2n^2 J_w$$

$$C = ML^2 + J_\psi + 2n^2 J_m$$

$$D = \frac{1}{2} m W^2 + J_\phi + \frac{W^2}{2R^2} (J_w + n^2 J_m) + ML^2 \sin^2 \psi$$



The derivatives of the state variables without linearization are expressed by

$$\dot{\theta} = \frac{d\theta}{dt}$$

$$\dot{\psi} = \frac{d\psi}{dt}$$

$$\dot{\phi} = \frac{d\phi}{dt}$$

$$\ddot{\theta} = \frac{(F_{\theta} + MLR\dot{\psi}^2 \sin \psi)(ML^2 + J_{\psi} + 2n^2 J_m) - B[F_{\psi} + MgL \sin \psi + ML^2 \dot{\phi}^2 \sin \psi \cos \psi]}{AC - B^2}$$

$$\ddot{\psi} = \frac{A[F_{\psi} + MgL \sin \psi + ML^2 \dot{\phi}^2 \sin \psi \cos \psi] - B[F_{\theta} + MLR\dot{\psi}^2 \sin \psi]}{AC - B^2}$$

$$\ddot{\phi} = \frac{\frac{W}{2R}\alpha(v_r - v_1) - \frac{W^2}{2R^2}(\beta + f_w) - 2ML^2 \sin \psi \cos \psi \dot{\phi} \dot{\psi}}{D}$$

From the above expressions Jacobian is given by  $J =$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & J(1,1) & 0 & J(1,2) & J(1,3) & J(1,4) \\ 0 & J(2,1) & 0 & J(2,2) & J(2,3) & J(2,4) \\ 0 & J(3,1) & 0 & 0 & J(3,2) & J(3,3) \end{bmatrix}$$

$$J(3,1) = \frac{-ML^2 D \dot{\psi} \dot{\phi}^2 \cos 2\psi - (\text{numerator}(\ddot{\phi})) (ML^2 \sin 2\psi)}{D^2}$$

$$J(3,2) = \frac{-2ML^2 \dot{\phi} \sin \psi \cos \psi}{D}$$

$$J(3,3) = \frac{-W^2}{2R^2}(\beta + f_w) - 2ML^2 \dot{\psi} \cos \psi \sin \psi / D$$

$$J(2,4) = \frac{2AML^2 \dot{\phi} \sin \psi \cos \psi}{AC - B^2}$$

$$J(2,3) = \frac{-2\beta(A+B) - 2BMLR\dot{\psi} \sin \psi}{AC - B^2}$$

$$J(2,2) = \frac{2B(\beta + f_w) + 2A\beta}{AC - B^2}$$

$$J(1,2) = \frac{-2(\beta + f_w)C - 2\beta B}{AC - B^2}$$

$$J(1,3) = \frac{(2\beta + 2MLR\dot{\psi} \sin \psi)C + 2\beta B}{AC - B^2}$$

$$J(1,4) = \frac{-2ML^2 \sin \psi \cos \psi \dot{\phi} B}{AC - B^2}$$

$$J(2,1) = ((AC - B^2)[A(MgL \cos \psi - 4ML^2 \dot{\phi}^2 \cos 2\psi) - B(MLR\dot{\psi}^2 \cos \psi) + [F_{\theta} + MLR\dot{\psi}^2 \sin \psi]][MLR \sin \psi] - (\text{numerator}(\ddot{\psi}))(2B)(MLR \sin \psi)) / [AC - B^2]^2$$

$$J(1,1) = (AC - B^2) \left[ MLR\dot{\psi} \cos^2 \psi (ML^2 + J_{\psi} + 2n^2 J_m) - B[MgL \cos \psi + 2ML^2 \dot{\phi}^2 \cos(2\psi)] + [F_{\psi} + MgL (\text{numerator}(\ddot{\theta}))(2BMLR \sin \psi)] / [AC - B^2]^2 \right]$$

### Linearization and State-Space Representation

The derivatives of the state variables are equated to zero to find the equilibrium points. The



equilibrium points found are when pitch angle  $\psi$  is 0 or  $\pi$  from the positive z axis and the derivatives of the state variables are 0.

Since point  $\psi = 0$  from vertical is unstable equilibrium, Jacobian is linearized around this point. After substituting the equilibrium points in the Jacobian, the following state space representation is obtained.

$$\dot{X} = A_0 X + B_0 U$$

$$\text{where, } X = \begin{bmatrix} \theta & \psi & \phi & \dot{\theta} & \dot{\psi} & \dot{\phi} \end{bmatrix}^\top$$

$$U = \begin{bmatrix} v_l & v_r \end{bmatrix}^\top$$

$$E = \begin{bmatrix} (2m + M)R^2 + 2J_v + 2n^2 J_m & MLR - 2n^2 J_m \\ MLR - 2n^2 J_m & ML^2 + J_\psi + 2n^2 J_m \end{bmatrix}$$

$$A_0 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & x(1,1) & 0 & x(1,2) & x(1,3) & 0 \\ 0 & x(2,1) & 0 & x(2,2) & x(2,3) & 0 \\ 0 & 0 & 0 & 0 & 0 & x(3,1) \end{bmatrix}$$

$$x(1,1) = \frac{-gMLE(1,2)}{\det(E)}$$

$$x(1,2) = \frac{-2[(\beta + f_w)E(2,2) + \beta E(1,2)]}{\det(E)}$$

$$x(1,3) = \frac{2\beta[E(2,2) + E(1,2)]}{\det(E)}$$

$$x(2,1) = \frac{gMLE(1,1)}{\det(E)}$$

$$x(2,2) = \frac{2[(\beta + f_w)E(1,2) + \beta E(1,1)]}{\det(E)}$$

$$x(2,3) = \frac{-2\beta[E(1,1) + E(1,2)]}{\det(E)}$$

$$x(3,1) = \frac{\frac{W^2}{2R^2}(\beta + f_w)}{\frac{1}{2}mW^2 + J_\phi + \frac{W^2}{2R^2}(J_w + n^2 J_m)}$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ B_0(4) & B_0(4) \\ B_0(5) & B_0(5) \\ -B_0(6) & B_0(6) \end{bmatrix}$$

$$B_0(4) = \frac{\alpha[E(2,2) + E(1,2)]}{\det(E)}$$

$$B_0(5) = \frac{-\alpha[E(1,1) + E(1,2)]}{\det(E)}$$

$$B_0(6) = \frac{W\alpha}{2R \left[ \frac{1}{2}mW^2 + J_\phi + \frac{W^2}{2R^2}(J_w + n^2 J_m) \right]}$$

**Q 3.** Equation 1 represents a continuous-time system. The equivalent discrete time system is represented as:

$$X(k+1) = A_d X(k) + B_d U(k) \quad (2)$$

Where  $X(k)$  is a measure of the states at  $k_{th}$  sampling instant, i.e.,  $X(k) = [x_1(k), x_2(k), \dots, x_n(k)]^T$ , and  $U(k)$  is the vector of input to the system at  $k_{th}$  sampling instant, i.e.  $U(k) = [u_1(k), u_2(k), \dots, u_m(k)]^T$ .  $A_d$  is the Discrete State Matrix &  $B_d$  is the Discrete Input Matrix.

What should be the position of eigen values of  $A_d$  for system to be stable.

**Hint:** In frequency domain, continuous-time system is represented with Laplace transform and discrete-time system is represented with Z transform. [5]

**A 3.** The Laplace transform in frequency domain is used to construct any aperiodic finite signal using growing or damped complex exponentials of continuous frequencies from zero to infinity. The laplace tranform is given by

$$F(s) = \int_0^\infty f(t)e^{-st} dt$$

where,  $s = \sigma + j\omega$

We can find the system transfer function using the state space equations under zero state conditions. The real part of the roots of the system transfer function should be negative, so that after convolution the output of the system is finite thereby meaning stable.

In z domain it is similar case but the continuous time signal is sampled with a period  $T_s$ , but  $z = re^{j\omega}$ , so here the  $\text{abs}(z)$  should be lesser than one so that system is stable.

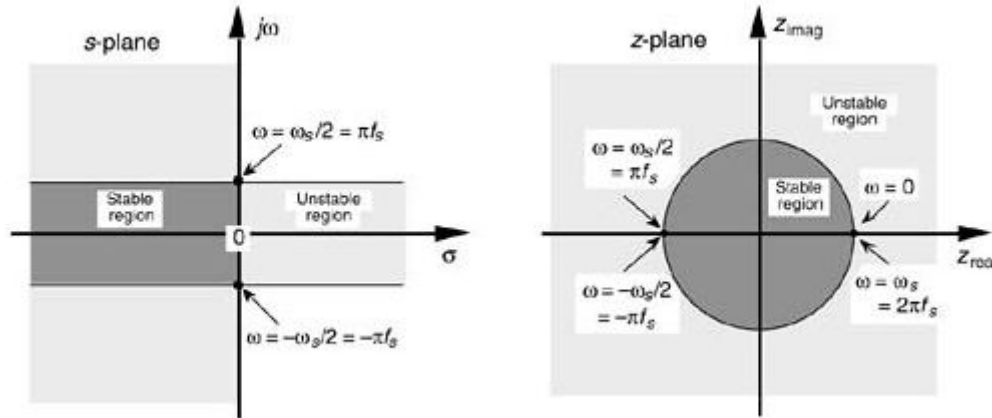
$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Hence the eigenvalues represent the s and z values of continuous and discrete case respectively of the system transfer function under zero state condition. Therefore for discrete case, the  $\text{abs}(\text{eigenvalues})$  should be lesser than one for the system to be stable.

If the real part of the s parameter is zero then it corresponds to the Y axis in the  $\sigma - j\omega$  plane which corresponds to the unit circle in the z plane.

The system is stable if  $T_s > 0$  and the eigenvalues are within the unit circle, as determined by this statement:

$\text{all}(\text{abs}(\text{eig}(Ad))) < 1$  where  $Ad = TA_0 + I$  considering Euler's transformation.



**Q 4.** Will LQR control always works? If No, then why not? and if Yes, Justify your answer.

**Hint:** Take a look at definition of Controllable System. What is controllability? [5]

**A 4.** No. LQR controllers are controllers that provide input that can steer a particular system from some initial point at  $x_0$  at  $t=0$  to any state  $x_{final}$  at  $t=n$ . If the controllability matrix has rank less than the number of state variables then the system is said to be uncontrollable. Hence if system is controllable then only can we use LQR controllers. Also when the the specifications of the components do not meet the requirement of the LQR, LQR might work but the desired state cannot be obtained and hence it will become uncontrollable.

**Q 5.** For balancing robot on two wheel i.e. as inverted pendulum, the center of mass should be made high or low? Justify your answer. [5]

**A 5.** • The torque equation is given by  $\tau = MgL \sin \theta$ .

- Here  $Mg$  is weight of the body and  $L$  is the distance of the centre of mass from the wheel axis.
- According to the equation when  $L$  increases, torque applied by the body increases, to counter this torque, the torque requirement of motor increases. Hence to reduce the torque requirement the position of centre of mass is made low in our design.
- Also from the equation,  $\theta$  can be more as  $L$  is made less, so the bot can balance itself for large angular displacement.

**Q 6.** Why do we require filter? Do we require both the gyroscope and the accelerometer for measuring the tilt angle of the robot? Why? [10]

**A 6.** We require filter to remove noise signals which is induced by the mechanical vibrations and also to complement for the disadvantage present in acquiring signals in accelerometer and gyroscope.

### Accelerometer

An accelerometer is used to measure the inertial force which is directed in the opposite direction of the acceleration vector. It measures force and not acceleration but it happens that acceleration also causes an inertial force which is captured by the force detecting mechanism of the accelerometer. We need to calculate pitch using the accelerometer which can be done using the formula

$$\text{pitch} = \text{atan2}(\text{accx}, \text{sqrt}(\text{accy}^2 + \text{accz}^2))$$

### Disadvantages.

1. Pure yaw angles cannot be calculated from just a accelerometer because Z will be perfectly perpendicular to the plane of motion.
2. Accelerometer experiences translational accelerations too. These will add additional acceleration values and so the orientation values obtained won't be the actual one.
3. Accelerometer alone used to measure the pitch angle will be very noisy. The platform is horizontal, but the motors are causing it to accelerate forward. The accelerometer cannot distinguish this from gravity.

Hence we use a low pass filter to filter out the short duration horizontal acceleration so that only the long term acceleration is captured that is gravity. However, the con of this is that Angle measurement will lag due to the averaging. The more it is filtered, the more it will lag. Hence a sudden change or a large change cannot be detected easily since it is averaged before.

### Gyrometer

The gyroscope measures the angular velocity of the body frame with respect to the inertial frame, expressed in the body frame. Hence to find the angular displacement, we have to integrate the angular velocity.

### Advantages and disadvantages

1. The gyrometer provides the orientation estimates at high sampling rates which are accurate on a short time scale but drift over longer time scales due to integration drift.

- If we measure a quantity which is zero using a non-perfect sensor, in case our measurements are corrupted by a constant bias, integration of these measurements will lead to a signal which grows linearly with time. If the sensor instead measures a zero-mean white noise signal, the expected value of the integrated measurements would be zero, but the variance would grow with time.

If we pass the angle obtained through a high pass filter we will obtain value for the angle measurement but it will be plagued by the variance due noise.

Hence we use a complementary filter where we pass the angle estimated by the accelerometer through a low pass filter (good estimation of the static conditions) and gyrometer through a high pass filter (good estimation of the dynamic conditions) and combine the results.

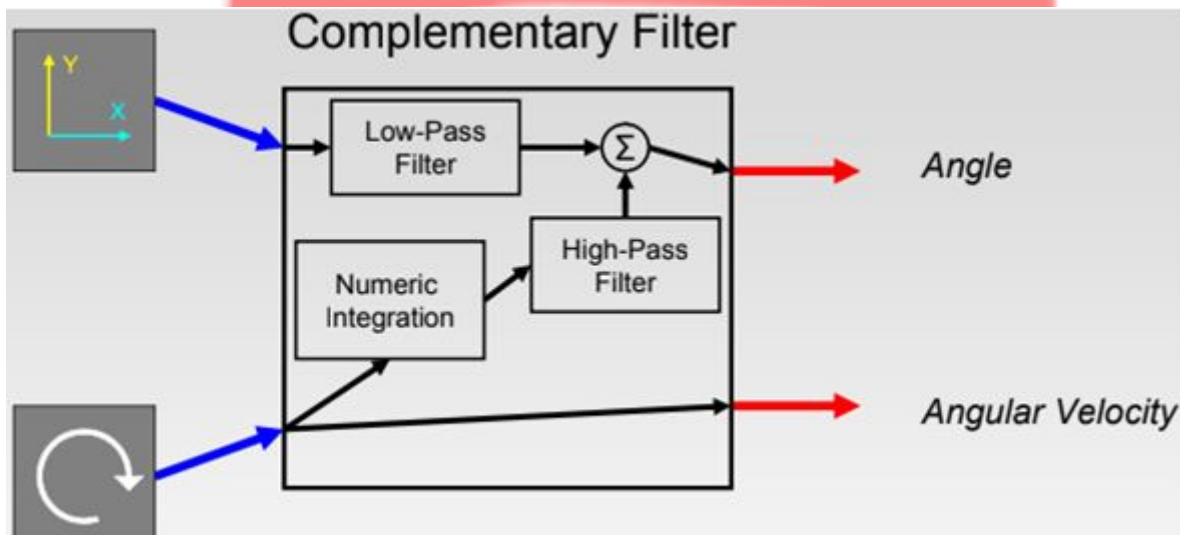
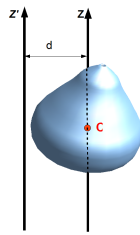
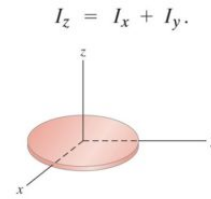


Figure 5: Complementary filter to obtain angular displacement and velocity

**Q 7.** What is Perpendicular and Parallel axis theorem for calculation of Moment of Inertia? Do you require this theorem for modelling the Medbot? Explain Mathematically. [15]



(a) Parallel Axis theorem



(b) Perpendicular Axis Theorem

Figure 6

### A 7. Parallel Axis Theorem

Suppose a body of mass  $m$  is rotated about an axis  $z$  passing through the body's centre of gravity. The body has a moment of inertia  $I_{cm}$  with respect to this axis. The parallel axis theorem states that if the body is made to rotate instead about a new axis  $z'$  which is parallel to the first axis and displaced from it by a distance  $d$ , then the moment of inertia  $I$  with respect to the new axis is related to  $I_{cm}$  by

$$I = I_{cm} + md^2$$

Explicitly,  $d$  is the perpendicular distance between the axes  $z$  and  $z'$

### Perpendicular Axis Theorem

The perpendicular axis theorem states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about the two axes at right angles to each other, in its own plane intersecting each other at the point where the perpendicular axis passes through it.

Define perpendicular axes  $x, y$ , and  $z$  (which meet at origin  $O$ ) so that the body lies in the  $xy$  plane, and the  $z$  axis is perpendicular to the plane of the body. Let  $I_x, I_y$  and  $I_z$  be moments of inertia axis  $x, y, z$  respectively, the perpendicular axis theorem states that <sup>[1]</sup>

$$I_z = I_x + I_y$$

Yes, we use parallel axis theorem in the derivation.

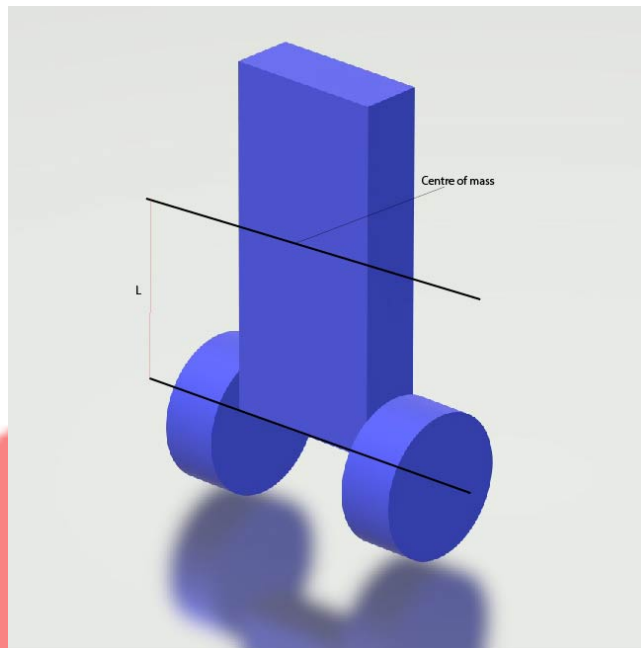


Figure 7: Medbot

1.  $J_{\psi} = ML^2/3$ , this is the moment of inertia of the body about its center of mass when axis is orthogonal to the height. Here the length of the body (H) is considered to be (2L).
2. The moment of inertia of a rectangular plane about its center of mass when axis is orthogonal to its height is  $MH^2/12$ , but  $H=2L$ .
3. Therefore moment of inertia was found to be  $M(2L)^2/12 = ML^2/3$ .

As seen in 'C' term in answer of question 2 we get  $ML^2 + J_{\psi} + 2n^2J_m$ . Here  $J_{\psi}$  term is getting added to  $ML^2$  where L is the distance of the body's centre of mass from the wheel axle.

Wheel axle is the pitch axis of rotation of the Medbot.

Also  $J_{\psi}$  is moment of inertia of the body when the axis of rotation passes through the centre of mass orthogonal to its height. Therefore we use parallel axis theorem to bring the axis of rotation to the wheel axle by adding the moment of inertia about the center of mass  $J_{\psi}$  with  $ML^2$ .

**Q 8.** What will happen in the following situations:

- (a) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]
- (b) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit falls outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]



- (c) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall inside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]
- (d) Medbot picks a First-Aid Kit from the shelf of Medical Store but the First-Aid Kit and the Medbot both fall outside the store. Will there be any penalty imposed, points awarded? Will the First-Aid Kit be repositioned? [2]

**A 8.** What will happen in the following situations:

- (a) No, Penalty is not imposed. First-Aid Kit is repositioned.
- (b) Penalty is not imposed. A pick-up is successful if the item is carried out of the store, since FAK kit fell outside the store, this is considered successful pick-up and 20 points are awarded. First-Aid Kit is repositioned.
- (c) A fall penalty of 50 points are imposed. Points are not awarded. First-Aid kit is repositioned.
- (d) A fall penalty of 50 points are imposed. This is a successful pick-up, so 20 points are awarded. First-Aid kit is repositioned.

**Q 9.** What will be the points awarded if Medbot picks only one of the item from the medical store and repeatedly moves back and forth around the gravel pathway or the bridge for the entire run. [4]

**A 9.**

- Initially if the Medbot goes to store to pick-up an item through gravel pathway
  - Marks awarded for traversing the bridge,  $M_B = 70 * (1) = 70$ .
  - Marks awarded for traversing the gravel pathway,  $M_G = 50 * (0.5 * 1 + 1) = 75$ .
  - Marks awarded for Successful pick-up = 20
  - Total = 165.
- Initially if the Medbot goes to store to pick-up an item by crossing the bridge.
  - Marks awarded for traversing the bridge,  $M_B = 70 * (0.5 * 1 + 1) = 105$ .
  - Marks awarded for traversing the gravel pathway,  $M_G = 50 * (1) = 50$ .
  - Marks awarded for Successful pick-up = 20
  - Total = 175.
- Initially if the Medbot goes to store to pick-up an item by passing through the pathway

where rooms are present.

- Marks awarded for traversing the bridge,  $M_B = 70 * (1) = 70$
- Marks awarded for traversing the gravel pathway,  $M_G = 50 * (1) = 50$ .
- Marks awarded for Successful pick-up = 20
- Total = 140.

**Q 10.** What are the different communication protocols you'll be using? Name the hardware interfaced related to each of the communication protocols. Explain how these communication protocols works and what are the differences between them. [8]

**A 10.** The different communication protocols we use for the Medbot include:

1. **Universal Asynchronous Receiver/Transmitter (UART)** for serial communication between the Xbee S2C module and the Arduino Mega.
2. **Inter-Integrated Circuit (I2C)** communication protocol for communication between the GY-87 module and the Arduino Mega.
3. The communication protocol used to communicate between the two Xbee modules is based upon IEEE 802.15.4 standard.

### Working

1. It is used for asynchronous serial communication in which the data format and transmission speeds are configurable. The electric signaling levels and methods are handled by a driver circuit external to the UART. A UART is usually an individual (or part of an) integrated circuit ( $I^2C$ ) used for serial communications over a computer or peripheral device serial port. There are three hardware serial ports in Arduino Mega.

### Data frame of UART

**Start bit:** The UART data transmission line is normally held at a high voltage level when it's not transmitting data. To start the transfer of data, the transmitting UART pulls the transmission line from high to low for one clock cycle. When the receiving UART detects the high to low voltage transition, it begins reading the bits in the data frame at the frequency of the baud rate.

**Data Frame:** The data frame contains the actual data being transferred. It can be 5 bits up to 8 bits long if a parity bit is used. If no parity bit is used, the data frame can be 9 bits long. In most cases, the data is sent with the least significant bit first.

**Parity:** If the parity bit is a 0, and the total is odd; or the parity bit is a 1, and the total is even, the UART knows that bits in the data frame have changed.

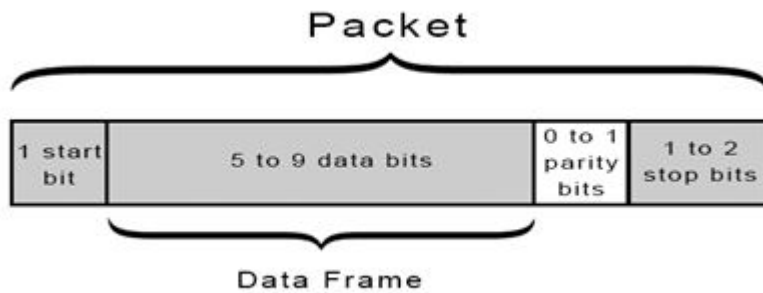


Figure 8: Data frame of UART

**Stop bits:** To signal the end of the data packet, the sending UART drives the data transmission line from a low voltage to a high voltage for at least two bit durations.

2.  $I^2C$  is a synchronous, multi-master, multi-slave, packet switched, single-ended, serial computer bus. In  $I^2C$ , there can be more than one master, only upper bus speed is defined and only two wires with pull-up resistors are needed to connect almost unlimited number of  $I^2C$  devices.  $I^2C$  can use even slower microcontrollers with general-purpose I/O pins since they only need to generate correct Start and Stop conditions in addition to functions for reading and writing a byte. Each slave device has a unique address. Transfer from and to master device is serial and it is split into 8-bit packets. The initial  $I^2C$  specifications defined maximum clock frequency of 100 kHz. This was later increased to 400 kHz as Fast mode.

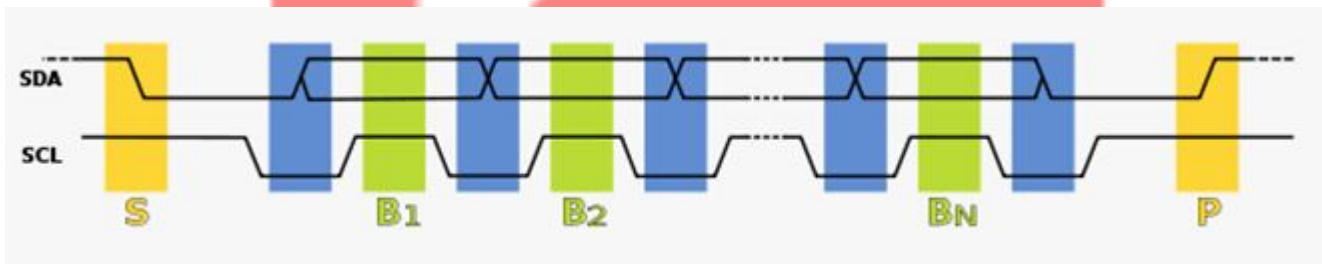


Figure 9:  $I_2C$  Protocol

- In normal state both lines (SCL and SDA) are high. The communication is initiated by the master device. It generates the Start condition (S) followed by the address of the slave device (B1).
- If the bit 0 of the address byte was set to 0 the master device will write to the slave device (B2). Otherwise, the next byte will be read from the slave device.
- Once all bytes are read or written (Bn) the master device generates Stop condition (P). This signals to other devices on the bus that the communication has ended and another

device may use the bus.

Most I2C devices support repeated start condition. This means that before the communication ends with a stop condition, master device can repeat start condition with address byte and change the mode from writing to reading.

3.

- IEEE 802.15.4 is a technical standard which defines the operation of low-rate wireless personal area networks (LR-WPANs).
- The 802.15.4 standard specifies that communication should occur in 5 MHz channels ranging from 2.405 to 2.480 GHz. Only approximately 2 MHz of the channel is consumed with the occupied bandwidth.
- In the 2.4 GHz band, a maximum over-the-air data rate of 250 kbps is specified, but due to the overhead of the protocol the actual theoretical maximum data rate is approximately half of that.
- ZigBee is a higher latency, lower bandwidth, asynchronous protocol that uses the 802.15.4 standard as a baseline and adds additional routing and networking functionality. We use the Xbee in API mode with sample rate of 10ms to obtain the information from the remote at the Medbot.

#### Differences

1. UART is typically for a point to point connection, I2C is a bus protocol
2. UART is (can be) full duplex, I2C is not
3. UART does not have a master/slave principle (no protocol), I2C has
4. UART can be used for transmission over distance, I2C is not meant for that.

**Q 11.** Why do we require IRF540N? Provide circuit diagram for interfacing IRF540N with the microcontroller. [5+5]

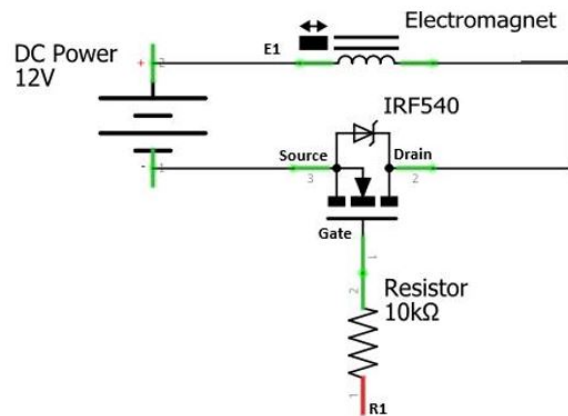


Figure 10: Electromagnet and MOSFET connection circuit.

- A 11.**
- Electromagnet needs good amount of current for it produce good amount of magnetic field. But Arduino pins are capable of delivering a maximum current of 40mA which is quite less.
  - Hence the electromagnet is powered by the LiPo battery source directly and use IRF540N(MOSFET) as a voltage controlled switch which is capable of delivering 23A at  $100^{\circ}C$  and  $V_{GS} = 10V$  with the gate as the pin from the Arduino to control the current.

IRF540N has high power dissipation capability since it's a power N channel MOSFET .If connected directly, the current drawn will be near maximum and there maybe unnecessary resets in the Arduino due to it. When the  $V_{GS}$  is high(control input) the MOSFET acts as short circuit and the Electromagnet is turned ON. When the  $V_{GS}$  is low the MOSFET acts as open circuit and the entire voltage drop is across the transistor and the Electromagnet is turned OFF. In the above figure R1 is connected to the microcontroller.