Calculate conditional probability for dependent events

Recently, you learned that **conditional probability** refers to the probability of an event occurring given that another event has already occurred. Conditional probability allows you to describe the relationship between dependent events, or how the occurrence of the first event affects the likelihood of the second event.

In this reading, you'll learn how to calculate conditional probability for two or more dependent events. Before we discuss calculating conditional probability, we'll go over the concept of dependence.

Conditional probability

Previously, you calculated probability for a single event, and for two or more independent events, such as two consecutive coin flips. Conditional probability applies to two or more dependent events.

Dependent events

Earlier, you learned two events are **independent** if the first event does not affect the outcome of the second event, or change its probability. For example, two consecutive coin tosses are independent events. Getting heads on the first toss doesn't affect the outcome of the second toss.

In contrast, two events are **dependent** if the occurrence of one event changes the probability of the other event. This means that the first event affects the outcome of the second event.

For instance, if you want to get a good grade on an exam, you first need to study the course material. Getting a good grade depends on studying. If you want to eat at a popular restaurant without waiting for a table, you have to arrive early. Avoiding a wait depends on arriving early. In each instance, you can say that the second event is dependent on, or conditional on, the first event.

Now that you have a better understanding of dependent events, let's return to conditional probability and review the formula.

Formula for conditional probability

The formula says that for two dependent events A and B, the probability of event A and event B occurring equals the probability of event A occurring, multiplied by the probability of event B occurring, given event A has already occurred.

Conditional probability

P(A and B) = P(A) * P(B|A)

In probability notation, the vertical bar between the letters B and A indicates dependence, or that the occurrence of event B depends on the occurrence of event A. You can say this as "the probability of B given A."

The formula can also be expressed as the probability of event B given event A equals the probability that both A and B occur divided by the probability of A.

Conditional probability

P(B|A) = P(A and B) / P(A)

These are just two ways of representing the same equation. Depending on the situation, or what information you are given up front, it may be easier to use one or the other.

Note: The conditional probability formula also applies to independent events. When A and B are independent events, P(B|A) = P(B). So, the formula becomes P(A and B) = P(A) * P(B). This formula is also the multiplication rule that you learned about earlier in the course.

Example: playing cards

Let's explore an example of conditional probability that deals with a standard deck of 52 playing cards.

Imagine two events:

- The first event is drawing a heart from the deck of cards.
- The second event is drawing another heart from the same deck.

Say you want to find out the probability of drawing two hearts in a row. These two events are dependent because getting a heart on the first draw changes the probability of getting a heart on the second draw.

A standard deck includes four different suits: hearts, diamonds, spades, and clubs. Each suit has 13 cards. For the first draw, the chance of getting a heart is 13 out of 52, or 25%. For the second draw, the probability of getting a heart changes because you've already picked a heart on the first draw. Now, there are 12 hearts in a deck of 51 cards. For the second draw, the chance of getting a heart is 12 out of 51, or about 23.5%. Getting a heart is now less likely—the probability has gone from 25% to 23.5%.

Now, let's apply the conditional probability formula:

P(A and B) = P(A) * P(B|A)

You want to calculate the probability of both event A and event B occurring. Let's call event A *1st heart,* which refers to getting a heart on the first draw. Let's call event B *2nd heart,* which refers to getting a heart on the second draw, given a heart was drawn the first time. The probability of event A is 13/52, or 25%. The probability of event B is 12/51, or 23.5%.

Let's enter these numbers into the formula:

P(1st heart and 2nd heart) = P(1st heart) * P(2nd heart | 1st heart) = 13/52 * 12/51 = 1/17 = 0.0588, or about 5.9%

So, there is a 5.9% chance of drawing two hearts in a row from a standard deck of playing cards.

Example: online purchases

Let's explore another example. Imagine you are a data professional working for an online retail store. You have data that tells you 20% of the customers who visit the store's website make a purchase of $100 or more. If a customer spends $100, they are eligible to receive a free gift card. The store randomly awards gift cards to 10% of the customers who spend at least $100.

You want to calculate the probability that a customer spends $100 and receives a gift card. Receiving a gift card depends on first spending $100. So, this is a conditional probability because it deals with two dependent events.

Let's apply the conditional probability formula:

P(A and B) = P(A) * P(B|A)

You want to calculate the probability of both event A and event B occurring. Let's call event A *$100* and event B *gift card.* The probability of event A is 0.2, or 20%. The probability of event B is 0.1, or 10%.

P($100 and gift card) = P($100) * P(gift card given $100) = 0.2 * 0.1 = 0.02, or 2%

So, the probability of a customer spending $100 or more and receiving a free gift card is 0.2 * 0.1 = 0.02, or 2%.

Key takeaways

Conditional probability helps you describe the relationship between dependent events. Data professionals often use conditional probability in a business context. For example, they might use conditional probability to predict how an event like a new ad campaign will impact sales revenue. This helps stakeholders make intelligent decisions about the best way to invest their company's resources.

Resources for more information

To learn more about conditional probability, refer to the following resource:

- This article from Investopedia discusses conditional probability in a business context.

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