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# Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec



# Good and bad uncertainty: Macroeconomic and financial market implications



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### ARTICLE INFO

Article history: Received 13 January 2014 Received in revised form 14 July 2014 Accepted 13 November 2014 Available online 28 May 2015

JEL classification: G12 E20

C58

Keywords: Uncertainty Economic growth Asset prices Recursive utility

#### ABSTRACT

Does macroeconomic uncertainty increase or decrease aggregate growth and asset prices? To address this question, we decompose aggregate uncertainty into 'good' and 'bad' volatility components, associated with positive and negative innovations to macroeconomic growth. We document that in line with our theoretical framework, these two uncertainties have opposite impact on aggregate growth and asset prices. Good uncertainty predicts an increase in future economic activity, such as consumption, output, and investment, and is positively related to valuation ratios, while bad uncertainty forecasts a decline in economic growth and depresses asset prices. Further, the market price of risk and equity beta of good uncertainty are positive, while negative for bad uncertainty. Hence, both uncertainty risks contribute positively to risk premia, and help explain the cross-section of expected returns beyond cash flow risk.

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#### 1. Introduction

How do changes in economic uncertainty affect macroeconomic quantities and asset prices? We show that the answer to this question hinges on the type of uncertainty one considers. 'Bad' uncertainty is the volatility that is associated with negative innovations to macroeconomic quantities (e.g., output, consumption, earnings), and with lower prices and investment, while 'good' uncertainty is the volatility that is associated with positive shocks to these variables, and with higher asset prices and investment.

To illustrate these two types of uncertainties, it is instructive to consider two episodes: (i) the high-tech revolution of early-mid 1990s, and (ii) the recent collapse of Lehman Brothers in the fall of 2008. In the first case, and with the introduction of the world-wide-web, a common view was that this technology would provide many positive growth opportunities that would enhance the economy, yet it was unknown by *how much*? We refer to such a situation as 'good' uncertainty. Alternatively, the second

<sup>\*</sup>We thank the editor (Bill Schwert), an anonymous referee (Timothy McQuade), and participants and the discussants at 2014 AEA Meeting, 7th Annual SoFiE Conference, 2014 Brazilian Finance Society Meeting, 2014 CIREQ Montreal Econometrics Conference, 2013 Minnesota Macro-Asset Pricing Conference, 2014 SED Meeting, 2014 NBER Summer Institute, 2014 NBER's Universities Research Conference, 2014 Tel Aviv Finance Conference, 2013 Tepper-LAEF Conference, 2014 UBC Winter Finance Conference, 2014 WFA Meeting, University of Chicago Conference Honoring Lars Hansen, Bl Norwegian Business School, Boston College, CKGSB, Federal Reserve Board, IDC, LBS, LSE, OSU, Princeton, SAIF-Tsinghua, University of Frankfurt, University of Miami, University of Notre-Dame, and Wharton for their comments and suggestions. Shaliastovich and Yaron thank Jacobs Levy Equity Management Center for Quantitative Financial Research and the Rodney White Center, and Shaliastovich thanks the Cynthia and Bennett Golub Endowment for financial support.

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case marked the beginning of the global financial crisis, and with many of the ensuing bankruptcy cases one knew that the state of economy was deteriorating—yet, again, it was not clear by *how much*? We consider this situation as a rise in 'bad' uncertainty. In both cases, uncertainty level rises relative to its long-run steady-state level, yet, the first case coincides with an optimistic view, and the second with a pessimistic one.

In this paper, we demonstrate that variations in good and bad uncertainty have separate and significant opposing impacts on the real economy and asset prices. We use an extended version of the long-run risks model of Bansal and Yaron (2004) to theoretically show conditions under which good and bad uncertainty have different impacts on prices. To make a meaningful distinction between good and bad uncertainty, we decompose, within the model, the overall shocks to consumption into two separate zeromean components (e.g., jumps) which capture positive and negative growth innovations. The volatilities of these two shocks are time varying, and capture uncertainty fluctuations associated with the positive and negative parts of the distribution of consumption growth. Thus, in the model, valuation ratios are driven by three state variables: predictable consumption growth, good uncertainty, and bad uncertainty. Consequently, the stochastic discount factor, and therefore risk premia, are determined by three sources of risk: cash flow, good uncertainty, and bad uncertainty risks.

We show that with a preference for early resolution of uncertainty, the direct impact of both types of uncertainty shocks is to reduce prices, though, prices respond more to bad than to good uncertainty. For prices to rise in response to a good uncertainty shock there has to be an explicit positive link between good uncertainty and future growth prospects—a feature that we impose in our benchmark model.<sup>2</sup> We further show that the market price of good uncertainty risk and its equity beta have the same (positive) sign. Thus, even though prices can rise in response to good uncertainty, it commands a positive risk premium.

Overall, the model's key empirical implications include: (i) good uncertainty positively predicts future measures of economic activity, while bad uncertainty negatively forecasts future economic growth; (ii) good uncertainty fluctuations are positively related to asset valuations and to the real risk-free rate, while an increase in bad uncertainty depresses asset prices and the riskless yield; and (iii) the shocks to good and bad uncertainty carry positive and negative market prices of risk, respectively, yet both contribute positively to the risk premium.<sup>3</sup>

We evaluate our model's empirical implications by utilizing a novel econometric approach to identify good and bad uncertainty from higher-frequency realized variation in the variables of interest (see Barndorff-Nielsen,

Kinnebrock, and Shephard, 2010). Empirically, we use the ex ante predictable components of the positive and negative realized semivariances of industrial production growth rate as the respective proxies for good and bad uncertainty. In its limiting behavior, positive (negative) semivariance captures one-half of the variation in any Gaussian symmetric movements in the growth rate of the variable of interest, as well as the variation of any non-Gaussian positive (negative) component in it. Thus, in our empirical work the positive (negative) semivariance captures the volatility component that is associated with the positive (negative) part of the total variation of industrial production growth, and its predictive component corresponds to the model concept for good (bad) uncertainty.

Consistent with the model, we document in the data that across various macroeconomic growth rates, and across various horizons, good economic uncertainty positively predicts future growth. This evidence includes growth for horizons of one to five years in consumption, output, investment, research and development (R&D), market earnings, and dividends. Similarly, we find a negative relationship between bad uncertainty and future growth rates of these macro variables. Together, these findings support the model feedback channel from macroeconomic uncertainty to future growth rates. Quantitatively, the impact of uncertainty has a large economic effect on the macro variables. For example, the private gross domestic product (GDP) growth increases by about 2.5% one year after a one standard deviation shock to good uncertainty, and this positive effect persists over the next three years. On the other hand, bad uncertainty shocks decrease output growth by about 1.3% one year after and their effects remain negative for several years. The responses of investment and R&D to these shocks are even stronger. Both capital and R&D investment significantly increase with good uncertainty and remain positive five years out, while they significantly drop with a shock to bad uncertainty. An implication of the offsetting responses to good and bad uncertainty is that the measured responses to overall uncertainty are going to be muted. Indeed, GDP growth declines only by about 0.25% after a shock to total uncertainty. The response to total uncertainty is significantly weaker than that to bad uncertainty, which underscores the potential importance of decomposing uncertainty into good and bad components.

The empirical evidence in the data is further consistent with the model's key asset-pricing implications. We document that the market price-dividend ratio and the risk-free rate appreciate with good uncertainty and decline with bad uncertainty. Quantitatively, the market log price-dividend ratio rises by about 0.07 one year out in response to a one standard deviation shock to good uncertainty and remains positive ten years afterward. Bad uncertainty shock depresses the log price-dividend ratio by 0.06 on impact and remains negative for ten years out. Similar to the macroeconomic growth rates, the response of the price-dividend ratio to total uncertainty is negative, but

<sup>&</sup>lt;sup>2</sup> Backus, Routledge, and Zin (2010) also feature a direct feedback from volatility to future growth. However, they focus on total volatility and show the importance of this feedback for reconciling various lead-lag correlations between consumption growth and market returns.

<sup>&</sup>lt;sup>3</sup> Although both uncertainties carry positive risk premium, their covariance, which may capture a common component, could contribute negatively to the risk premium.

<sup>&</sup>lt;sup>4</sup> We use industrial production because high-frequency real consumption data are not available for the long sample.

is understated relative to the response to bad uncertainty. The evidence for the response of the price–earnings ratio is very similar to that of the price–dividend ratio. In addition, consistent with the model, we show that both bad and good uncertainty positively predict future excess returns and their volatility.

Finally, we estimate the market prices of good and bad volatility risks using the cross-section of asset returns that includes the market return, 25 equity portfolios sorted on book-to-market ratio and size, and two bond portfolios (Credit and Term premium portfolios). We show that the market price of risk is positive for good uncertainty, while it is negative for bad uncertainty. Moreover, asset returns have a positive exposure (beta) to good uncertainty risk, and a negative exposure to bad uncertainty risk. Consequently, both good and bad uncertainty command a positive risk premium, although the interaction of their shocks can contribute negatively to the total risk compensation, since the good and bad uncertainty shocks are positively correlated. The market risk premium is 7.2% in the data relative to 8.2% in the model. In the data, the value spread is 4.38%, which is comparable to 3.34% in the model. The size spread is 4.39%, relative to 5.21% in the model. For the Credit premium portfolio the risk premium is 1.98% in the data and 2.15% in the model, and the Term premium is 1.82% in the data relative to 0.64% in the model.

#### 1.1. Related literature

Our paper is related to a growing theoretical and empirical literature that documents the connection between economic uncertainty, aggregate quantities, and asset prices. Our concept of economic uncertainty refers to the time series volatility of shocks to economic quantity variables of interest (e.g., consumption and GDP growth). This is distinct from other aspects of uncertainty, such as parameter uncertainty, learning, robust-control, and ambiguity (see discussions in Pastor and Veronesi, 2009; Hansen and Sargent, 2010; Epstein and Schneider, 2010). While there is a longstanding and voluminous literature on the time-varying second moments in asset returns, the evidence for time variation in the second moments of macro aggregates, such as consumption, dividends, earnings, investment, and output, is more limited and recent. Kandel and Stambaugh (1991) is an early paper providing evidence for stochastic volatility in consumption growth. More recently, McConnell and Perez-Quiros (2000), Stock and Watson (2003), and Bansal, Khatchatrian, and Yaron (2005) provide supporting evidence that volatility measures based on macro aggregates feature persistent predictable variation.

The evidence on time-varying volatility of macro aggregates has also instilled recent interest in examining the role of uncertainty in dynamic stochastic general equilibrium (DSGE) production models. Bloom (2009) shows that increased volatility, measured via VIX, leads to an immediate drop in consumption and output growth rates as firms delay their investment decisions. Generally, the literature has emphasized a negative relationship between growth and uncertainty—see Ramey and Ramey (1995), Gilchrist, Sim, and Zakrajsek (2014), Fernandez-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2011), and

Basu and Bundick (2012), to name a few. Other papers, such as Gilchrist and Williams (2005), Jones, Manuelli, Siu, and Stacchetti (2005), Malkhozov (2014), and Kung and Schmid (2015) feature alternative economic channels which can generate a positive relationship between uncertainty and investment and thus growth. In addition, Croce, Nguyen, and Schmid (2012) and Pastor and Veronesi (2012) highlight the negative impact of government policy uncertainty on prices and growth.

In terms of asset prices, Bansal and Yaron (2004) show that with Epstein and Zin (1989) recursive preferences and an intertemporal elasticity of substitution (IES) larger than one, economic uncertainty is a priced risk, and is negatively related to price-dividend ratios. More recently, Bansal, Kiku, Shaliastovich, and Yaron (2014) examine the implications of macroeconomic volatility for the time variation in risk premia, for the return on human capital. and for the cross-section of returns. They develop a dynamic capital asset-pricing model (CAPM) framework for which one of the factors, in addition to the standard cash flow and discount rate risks, is aggregate volatility. Campbell, Giglio, Polk, and Turley (2012) also analyze the role of uncertainty in an extended version of the intertemporal capital asset-pricing model (ICAPM). While both papers document a significant role for uncertainty, Bansal, Kiku, Shaliastovich, and Yaron (2014) find both the betas and market price of uncertainty risk to be negative, and thus uncertainty to positively contribute to equity risk premia, whereas the evidence in Campbell, Giglio, Polk, and Turley (2012) is more mixed in terms of whether assets have negative or positive exposure (beta) to volatility. The empirical framework in this paper, allowing for two types of uncertainties, can in principle accommodate several of these uncertainty effects.

Our framework features two types of macroeconomic uncertainties. In terms of estimating two types of uncertainties, the literature has mainly focused on return-based measures. Patton and Sheppard (2015), Feunou, Jahan-Parvar, and Tédongap (2013), and Bekaert, Engstrom, and Ermolov (2015) use return data to capture fluctuations in good and bad volatilities, and study their effects on the dynamics of equity returns. Specifically, Patton and Sheppard (2015) and Feunou, Jahan-Parvar, and Tédongap (2013) use realized semivariance measures to construct the two volatilities, whereas we construct bad and good uncertainty measures directly from the macro aggregates.

Our framework is also related to a recent literature which highlights non-Gaussian shocks in the fundamentals. One analytically convenient specification that our framework accommodates and which is widely used features Poisson jumps in consumption dynamics (see, e.g., Eraker and Shaliastovich, 2008; Benzoni, Collin-Dufresne, and Goldstein, 2011; Drechsler and Yaron, 2011; Tsai and Wachter, 2014 for recent examples). In another specification, which again can be accommodated within our framework, the cash flow shocks are drawn from a Gamma distribution with a time-varying shape parameter, in which case the consumption shock dynamics follow the good and bad environment specification in Bekaert and Engstrom (2009). Finally, an alternative approach for generating time variation in higher-order

moments is provided in Colacito, Ghysels, and Meng (2013). They model shocks to expected consumption as drawn from a skew-normal distribution with time-varying parameters and allow for a separate process for stochastic volatility. Our modeling approach focuses on bad and good volatility as the key driving forces for time variation in consumption growth distribution, and is largely motivated by our empirical analysis.

There is also a voluminous literature on the implications of time-varying higher-order moments of returns for risk pricing. For example, Bansal and Viswanathan (1993) develop a nonlinear pricing kernel framework and show its improvement in explaining asset prices relative to a linear arbitrage pricing theory (APT) model, while Chabi-Yo (2012) develops an intertemporal capital asset pricing model in which innovations in higher moments are priced. The empirical literature identifies these risks based on financial market data, and generally finds that left-tail risk is important for explaining the time series and cross-section of returns above and beyond the market volatility risk; see, e.g., Kapadia (2006), Adrian and Rosenberg (2008), Harvey and Siddique (2000), Chang, Christoffersen, and Jacobs (2013), and Conrad, Dittmar, and Ghysels (2013).

The rest of this paper is organized as follows. In Section 2 we provide a theoretical framework for good and bad uncertainty and highlight their role for future growth and asset prices. Section 3 discusses our empirical approach to construct good and bad uncertainty in the macroeconomic data. In Section 4 we show our empirical results for the effect of good and bad uncertainties on aggregate macro quantities and aggregate asset prices, and the role of uncertainty risks for the market return and the cross-section of risk premia. Section 5 discusses the robustness of our key empirical results, and the last section provides concluding comments.

#### 2. Economic model

To provide an economic structure for our empirical analysis, in this Section we lay out a version of the long-run risks model that incorporates fluctuations in good and bad macroeconomic uncertainties. We use our economic model to highlight the roles of the good and bad uncertainties for future growth and the equilibrium asset prices.

#### 2.1. Preferences

We consider a discrete-time endowment economy. The preferences of the representative agent over the future consumption stream are characterized by the Kreps and Porteus (1978) recursive utility of Epstein and Zin (1989) and Weil (1989):

$$U_{t} = \left[ (1 - \beta)C_{t}^{(1 - \gamma)/\theta} + \beta (E_{t}U_{t+1}^{1 - \gamma})^{1/\theta} \right]^{\theta/(1 - \gamma)}, \tag{1}$$

where  $C_t$  is consumption,  $\beta$  is the subjective discount factor,  $\gamma$  is the risk-aversion coefficient, and  $\psi$  is the elasticity of

intertemporal substitution (IES). For ease of notation, the parameter  $\theta$  is defined as  $\theta \equiv (1-\gamma)/(1-1/\psi)$ . Note that when  $\theta = 1$ , that is,  $\gamma = 1/\psi$ , the recursive preferences collapse to the standard case of expected power utility, in which case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk aversion exceeds the reciprocal of IES  $(\gamma > 1/\psi)$ , the agent prefers early resolution of uncertainty of consumption path, otherwise, the agent has a preference for late resolution of uncertainty.

As is shown in Epstein and Zin (1989), the logarithm of the intertemporal marginal rate of substitution implied by these preferences is given by

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) r_{c,t+1},$$
 (2)

where  $\Delta c_{t+1} = \log(C_{t+1}/C_t)$  is the log growth rate of aggregate consumption, and  $r_{c,t}$  is a log return on the asset which delivers aggregate consumption as dividends (the wealth portfolio). This return is different from the observed return on the market portfolio as the levels of market dividends and consumption are not the same. We solve for the endogenous wealth return and the equilibrium stochastic discount factor in (2) using the dynamics for the endowment process and the standard Euler equation,

$$E_t[\exp\{m_{t+1}\}R_{i,t+1}] = 1, (3)$$

which hold for the return on any asset in the economy,  $R_{i,t+1}$ , including the wealth portfolio.

#### 2.2. Consumption dynamics

Our specification of the endowment dynamics incorporates the underlying channels of the long-run risks model of Bansal and Yaron (2004), such as the persistent fluctuations in expected growth and the volatility of consumption process. The novel ingredients of our model include: (i) the decomposition of the total macroeconomic volatility into good and bad components associated with good and bad consumption shocks, respectively, and (ii) the direct effect of macroeconomic volatilities on future economic growth. We show that these new model features are well-motivated empirically and help us interpret the relation between the good and bad uncertainties, the economic growth, and the asset prices in the data.

Our benchmark specification for the consumption dynamics is written as follows:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_c(\varepsilon_{g,t+1} - \varepsilon_{b,t+1}), \tag{4}$$

$$X_{t+1} = \rho X_t + \tau_g V_{gt} - \tau_b V_{bt} + \sigma_X (\varepsilon_{g,t+1} - \varepsilon_{b,t+1}),$$
(5)

where  $x_t$  is the predictable component of next-period consumption growth, and  $\varepsilon_{gt+1}$  and  $\varepsilon_{bt+1}$  are two meanzero consumption shocks which for parsimony affect both the realized and expected consumption growth.<sup>6</sup> The shocks  $\varepsilon_{gt+1}$  and  $\varepsilon_{bt+1}$  separately capture positive and negative shocks in consumption dynamics, respectively,

<sup>&</sup>lt;sup>5</sup> See also a related literature on market downside risk, e.g., Ang, Chen, and Xing (2006) and Lettau, Maggiori, and Weber (2014), which emphasizes the importance of market left-tail risk.

<sup>&</sup>lt;sup>6</sup> It is straightforward to extend the specification to allow for separate shocks in realized and expected consumption growth rates and break the perfect correlation of the two. This does not affect our key results, and so we do not entertain this case to ease the exposition.

and are modeled as

$$\varepsilon_{i,t+1} = \tilde{\varepsilon}_{i,t+1} - \mathbb{E}_t \tilde{\varepsilon}_{i,t+1} \quad \text{for } i = \{g, b\},$$
 (6)

where the underlying shocks  $\tilde{\varepsilon}_{i,t+1}$  have a positive support, namely,  $\tilde{\varepsilon}_{i,t+1} > 0$  for  $i = \{g,b\}$ . This ensures that the consumption shocks  $\varepsilon_{gt+1}$  and  $\varepsilon_{bt+1}$  are conditionally mean zero, and are driven by positive and negative shocks to consumption growth, respectively.

We assume that the volatilities of consumption shocks are time varying and driven by the state variables  $V_{gt}$  and  $V_{ht}$ ; in particular,

$$Var_t \ \varepsilon_{g,t+1} = Var_t \ \tilde{\varepsilon}_{g,t+1} \equiv V_{gt},$$
  
 $Var_t \ \varepsilon_{h,t+1} = Var_t \ \tilde{\varepsilon}_{h,t+1} \equiv V_{ht}.$ 

This allows us to interpret  $V_{gt}$  and  $V_{bt}$  as good and bad macroeconomic uncertainties, that is, uncertainties regarding the right and left tail movements in consumption growth. In our specification, the good and bad uncertainties follow separate AR(1) processes,

$$V_{g,t+1} = (1 - \nu_g)V_{g0} + \nu_g V_{gt} + \sigma_{gw} W_{g,t+1}, \tag{7}$$

$$V_{b,t+1} = (1 - \nu_b)V_{b0} + \nu_b V_{bt} + \sigma_{bw} W_{b,t+1}, \tag{8}$$

where for  $i = \{g, b\}$ ,  $V_{i0}$  is the level,  $\nu_i$  the persistence, and  $w_{i,t+1}$  the shock in the uncertainty. For simplicity, the volatility shocks are Normally distributed, and we let  $\alpha$  denote the correlation between the good and bad volatility shocks.

By construction, the macro volatilities govern the magnitude of the good and bad consumption innovation. In addition to that, our feedback specification in (5) also allows for a direct effect of good and bad macro uncertainty on future levels of economic growth. Backus, Routledge, and Zin (2010) use a similar feedback specification from a single (total) volatility to future growth. Our specification features two volatilities (good and bad), and for  $\tau_g > 0$  and  $\tau_b > 0$ , an increase in good volatility raises future consumption growth rates, while an increase in bad volatility dampens future economic growth. The twovolatility specification captures, in a reduced-form way, an economic intuition that good uncertainty, through the positive impact of new innovation on growth opportunities, would increase investment and hence future economic growth, while bad uncertainty, due to the unknown magnitude of adverse news and its impact on investment, would result in lower growth in the future. While we do not provide the primitive micro-foundation for this channel, we show direct empirical evidence to support our volatility feedback specification. Further, we show that the volatility feedback for future cash flows also leads to testable implications for the asset prices which are supported in the data.

It is important to note that our specification for consumption growth displays non-Gaussian dynamics with time-varying mean, volatility, and higher-order moments. Specifically, total consumption volatility is equal to the sum of the good and bad uncertainties,  $V_{gt} + V_{bt}$ , whereas skewness, kurtosis, and all other higher moments are functions of the underlying volatility variables  $V_{gt}$  and  $V_{bt}$ . The specific way in which  $V_{gt}$  and  $V_{bt}$  affect those higher moments depends on the underlying distribution

for  $\tilde{\epsilon}_{i,t+1}$ ,  $i = \{b,g\}$ . One specification that is analytically convenient and widely used features Poisson jumps in the consumption fundamentals, in which case, skewness is directly related to fluctuations in the intensity of jumps. In this case, the time variation in jump intensity affects separately the left and right tails of the consumption distribution, and hence the movements in good and bad volatility and higher-order moments. Another specification is one in which  $ilde{\epsilon}_{i,t+1}$  are drawn from a Gamma distribution with a scale parameter 1 and a time-varying shape parameter, in which case the consumption shocks dynamics follow the good and bad environment specification in Bekaert and Engstrom (2009). The time-varying shape parameters governing the Gamma distribution drive the variance and higher-order moments of consumption growth distribution. An alternative approach for generating time variation in higher-order moments is given in Colacito, Ghysels, and Meng (2013). They model shocks to expected consumption as drawn from a skew-normal distribution with time-varying parameters and a separate process for stochastic volatility which leads to separate movements in consumption volatility and skewness. Our modeling approach focuses on bad and good volatility as the key driving forces for time variation in consumption growth distribution, which is largely motivated by our empirical analysis.

#### 2.3. Equilibrium asset prices

To get closed-form expressions for the equilibrium asset prices, we consider the consumption shock distribution for which the log moment-generating function is linear in the underlying variances  $V_{g,t}$  and  $V_{h,t}$ . That is,

$$\log E_t e^{u\varepsilon_{i,t+1}} = f(u)V_{i,t} \quad \text{for } i = \{g, b\}, \tag{9}$$

and the function f(u) captures the shape of the momentgenerating function of the underlying consumption shocks. As discussed earlier, prominent examples of such distributions include compound Poisson jump distribution and Gamma distribution. As shown in Appendix B, for this class of distributions the function  $f(\cdot)$  is non-negative, convex, and asymmetric, that is, f(u) > f(-u) for u > 0.

We use a standard log-linearization approach to obtain analytical solutions to our equilibrium model. Below we show a summary of our key results, and all the additional details are provided in Appendix B.

In equilibrium, the solution to the log price-consumption ratio on the wealth portfolio is linear in the expected growth and the good and bad uncertainty states:

$$pc_{t} = A_{0} + A_{x}x_{t} + A_{gy}V_{gt} + A_{hy}V_{ht}.$$
(10)

The slope coefficients are given by

$$A_{x} = \frac{1 - \frac{1}{\psi}}{1 - \kappa_{1} \rho},$$

$$A_{gv} = \tilde{A}_{gv} + \tau_g \frac{\kappa_1 A_x}{1 - \kappa_1 \nu_g}, \quad \tilde{A}_{gv} = \frac{f\left(\theta\left(\left(1 - \frac{1}{\psi}\right)\sigma_c + \kappa_1 A_x \sigma_x\right)\right)}{\theta(1 - \kappa_1 \nu_g)}$$

$$A_{bv} = \tilde{A}_{bv} - \tau_b \frac{\kappa_1 A_x}{1 - \kappa_1 \nu_b}, \quad \tilde{A}_{bv} = \frac{f\left(-\theta\left(\left(1 - \frac{1}{\psi}\right)\sigma_c + \kappa_1 A_x \sigma_x\right)\right)}{\theta(1 - \kappa_1 \nu_g)},$$
(11)

where the parameter  $\kappa_1 \in (0,1)$  is the log-linearization coefficient, and the  $\tilde{A}$ 's are the uncertainty loadings on the price-consumption ratio that would be obtained if the consumption dynamics did not include a direct feedback from uncertainty to growth prospects, namely, if  $\tau_b = \tau_g = 0$ .

As can be seen from the above equations, the response of the asset valuations to the underlying macroeconomic states is pinned down by the preference parameters and model parameters which govern the consumption dynamics. The solution to the expected growth loading  $A_x$  is identical to Bansal and Yaron (2004), and implies that when the substitution effect dominates the wealth effect ( $\psi > 1$ ), asset prices rise with positive growth prospects;  $A_x > 0$ .

The expressions for the uncertainty loadings are more general than the ones in the literature and take into account our assumptions on the volatility dynamics. First, our specification separates positive and negative consumption innovations which have their own good and bad volatility, respectively. The impact of this pure volatility channel on asset prices is captured by the first components of the volatility loadings in (11),  $\tilde{A}_{gv}$  and  $\tilde{A}_{bv}$ . In particular, when both  $\gamma$  and  $\psi$  are above one, these two loadings are negative:  $\tilde{A}_{gv}, \tilde{A}_{bv} < 0$ . That is, with a strong preference for early resolution of uncertainty, the agent dislikes volatility, good or bad, so the direct effect of an increase in uncertainty about either positive or negative tail of consumption dynamics is to decrease equilibrium equity prices. In the absence of the cash flow effect, both good and bad uncertainties depress asset valuations, albeit by a different amount, Indeed, due to a positive skewness of underlying consumption shocks, an increase in good (bad) uncertainty asymmetrically raises the right (left) tail of the future consumption growth distribution, and this asymmetry leads to a quantitatively larger negative response of the asset prices to bad uncertainty than to good uncertainty:  $|A_{bv}| > |A_{gv}|$ .

In addition to the direct volatility effect, in our model the good and bad uncertainties can also impact asset prices through their feedback on future cash flows (see Eq. (5)). For  $\tau_b > 0$ , the negative effect of bad uncertainty on future expected growth further dampens asset valuations, and as shown in (11), the bad volatility coefficient  $A_{bv}$  becomes even more negative. On the other hand, when good uncertainty has a positive and large impact on future growth, the cash flow effect of the good uncertainty can exceed its direct volatility effect, and as a result the total asset-price response to good uncertainty becomes positive:  $A_{gv} > 0$ . Hence, in our framework, good and bad uncertainties can have opposite impact on equity prices, with bad uncertainty shocks decreasing and good uncertainty shocks increasing asset valuations, which we show is an important aspect of the economic data.<sup>7</sup>

The aforementioned effect of uncertainty on asset valuations is related to several recent studies. In the context of long-run risks models with preferences for early resolution of uncertainty, Eraker and Shaliastovich (2008) and Drechsler and Yaron (2011) entertain jumps in cash flows and show that asset valuations drop with increase in jump intensity, and in particular, are sensitive to jumps which affect the left tail of consumption distribution. This effect on prices is also reflected in Colacito, Ghysels, and Meng (2013) who show that asset valuations decline when skewness becomes more negative. Tsai and Wachter (2014) consider a specification that incorporates timevarying rare disasters and booms. As their Poisson jump shocks are uncompensated, the intensities of booms and disasters have a direct impact on expected growth and thus capture the differential  $\tau$  effects highlighted above, which leads to a differential impact of jump intensity on prices. Finally, in the context of the habits model in Bekaert and Engstrom (2009), prices decline at times of high expected growth and increase at times of good or bad variance of Gamma-distributed consumption growth shocks. The difference in the response of prices to uncertainty relative to our specification is due to the preference structure, and in particular, the preference for early resolution of uncertainty.

In the model, the good and bad uncertainty can also have different implications on equilibrium risk-free rates. Using a standard Euler equation (3), the solutions to equilibrium yields on *n*-period real bonds are linear in the underlying state variables:

$$y_{t,n} = \frac{1}{n} (B_{0,n} + B_{x,n} x_t + B_{gv,n} V_{gt} + B_{bv,n} V_{bt}), \tag{12}$$

where  $B_{x,n}$ ,  $B_{gv,n}$ , and  $B_{bv,n}$  are the bond loadings to expected growth, good, and bad uncertainty factors, whose solutions are provided in Appendix B. As shown in the literature, real bond yields increase at times of high expected growth, and the bond loading  $B_{x,n}$  is positive. Further, an increase in either good and bad uncertainty raises the precautionary savings motive for the representative agent, so the direct impact of either uncertainty on risk-free rates is negative. However, in addition to the direct volatility effect, in our framework good and bad uncertainties also have an impact on future economic growth. Similar to the discussion of the consumption claim, bad uncertainty reduces future growth rates which further dampens real rates, so  $B_{bv,n}$  becomes more negative. On the other hand, the positive cash flow impact of good volatility can offset the precautionary savings motive at longer maturities and can lead to a positive response of interest rates to good uncertainty. Thus, due to the volatility feedback, in our framework good and bad uncertainties can have opposite effects on the risk-free rates, which we show is consistent with the data.

 $<sup>^7</sup>$  Note that in our simple endowment economy, welfare is increasing in the value of the consumption claim. When  $A_{\rm gv}$  is positive, the implication is that good uncertainty shock increases welfare. This is not surprising since for  $A_{\rm gv}$  to be positive there must be a significant positive

<sup>(</sup>footnote continued)

#### 2.4. Risk compensation

Using the model solution to the price-consumption ratio in (10), we can provide the equilibrium solution to the stochastic discount factor in terms of the fundamental states and the model and preference parameters. The innovation in the stochastic discount factor, which characterizes the sources and magnitudes of the underlying risk in the economy, is given by

$$m_{t+1} - E_{t}[m_{t+1}] = -\lambda_{x} \sigma_{x} (\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) - \lambda_{gv} \sigma_{gw} W_{g,t+1} - \lambda_{bv} \sigma_{bw} W_{b,t+1},$$
(13)

and  $\lambda_x, \lambda_{gv}$ , and  $\lambda_{bv}$  are the market prices of risk of growth, good volatility, and bad volatility risks. Their solutions are given by

$$\lambda_{x} = (1 - \theta)\kappa_{1}A_{x} + \gamma \frac{\sigma_{c}}{\sigma_{x}}$$
(14)

$$\lambda_{gv} = (1 - \theta)\kappa_1 A_{gv},\tag{15}$$

$$\lambda_{bv} = (1 - \theta)\kappa_1 A_{bv}. \tag{16}$$

When the agent has a preference for early resolution of uncertainty, the market price of consumption growth risk  $\lambda_x$  is positive:  $\lambda_x > 0$ . Consistent with our discussion of the priceconsumption coefficients, the market prices of the volatility risks depend on the strength of the volatility feedback for future cash flow. When the good and bad uncertainties have no impact on future growth ( $\tau_g = \tau_h = 0$ ), the market prices of both volatility risks are negative. Indeed, with preference for early resolution of uncertainty, the agent dislikes volatility, good or bad, and thus high uncertainties represent high risk states for the investor. The market prices of uncertainty risks change when we introduce volatility feedback for future growth. When bad volatility predicts lower future growth, it makes bad volatility fluctuations even riskier, which increase, in absolute value, the market price of bad uncertainty risk, so  $\lambda_{bv}$  < 0. On the other hand, when good uncertainty positively impacts future economic growth, the market price of good uncertainty can become positive:  $\lambda_{gv} > 0$ . Thus, in our framework, bad and good uncertainty can have opposite market prices of risk.

To derive the implications for the risk premium, we consider an equity claim whose dividends represent a levered claim on total consumption, similar to Abel (1990) and Bansal and Yaron (2004). Specifically, we model the dividend growth dynamics as follows:

$$\Delta d_{t+1} = \mu_d + \phi_x x_t + \sigma_d u_{dt+1},\tag{17}$$

where  $\phi_x > 0$  is the dividend leverage parameter which captures the exposure of equity cash flows to expected consumption risks, and  $u_{d,t+1}$  is a Normal dividend-specific shock which for simplicity is homoskedastic and independent from other economic innovations. Using the

dividend dynamics, we solve for the equilibrium return on the equity claim,  $r_{d,t+1}$ , in an analogous way to the consumption asset. The return dynamics satisfies

$$r_{d,t+1} = \mathbb{E}_{t}[r_{d,t+1}] + \beta_{x} \sigma_{x} (\varepsilon_{g,t+1} - \varepsilon_{b,t+1})$$

$$+ \beta_{gv} \sigma_{gw} w_{g,t+1} + \beta_{bv} \sigma_{bw}$$

$$+ \sigma_{d} u_{d,t+1},$$

$$(18)$$

where  $\beta_x$ ,  $\beta_{gv}$ , and  $\beta_{bv}$  are the equity betas which reflect the response of the asset valuations to the underlying expected growth, good, and bad volatility risks, respectively. Similar to the consumption asset case, the equity betas to growth risks and good volatility risks are positive, while the equity beta to bad uncertainty risks is negative:  $\beta_x > 0$ ,  $\beta_{gv} > 0$ ,  $\beta_{bv} < 0$ . Further, since the volatilities of  $\epsilon_{b,t+1}$  and  $\epsilon_{g,t+1}$  are driven by  $V_{b,t}$  and  $V_{g,t}$ , it immediately follows from Eq. (18) that the conditional variance of returns is time varying and increasing in good and bad uncertainties (see Appendix B for details).

In equilibrium, the risk compensation on equities depends on the exposure of the asset to the underlying sources of risk, the market prices of risks, and the quantity of risk. Specifically, the equity risk premium is given by

$$\begin{aligned} & \mathbf{E}_{t} R_{d,t+1} - R_{f,t} \approx \log \mathbf{E}_{r} e^{r_{d,t+1} - r_{f,t}} \\ &= \left[ f(-\lambda_{x} \sigma_{x}) - f((\beta_{x} - \lambda_{x}) \sigma_{x}) + f(\beta_{x} \sigma_{x}) \right] V_{gt} \\ &+ \left[ f(\lambda_{x} \sigma_{x}) - f((\lambda_{x} - \beta_{x}) \sigma_{x}) + f(-\beta_{x} \sigma_{x}) \right] V_{bt} \\ &+ \beta_{gv} \lambda_{gv} \sigma_{gw}^{2} + \beta_{gv} \lambda_{bv} \sigma_{bw}^{2} \\ &+ \alpha \sigma_{bw} \sigma_{gw} (\beta_{gv} \lambda_{bv} + \beta_{bv} \lambda_{gv}). \end{aligned}$$

$$(19)$$

In our model, all three sources of risks contribute to the risk premia, and the direct contribution of each risk to the equity risk premium is positive. The first two components of the equity premium above capture the contribution of the non-Gaussian growth risk, which is time varying and driven by the good and bad volatilities. When  $\gamma > 1$  and  $\psi > 1$ , the market price of growth risk  $\lambda_x$  and the equity exposure to growth risk  $\beta_x$  are both positive. As we show in Appendix B, this implies that the equity premium loadings on both good and bad volatilities are positive, so that the growth risks receive positive risk compensation unconditionally, and this risk compensation increases at times of high good or bad volatility. The remaining constant components in the equity risk premia equation capture the contributions of the Gaussian volatility shocks. As the market prices of volatility risks and equity exposure to volatility risks have the same sign, the volatility risks receive positive risk compensation in equities. The last term in the decomposition above captures the covariance between good and bad uncertainty risk, and is negative when the two uncertainties have positive correlation ( $\alpha > 0$ ).

To get further intuition for the nature of the risk compensation, we consider a Taylor expansion of the equity risk premium:

$$E_t R_{d,t+1} - R_{f,t} \approx \text{const} + \beta_x \lambda_x \sigma_x^2 (V_{gt} + V_{bt})$$

<sup>&</sup>lt;sup>8</sup> It is straightforward to generalize the dividend dynamics to incorporate stochastic volatility of dividend shocks, correlation with consumption shocks, and the feedback effect of volatility to expected dividends (see, e.g., Bansal, Kiku, and Yaron, 2012; Schorfheide, Song, and Yaron, 2013). As our focus is on aggregate macroeconomic uncertainty, these extensions do not affect our key results, and for simplicity are not

<sup>(</sup>footnote continued)

entertained. However, it is worth noting that, by convexity, separate idiosyncratic dividend volatility can be positively related to equity prices (see, e.g., Pastor and Veronesi, 2006; Ai and Kiku, 2012; Johnson and Lee, 2014).

$$-\lambda_x \beta_y \sigma_y^3 (\lambda_x - \beta_y) (V_{gt} - V_{ht}) + \cdots$$
 (20)

The constant in this equation captures constant contribution of volatility risks to the risk premia. Subsequent terms pick out the second- and third-order components in the decomposition of the non-Gaussian growth risk premia; for simplicity, we omit higher-order terms. The secondorder component is standard, and is equal to the negative of the covariance of log returns and log stochastic discount factor. This component is driven by the quantity of total consumption variance,  $V_{gt} + V_{bt}$ . An increase in either good or bad volatility directly raises total consumption variance, and hence increases equity risk premia ( $\beta_x$  and  $\lambda_x$  are both positive). The third-order component is driven by the quantity of consumption skewness,  $V_{gt} - V_{bt}$ . Under typical parameter calibration of the model,  $\lambda_x > \beta_x$ . This implies that when  $V_{bt}$  increases relative to  $V_{gt}$  and the skewness of consumption shocks decreases (becomes more negative), the equity premium goes up. Hence, the total risk premium increases at times of high good or bad volatility, but the bad volatility has a larger effect capturing the importance of the left tails.

The quantities of total consumption variance and skewness risk are time varying themselves, and directly contribute to the equity risk premium. In our model, the total variance and skewness are linearly related to the good and bad volatilities, so that the risk compensation for the variance and skewness risk are components of the constant risk compensation for good and bad volatility risks in (19)–(20). We show the implied market prices and equity betas to variance and skewness risk in Appendix B. In particular, in our framework agents dislike states with low consumption growth skewness (larger left tails), thus leading asset prices to fall in those states.

#### 3. Data and uncertainty measures

#### 3.1. Data

In our benchmark analysis we use annual data from 1930 to 2012. Consumption and output data come from the Bureau of Economic Analysis (BEA) National Income and Product Accounts (NIPA) tables. Consumption corresponds to the real per capita expenditures on non-durable goods and services and output is real per capita gross domestic product minus government consumption. Capital investment data are from the NIPA tables; R&D investment is available at the National Science Foundation (NSF) for the 1953 to 2008 period, and the R&D stock data are taken from the BEA Research and Development Satellite Account for the 1959 to 2007 period. To measure the fluctuations in macroeconomic volatility, we use monthly data on industrial production from the Federal Reserve Bank of St. Louis.

#### Table 1

Data summary statistics.

The table shows summary statistics for the macroeconomic variables (Panel A), aggregate asset prices (Panel B), and the realized variance measures (Panel C). Consumption, private GDP, as well as capital and R&D investment series are real and per capita. Dividends, earnings, stock prices, and returns are computed for a broad market portfolio. The real risk-free rate corresponds to a 3-month T-bill rate minus expected inflation. Default spread is the difference between the yields on BAAand AAA-rated corporate bonds. The total realized variance, RV, is based on the sum of squared observations of demeaned monthly industrial production growth over one year, re-scaled to match the unconditional variance of consumption growth. The positive and negative realized semivariances,  $RV_p$  and  $RV_n$ , decompose the total realized variance into the components pertaining to only positive and negative movements in industrial production growth, respectively. All growth rates and returns are in percentages, and the realized variances are multiplied by 10,000. Data on R&D investment are annual from 1954 to 2008, and all the other data are annual from 1930 to 2012.

Panel A: Macro growth rates	Mean	Std. dev.	AR(1)
Consumption growth GDP growth Earnings growth Market dividend growth Capital investment growth	1.84 2.04 1.77 1.27 1.75	2.16 12.91 26.11 11.32 14.80	0.50 0.41 0.01 0.20 0.42
R&D investment growth  Panel B: Asset prices  Market return	3.51 5.79	19.85	- 0.01
Market price-dividend ratio Real risk-free rate Default spread Panel C: Realized volatility	3.39 0.34 1.21	0.45 2.55 0.81	0.88 0.73 0.72
RV <sub>p</sub> RV <sub>n</sub> RV	2.34 2.27 4.61	7.37 5.68 10.91	0.24 0.29 0.44

Our aggregate asset-price data include 3-month Treasury bill rate, the stock price and dividend on the broad market portfolio from the Center for Research in Security Prices (CRSP), and aggregate earnings data from Robert Shiller's website. We adjust nominal short-term rate by the expected inflation to obtain a proxy for the real riskfree rate. Additionally, we collect data on equity portfolios sorted on key characteristics, such as book-to-market ratio and size, from the Fama-French Data Library. Our bond portfolios, as in Ferson, Nallareddy, and Xie (2013), include the excess returns of low- over high-grade corporate bonds (Credit premium portfolio), and the excess returns of long- over short-term Treasury bonds (Term premium portfolio).<sup>10</sup> To measure the default spread, we use the difference between the BAA and AAA corporate yields from the Federal Reserve Bank of St. Louis.

The summary statistics for the key macroeconomic variables are shown in Panel A of Table 1. Over the 1930 to 2012 sample period the average consumption growth is 1.8% and its volatility is 2.2%. The average growth rates in

 $<sup>^9</sup>$  In the model,  $\lambda_x = (1-\theta)\kappa_1 A_x + \gamma \sigma_c / \sigma_x$ , and  $\beta_x = \kappa_{1.d} H_x$ . The term  $(1-\theta)$  is positive under early resolution of uncertainty, and amounts to 28 under a typical calibration of  $\gamma = 10$ ,  $\psi = 1.5$ . The equity price response to growth news  $H_x$  is magnified relative to consumption asset-price response  $A_x$  by the leverage of the dividend stream  $\phi_x$ , so that  $H_x/A_x$  is around 3–5. The log-linearization parameters  $\kappa_1 \approx \kappa_{1.d} \approx 1$ . In all, this provides,  $\lambda_x > \beta_x$ .

<sup>&</sup>lt;sup>10</sup> We thank Wayne Ferson for providing us data on these bond portfolios which we extend till 2012 using long-term government data and corporate bond data from Barclays.

output, capital investment, market dividends, and earnings are similar to that in consumption, and it is larger for the R&D investment (3.5%) over the 1954 to 2008 period. As shown in the table, many of the macroeconomic variables are quite volatile relative to consumption: the standard deviation of earnings growth is 26%, of capital investment growth is almost 15%, and of the market dividend growth is 11%. Most of the macroeconomic series are quite persistent with an AR(1) coefficient of about 0.5.

Panel B of Table 1 shows the summary statistics for the key asset-price variables. The average real log market return of 5.8% exceeds the average real rate of 0.3%, which implies an equity premium (in logs) of 5.5% over the sample. The market return is also quite volatile relative to the risk-free rate, with a standard deviation of almost 20% compared to 2.5% for the risk-free rate. The corporate yield on BAA firms is on average 1.2% above that for the AAA firms, and the default spread fluctuates significantly over time. The default spread, real risk-free rate, and the market price–dividend ratio are very persistent in the sample, and their AR(1) coefficients range from 0.72 to 0.88.

#### 3.2. Measurement of good and bad uncertainties

To measure good and bad uncertainty in the data, we follow the approach in Barndorff-Nielsen, Kinnebrock, and Shephard (2010) to decompose the usual realized variance into two components that separately capture positive and negative (hence, "good" and "bad") movements in the underlying variable, respectively. While we focus on the variation in the aggregate macroeconomic variables, Feunou, Jahan-Parvar, and Tédongap (2013) and Patton and Sheppard (2015) entertain a similar type of semivariance measures in the context of stock market variation.<sup>11</sup>

Specifically, consider an aggregate macroeconomic variable y (e.g., industrial production, earnings, consumption), and let  $\Delta y$  stand for the demeaned growth rate in y. Then, we define the positive and negative realized semivariances,  $RV_p$  and  $RV_p$ , as follows:

$$RV_{p,t+1} = \sum_{i=1}^{N} \mathbb{I}\left(\Delta y_{t+i/N} \ge 0\right) \Delta y_{t+i/N}^{2},\tag{21}$$

$$RV_{n,t+1} = \sum_{i=1}^{N} \mathbb{I}\left(\Delta y_{t+i/N} < 0\right) \Delta y_{t+i/N}^{2}, \tag{22}$$

where  $\mathbb{I}(\cdot)$  is the indicator function and N represents the number of observations of y available during one period (a year in our case). It is worth noting that  $RV_p$  and  $RV_n$  add up to the standard realized variance measure, RV, that is,

$$RV_{t+1} = \sum_{i=1}^{N} \Delta y_{t+i/N}^2 = RV_{n,t+1} + RV_{p,t+1}.$$

Barndorff-Nielsen, Kinnebrock, and Shephard (2010) show that in the limit the positive (negative) semivariance captures one-half of the variation of any Gaussian

symmetric shifts in  $\Delta y$ , plus the variation of non-Gaussian positive (negative) fluctuations; see Appendix A for further details. Notably, the result in this paper implies that asymptotically, the semivariances are unaffected by movements in the conditional mean; however, given the finite-sample considerations, we confirm the robustness of our results removing the fluctuations in conditional mean.

The positive and negative semivariances are informative about the realized variation associated with movements in the right and left tail, respectively, of the underlying variable. Positive (negative) semivariance therefore corresponds to good (bad) realized variance states of the underlying variable and thus, we use the predictable component of this measure as the empirical proxy for ex ante good (bad) uncertainty. To construct the predictive components, we project the logarithm of the future average h-period realized semivariance on the set of time t predictors  $X_t$ :

$$\log\left(\frac{1}{h}\sum_{i=1}^{h}RV_{j,t+i}\right) = \operatorname{const}_{j} + \nu_{j}'X_{t} + \operatorname{error}, \quad j = \{p, n\}, \quad (23)$$

and take as the proxies for the ex ante good and bad uncertainty  $V_g$  and  $V_b$  the exponentiated fitted values of the projection above:

$$V_{g,t} = \exp\left(\operatorname{const}_p + \nu_p' X_t\right), \quad V_{b,t} = \exp\left(\operatorname{const}_n + \nu_n' X_t\right).$$
 (24)

The log transformation ensures that our ex ante uncertainty measures remain strictly positive.

In addition to measuring the ex ante uncertainties, we use a similar approach to construct a proxy for the expected consumption growth rate,  $x_t$  which corresponds to the fitted value of the projection of future consumption growth on the same predictor vector  $X_t$ :

$$\frac{1}{h}\sum_{i=1}^{h} \Delta c_{j,t+i} = \text{const}_c + \nu_c X_t + \text{error},$$

 $x_t = \text{const}_c + \nu'_c X_t$ .

In our empirical applications we let y be industrial production, which is available at monthly frequency, and use that to construct realized variance at the annual frequency. As there are 12 observations of industrial production within a year, our measurement approach is consistent with the model setup which allows for multiple good and bad shocks within a period (a year). To reduce measurement noise in constructing the uncertainties, in our benchmark empirical implementation we set the forecast window h to three years. Finally, the set of the benchmark predictors  $X_t$  includes positive and negative realized semivariances  $RV_p$ ,  $RV_n$ , consumption growth  $\Delta c$ , the real-market return  $r_d$ , the market price-dividend ratio pd, the real risk-free rate  $r_p$  and the default spread  $def_t^{12}$ 

<sup>&</sup>lt;sup>11</sup> The use of semivariance in finance goes back to at least Markowitz (1959), and more recent applications include, for example, Hogan and Warren (1974) and Lewis (1990).

 $<sup>^{12}</sup>$  As shown in Section 5, our results are robust to using standard ordinary least squares (OLS) regression instead of the log, the use of alternative predictors, different forecast windows h, removing the conditional mean in constructing the semivariance measures, and using other measures for y.

Panel C of Table 1 reports the key summary statistics for our realized variance measures. The positive and negative semivariances contribute about equally to the level of the total variation in the economic series, and the positive semivariance is more volatile than the negative one. The realized variation measures co-move strongly together: the contemporaneous correlation between total and negative realized variances is 80%, and the correlation between the positive and negative realized variance measures is economically significant, and amounts to 40%.

Fig. 1 shows the plot of the total realized variance, smoothed over the three-year window to reduce measurement noise. As can be seen from the graph, the overall macroeconomic volatility gradually declines over time, consistent with the evidence in McConnell and Perez-Quiros (2000) and Stock and Watson (2003), as well as Bansal, Khatchatrian, and Yaron (2005), Lettau, Ludvigson, and Wachter (2008), and Bansal, Kiku, Shaliastovich, and Yaron (2014). Further, the realized variance is strongly countercyclical: indeed, its average value in recessions is twice as large as in expansions. The most prominent increases in the realized variance occur in the recessions of the early and late 1930s, the recession in 1945, and more recently, in the Great Recession in the late 2000s. Not surprisingly, the countercyclicality of the total variance is driven mostly by the negative component of the realized variance. To highlight the difference between the positive and negative variances, we show in Fig. 2 the residual positive variance (smoothed over the three-year window) which is orthogonal to the negative variance. This residual is computed from the projection of the positive realized variance onto the negative one. As shown on the graph, the residual positive variance sharply declines in recessions, and the largest post-war drop in the residual positive variance occurs in the recession of 2008–2009.

We project the logarithms of the future three-year realized variances and the future three-year consumption growth rates on the benchmark predictor variables to

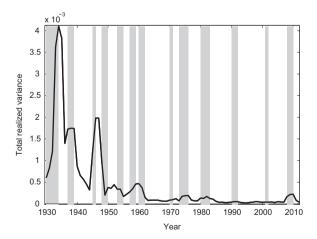
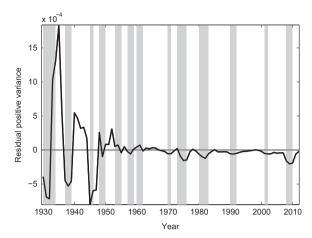


Fig. 1. Total realized variance. The figure shows the time series plot of the total realized variance smoothed over a 3-year window. The total realized variance is based on the sum of squared observations of demeaned monthly industrial production growth over 1-year, re-scaled to match the unconditional variance of consumption growth. The shaded areas represent National Bureau of Economic Research (NBER) recessions.



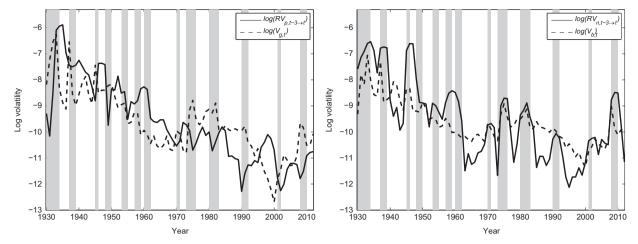
**Fig. 2.** Residual positive variance. The figure shows the time series plot of the residual positive variance, smoothed over a 3-year window, which is orthogonal to the negative variance. The positive and negative realized semivariances decompose the total realized variance into the components pertaining only to positive and negative movements in industrial production growth, respectively. The residual positive variance is computed from the projection of the positive realized semivariance onto the negative one. The shaded areas represent NBER recessions.

construct the ex ante uncertainty and expected growth measures. It is hard to interpret individual slope coefficients due to the correlation among the predictive variables, so for brevity we do not report them in the paper; typically, the market variables, such as the market pricedividend ratio, the market return, the risk-free rate, and the default spread, are significant in the regression, in addition to the lags of the realized variance measures themselves. The  $R^2$  in these predictive regressions ranges from 30% for the negative variance and consumption growth to 60% for the positive variance.

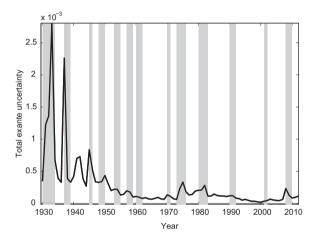
We show the fitted values from these projections alongside the realized variance measures on Fig. 3. The logs of the realized variances are much smoother than the realized variances themselves (see Fig. 1), and the fitted values track well both the persistent declines and the business-cycle movements in the underlying uncertainty. We exponentiate the fitted values to obtain the proxies for the good and bad ex ante uncertainties. Figs. 4 and 5 show the total uncertainty and the residual ex ante good uncertainty which is obtained from the projection of the good uncertainty on the bad uncertainty. Consistent with our discussion for the realized quantities, the total uncertainty gradually decreases over time, and the residual good uncertainty generally goes down in bad times. Indeed, in the recent period, the residual good uncertainty increases in the 1990s, and then sharply declines in 2008. Notably, the ex ante uncertainties are much more persistent than the realized ones: the AR(1) coefficients for good and bad uncertainties are about 0.5, relative to 0.2-0.3 for the realized variances.

#### 4. Empirical results

In this section we empirically analyze the implications of good and bad uncertainty along several key dimensions.



**Fig. 3.** Realized and predictive log volatilities. The figure shows the time series plots of the log positive (left panel) and negative (right panel) realized variances and their predictive values from the projection. The shaded areas represent NBER recessions. The benchmark predictive variables in the projection include positive and negative realized semivariances, consumption growth rate, the real-market return, the market price-dividend ratio, the real risk-free rate, and the default spread.

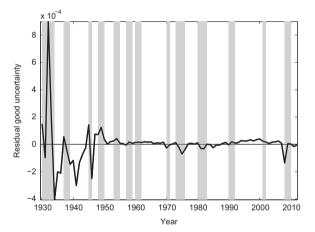


**Fig. 4.** Total ex ante uncertainty. The figure shows the time series plot of the total ex ante uncertainty. The total ex ante uncertainty is constructed from the predictive regressions of future overall realized variance. The shaded areas represent NBER recessions.

In Section 4.1 we analyze the effects of uncertainty on aggregate macro quantities such as output, consumption, and investment. In Section 4.2 we consider the impact of uncertainties on aggregate asset prices such as the market price–dividend ratio, the risk-free rate, and the default spread. In Section 4.3 we examine the role of uncertainty for the market and cross-section of risk premia. Our benchmark analysis is based on the full sample from 1930–2012 and in the robustness Section we show that the key results are maintained for the post-war period.

## 4.1. Macroeconomic uncertainties and growth

Using our empirical proxies for good and bad uncertainty,  $V_{gt}$  and  $V_{bt}$ , we show empirical support that good uncertainty is associated with an increase in future output growth, consumption growth, and investment, while bad uncertainty is associated with lower growth rates for these



**Fig. 5.** Residual good uncertainty. The figure shows the time series plot of the residual good uncertainty which is orthogonal to the bad uncertainty. The good and bad uncertainties are constructed from the predictive regressions of future realized positive and negative variances, respectively. The residual good uncertainty is computed from the projection of good uncertainty onto bad uncertainty. The shaded areas represent NBER recessions.

macro quantities. This is consistent with our cash flow dynamics in the economic model specification shown in Eq. (5).

To document our predictability evidence, we regress future growth rate for horizon h years on the current proxies for good and bad uncertainty and the expected growth, that is, we run a predictive regression

$$\frac{1}{h} \sum_{i=1}^{h} \Delta y_{t+j} = a_h + b'_h [x_t, V_{gt}, V_{bt}] + \text{error},$$

for the key macroeconomic variables of interest y and forecast horizons h from one to five years. Table 2 reports the slope coefficients and the  $R^2$  for the regressions of consumption growth, private GDP, corporate earnings, and

 Table 2

 Macroeconomic uncertainties and aggregate growth.

The table shows the predictability evidence from the projection of future macroeconomic growth rates on the current expected consumption growth x, good uncertainty  $V_g$ , and bad uncertainty  $V_b$ :  $(1/h)\sum_{j=1}^h \Delta y_{t+j} = a_h + b_h'[x_t, V_{gt}, V_{bt}] + \text{error}$ . The table reports the slope coefficients  $b_h$ , t-statistics, and the adjusted  $R^2$ 's for the regression horizons of h=1, 3, and 5 years for the corresponding aggregate series y. The p-values are computed for the Wald test for the joint significance of good and bad uncertainty,  $H_0$ :  $b_{gy} = b_{by} = 0$ . Standard errors are Newey-West adjusted. The notations  $\dagger$ ,  $\ast$ , and  $\diamond$  indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses  $b_x > 0$ ,  $b_{gy} > 0$ , and  $b_{hy} < 0$ . The data are annual from 1930 to 2012.

	х	$V_b$	$V_g$	$Adj-R^2$	<i>p</i> -Value
Consumption growth:					
1Y Ahead	1.98°	$-64.76^{\dagger}$	12.97	0.51	0.25
	[4.98]	[-1.42]	[0.83]		
3Y Ahead	1.07°	-22.56	12.67	0.33	0.05
	[2.98]	[-0.68]	[1.21]		
5Y Ahead	0.46°	-2.20	6.81	0.18	< 0.01
	[2.93]	[80.0-]	[0.73]		
GDP growth:					
1Y Ahead	4.87°	$-733.08^{\dagger}$	277.73 <sup>†</sup>	0.07	0.25
	[7.37]	[-1.62]	[1.44]		
3Y Ahead	2.53*	$-410.36^{\dagger}$	180.07 <sup>†</sup>	0.04	0.28
	[2.13]	[-1.29]	[1.44]		
5Y Ahead	1.46°	- 142.27 <sup>*</sup>	66.85°	0.01	0.02
	[2.53]	[-1.72]	[2.51]		
Market dividend growth:					
1Y Ahead	8.93°	-474.89°	55.04	0.41	< 0.01
	[4.46]	[-2.41]	[0.84]		
3Y Ahead	2.89 <sup>†</sup>	- 107.83	60.23	0.08	0.16
	[1.45]	[-0.66]	[1.17]		
5Y Ahead	1.22	- 182.40*	79.83°	0.04	0.01
	[1.52]	[-2.02]	[2.67]		
Earnings growth:		. ,	. ,		
1Y Ahead	12.34°	$-682.77^{\dagger}$	134.02	0.10	< 0.01
	[3.59]	[-1.29]	[0.66]		
3Y Ahead	0.78	60.55	21.86	-0.02	0.46
	[0.28]	[0.19]	[0.19]		
5Y Ahead	0.85	- 155.41	98.54*	0.01	< 0.01
	[0.78]	[-0.97]	[1.84]		

market dividend growth, and Table 3 shows the evidence for capital investment and R&D measures.

It is evident from these two tables that across the various macroeconomic growth rates and across all the horizons, the slope coefficient on good uncertainty is always positive. This is consistent with the underlying premise of the feedback channel of good uncertainty on macro growth rates. Further, except for the three-year horizon for earnings, all slope coefficients for bad uncertainty are negative, which implies, consistently with the theory, that a rise in bad uncertainty would lead to a reduction in macro growth rates. Finally, in line with our economic model, the expected growth channel always has a positive effect on the macro growth rates as demonstrated by the positive slope coefficients across all the predicted variables and horizons.

The slope coefficients for all three predictive variables are economically large and in many cases are also statistically significant. All our tables include the usual Newey-West *t*-statistics for all the estimated coefficients. Additionally, to facilitate the comparison of the empirical results with our economic model, we also indicate the significance of the coefficients against the economically motivated alternative one-sided hypotheses. For the

growth predictability regressions, our hypotheses are that growth and good uncertainty have a positive impact, while bad uncertainty has a negative impact, respectively. As shown in Tables 2 and 3, the expected growth (cash flow) channel is almost always significant, while the significance of good and bad uncertainty varies across predicted variables and maturities, although they tend to be significant in one-sided tests. Because, the uncertainty measures are quite correlated, the evaluation of individual significance may be difficult to assess. Therefore, in the last column of these tables we report the p-value of a Wald test for the joint significance of good and bad uncertainty. For the most part the tests reject the joint hypothesis that the loadings on good and bad uncertainty are zero. In particular, at the five-year horizon, all of the p-values are below 5%, and they are below 1% for all the investment series at all the horizons.

It is worth noting that the adjusted  $R^2$ 's for predicting most of the future aggregate growth series are quite substantial. For example, the consumption growth  $R^2$  is 50% at the one-year horizon, and the  $R^2$  for the market dividends reaches 40%, while it is about 10% for earnings and private GDP. For the investment and R&D series the  $R^2$ 's at the one-year horizon are also substantial and range

**Table 3**Macroeconomic uncertainties and investment.

The table shows the predictability evidence from the projection of future investment growth rates on the current expected consumption growth x, good uncertainty  $V_g$ , and bad uncertainty  $V_b$ :  $(1/h)\sum_{j=1}^h \Delta y_{t+j} = a_h + b_h'[x_t, V_{gt}, V_{bt}] + \text{error}$ . The table reports the slope coefficients  $b_h$ , t-statistics, and the adjusted  $R^2$ 's for the regression horizons of h=1, 3, and 5 years for the corresponding investment series y. The p-values are computed for the Wald test for the joint significance of good and bad uncertainty,  $H_0$ :  $b_{gy} = b_{by} = 0$ . Standard errors are Newey-West adjusted. The notations  $\dagger$ ,  $\ast$ , and  $\diamond$  indicate the significance of the coefficients at 10%, 5%, and 1%, levels respectively, against the economically motivated, alternative one-sided hypotheses  $b_x > 0$ ,  $b_{gy} > 0$ , and  $b_{hy} < 0$ . R&D investment data are from 1954 to 2008, R&D stock data are from 1960 to 2007, and all the other data are annual from 1930 to 2012.

	x	$V_b$	$V_g$	$Adj-R^2$	<i>p</i> -Value
Gross private capital investment growth:					
1Y Ahead	24.85°	$-2309.41^{\circ}$	912.46°	0.40	< 0.01
	[4.42]	[-2.85]	[3.41]		
3Y Ahead	7.76°	-891.16*	542.32°	0.28	< 0.01
	[2.61]	[-2.18]	[3.60]		
5Y Ahead	3.53°	-399.32*	287.53°	0.29	< 0.01
	[2.46]	[-2.17]	[4.31]		
Nonresidential capital investment growth:					
1Y Ahead	13.81°	-789.80*	226.19 <sup>†</sup>	0.45	0.07
	[6.74]	[-1.83]	[1.51]		
3Y Ahead	5.72°	-272.28	167.58*	0.22	< 0.01
	[3.04]	[-1.26]	[2.11]		
5Y Ahead	2.90°	- 124.54	93.97*	0.18	0.01
	[3.44]	[-1.01]	[2.15]		
R&D investment growth:					
1Y Ahead	4.45°	$-822.83^{\circ}$	571.37*	0.28	0.05
	[4.05]	[-2.43]	[2.16]		
3Y Ahead	1.53°	-980.22°	885.88°	0.23	< 0.01
	[2.59]	[-2.59]	[4.76]		
5Y Ahead	0.59 <sup>†</sup>	-847.67°	775.23°	0.24	< 0.01
	[1.59]	[-2.88]	[4.86]		
R&D stock growth:	. ,	. ,	. ,		
1Y Ahead	1.13°	-983.80°	308.73*	0.55	< 0.01
	[3.83]	[-3.31]	[1.74]		
3Y Ahead	1.05°	-950.27°	342.17 <sup>†</sup>	0.46	< 0.01
	[3.60]	[-2.86]	[1.57]		
5Y Ahead	0.68*	-998.32°	428.55*	0.41	< 0.01
	[2.54]	[-2.86]	[1.81]		
Utility patents count growth:	1 1	,			
1Y Ahead	2.57*	-209.98	13.11	0.11	0.11
	[1.72]	[-1.01]	[0.15]		
3Y Ahead	2.40*	- 158.15*	18.55	0.13	0.02
	[1.88]	[-1.78]	[0.64]		
5Y Ahead	1.54*	- 159.60*	26.64	0.14	< 0.01
o i i meda	[1.96]	[-1.94]	[0.92]	J 1	< 0.01

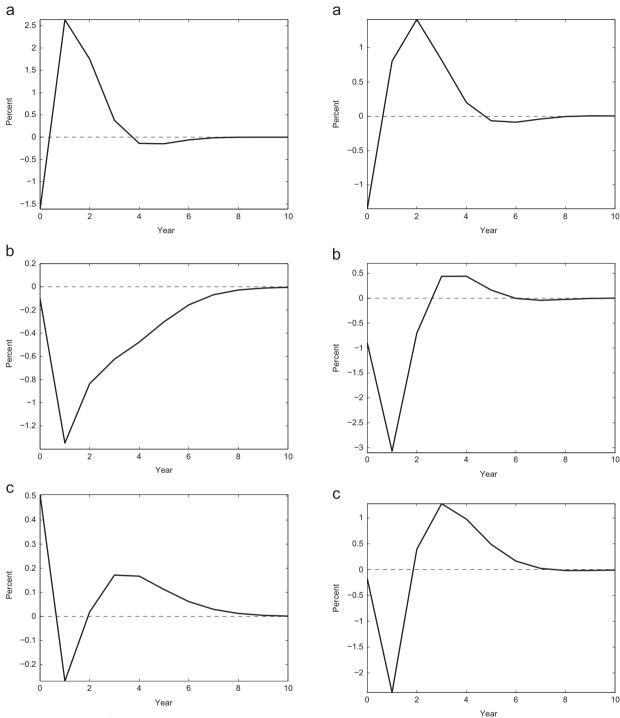
from 28% to 55%. The  $R^2$ 's generally decline with the forecast horizon but for many variables, such as consumption and investment, they remain quite large even at five years.

To further illustrate the economic impact of uncertainty, Figs. 6–8 provide impulse responses of the key economic variables to good and bad uncertainty shocks. The impulse response functions are computed from a first-order vector autoregression (VAR(1)) that includes bad uncertainty, good uncertainty, predictable consumption growth, and the macroeconomic variable of interest. Each figure provides three panels containing the responses to a one standard deviation shock in good, bad, and total uncertainty, respectively.

Fig. 6 provides the impulse response of private GDP growth to uncertainty. Panel A of the figure demonstrates that output growth increases by about 2.5% after one year due to a good uncertainty shock, and this positive effect persists over the next three years. Panel B shows that bad uncertainty decreases output growth by about 1.3% after one year, and remains negative even ten years out. Panel C shows that

output response to overall uncertainty mimics that of bad uncertainty but the magnitude of the response is significantly smaller—output growth is reduced by about 0.25% one year after the shock, and becomes positive after the second year. Recall that good and bad uncertainty have opposite effects on output yet they tend to comove, and therefore the response to total uncertainty becomes less pronounced.

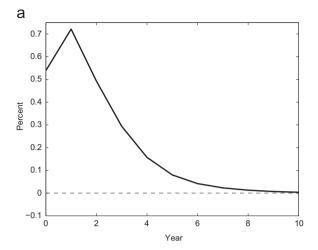
Fig. 7 provides the impulse response of capital investment to bad, good, and total uncertainty, while Fig. 8 shows the response of R&D investment to these respective shocks. The evidence is even sharper than that for GDP. Both investment measures significantly increase with good uncertainty and remain positive till about five years out. These investment measures significantly decrease with a shock to bad uncertainty and total uncertainty several years out. Total uncertainty is a muted version of the impulse response to bad uncertainty and is consistent with the finding in Bloom (2009) who shows a significant shortrun reduction of total output in response to uncertainty shock, followed by a recovery and overshoot. Comparing Panels B and C of the figures highlights a potential bias in

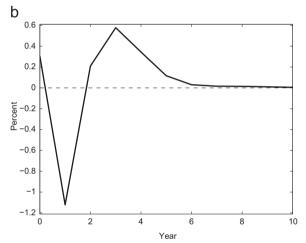


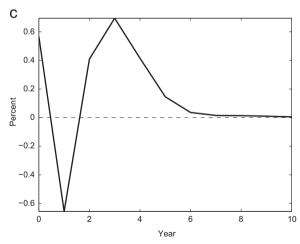
**Fig. 6.** Impulse response of GDP to macro uncertainties. The figure shows impulse responses of private GDP growth to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and GDP growth rate. Data are annual from 1930 to 2012: (a) GDP growth response to good uncertainty shock; (b) GDP growth response to bad uncertainty shock; and (c) GDP growth response to total uncertainty shock.

**Fig. 7.** Impulse response of capital investment to macro uncertainties. The figure shows impulse responses of capital investment growth to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and capital investment growth rate. Data are annual from 1930 to 2012: (a) capital investment growth response to good uncertainty shock; (b) capital investment growth response to bad uncertainty shock; and

(c) capital investment growth response to overall uncertainty shock.







**Fig. 8.** Impulse response of R&D investment to macro uncertainties. The figure shows impulse responses of R&D investment growth to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the lars panel), expected consumption growth, and R&D investment growth rate. Data are annual from 1954 to 2008: (a) R&D investment growth response to good uncertainty shock; (b) R&D investment growth response to bad uncertainty shock; and (c) R&D investment growth response to overall uncertainty shock;

the magnitude of the decline in investment (and other macro quantities) in response to uncertainty when total uncertainty is used rather than bad uncertainty. For example, for capital investment the maximal decline is about 2.3% for total uncertainty and 3% for bad uncertainty, and for the R&D investment the maximal response is 0.6% for total uncertainty while it is 1.1% for bad uncertainty, which indicates that the response differences are economically significant. Thus, decomposing uncertainty to good and bad components allows for a cleaner and sharper identification of the impact of uncertainty on growth.

Finally, we have also considered the impact of good and bad uncertainty on aggregate employment measures. Consistent with our findings for economic growth rates, we find that high good uncertainty predicts an increase in future aggregate employment and hours worked and a reduction in future unemployment rates, while high bad uncertainty is associated with a decline in future employment and an increase in unemployment rates. In the interest of space, we do not report these results in the tables.

#### 4.2. Macroeconomic uncertainties and aggregate prices

We next use our good and bad uncertainty measures to provide empirical evidence that good uncertainty is associated with an increase in stock market valuations and decrease in the risk-free rates and the default spreads, while bad uncertainty has an opposite effect on these asset prices. This is consistent with the equilibrium asset-price implications in the model specification in Section 2.

To document the link between asset prices and uncertainties, we consider contemporaneous projections of the market variables on the expected growth and good and bad uncertainties, which we run both in levels and in first differences, that is:<sup>13</sup>

$$\begin{aligned} y_t &= a + b'[x_t, V_{gt}, V_{bt}] + \text{error}, \\ \Delta y_t &= a + b'[\Delta x_t, \Delta V_{gt}, \Delta V_{bt}] + \text{error}, \end{aligned}$$

where now, y refers to the dividend yield, risk-free rate, or default spread.

Table 4 shows the slope coefficients and the  $R^2$ 's in these regressions for the market price-dividend ratio, the real risk-free rate, and the default spread. As is evident from the table, the slope coefficients on bad uncertainty are negative for the market price-dividend ratio and the real risk-free rate, and they are positive for the default spread. The slope coefficients are of the opposite sign for the good uncertainty, and indicate that market valuations and interest rates go up and the default spread falls at times of high good uncertainty. Finally, the price-dividend ratio and the risk-free rates increase, while the default spread falls at times of high expected growth. Importantly, all these empirical findings are consistent with the implication of our model, outlined in Section 2, that high expected growth, high good volatility, and low bad volatility are good economic states.

<sup>&</sup>lt;sup>13</sup> Instead of the first difference, we have also run the regression on the innovations into the variables, and the results are very similar.

**Table 4**Macroeconomic uncertainties and aggregate prices.

The table reports the evidence from the projections of the aggregate asset-price variables on the contemporaneous expected consumption growth x, and the good and bad uncertainty variables,  $V_g$  and  $V_b$ . Panel A shows the regression results based on the levels of the variables, and Panel B shows the output for the first differences. The table reports the slope coefficients, t-statistics, the adjusted  $R^2$ 's, and the p-values for the Wald test for the joint significance of good and bad uncertainty,  $H_0$ :  $b_{gv} = b_{bv} = 0$ . Standard errors are Newey-West adjusted. The notations \* and  $\diamond$  indicate the significance at 5% and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses  $b_x > 0$ ,  $b_{gv} > 0$ , and  $b_{bv} < 0$  for the market price-dividend ratio and the real risk-free rate, and  $b_x < 0$ ,  $b_{gv} < 0$ , and  $b_{bv} > 0$  for the default spread. The data are annual from 1930 to 2012.

Panel A: Level-based project	ion				
	x	$V_b$	$V_{ m g}$	$Adj-R^2$	<i>p</i> -Value
Market price-	8.82	-2313.28°	279.27	0.21	< 0.01
dividend ratio	[0.94]	[-2.67]	[0.93]		
Real risk-free rate	0.05	-222.24°	80.50°	0.21	< 0.01
	[0.08]	[-2.36]	[2.74]		
Default spread	-0.36*	50.54°	-3.80	0.47	< 0.01
	[-1.81]	[2.99]	[-0.52]		
Panel B: First difference-base	ed projection				
	$\Delta x$	$\Delta {V}_{b}$	$\Delta V_g$	$Adj-R^2$	<i>p</i> -Value
ΔMarket price-	18.57°	-1353.26°	448.49°	0.61	< 0.01
dividend ratio	[9.97]	[-4.21]	[3.11]		
ΔReal risk-free rate	0.01	<b>- 107.47*</b>	31.75	0.16	< 0.01
	[0.04]	[-1.65]	[1.19]		
∆Default spread	-0.26°	40.46	- 10.64*	0.30	0.01
•	[-2.61]	[2.84]	[-1.98]		

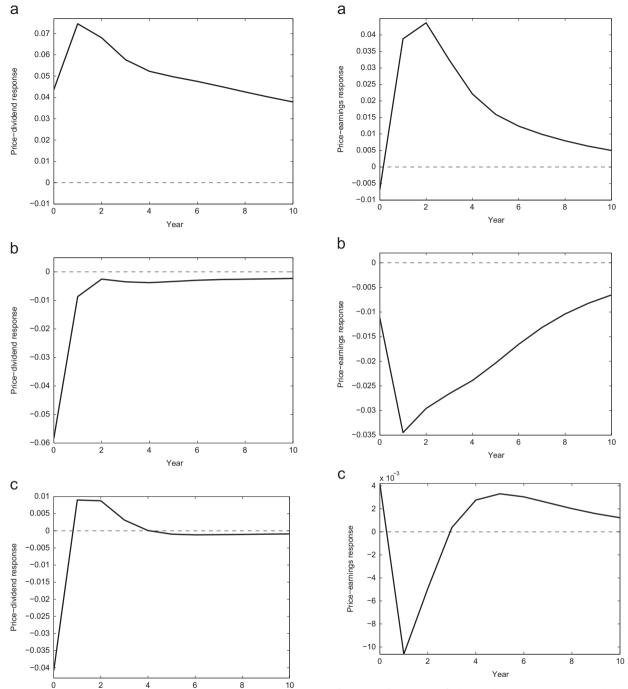
The slope coefficients for our three state variables are economically large. In most cases, the volatility slope coefficients are statistically significant using economically motivated one-sided tests. These tests specify that good (bad) volatility has a positive (negative) impact on pricedividend ratio and risk-free rate, and opposite for the default spreads. Jointly, the two uncertainty variables are always significant with a p-value of 1% or below. The statistical significance is especially pronounced for the first difference projections. Recall that the asset-price variables that we use are very persistent and may contain slowmoving near-unit root components which can impact statistical inference. First difference (or alternatively, using the innovations into the variables) substantially reduces the autocorrelation of the series and allows us to more accurately measure the response of the asset prices to the underlying shocks in macroeconomic variables.

It is also worth noting that our three macroeconomic factors can explain a significant portion of the variation in asset prices. The  $R^2$  in the regressions is 20% for the level of the price–dividend ratio and 60% for the first difference. For the real rate, the  $R^2$ 's are about 20%, and it is 50% for the level of the default spread and 30% for the first difference.

Figs. 9 and 10 further illustrate the impact of uncertainties on asset prices and show the impulse responses of the price-dividend and price-earnings ratio to a one-standard deviation uncertainty shock from the VAR(1). Panel A of Fig. 9 documents that the price-dividend ratio increases by 0.07 one year after a good uncertainty shock and remains positive ten years out. Similarly, the price-earnings ratio increases to about 0.04 in the first two years and its response is also positive at ten years, as depicted in Panel A of Fig. 10. Bad uncertainty shocks depress both immediate and future asset valuations. Price-dividend ratio drops by 0.06 on the impact, while price-earnings

ratio declines by about 0.04 one year after, and all the impulse responses are negative ten years after the shock. The response of the asset prices to the total uncertainty shock is significantly less pronounced than the response to bad uncertainty: the price-dividend ratio decreases immediately by only 0.04 on the impact of the total uncertainty shock, and the response reaches a positive level of 0.01 at one year and goes to zero after three years. Similarly, price-earnings ratio decreases by 0.01 one year after the impact, and the response becomes positive after three years. This weaker response of prices to total uncertainty is consistent with the analysis in Section 2, where it is shown that asset prices' react less to good uncertainty than they do to bad uncertainty even when there was no feedback effect from good uncertainty to expected growth and asset prices reaction to both uncertainties were negative. In the model and in the data, total uncertainty is a combination of the correlated bad and good uncertainty components, which have opposite effects on the asset prices, and it therefore immediately follows that the response of asset prices to the total uncertainty shock is less pronounced. This muted response of asset prices to the total uncertainty masks the significant but opposite effects that different uncertainty components can have on asset valuations, and motivates our decomposition of the total uncertainty into the good and bad parts.

As a final assessment of the model implications for the market return, we evaluate the impact of our macroeconomic uncertainty measures on future level and realized variation in excess returns. In our framework  $V_{gt}$  and  $V_{bt}$  are the key state variables which drive fluctuations in risk premia and volatility of returns, and in particular, the model-implied loadings of the risk premia and volatility on both  $V_{gt}$  and  $V_{bt}$  are positive. Panel A in Table 5 provides the regression results for predicting excess returns for one, three, and five years. At one- and three-year horizons, the



**Fig. 9.** Impulse response of price-dividend ratio to macro uncertainties. The figure shows impulse responses of the market price-dividend ratio to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and the market price-dividend ratio. Data are annual from 1930 to 2012: (a) price-dividend ratio response to good uncertainty shock; (b) price-dividend ratio response to bad uncertainty shock; and (c) price-dividend ratio response to overall uncertainty shock.

Year

**Fig. 10.** Impulse response of price–earnings ratio to macro uncertainties. The figure shows impulse responses of the market price–earnings ratio to one-standard deviation good, bad, and total uncertainty shocks. The impulse responses are computed from a VAR(1) which includes macroeconomic uncertainty measures (bad and good uncertainty for the first two panels, and total uncertainty for the last panel), expected consumption growth, and the market price–earnings ratio. Data are annual from 1930 to 2012: (a) price–earnings ratio response to good uncertainty shock; (b) price–earnings ratio response to bad uncertainty shock; and (c) price–earnings ratio response to overall uncertainty shock.

 Table 5

 Macroeconomic uncertainties and equity returns.

The table reports the evidence from the projections of future excess returns (Panel A) and realized variance of returns (Panel B) on expected consumption growth x, and the good and bad uncertainty variables,  $V_g$  and  $V_b$ . The table reports the slope coefficients, t-statistics, the adjusted  $R^2$ 's, and the p-values for the Wald test for the joint significance of good and bad uncertainty,  $H_0$ :  $b_g = b_{bv} = 0$ . Standard errors are Newey-West adjusted. The notations  $\dagger$  and  $\ast$  indicate the significance at 10% and 5% levels, respectively, against the economically motivated alternative hypotheses  $b_x \neq 0$ ,  $b_g > 0$ , and  $b_{bv} > 0$ . The data are annual from 1930 to 2012.

	X	$V_b$	$V_{g}$	$Adj-R^2$	<i>p</i> -Value
Panel A: Excess retur	n projection				
1Y Ahead	- 1.91 [-0.45]	198.57 [0.45]	19.85 [0.12]	-0.01	0.05
3Y Ahead	- 1.41 [ - 1.15]	82.02 [0.37]	69.42 [0.90]	0.09	< 0.01
5Y Ahead	- 2.04* [-2.44]	227.97 <sup>†</sup> [1.54]	- 30.34 [ - 0.74]	0.08	0.14
Panel B: Return volat	ility projection				
1Y Ahead	$-1.94^{\dagger}$ [-1.66]	24.39 [0.34]	49.10* [2.19]	0.34	< 0.01
3Y Ahead	- 1.39 [-1.15]	64.42 [0.87]	3.35 [0.16]	0.16	0.02
5Y Ahead	- 0.63 [ - 0.80]	34.51 [0.63]	10.75 [0.75]	0.15	< 0.01

loading coefficients are positive on both measures of ex ante uncertainty and jointly statistically significant. At the five-year horizon the loading on  $V_{bt}$  is positive while that on  $V_{gt}$  is negative although both coefficients are statistically insignificant. The R2's for the three- and fiveyear horizons are non-negligible at about 10%. Similarly, Panel B of Table 5 shows that return volatility loadings on good and bad uncertainty are positive and jointly statistically significant at all horizons with economically significant  $R^2$ 's of 15–30%. These findings are in line with the model implications. It is also interesting to note that the coefficient on  $V_{gt}$  is smaller than that of  $V_{bt}$ , consistent with the notion that the effect of bad uncertainty is more important for asset pricing than that of good uncertainty. This is also consistent with the findings in Colacito, Ghysels, and Meng (2013) for the importance of time variation in skewness and left tail.

# 4.3. Macroeconomic uncertainties and cross-section of returns

Using our empirical measures in the data, we show the implications of macroeconomic growth and good and bad uncertainties for the market and a cross-section of asset returns. Our empirical analysis yields the following key results. First, the risk exposures (betas) to bad uncertainty are negative, and the risk exposures to good uncertainty and expected growth are positive for the market and across the considered asset portfolio returns. This is consistent with our empirical evidence on the impact of growth and uncertainty fluctuations for the market valuations in Section 4.2, and with the equilibrium implications of the model in Section 2. Second, in line with the theoretical model, we document that bad uncertainty has a negative market price of risk, while the market prices of good uncertainty and expected growth risks are positive in the data. Hence, the high-risk states for the investors are

those associated with low expected growth, low good uncertainty, and high bad uncertainty. We show that the risk premia for all the three macroeconomic risk factors are positive, and the uncertainty risk premia help explain the cross-section of expected returns beyond the cash flow channel.

Specifically, following our theoretical model, the portfolio risk premium is given by the product of the market prices of fundamental risks  $\Lambda = (\lambda_x, \lambda_{gv}, \lambda_{bv})$ , the variance–covariance matrix  $\Omega$  which captures the quantity of risk, and the exposure of the assets to the underlying macroeconomic risk  $\beta_i^{14}$ :

$$E[R_{i,t+1} - R_{f,t}] = \Lambda' \Omega \beta_i. \tag{25}$$

Given the innovations to the portfolio returns and to our aggregate risk factors, we can estimate the equity exposures and the market prices of expected growth and bad and good uncertainty risks using a standard Fama and MacBeth (1973) procedure.<sup>15</sup> We first obtain the return betas by running a multivariate regression of each portfolio return innovation on the innovations to the three factors:

$$r_{i,t+1} - E_t r_{i,t+1} = \text{const} + \beta_{i,x} (x_{t+1} - E_t [x_{t+1}])$$

$$+ \beta_{i,gv} (V_{g,t+1} - E_t [V_{g,t+1}])$$

$$+ \beta_{i,bv} (V_{b,t+1} - E_t [V_{b,t+1}])$$
+ error. (26)

<sup>&</sup>lt;sup>14</sup> In our model growth shocks are non-Gaussian and therefore the risk premia may include higher-order terms associated with expected growth risk. The volatility risk premia are still linear in the volatility risk exposures. As the focus of our paper is on volatility risk, we maintain a standard linear framework for cross-section evaluation.

<sup>&</sup>lt;sup>15</sup> We have also considered an alternative econometric approach to measure return innovations similar to Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), and Bansal, Kiku, Shaliastovich, and Yaron (2014). The results are similar to our benchmark specification.

The slope coefficients in the above projection,  $\beta_{i,x}$ ,  $\beta_{i,gv}$ , and  $\beta_{i,bv}$ , represent the portfolio exposures to expected growth, good uncertainty, and bad uncertainty risk, respectively. Next we obtain the factor risk premia  $\tilde{\Lambda}$  by running a cross-sectional regression of average returns on the estimated betas:

$$\overline{R_i - R_f} = \tilde{\lambda}_x \beta_{i,x} + \tilde{\lambda}_{gv} \beta_{i,gv} + \tilde{\lambda}_{bv} \beta_{i,bv} + \text{error.}$$
(27)

We impose a zero-beta restriction in the estimation and thus run the regression without an intercept. The implied factor risk premia,  $\tilde{\Lambda} = (\tilde{\lambda}_x, \tilde{\lambda}_{gv}, \tilde{\lambda}_{bv})$ , encompass both the vector of the underlying prices of risks  $\Lambda$  and the quantity of risks  $\Omega$ :

$$\tilde{\Lambda} = \Omega \Lambda$$

To calculate the underlying prices of expected growth, good and bad uncertainty risk  $\Lambda$ , we pre-multiply the factor risk premia  $\tilde{\Lambda}$  by the inverse of the quantity of risk  $\Omega$ , which corresponds to the estimate of the unconditional variance of the factor innovation in the data. To compute standard errors, we embed the two-state procedure into Generalized Method of Moments (GMM), which allows us to capture statistical uncertainty in estimating jointly asset exposures and market prices of risk.

In our benchmark implementation, we use the market return, the cross-section of 25 equity portfolios sorted on book-to-market ratio and size, as well as two bond portfolios (Credit premium and Term premium portfolios). 16 Table 6 shows our key evidence concerning the estimated exposures of these portfolios to expected growth and uncertainty risks and the market prices of risks. Panel A of the table documents that our macroeconomic risk factors are priced in the cross-section, and the market prices of expected growth and good uncertainty risks are positive, while the market price of bad uncertainty risk is negative. This indicates that the adverse economic states for the investor are those with low growth, high bad uncertainty, and low good uncertainty, consistent with the theoretical model. Using one-sided tests against these economically motivated alternatives, the market price of the expected growth risk is significant at a 1% level, while the market prices of volatility risks are significant at a 5% level.

Panel B of the table shows that the equity and bond returns are exposed to these three sources of risks. In particular, all assets have a positive exposure to expected growth risk. The estimated exposures to bad uncertainty risks are negative, while the betas to good uncertainty risks are positive for all the considered asset portfolios. Thus, consistent with our economic model, our evidence indicates that asset returns increase at times of high expected growth and high good uncertainty and decrease at times of high bad uncertainty, and the magnitudes of the response vary in the cross-section. Nearly all of the estimated exposures are significant at a 1% level.

We combine the estimated market prices of risk, quantity of risk, and the equity and bond betas to evaluate the cross-sectional risk premia implications of our model, and report these empirical results in Table 7. As shown in the table, our estimated model can match quite well the level and the dispersion of the risk premia in the cross-section of assets. The market risk premium is 7.2% in the data relative to 8.2% in the model. To help compare the implications of the model to the data, we aggregate the reported average returns into value and size spreads. We define the value spread as the difference between the weighted average returns on the highest and lowest book-to-market portfolios across the five sizes. Similarly, the size spread is defined as the difference between the weighted average returns on the biggest and smallest size portfolios across the book-to-market sorts. In the data, the value spread is 4.38%, which is comparable to 3.34% in the model. The size spread is 4.39%. relative to 5.21% in the model. For the Credit premium portfolio the risk premium is 1.98% in the data and 2.15% in the model. The Term premium is 1.82% in the data relative to 0.64% in the model.<sup>17</sup> We further use the risk premium condition (25) to decompose the model risk premia into the various risk contributions. Because our risk factors are correlated, in addition to the own risk compensations for individual shocks (i.e., terms involving the variances on the diagonal of  $\Omega$ ) we also include the risk components due to the interaction of different shocks (i.e., the covariance elements off the diagonal). As shown in the table, the direct risk compensations for the expected growth and good and bad uncertainty shocks are positive for all the portfolios. This is an immediate consequence of our empirical finding that the equity and bond betas and the market prices of risks are of the same sign, so the direct contribution of each source of risk to the total risk premium is positive. On the other hand, the risk premia interaction terms can be negative and quite large, e.g., the risk premia due to the covariance of good and bad uncertainty. While it is hard to assess separate risk contributions of each risk factor due to the nonnegligible covariance interactions, our results suggest that both good and bad uncertainties have considerable impact on the level and the cross-section of returns.

Overall, our findings for the expected growth risk channel are in line with Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), and Bansal, Kiku, Shaliastovich, and Yaron (2014) who show the importance of growth risk for the cross-section of expected returns. Our evidence for the bad uncertainty is further consistent with Bansal, Kiku, Shaliastovich, and Yaron (2014), who document that total macroeconomic volatility has a negative market price of risk and depresses asset valuations in the cross-section. On the other hand, our finding for the separate role of the good uncertainty for the stock returns, above and beyond the expected growth and total uncertainty channel, is a novel contribution of this paper.

<sup>&</sup>lt;sup>16</sup> In a related literature, Chen (2010), Bhamra, Kuehn, and Strebulaev (2010), and McQuade (2014) develop economic models to study defaults and corporate bond spreads, and Bansal and Shaliastovich (2013) and Piazzesi and Schneider (2007) consider model implications for nominal bond yields.

<sup>&</sup>lt;sup>17</sup> The model excludes an inflation factor which is well known to be important for explaining the term premia.

Table 6

Cross-sectional implications.

The table shows the estimates of the market prices of risks (Panel A) and asset exposures (Panel B) to expected growth, good uncertainty, and bad uncertainty risks. The cross-section includes the market (MKT), 25 portfolios sorted on size (S) and book-to-market (BM), and Credit (DEF) and Term premium (TERM) bond portfolios. The reported betas and the market prices of risks are divided by 100. *T*-statistics are in brackets, and are based on Newey-West standard errors from GMM estimation. The notations  $\dagger$ ,  $\ast$ , and  $\diamond$  indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses that  $\lambda_x$ ,  $\beta_x$ ,  $\lambda_g v$ , and  $\beta_g v$  are positive, and  $\lambda_b v$  are negative. Data are annual from 1930 to 2012.

Panel A: Market	t-prices of risk (Λ/10	00)					
λ	$\lambda_{x}$	$\lambda_{b  u}$	$\lambda_{g_{\mathcal{V}}}$				
	0.95° [4.37]	-66.06* [-1.82]	38.58* [2.13]	_			
Panel B: Exposu	res to risks (β/100)						
	$eta_{x}$	$eta_{b u}$	$eta_{ m gv}$	_	$\beta_{x}$	$eta_{b u}$	$\beta_{\rm gv}$
MKT	25.08°	- 1537.04°	613.89°	S3BM4	33.53°	– 1591.13°	707.77
	[15.33]	[-4.87]	[3.95]		[13.47]	[-3.45]	[3.16]
S1BM1	34.49°	-2888.64°	1016.18°	S3BM5	37.73°	- 1493.40°	696.40
	[6.16]	[-4.24]	[2.99]		[13.10]	[-2.89]	[2.97]
S1BM2	47.06°	- 1351.55 <sup>*</sup>	939.72°	S4BM1	25.92°	- 1809.04°	732.72
	[10.37]	[-1.72]	[2.72]		[9.56]	[-5.70]	[4.47]
S1BM3	40.52°	-2058.56°	919.29°	S4BM2	28.82°	- 1585.84°	757.34
	[14.79]	[-3.90]	[3.79]		[12.48]	[-3.89]	[4.10]
S1BM4	42.95°	- 1919.89°	1001.33°	S4BM3	32.46°	- 1790.46°	802.42
	[12.81]	[-3.88]	[4.54]		[14.91]	[-4.30]	[3.97
S1BM5	46.40°	- 1943.51°	1049.88°	S4BM4	35.95°	-2178.10°	975.61
	[14.18]	[-3.66]	[4.38]		[22.20]	[-8.36]	[7.03
S2BM1	35.86°	- 1351.02*	615.07*	S4BM5	39.92°	- 1871.59°	1085.74°
	[10.85]	[-2.18]	[2.10]		[12.09]	[-3.91]	[4.38
S2BM2	37.06°	- 1225.38*	588.72°	S5BM1	23.11°	- 1592.88°	563.17°
	[15.11]	[-2.30]	[2.44]		[9.95]	[-4.26]	[3.01]
S2BM3	37.04°	- 1637.18°	775.61°	S5BM2	24.51°	- 1684.39°	683.83
	[12.08]	[-2.56]	[2.79]		[14.23]	[-6.77]	[5.74]
S2BM4	38.42°	- 1672.86°	863.86°	S5BM3	27.41°	- 1617.78°	733.52
	[15.77]	[-3.81]	[4.19]		[11.65]	[-6.94]	[6.90
S2BM5	36.69°	-2316.27°	1000.77°	S5BM4	31.47°	-1503.41°	734.06
	[13.68]	[-5.37]	[4.45]		[11.36]	[-4.17]	[4.26
S3BM1	33.92°	- 1754.57°	812.16°	S5BM5	27.62°	-3842.95°	1570.43
	[12.46]	[-3.32]	[3.49]		[11.72]	[-15.80]	[15.55
S3BM2	31.04°	- 1648.15°	634.32°	DEF	8.36°	-388.33°	33.08
	[13.45]	[-3.87]	[3.32]		[6.60]	[-3.15]	[0.57
S3BM3	31.62°	- 1542.86°	640.69°	TERM	1.80 <sup>†</sup>	-227.44°	89.18
	[17.24]	[-3.53]	[3.14]		[1.46]	[-2.46]	[1.91]

#### 5. Robustness

Our benchmark empirical results are based on the predictive uncertainty measures which are constructed from industrial production data, and which span the full sample period from 1930 to 2012. In this Section, we show that our main conclusions are not mechanical and are robust to alternative proxies for the realized variation measures, the construction of the ex ante uncertainties, and using the post-war period.

#### 5.1. Simulation analysis

We use a calibrated model to conduct a Monte-Carlo simulation analysis of the realized semivariances and verify that our empirical results are not driven by the mechanics of the constructed estimators. Specifically, we consider a long-run risks model of Bansal and Yaron (2004) which features conditionally Gaussian consumption shocks, a single stochastic volatility process, and no

volatility feedback into the expected growth. Hence, under the null of the model, there are no separate movements in good and bad volatilities and no effect of volatility on future growth. This allows us to assess whether the mechanics of the construction of the semivariances or the finite-sample considerations can spuriously generate our empirical findings. The model setup and calibration are described in Appendix C and follow Bansal, Kiku, and Yaron (2012). We simulate the model on monthly frequency, and use the same approach as in Section 3.2 to construct realized positive and negative variances based on the simulated consumption data. The ex ante expectations of the quantities are determined from the projections on the model predictive variables, which include positive and negative semivariances, consumption growth, market return, the market price-dividend ratio, and the riskfree rate.

Tables 8 and 9 show the model evidence for the projections of consumption and dividend growth rates, for horizons of one to five years, on the extracted expected

**Table 7** Risk premia decomposition.

The table shows the estimates of risk premia in the data and in the model, and the decomposition of the model risk premia into the compensations for expected growth, good uncertainty, bad uncertainty, and the covariance components. The cross-section includes the market (MKT), 25 portfolios sorted on size (S) and book-to-market (BM), and Credit (DEF) and Term premium (TERM) bond portfolios. Data are annual from 1930 to 2012.

	Tot	tal			Model d	ecomposition		
	Model	Data	$RP_{x,x}$	$RP_{bv,bv}$	$RP_{g\nu,g\nu}$	$RP_{x,bv}$	$RP_{x,gv}$	$RP_{bv,gv}$
MKT	8.15	7.17	10.27	9.92	12.98	1.12	-5.64	-20.51
S1BM1	11.04	5.48	14.12	18.65	21.49	1.81	-8.35	-36.68
S1BM2	16.15	10.78	19.27	8.72	19.87	1.58	-9.84	-23.46
S1BM3	13.35	13.93	16.59	13.29	19.44	1.67	-8.85	-28.79
S1BM4	14.55	16.43	17.59	12.39	21.17	1.68	-9.48	-28.80
S1BM5	15.74	18.68	19.00	12.55	22.20	1.77	-10.13	-29.65
S2BM1	11.52	8.47	14.68	8.72	13.01	1.32	<b>-7.15</b>	-19.05
S2BM2	11.91	11.97	15.17	7.91	12.45	1.30	-7.22	-17.70
S2BM3	12.21	13.71	15.17	10.57	16.40	1.44	-7.87	-23.50
S2BM4	12.94	14.59	15.73	10.80	18.27	1.49	-8.37	-24.98
S2BM5	12.27	15.90	15.03	14.95	21.16	1.67	-8.60	-31.94
S3BM1	11.31	9.40	13.89	11.33	17.17	1.41	-7.56	-24.93
S3BM2	9.85	11.39	12.71	10.64	13.41	1.30	-6.54	-21.67
S3BM3	10.17	12.09	12.95	9.96	13.55	1.28	-6.64	-20.92
S3BM4	10.95	13.28	13.73	10.27	14.97	1.34	-7.14	-22.22
S3BM5	12.25	14.73	15.45	9.64	14.72	1.41	-7.69	-21.29
S4BM1	8.58	8.54	10.61	11.68	15.49	1.24	-6.16	-24.28
S4BM2	9.78	9.18	11.80	10.24	16.01	1.23	-6.66	-22.85
S4BM3	10.80	11.28	13.29	11.56	16.97	1.39	-7.32	-25.08
S4BM4	12.10	12.46	14.72	14.06	20.63	1.60	-8.41	-30.50
S4BM5	14.07	12.91	16.35	12.08	22.96	1.59	-9.35	-29.57
S5BM1	7.30	7.07	9.46	10.28	11.91	1.10	-5.18	-20.27
S5BM2	8.10	7.17	10.04	10.87	14.46	1.16	-5.80	-22.63
S5BM3	9.23	8.27	11.22	10.44	15.51	1.21	-6.38	-22.78
S5BM4	10.55	8.54	12.89	9.70	15.52	1.26	-6.95	-21.88
S5BM5	10.23	11.03	11.31	24.81	33,21	1.98	-9.30	-51.77
DEF	2.15	1.98	3.42	2.51	0.70	0.33	-1.28	-3.53
TERM	0.64	1.82	0.74	1.47	1.89	0.12	-0.56	-3.01

growth, and the good and bad uncertainties. We report model evidence for finite samples of 83 years each, and population values based on a long simulation of 1,000,000 years. The top panels in the tables report the findings under the benchmark specification. Consistent with our empirical robustness analysis (see below), we also consider two modifications of the benchmark specification, where the predictive uncertainties are based on straight OLS rather than log of the variances, and where we use the AR(1) fit to the monthly consumption growth to remove the fluctuations in the conditional mean.

Table 8 reports the slopes and the  $R^2$ 's for the consumption and dividend projections. As shown in the table, the slope coefficients on bad (good) volatility are generally positive (negative), at least for one- and three-year horizons, and these coefficients decrease (increase) with the horizon of the regressions. The evidence is especially pronounced in the population; indeed, in benchmark simulation specifications all the bad (good) volatility slopes from one to five years to maturity are positive (negative). This is opposite of what we find in the data, where the coefficients on bad (good) volatility are generally most negative (positive) at short horizons, and tend to increase (decrease) with the horizon. The table also shows a considerable amount of noise in estimating the ex ante uncertainties in small samples, as all the small-sample volatility loadings are insignificant.

In Table 9 we assess the joint probability for finding the same volatility signs as in the data combining the evidence across the horizons and across the consumption and dividend regressions. The table documents that across one- to five-year horizon consumption growth regressions, all bad volatility loadings turn out negative and all good volatility turn out positive in 3% of the cases. For dividend regressions, this number is 9%. Finally, combining the evidence from both the consumption and dividend predictability regression, the probabilities of finding the same signs in simulations as in the data are less than 1%. Thus, the simulation evidence clearly shows that the patterns in volatility loadings we find in the data cannot be simply attributed to the mechanics of construction of the realized variance estimators or the finite-sample considerations.

#### 5.2. Empirical analysis

We consider various robustness checks concerning the construction of the realized variances in the data. For the first round of robustness checks, we maintain the industrial production growth data to measure the realized variances and modify the construction of ex ante good and bad uncertainties in several dimensions. First, to mitigate potential small-sample concerns with the realized variance estimators, we consider removing the conditional (rather than

**Table 8**Simulation analysis of macroeconomic uncertainties and aggregate growth.

The table shows the Monte-Carlo predictability evidence for the projection of future consumption and dividend growth rates on the current expected consumption growth x, good uncertainty  $V_g$ , and bad uncertainty  $V_b$ :  $(1/h)\sum_{j=1}^h \Delta y_{t+j} = a_h + b'_h | x_t, V_{gt}, V_{bt}| + \text{error}$ . The table reports the population and small-sample estimates (corresponding to 5%, 50%, and 95% percentile of the distribution in simulations) of the slope coefficients and  $R^2$ 's. The consumption, dividends, and asset prices are simulated on monthly frequency and aggregated to annual horizon under the long-run risks, single-volatility model configuration of Bansal, Kiku, and Yaron (2012). Realized positive and negative variances are constructed from the model-simulated demeaned monthly consumption growth rate over the year. The ex ante uncertainty measures correspond to the projections of the log realized variances on the set of predictors, such as realized positive and negative variances, consumption growth, market return, the market price-dividend ratio, and the risk-free rate. Small-sample evidence is based on 100,000 simulations of 83 years of monthly data; the population estimates are based on a long simulation of 1,000,000 years of data.

		х				$V_l$	,			$V_g$				R	2	
	Pop	5%	Med	95%	Pop	5%	Med	95%	Pop	5%	Med	95%	Pop	5%	Med	95%
							Benc	hmark mo	odel:							
Consumpti	on:															
1Y Ahead	1.57	1.22	2.00	3.00	8.53	-10.58	53.56	144.74	-9.56	-147.41	-56.91	8.67	0.41	0.24	0.42	0.59
3Y Ahead	1.03	0.64	1.02	1.41	1.19	-35.10	1.57	39.67	-1.61	-38.30	-2.01	33.39	0.36	0.16	0.38	0.60
5Y Ahead	0.78	0.24	0.67	1.13	0.24	-45.65	-2.81	40.08	-0.53	-38.34	2.80	44.23	0.29	0.09	0.31	0.56
Dividend:																
1Y Ahead	3.95	-0.47	5.37	12.76	19.89	-480.68	106.35	788.62	-51.52	-1087.24	-282.10	254.73	0.09	0.01	0.14	0.33
3Y Ahead	2.57	-0.89	2.68	6.42	1.71	-357.82	-2.84	354.48	-12.72	-428.53	-52.35	277.72	0.09	-0.01	0.12	0.35
5Y Ahead	1.96	-1.21	1.75	4.82	-0.20	-304.46	-11.11	277.78	-6.72	-319.33	-21.01	257.18	0.08	-0.02	0.12	0.38
							Straig	ht OLS m	odel:							
Consumpti	on:															
1Y Ahead	3.15	1.78	2.60	3.71	132.97	35.13	115.83	241.12	-132.80	-240.56	-116.93	-37.91	0.46	0.26	0.45	0.62
3Y Ahead	1.00	1.00	1.00	1.00	0.00	-0.00	0.00	0.00	0.00	-0.00	0.00	0.00	0.36	0.15	0.37	0.59
5Y Ahead	0.58	0.14	0.57	0.95	-14.54	-56.22	-13.27	24.80	14.52	-23.91	13.59	55.94	0.30	0.08	0.30	0.56
Dividend:																
1Y Ahead	12.29	-2.05	7.76	18.49	660.74	-782.68	336.74	1518.28	-707.98	-1753.33	-545.05	528.96	0.13	0.01	0.16	0.36
3Y Ahead	3.96	-1.90	2.86	7.67	109.97	-488.25	9.05	503.43	-126.15	-575.38	-74.37	402.16	0.09	-0.01	0.11	0.34
5Y Ahead	2.33	-2.06	1.60	5.21	29.93	-406.45	-28.67	329.97	-39.86	-370.39	-8.94	355.17	0.08	-0.02	0.11	0.37
							AR(1) Ac	ljustment	model:							
Consumpti	on:															
1Y Ahead	1.57	1.22	2.00	2.99	8.88	-9.97	54.65	147.90	-9.83	-149.61	-58.09	8.32	0.41	0.23	0.42	0.59
3Y Ahead	1.02	0.64	1.01	1.41	1.12	-36.22	1.43	39.92	-1.50	-38.38	-1.80	34.87	0.36	0.16	0.38	0.60
5Y Ahead	0.78	0.24	0.67	1.13	0.12	-47.10	-3.13	40.17	-0.30	-38.21	3.09	45.67	0.29	0.09	0.31	0.56
Dividend:																
1Y Ahead	3.91	-0.48	5.35	12.71	19.92	-492.75	110.35	801.79	-50.16	-1098.71	-285.97	267.96	0.09	0.01	0.14	0.33
3Y Ahead	2.54	-0.88	2.67	6.42	1.95	-365.16	-3.93	360.27	-12.12	-431.43	-52.98	286.43	0.09	-0.01	0.12	0.35
5Y Ahead	1.92	- 1.20	1.75	4.81	-0.35	-310.83	- 12.29	282.25	-5.62	-322.12	-20.90	262.79	0.08	-0.02	0.12	0.38

the unconditional) mean of industrial production growth in constructing good and bad realized variances. We do so by using the residuals based on fitting an AR(1) to industrial production growth. The summary of the key results for this specification is reported in Table 10. By and large, the findings are qualitatively and quantitatively similar to those reported in the benchmark specification. It is worth emphasizing that asymptotically, the conditional mean dynamics do not impact the properties of the realized variance. Our empirical results indicate the conditional mean dynamics also do not affect the realized variance in our finite sample.

Next, we consider changing the cutoff point for defining good and bad uncertainty. Instead of using the unconditional mean, now the good variance state is defined for the states in which industrial production is above its 75 percentile. Table 11 provides key results for this case and shows that the main findings for our benchmark specification are intact. Further, instead of taking the logs of the realized variances and exponentiating the fitted values, we run standard OLS regressions on the levels of the positive and negative realized variances and use directly the fitted values from these regressions as proxies for good and bad uncertainties, respectively. Alternatively, while in our

benchmark approach we predict the realized variances over a three-year forecast window, for robustness, we also consider shorter and longer horizons, such as one- and five-year window specifications. We also expand the set of the predictive variables and include the term spread, defined as the difference between the 10-year and 3month Treasury yield, to the benchmark set of predictors. We also experimented with removing some of the variables (e.g., default spread, price-dividend ratio, risk-free rate) from the benchmark set of predictive variables. Finally, we also consider using the cross-section of industry portfolios instead of size and book-to-market to identify the betas and market prices of risk. In the interest of space we do not report these additional tables but note that across all of these modifications of the benchmark specification, we confirm our key empirical results regarding: (i) the relation between good and bad uncertainties and the future macroeconomic growth rates, (ii) the relation between the two uncertainties and the aggregate asset prices, and (iii) the market prices and exposures to the three underlying risks.

For the second set of robustness checks, we consider monthly earnings data, instead of industrial production

#### Table 9

Model-implied significance of the volatility coefficients.

The table shows the Monte-Carlo predictability evidence for the projection of future consumption and dividend growth rates on the current expected consumption growth x, good uncertainty  $V_g$ , and bad uncertainty  $V_b$ :  $(1/h) \sum_{j=1}^h \Delta y_{t+j} = a_h + b'_h[x_t, V_{gt}, V_{bt}] + \text{error}$ . The table reports the fraction of samples in which bad (good) uncertainty loadings at 1-, 3-, and 5-year maturities are all negative (positive). The data are simulated on monthly frequency and aggregated to annual horizon under the long-run risks, single-volatility model configuration of Bansal, Kiku, and Yaron (2012). Realized positive and negative variances are constructed from the model-simulated demeaned monthly consumption growth rate over the year. The ex ante uncertainty measures correspond to the projections of the log realized variances on the set of predictors, such as realized positive and negative variances, consumption growth. market return, the market price-dividend ratio, and the risk-free rate. Small-sample evidence is based on 100,000 simulations of 83 years of monthly data.

	$Pr(b_{bv} < 0)$	$Pr(b_{gv} > 0)$	$Pr(b_{bv<0}\&b_{gv}>0)$						
Benchmark model:									
Consumption growth:	0.05	0.04	0.03						
Dividend growth:	0.24	0.12	0.09						
Joint:	0.02	0.01	0.01						
	Straight OLS	model:							
Consumption growth:	0.004	0.004	0.002						
Dividend growth:	0.21	0.13	0.11						
Joint:	0.002	0.001	0.001						
AF	R(1) Adjustm	ent model:							
Consumption growth:	0.05	0.04	0.03						
Dividend growth:	0.24	0.12	0.09						
Joint:	0.02	0.01	0.01						

data, to construct realized variances. Table 12 shows a summary of the key macroeconomic and asset-pricing implications of the good and bad uncertainty using these alternative measures of volatility. The table shows that the earnings-based uncertainty measures deliver very similar implications to the industrial-production-based ones. Indeed, as shown in Panel A, with a single exception of R&D investment growth, all future macroeconomic growth rates increase following positive shocks to expected growth, positive shocks to good uncertainty, and negative shocks to bad uncertainty. As shown in Panel B of the table, the contemporaneous responses of aggregate asset prices to uncertainty based on earnings volatility measures are very similar to those based on industrial production measures of volatility. With the exception of the risk-free rate projection, this evidence again is consistent with interpreting the high expected growth, high good uncertainty, and low bad uncertainty as good states for asset valuations. This conclusion is confirmed in Panel C which documents that the market prices of expected consumption and good uncertainty risks are positive, and that of bad uncertainty is negative. As in the benchmark specification, the estimated equity exposures to these risk factors have the same sign as the market prices of risks, so the direct contribution of each macroeconomic risk to the equity risk premium is positive.

Using the estimated expected growth and uncertainty measures we verify whether the results are robust to the post-war sample. As shown in Table 13, for the majority of the considered projections, our benchmark conclusions for

the relation of the macroeconomic volatilities to growth and asset prices are unchanged.

#### 6. Conclusion

In this paper we present an economic framework and empirical measures for studying good and bad aggregate uncertainty. We define good and bad uncertainty as the variance associated with the respective positive and negative innovations of an underlying macroeconomic variable. We show that in the model and in the data, fluctuations in good and bad macroeconomic uncertainty have a significant and opposite impact on future growth and asset valuations.

We develop a version of the long-run risk model which features separate volatilities for good and bad consumption shocks, and feedback from volatilities to future growth. We show that the equity prices decline with bad uncertainty and rise with good uncertainty, provided there is a sufficiently large feedback from good uncertainty to future growth. Moreover, we show that the market price of risk and equity beta are both positive for good uncertainty, while they are both negative for bad uncertainty. This implies that both good and bad uncertainty risks contribute positively to the risk premia.

Empirically, we use the realized semivariance measures based on the industrial production data to construct good and bad uncertainties, and show the model implications are consistent with the data. Specifically, future economic growth, such as consumption, dividend, earnings, GDP, and investment, rise with good uncertainty, while they fall with bad uncertainty. Consistent with the model, equity prices and interest rates increase (decrease) with good (bad) uncertainty. Finally, using the cross-section of assets we estimate a positive market price of good uncertainty risk, and a negative one for bad uncertainty risk. In all, our theoretical and empirical evidence shows the importance of separate movements of good and bad uncertainty for economic growth and asset prices. We leave it for future work to provide explicit economic channels, linking good and bad uncertainty risks with technological aspects of production, investment, and financing opportunities.

## Appendix A. Realized variance asymptotics

Consider a jump-diffusion process  $y_t$ :

$$y_t = \int_0^t \mu_s \, ds + \int_0^t \sigma_s \, dW_s + J_t,$$
 (A.1)

where  $\mu_s$  is a locally bounded predictable drift process,  $\sigma_s$  is a strictly positive càdlàg volatility process,  $J_t$  is a finite activity jump process, and  $\mu_s$ ,  $\sigma_s$ , and  $J_t$  are adapted to some common filtration  $\mathcal{F}_t$ .

The realized semivariances are defined as follows:

$$RV_{p,t+1} = \sum_{i=1}^{N} \mathbb{I}(\Delta y_{t+i/N} \ge 0) \Delta y_{t+i/N}^{2},$$
  

$$RV_{n,t+1} = \sum_{i=1}^{N} \mathbb{I}(\Delta y_{t+i/N} < 0) \Delta y_{t+i/N}^{2}.$$

 Table 10

 Conditionally demeaned industrial production-based uncertainties.

The table presents a summary of the macroeconomic and asset-price evidence using alternative measures of good and bad uncertainty based on monthly, conditionally demeaned, industrial-production data. The conditional mean is estimated based on an AR(1) model of industrial production growth. Panel A documents the slope coefficients, t-statistics, and the  $R^2$  in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty  $V_b$ , and bad uncertainty  $V_b$ . Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations  $\dagger$ ,  $\star$ , and  $\diamond$  indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1930 to 2012 (post-war for R&D).

	X	$V_b$	$V_g$	$Adj-R^2$
Panel A: Aggregate growth rate predictab	ility			
Consumption growth	2.11° [4.80]	-63.49 [-1.03]	21.41 [0.79]	0.48
GDP growth	7.22 ° [2.69]	-910.48 <sup>†</sup> [-1.39]	460.51 <sup>†</sup> [1.29]	0.13
Market dividend growth	7.26° [3.41]	76.37 [0.29]	- 154.09 [ - 1.18]	0.29
Earnings growth	13.05° [2.69]	-401.02 [-0.67]	75.59 [0.26]	0.09
Capital investment growth	24.56° [3.65]	- 1574.59 <sup>†</sup>   - 1.59	885.52* [2.01]	0.32
R&D investment growth	4.11° [4.92]	- 1046.42* [ - 1.95]	594.90* [1.77]	0.22
Panel B: Aggregate asset prices Level-based projections:	[4.52]	[-1.55]	[1.77]	
Market price-dividend ratio	3.60 [0.43]	-588.21 [-0.47]	-470.52 [ $-1.09$ ]	0.17
Real risk-free rate	-0.44 [-0.71]	− 106.17 <sup>†</sup> [ − 1.45]	39.29 <sup>†</sup> [1.53]	0.07
Default spread	-0.44° [-2.43]	63.02° [2.66]	– 13.20 [ – 1.15]	0.45
First difference-based projections:	1 1			
ΔMarket price-dividend ratio	21.25° [9.35]	-706.94* [-2.23]	367.33° [3.22]	0.52
ΔReal risk-free rate	0.36 <sup>†</sup> [1.53]	- 111.12* [ - 2.17]	41.35 <sup>†</sup> [1.57]	0.15
ΔDefault spread	-0.33° [-2.83]	33.99° [2.52]	-11.47* [-2.28]	0.22
Panel C: Asset-pricing implications				
Prices of risk $(\Lambda/100)$	1.04° [3.53]	- 18.65 [ - 0.52]	36.98 <sup>†</sup> [1.58]	
Market exposures ( $\beta/100$ )	30.92° [15.11]	- 1452.43° [ - 3.82]	811.16° [6.33]	

Barndorff-Nielsen, Kinnebrock, and Shephard (2010) derive the behavior of this statistic under in-fill asymptotics, and in particular, they show that

$$\begin{split} RV_{p,t+1} & \stackrel{p}{\to} \frac{1}{2} \int_t^{t+1} \sigma_s^2 \, ds + \sum_{t \leq s \leq t+1} \mathbb{I}\left(\Delta J_s \geq 0\right) \Delta J_s^2, \\ RV_{n,t+1} & \stackrel{p}{\to} \frac{1}{2} \int_t^{t+1} \sigma_s^2 \, ds + \sum_{t \leq s \leq t+1} \mathbb{I}\left(\Delta J_s < 0\right) \Delta J_s^2. \end{split}$$

Intuitively, the positive (negative) semivariances are informative about positive (negative) squared jumps.

While the above theory considers jump-diffusion processes, Diop, Jacod, and Todorov (2013) provide asymptotic convergence results and Central Limit theorems for the quadratic variations of pure jump Itô semimartingales.

#### Appendix B. Benchmark model solution

In case when  $\epsilon_{i,t+1}$ ,  $i = \{g, b\}$ , follows a compensated compound Poisson distribution with time-varying intensity

 $l_t$ , its log moment-generating function is given by

$$\log E_t e^{u\epsilon_{i,t+1}} = l_t(\alpha(u) - u\alpha'(0) - 1), \tag{B.1}$$

where  $\alpha(u) = \mathsf{E}_t e^{\iota y_{l,t+1}}$  denotes the moment-generating function of the underlying positive jumps. The conditional variance of the compound Poisson is given by  $V_{i,t} = Var_t \; \epsilon_{i,t+1} = l_t \alpha^{"}(0)$ , which implies that its log moment-generating function is linear in its variance,

$$\log E_t e^{u\epsilon_{lt+1}} = V_t f(u) \quad \text{for } f(u) = \frac{\alpha(u) - u\alpha'(0) - 1}{\alpha'(0)}. \tag{B.2}$$

Because the underlying Poisson jumps are positive,  $(J_{i,t+1}>0)$ , f(u) is positive, convex, and asymmetric, so that f(u)>f(-u) for u>0.<sup>18</sup>

In case when  $\epsilon_{i,t+1}$ ,  $i = \{g, b\}$ , follows a demeaned Gamma distribution with a unit scale and time-varying shape parameter  $V_t$ , Bekaert and Engstrom (2009) show

<sup>&</sup>lt;sup>18</sup> To prove asymmetry, note that because jump distribution is positively skewed, f'(u) > 0. This implies that f(u) - f(-u) is increasing in u, so that f(u) - f(-u) > 0 for u > 0.

 Table 11

 Industrial production-based uncertainties with shifted cutoff.

The table presents a summary of the macroeconomic and asset-price evidence using alternative measures of good and bad uncertainty based on monthly industrial-production data, computed under a shifted cutoff for the good and bad uncertainty observations. The ex post positive (negative) semivariance is computed using observations above (below) the 75th percentile of industrial production growth. Panel A documents the slope coefficients, t-statistics, and the  $R^2$  in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty  $V_g$ , and bad uncertainty  $V_b$ . Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations  $\dagger$ ,  $\star$ , and  $\circ$  indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1930 to 2012 (post-war for R&D).

	X	$V_b$	$V_g$	$Adj-R^2$
Panel A: Aggregate growth rate predictab	ility			
Consumption growth	2.01°	$-80.25^{\dagger}$	16.91	0.52
	[5.14]	[-1.56]	[1.09]	
GDP growth	4.48°	$-680.39^{\dagger}$	229.82 <sup>†</sup>	0.05
	[6.60]	[-1.58]	[1.50]	
Market dividend growth	8.78°	-459.88*	38.09	0.40
	[4.37]	[-1.96]	[0.50]	
Earnings growth	12.26°	- <b>724.7</b> 1	129.06	0.10
	[3.58]	[-1.19]	[0.60]	
Capital investment growth	24.08°	-2319.51*	830.56°	0.38
	[4.28]	[-2.31]	[2.65]	
R&D investment growth	4.39°	-724.28°	582.91*	0.28
3	[4.02]	[-2.73]	[2.31]	
Panel B: Aggregate asset prices Level-based projections:				
Market price-dividend ratio	10.46	-3075.22°	$493.02^{\dagger}$	0.24
-	[1.16]	[-2.96]	[1.49]	
Real risk-free rate	-0.02	-226.10*	73.65°	0.19
	[-0.03]	[-2.14]	[2.51]	
Default spread	-0.41°	72.97°	$-10.93^{\dagger}$	0.52
•	[-2.42]	[4.30]	[-1.40]	
First difference-based projections:	. ,		. ,	
ΔMarket price-dividend ratio	18.54°	- 1588.31°	445.77°	0.61
•	[10.25]	[-4.72]	[3.58]	
ΔReal risk-free rate	-0.01	$-117.20^{\dagger}$	28.81	0.14
	[-0.06]	[-1.53]	[1.10]	
ΔDefault spread	-0.28*	57.12°	- 14.03*	0.39
	[-2.89]	[3.13]	[-2.21]	0.55
Panel C: Asset-Pricing Implications		,		
Prices of risk ( $\Lambda/100$ )	0.94°	-76 <b>.</b> 20*	37.39*	
111000 01 11011 (11, 100)	[4.32]	[-1.73]	[2.06]	
Market exposures ( $\beta/100$ )	24.39°	- 1642.30°	560.40°	
market exposures (p/100)	[16.86]	[-5.18]	[4.19]	

that its log moment-generating function satisfies

$$\log E_t e^{u\epsilon_{i,t+1}} = V_t f(u), \tag{B.3}$$

for  $f(u) = -(\log(1-u) + u)$ . The function f(u) is positive, convex, and asymmetric, so that f(u) > f(-u) for u > 0.

The solution of the model relies on a standard loglinearization of returns,

$$r_{c,t+1} \approx \kappa_0 + \kappa_1 p c_{t+1} - p c_t + \Delta c_{t+1}. \tag{B.4}$$

In equilibrium, the price-consumption ratio is linear in the expected growth and uncertainty factors, as shown by Eq. (10). The log-linearization parameter  $\kappa_1$  satisfies the equation

$$\log \kappa_{1} = \log \delta + \left(1 - \frac{1}{\psi}\right)\mu_{c} + A_{gv}(1 - \kappa_{1}\nu_{g})V_{g0} + A_{bv}(1 - \kappa_{1}\nu_{b})V_{b0} + \theta\kappa_{1}^{2}\left[\frac{1}{2}A_{gv}^{2}\sigma_{gw}^{2} + \frac{1}{2}A_{bv}^{2}\sigma_{bw}^{2} + \alpha A_{gv}A_{bv}\sigma_{gw}\sigma_{bw}\right]. \quad (B.5)$$

The real stochastic discount factor is equal to

$$m_{t+1} = m_0 + m_x x_t + m_{gv} V_{gt} + m_{bv} V_{bt}$$
$$-\lambda_x \sigma_x (\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) - \lambda_{gv} \sigma_{gw} w_{g,t+1}$$
$$-\lambda_{bv} \sigma_{bw} w_{b,t+1}, \tag{B.6}$$

where the market prices of risk are specified in Eqs. (14)–(16), and the loadings on the state variables are given by

$$m_x = -\gamma + (1-\theta)(1-\kappa_1\rho)A_x = -\frac{1}{\psi},$$

$$\begin{split} m_{gv} &= \left(1 - \theta\right) \left(A_{gv} \left(1 - \kappa_1 \nu_g\right) - \kappa_1 A_x \tau_g\right) \\ &= \frac{1 - \theta}{\theta} f\left(\theta\left(\left(1 - \frac{1}{\psi}\right) \sigma_c + \kappa_1 A_x \sigma_x\right)\right), \end{split}$$

$$\begin{split} m_{bv} &= \left(1 - \theta\right) (A_{bv} (1 - \kappa_1 \nu_b) + \kappa_1 A_x \tau_b) \\ &= \frac{1 - \theta}{\theta} f\left(\theta \left(-\left(1 - \frac{1}{\psi}\right) \sigma_c - \kappa_1 A_x \sigma_x\right)\right). \end{split}$$

**Table 12** Earnings-based uncertainties.

The table presents a summary of the macroeconomic and asset-price evidence using alternative measures of good and bad uncertainty based on monthly corporate earnings data. Panel A documents the slope coefficients, t-statistics, and the  $R^2$  in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty  $V_g$ , and bad uncertainty  $V_b$ . Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations  $\dagger$ ,  $\star$ , and  $\circ$  indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1930 to 2012 (post-war for R&D).

	x	$V_b$	$V_g$	Adj-R <sup>2</sup>
Panel A: Aggregate growth rate predictab	ility			
Consumption growth	1.86°	-160.58*	41.10*	0.53
	[7.10]	[-2.00]	[1.97]	
GDP growth	4.86*	−371.75°	98.78°	0.05
	[2.00]	[-2.73]	[2.84]	
Market dividend growth	6.28°	- 1448.37°	354.80°	0.37
_	[3.01]	[-3.99]	[3.85]	
Earnings growth	4.12	- 1319.52 <sup>†</sup>	166.00	0.21
	[0.67]	[-1.60]	[0.86]	
Capital investment growth	18.98	- 3498.01°	901.57	0.43
	[5.40]	[-3.17]	[3.21]	
R&D investment growth	2.63°	720.54°	- 195,27°	0.26
G	[2.72]	[2.55]	[-2.69]	
Panel B: Aggregate Asset Prices				
Level-based projections:				
Market price-dividend ratio	2.38	-5282.53°	1377.15°	0.13
	[0.37]	[-2.59]	[2.66]	
Real risk-free rate	-0.40	51.03	-24.48	-0.01
	[-0.39]	[0.25]	[-0.44]	
Default spread	-0.08	76.82*	- 15.42	0.19
	[-0.51]	[1.68]	[-1.26]	
First difference-based projections:				
ΔMarket price-dividend ratio	16.81°	-1688.33°	399.38°	0.61
	[7.89]	[-4.93]	[4.48]	
ΔReal risk-free rate	-0.44	−7 <b>4.</b> 01*	12.36	0.05
	[-0.96]	[-2.02]	[1.24]	
ΔDefault spread	-0.09	39.07 <sup>†</sup>	-6.31	0.44
	[-1.15]	[1.28]	[-0.81]	
Panel C: Asset-pricing implications	,			
Prices of risk $(\Lambda/100)$	0.46	-238.78°	69.48°	
., ,	[1.24]	[-3.32]	[3.19]	
Market exposures ( $\beta/100$ )	25.68°	- 1502.40°	401.75°	
r (/ / /	[13.54]	[-6.92]	[7.49]	

Note that we can alternatively rewrite the stochastic discount factor in terms of  $V_{gt} + V_{bt}$  and  $V_{gt} - V_{bt}$ , which capture the total variance and skewness of consumption dynamics:

$$\begin{split} m_{t+1} &= m_0 + m_x x_t + \frac{m_{gv} + m_{bv}}{2} (V_{gt} + V_{bt}) \\ &+ \frac{m_{gv} - m_{bv}}{2} (V_{gt} - V_{bt}) \\ &- \lambda_x \sigma_x (\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) \\ &- \frac{\lambda_{gv} + \lambda_{bv}}{2} (\sigma_{gw} w_{g,t+1} + \sigma_{bw} w_{b,t+1}) \\ &- \frac{\lambda_{gv} - \lambda_{bv}}{2} (\sigma_{gw} w_{g,t+1} - \sigma_{bw} w_{b,t+1}). \end{split} \tag{B.7}$$

The last two shocks are equal to the innovations into the total variance and skewness of consumption. As  $\lambda_{gv} > 0$  and  $\lambda_{bv} < 0$ , the market price of risk of skewness is

positive: agents dislike the states with low (negative) skewness.

The bond loadings satisfy the recursive equations:

$$B_{x,n} = \rho B_{x,n-1} - m_x,$$
 (B.8)

$$B_{gv,n} = \nu_g B_{gv,n-1} - m_{gv}$$
 
$$-f(-\sigma_x(\lambda_x + B_{x,n-1})) + \tau_g B_{x,n-1}, \tag{B.9}$$

$$B_{bv,n} = \nu_b B_{bv,n-1} - m_{bv}$$

$$-f(\sigma_{x}(\lambda_{x}+B_{x,n-1}))-\tau_{b}B_{x,n-1}, \tag{B.10}$$

for 
$$B_{x,0} = B_{gv,0} = B_{bv,0} = 0$$
.

Similarly, the return of the dividend-paying asset can be expressed by

$$r_{d,t+1} \approx \kappa_{0,d} + \kappa_{1,d} p d_{t+1} - p d_t + \Delta d_{t+1},$$
 (B.11)

**Table 13** Benchmark uncertainties: post-war sample.

The table presents a summary of the macroeconomic and asset-price evidence using the benchmark uncertainty measures in the post-war period. Panel A documents the slope coefficients, t-statistics, and the  $R^2$  in the projections of one-year-ahead macroeconomic growth rates on the expected growth x, good uncertainty  $V_B$ , and bad uncertainty  $V_B$ . Panel B shows the evidence from the contemporaneous regressions of the aggregate asset prices on these factors. Panel C shows the estimates of the market prices of risks and the market return exposures to expected growth, good uncertainty, and bad uncertainty risks. The notations  $\dagger$ ,  $\star$ , and  $\diamond$  indicate the significance of the coefficients at 10%, 5%, and 1% levels, respectively, against the economically motivated, alternative one-sided hypotheses, specified in earlier tables. Data are annual from 1947 to 2012.

	х	$V_b$	$V_{ m g}$	Adj-R <sup>2</sup>		
Panel A: Aggregate growth rate predictability						
Consumption growth	1.43° [6.27]	-275.72° [-3.57]	123.23° [2.88]	0.41		
GDP growth	2.27 <sup>†</sup> [1.33]	- 1127.75° [-2.41]	1174.93* [2.09]	0.44		
Market dividend growth	2.41 [1.26]	-362,83 [-0.59]	136.85 [0.51]	-0.01		
Earnings growth	11.39* [1.78]	- 1941.22 [-0.77]	666.54 [0.67]	0.02		
Capital investment growth	8.33° [3.45]	- 2231.30° [-4.51]	1813.75° [3.88]	0.42		
Panel B: Aggregate asset prices						
Level-based projections: Market price-dividend ratio	-5.92 [-0.65]	−3987.38 <sup>†</sup> [−1.59]	− 1011.95 [ − 0.75]	0.34		
Real risk-free rate	1.32° [3.04]	-440.50° [-2.35]	80.05 [1.12]	0.37		
Default spread	- 0.41° [ - 3.15]	113.33* [1.66]	-49.41* [-2.11]	0.33		
First difference-based projections:	[]	[]	1 1			
ΔMarket price-dividend ratio	20.93° [12.62]	$-2740.74^{\circ}$ [-4.31]	665.55° [2.70]	0.61		
ΔReal risk-free rate	0.37 [1.09]	-364.57° [-3.39]	174.82° [5.60]	0.46		
ΔDefault spread	-0.40° [-3.63]	122.58° [2.99]	-4.44 [-0.77]	0.61		
Panel C: Asset-pricing implications						
Prices of risk ( $\Lambda/100$ )	0.82° [4.87]	−74.95° [−2.81]	38.15° [3.43]			
Market exposures ( $\beta$ /100)	28.84° [11.35]	-2912.56° [-4.74]	938.32° [2.57]			

where  $\kappa_{0,d}$  and  $\kappa_{1,d}$  are the log-linearization parameters, and  $\kappa_{1,d}$  satisfies

$$\begin{split} \log \kappa_{1,d} &= m_0 + \mu_d + H_{gv} (1 - \kappa_{1,d} \nu_g) V_{g0} \\ &+ H_{bv} (1 - \kappa_{1,d} \nu_b) V_{b0} \\ &+ \kappa_{1,d}^2 \left[ \frac{1}{2} H_{gv}^2 \sigma_{gw}^2 + \frac{1}{2} H_{bv}^2 \sigma_{bw}^2 + \alpha H_{gv} H_{bv} \sigma_{gw} \sigma_{bw} \right]. \end{split} \tag{B.12}$$

The return dynamics can be expressed in the following way:

$$\begin{split} r_{d,t+1} &= \mathbb{E}_t[r_{d,t+1}] + \beta_\chi \sigma_\chi(\varepsilon_{g,t+1} - \varepsilon_{b,t+1}) \\ &+ \beta_{gv} \sigma_{gw} w_{g,t+1} + \beta_{bv} \sigma_{bw} w_{b,t+1} + \sigma_d u_{d,t+1}, \end{split} \tag{B.13}$$

where the equity betas are given by

$$\beta_x = \kappa_{1,d} H_x$$
,  $\beta_{gv} = \kappa_{1,d} H_{gv}$  and  $\beta_{bv} = \kappa_{1,d} H_{bv}$ . (B.14)

 $H_x$ ,  $H_{gv}$ , and  $H_{bv}$  are the equilibrium loadings of the pricedividend ratio on predictable consumption growth, good uncertainty, and bad uncertainty, respectively, and are given by

$$H_{X} = \frac{\phi_{X} + m_{X}}{1 - \kappa_{1,d} \rho},\tag{B.15}$$

$$H_{gv} = \frac{f(\kappa_{1,d} H_X \sigma_X - \lambda_X \sigma_X) + \kappa_{1,d} H_X \tau_g + m_{gv}}{1 - \kappa_{1,d} \nu_g},$$
 (B.16)

$$H_{bv} = \frac{f(-\kappa_{1,d} H_x \sigma_x + \lambda_x \sigma_x) - \kappa_{1,d} H_x \tau_b + m_{bv}}{1 - \kappa_{1,d} \nu_b}.$$
 (B.17)

Note that we can alternatively rewrite the return dynamics in terms of  $V_{gt} + V_{bt}$  and  $V_{gt} - V_{bt}$ , which capture the total variance and skewness of consumption dynamics:

$$\begin{split} r_{d,t+1} &= \mathsf{E}_{t} \big[ r_{d,t+1} \big] + \beta_{x} \sigma_{x} \big( \varepsilon_{g,t+1} - \varepsilon_{b,t+1} \big) \\ &+ \frac{\beta_{gv} + \beta_{bv}}{2} \sigma_{gw} \big( w_{g,t+1} + w_{b,t+1} \big) \\ &+ \frac{\beta_{gv} - \beta_{bv}}{2} \sigma_{bw} \big( w_{g,t+1} - w_{b,t+1} \big) + \sigma_{d} u_{d,t+1}. \end{split} \tag{B.18}$$

As  $\beta_{gv} > 0$  and  $\beta_{bv} < 0$ , equity exposure to skewness risk is positive: equities fall at times of low (negative) skewness.

It follows that the conditional variance of returns is time varying and driven by good and bad uncertainties:

$$Var_{t}r_{d,t+1} = \beta_{gv}^{2}\sigma_{gw}^{2} + \beta_{bv}^{2}\sigma_{bw}^{2} + \sigma_{d}^{2} + \beta_{x}^{2}\sigma_{x}^{2}(V_{gt} + V_{bt}).$$
 (B.19)

In particular, the variance of returns increases at times of high good or bad uncertainty.

#### Table R1

Model calibration

The table shows the calibrated parameters of the long-run risks model at monthly frequency. The parameter  $\delta$  is the subjective discount factor,  $\gamma$  is the coefficient of relative risk aversion, and  $\psi$  is the intertemporal elasticity of substitution.  $\mu$  is the unconditional expectation of consumption growth,  $\rho$  captures the persistence of expected consumption, and  $\phi_e$  governs the scale of expected consumption shocks. The parameters  $\sigma_c$ ,  $\nu$ , and  $\sigma_w$  represent the level, persistence, and the standard deviation of volatility shocks, respectively.  $\mu_d$  is the unconditional growth rate of dividends,  $\phi$  captures the exposure of dividends to expected consumption shocks, and  $\pi$  reflects the exposure of dividends to realized consumption shocks. The parameter  $\phi_d$  governs the volatility of the idiosyncratic dividend shock.

Preferences	δ	γ	Ψ	
	0.9987	10	2	
Consumption	μ	ρ	$\varphi_e$	
	0.0015	0.975	0.038	
Volatility	$\sigma_{c}$	ν	$\sigma_{\!\scriptscriptstyle W}$	
	0.0072	0.999	2.8e – 06	
Dividend	$\mu_d$	φ	$arphi_d$	π
	0.0015	2.5	5.96	2.6

In levels, the equity risk premium satisfies

$$\begin{split} & \mathbf{E}_{t} R_{d,t+1} - R_{f,t} \approx \log \mathbf{E}_{r} e^{r_{d,t+1} - r_{f,t}} \\ & = \left[ f(-\lambda_{x} \sigma_{x}) - f((\beta_{x} - \lambda_{x}) \sigma_{x}) + f(\beta_{x} \sigma_{x}) \right] V_{gt} \\ & + \left[ f(\lambda_{x} \sigma_{x}) - f((\lambda_{x} - \beta_{x}) \sigma_{x}) + f(-\beta_{x} \sigma_{x}) \right] V_{bt} \\ & + \beta_{gv} \lambda_{gv} \sigma_{gw}^{2} + \beta_{gv} \lambda_{bv} \sigma_{bw}^{2} \\ & + \alpha \sigma_{bw} \sigma_{gw} (\beta_{\sigma v} \lambda_{bv} + \beta_{bv} \lambda_{gv}). \end{split} \tag{B.20}$$

Under standard parameter configuration, the equity premium loadings on good and bad volatility are positive. Indeed, notice that these loadings can be written as f(a)+f(b)-f(a+b). As  $\lambda_x>0$  and  $\beta_x>0$ , a and b have opposite signs. Without loss of generality, let a>0 and b<0. Suppose a+b>0. Then, as f(u) is positive and increasing for u>0,  $a>a+b>0 \Rightarrow f(a)+f(b)>f(a)>f(a+b)$ . Alternatively, suppose a+b<0. Then, as f(u) is positive and decreasing for u<0,  $0>a+b>b \Rightarrow f(a)+f(b)>f(b)>f(b)>f(a+b)$ . In both cases, f(a)+f(b)-f(a+b)>0.

### Appendix C. Long-run risks model specification

In a standard long-run risks model, consumption dynamics satisfies

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1},\tag{C.1}$$

$$X_{t+1} = \rho X_t + \varphi_\rho \sigma_t \epsilon_{t+1}, \tag{C.2}$$

$$\sigma_{t+1}^2 = \sigma_c^2 + \nu(\sigma_t^2 - \sigma_c^2) + \sigma_w w_{t+1}, \tag{C.3}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{d,t+1}, \tag{C.4}$$

where  $\rho$  governs the persistence of expected consumption growth  $x_t$ , and  $\nu$  determines the persistence of the conditional aggregate volatility  $\sigma_t^2$ .  $\eta_t$  is a short-run consumption

shock,  $\epsilon_t$  is the shock to the expected consumption growth, and  $w_{t+1}$  is the shock to the conditional volatility of consumption growth; for parsimony, these three shocks are assumed to be independent and identically distributed (i.i.d.) Normal. The parameter configuration for consumption and dividend dynamics used in our model simulation is identical to Bansal, Kiku, and Yaron (2012), and is given in Table B1.

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