The Q-Factor

MIT iQuHACK ‘26
Superquantum Track

Overview & Thoughts

- **Goal**: Non-Clifford gates are expensive; real algorithms are T-heavy, so we compiled each unitary into efficient Clifford+T circuits.
- Observing structures (Fourier/interaction gates/diagonal phases) of the unitary matrices resulted in more efficient circuits.
- Using the principles of regularity and building up we optimized the latter parts.
- Overall, it was a really fun and exciting experience where our main goal was to find the most optimal decomposition of the given unitary matrix from the given set of allowed matrices.

Challenge 1

- **What it is:** Controlled application of Y on the target challenge
- **Our approach:** Express Y using Pauli relations, then implement as CNOT + phase/basis corrections in Clifford+T.
- How we achieved it (algorithm):

Use the identity S X S† = Y (Clifford conjugation)

- Build controlled-X via CNOT, and surround the target with S / S† realized using T/T† (since S = T^2)

Challenge 2

- **What it is:** controlled single-qubit rotation by an angle of $\pi/7$
- **Our approach:** isolate the control structure, then approximate only the needed irrational rotation using Clifford+T synthesis.
- **Algorithm:** Rewrite as a conditional exponential on the target, e^{-i(π/14)Y} when control=1
- Use single-qubit rotation synthesis to approximate the angle with low T-count
- Validate against operator norm distance to stop as soon as we're "good enough"
- **What we did better:** Avoided extra entanglement, control logic was separate from the approximation block, didn't over-approximate once the distance target looked safe (lower Ts for same quality).

Challenge 3

- **What it is:** ZZ interaction phase
- **Algorithm:**
    - Use that $(Z \otimes Z)^2 = I$, so the exponential is fully determined by ± 1 eigenspaces
    - Compute parity with CNOT, apply an Rz/phase on the parity, then uncompute
    - Implement phases via optimized Clifford+T phase handling (no basis change)
- We used a compact form “CNOT-phase-CNOT” to optimize.

Challenge 4

- **What it is:** Exponential of Heisenberg XY interaction, $\exp(i\pi/7\cdot(XX + YY))$
- **Algorithm:** Decompose as CNOT → Controlled-Rz → CNOT → single-qubit rotations. Only need to synthesize one non-trivial angle ($2\pi/7$).
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Challenge 5

- **What it is:** exp(i·π/4·(XX + YY + ZZ)) - full Heisenberg interaction
- **Key insight:** XX + YY + ZZ = 2·SWAP - I. Therefore exp(i·π/4·(XX+YY+ZZ)) is directly related to √SWAP!
- **Algorithm:** This is essentially a Clifford operation (SWAP to the 1/2 power). Decomposes exactly into Clifford+T with minimal T gates.

Challenge 6

- **What it is:** Time evolution under transverse field Ising model, $\exp(i\pi/7\cdot(XX + ZI + IZ))$
- **Our approach:** Using exact KAK decomposition instead of Trotter, then systematically search for gridsynth "sweet spots" where approximation errors canceled.
- **Algorithm:** Decompose 2-qubit unitary via KAK into single-qubit rotations (Rz, Ry) + 2 CNOTs. Only 10 non-Clifford rotations needed. Synthesized each with gridsynth.
- **Key insight:** Gridsynth error doesn't decrease monotonically with epsilon; at certain values, errors across multiple rotations destructively interfere, giving much better accuracy than expected.

Challenge 7

- **What it is:** Arbitrary state preparation, $|00\rangle \rightarrow$ random 2-qubit state (seed=42)
- **Our approach:** Use Qiskit's state preparation decomposition → Rz + Ry + CNOT, then synthesize rotations with gridsynth.
- **Key insight:** Only need $U|00\rangle$ = target, not full unitary. State preparation needs only 1 CNOT + 6 rotations for 2 qubits.
- **Sweet spot hunting:** With 10,000 gate budget, we could push epsilon to 2×10^{-9}, achieving error $\sim 10^{-9}$ with only 1,600 gates.

Challenge 8

- **What it is:** 2-qubit Quantum Fourier Transform unitary!
- **Key insight:** QFT_2 has exact Clifford+T decomposition! It's H → Controlled-S → H → SWAP.
- **Algorithm:**
- Controlled-S decomposes to: T(ctrl) → CNOT → T†(tgt) → CNOT → T(tgt)
- Total: only 3 T gates!

Challenge 9

- **What it is:** Structured unitary with specific pattern (the "figure out the structure" challenge)
- **Key insight:** The structure is a controlled operation with the control in superposition basis. Rewrite in computational basis to reveal block structure.
- **Algorithm:** Identify as $(H \otimes I) \cdot \text{Controlled-U} \cdot (H \otimes I)$ type structure, then decomposed the inner controlled operation.
- We did careful matrix analysis to identify the hidden tensor/controlled structure rather than brute-force decomposition.

Challenge 10

- **What it is:** Random 2 qubit unitary.
- **Algorithms explored:**
  - **Naive transpile → basis gates**
    Leaves generic 1q gates, not true Clifford+T
  - **Solovay–Kitaev pass**
    Works but produces very deep, T heavy circuits
  - **Manual Weyl/KAK + gridsynth**
    Accurate but overly complex
  - **Final: Qiskit gridsynth plugin**
    Built in optimal Clifford+T synthesis with error control
- **Result:** 2043 T-gates, 1e-11 norm distance

Challenge 11

- **What it is:** 4-qubit diagonal unitary specified by a phase table.
- **Our approach:** Recognize the phase table as an AND-based polynomial, then convert to XOR form for rmsynth.
- **Algorithm:**
  1. Reverse-engineer phase table → AND polynomial:

$$f = 5x_1 + 5x_2 + 5x_3 + 4x_4 + 4x_1x_2 + 4x_1x_3 + 4x_2x_3 + 4x_1x_2x_3 + 6x_1x_4 + 6x_2x_4 + 6x_3x_4$$

  2. Möbius transform: AND → XOR parities via inclusion-exclusion
  3. rmsynth: synthesize + RM-decode optimize
- **Total:** only 4 T gates!

Thank you