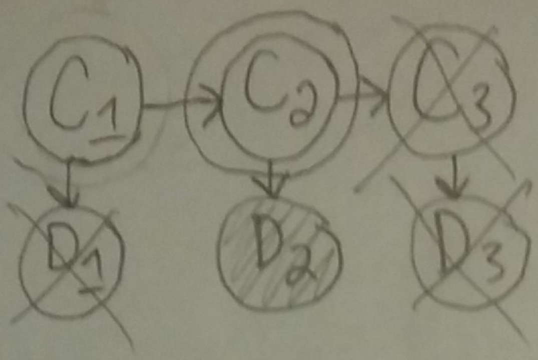
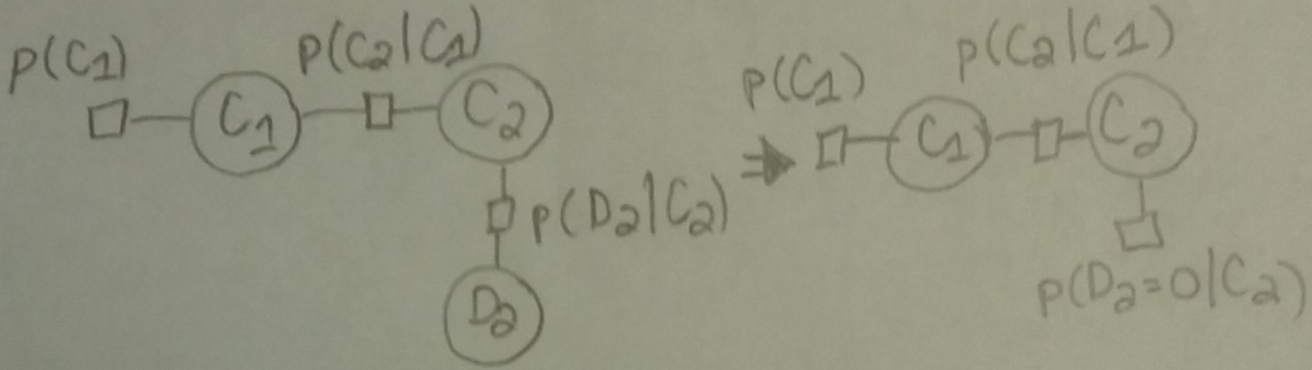
# 1a




$$P(C_2|D_2=0) \propto p(D_2=0|C_2)\sum_{C_1} p(C_1)p(C_2|C_1)$$

$$P(C_2=1|D_2=0) \propto p(D_2=0|C_2=1)\sum_{C_1} p(C_1)p(C_2=1|C_1)$$

$$= \eta * [(0.5)(\epsilon + 1 - \epsilon)] = \eta * (0.5)$$

$$P(C_2=0|D_2=0) \propto p(D_2=0|C_2=0)\sum_{C_1} p(C_1)p(C_2=0|C_1)$$

$$= (1-\eta) * [(0.5)(1-\epsilon+\epsilon)] = (1-\eta) * (0.5)$$

$$\boxed{P(C_2=1|D_2=0) = \frac{(0.5)\eta}{(0.5)(\eta+1-\eta)} = \eta}$$

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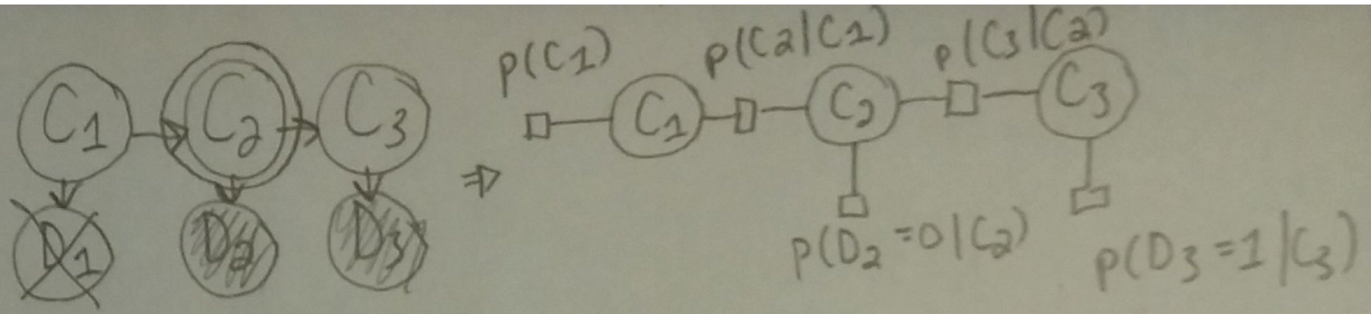
(1b)




$$\mathbb{P}(C_2=c_2 \mid D_2=0, D_3=1) \propto \sum_{C_1, C_3} p(C_1)p(C_2|C_1)p(C_3|C_2)p(D_2=0|C_2)\,p(D_3=1|C_3)$$

$$\mathbb{P}(C_2=c_2 \mid D_2=0, D_3=1) \propto p(D_2=0|C_2)\left(\sum_{C_1} p(C_1)p(C_2|C_1)\right)\left(\sum_{C_3} p(C_3|C_2)p(D_3=1|C_3)\right)$$

$$\mathbb{P}(C_2=1 \mid D_2=0, D_3=1) \propto \eta\left(0.5(\epsilon+1-\epsilon)\right) * \left[(\epsilon * \eta) + (1-\epsilon)*(1-\eta)\right]$$

$$= (0.5)\eta * \left[(\epsilon\eta) + (1-\epsilon)(1-\eta)\right]$$

$$\mathbb{P}(C_2=0 \mid D_2=0, D_3=1) \propto (0.5)(1-\eta) * \left[(1-\epsilon)*\eta + \epsilon * (1-\eta)\right]$$

$$\boxed{\mathbb{P}(C_2=1 \mid D_2=0, D_3=1) = \frac{(0.5)\eta * [\epsilon\eta + (1-\epsilon)(1-\eta)]}{(0.5)\eta * [\epsilon\eta + (1-\epsilon)(1-\eta)] + (0.5)(1-\eta) * [(1-\epsilon)*\eta + \epsilon(1-\eta)]}}$$

1C) $\epsilon = 0.1$ and $\eta = 0.2$

$\mathbb{P}(C_2 = 1 | D_2 = 0) = 0.2000$

$\mathbb{P}(C_2 = 1 | D_2 = 0, D_3 = 1) \propto (0.5)(0.2) * [(0.1)(0.2) + (0.9)(0.8)] =$

$.1 * (.02 + .72) = .0740$

$\mathbb{P}(C_2 = 0 | D_2 = 0, D_3 = 1) \propto (0.5)(0.8) * [(0.9)(0.2) + (0.1)(0.8)] =$

$.4 * (.18 + .08) = .1040$

$\mathbb{P}(C_2 = 1 | D_2 = 0, D_3 = 1) = \frac{.0740}{.0740 + .1040} = \frac{0.0740}{0.1780} = 0.4157$

i) $\mathbb{P}(C_2 = 1 | D_2 = 0) = 0.2000$ & $\mathbb{P}(C_2 = 1 | D_2 = 0, D_3 = 1) = 0.4157$

ii) $D_3 = 1$ increased the probability of the car advancing. In other words, the position of a car observed at time step $t$ is close to the position of a car at $t$ and thus increases the probability.

iii) Set $\epsilon = 0$, it factored out of $\mathbb{P}(C_1 = 1 | D_2 = 0)$.
$\epsilon = 0$ Also implies that the car moves to the next position with absolute certainty and thus the observation at $D_3$ is not noisy.

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(5a) $C_{11}$ and $C_{12}$ are independent.

$$\mathbb{P}(C_{11}, C_{12} | E_1 = e_1) = \mathbb{P}(C_{11} | E_1 = e_1) * \mathbb{P}(C_{12} | E_1 = e_1)$$

$$\mathbb{P}(C_{11}, C_{12} | E_1 = e_1) \propto P(C_{11}) P_N(e_{11}; \|a_1 - C_{11}\|, \sigma^2) * P(C_{12}) P_N(e_{12}; \|a_1 - C_{12}\|, \sigma^2)$$

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(5b) $$\mathbb{P}(C_{11} = c_{11}, \ldots, C_{1K} = c_{1K} | E_1 = e_1) = \prod_i^K \mathbb{P}(C_{1i} = c_{1i} | E_1 = e_1)$$

$$\propto P(C_{11})^K * \prod_i^K P_N(e_{1i}; \|a_1 - C_{1i}\| \sigma^2)$$

Since all $C_{1i}$ are unique and each value of $C_{1i}$ can take the value that maximizes the above probability and then $C_{1j}$ $i \neq j$ must take a value that is less than $\max(C_{1i})$, there are $K!$ assignments to $C_{1i}$ to reach the maximum.