Recursion Revisited

Chapter 8

Fundamentals

- A method is <u>recursive</u> if it calls itself.
- A recursive method should contain a <u>stopping case</u> or <u>base case</u> that is not recursive.

  (Without it, the method would be infinitely recursive, never ending.)
- The recursive call(s) should be for simpler versions of the same problem.

Activation Records

- When a method calls another method (even itself), an activation record is stored on the system stack (of the O.S.).
- An activation record contains:

  - where to return when the called method ends

  - parameter(s) passed to the called method

  - values of the method’s local variables
- When a method returns, it uses the top activation record on the system stack to restore the conditions before the method call.

Factorial

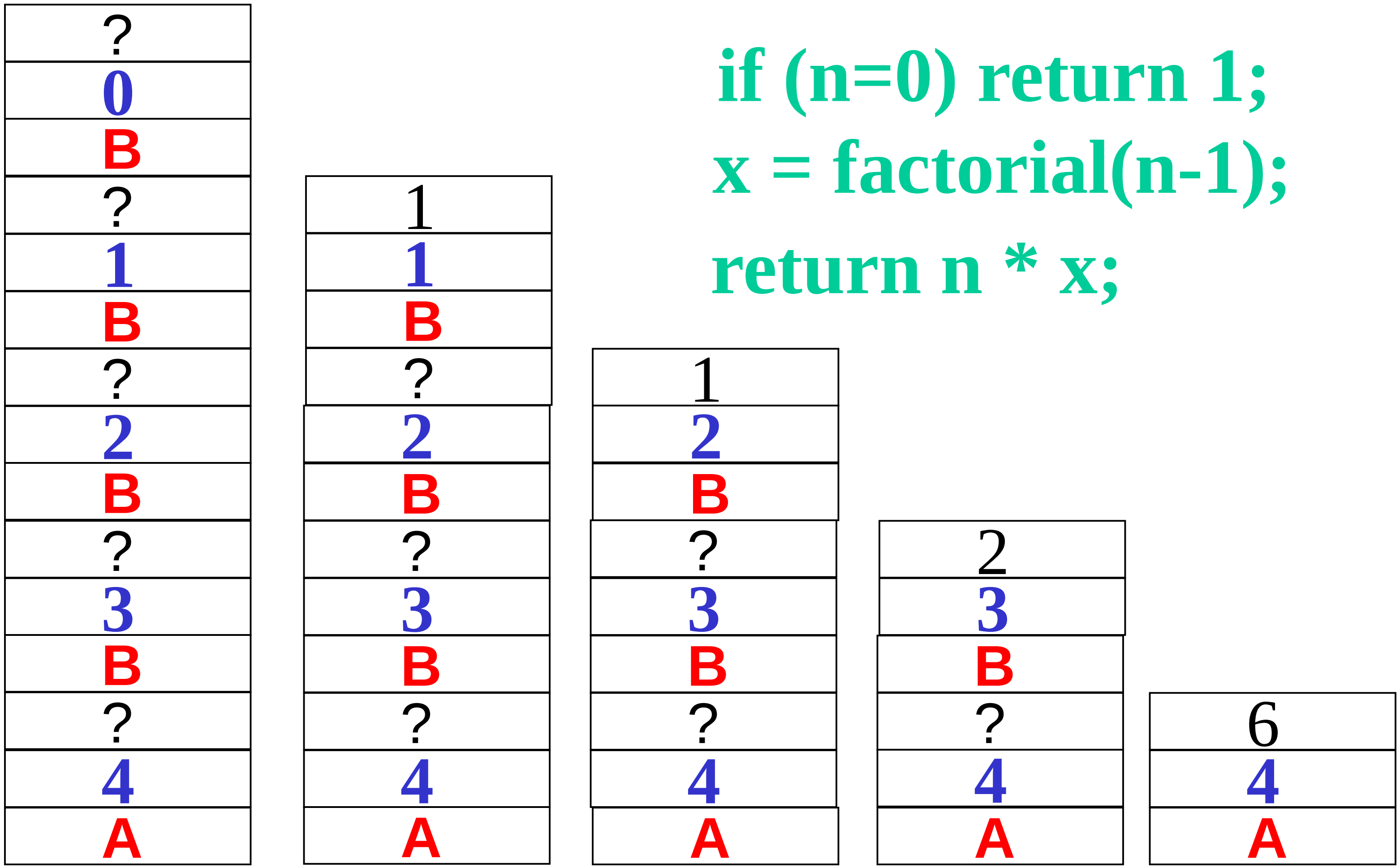
```
public int factorial(int n) {
  if (n == 0)
     return 1;
  int x = factorial(n-1);       ← return
                                  location B
  return n * x;
}

result = factorial(4); ← return location A
System.out.println(result);
```

Factorial

Activation Record

x
n
Return location

Trace(Activation Record)




```
if (n=0) return 1;
x = factorial(n-1);
return n * x;
```

Trace (factorial)

```
public int factorial(int n) {
    if (n == 0) return 1;
    int x = factorial(n-1);
    return n * x; }
```

factorial **4** — return **4** * **6** = 24

factorial **3** — return **3** * **2** = 6

factorial **2** — return **2** * **1** = 2

factorial **1** — return **1** * **1** = 1

factorial **0** — return 1

Activation Records(summary)

- Hold return location.
- Temporary storage for local variables including parameters, if any.
- Basis for re-entrant code.

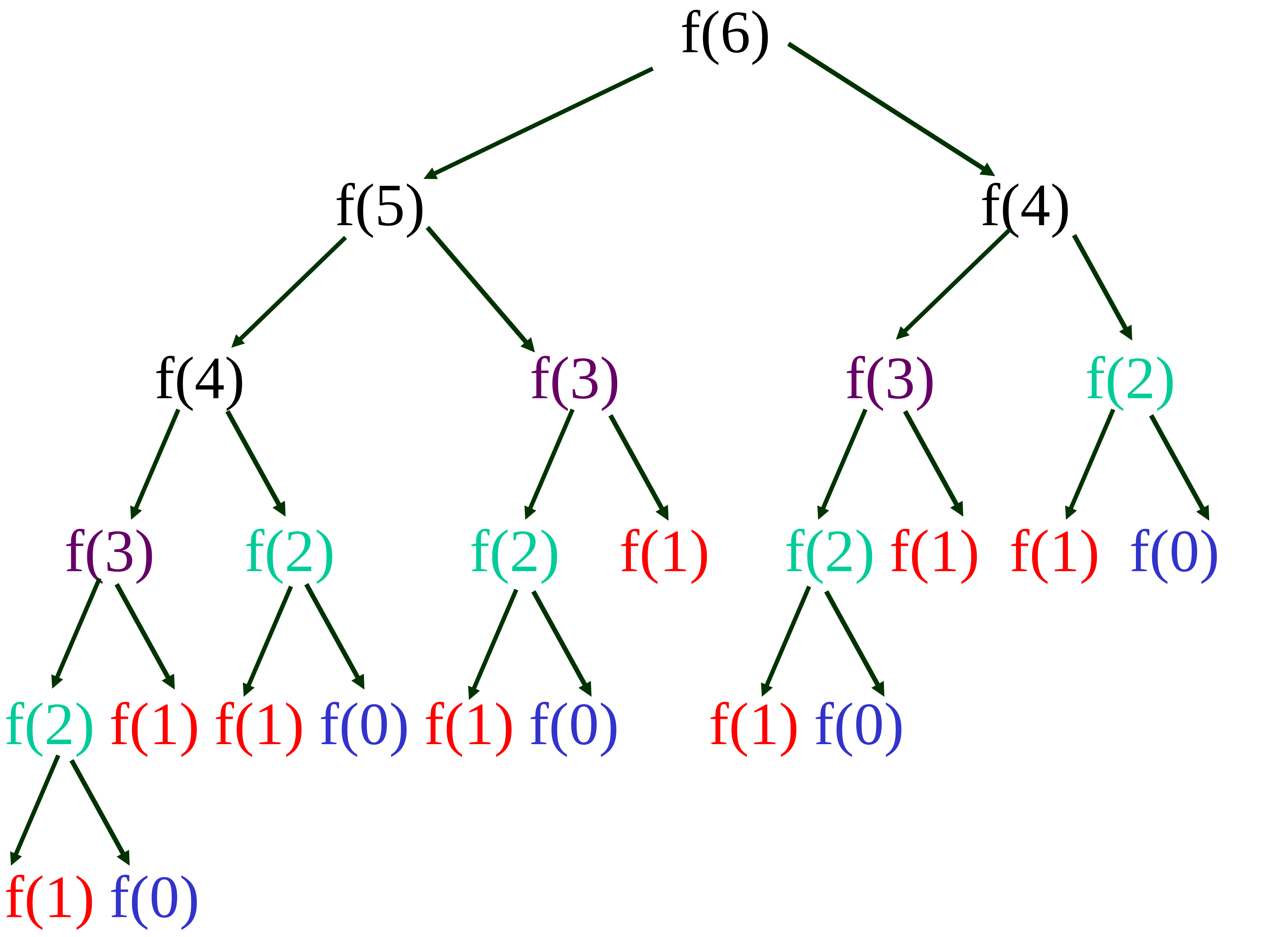
Fibonacci Numbers

```
public int fib(int n) {
  if (n == 0 || n == 1)
     return n;
  int x = fib(n-1);     ← return location B
  int y = fib(n-2);     ← return location C
  return (x + y);
}

result = fib(4);        ← return location A
System.out.println(result);
```

Fibonacci Numbers

Activation Record

y
x
n
Return location



f(6)
f(5)
f(4)
f(4)
f(3)
f(3)
f(2)
f(3)
f(2)
f(2)
f(1)
f(2)
f(1)
f(1)
f(0)
f(2)
f(1)
f(1)
f(0)
f(1)
f(0)
f(1)
f(0)
f(1)
f(0)

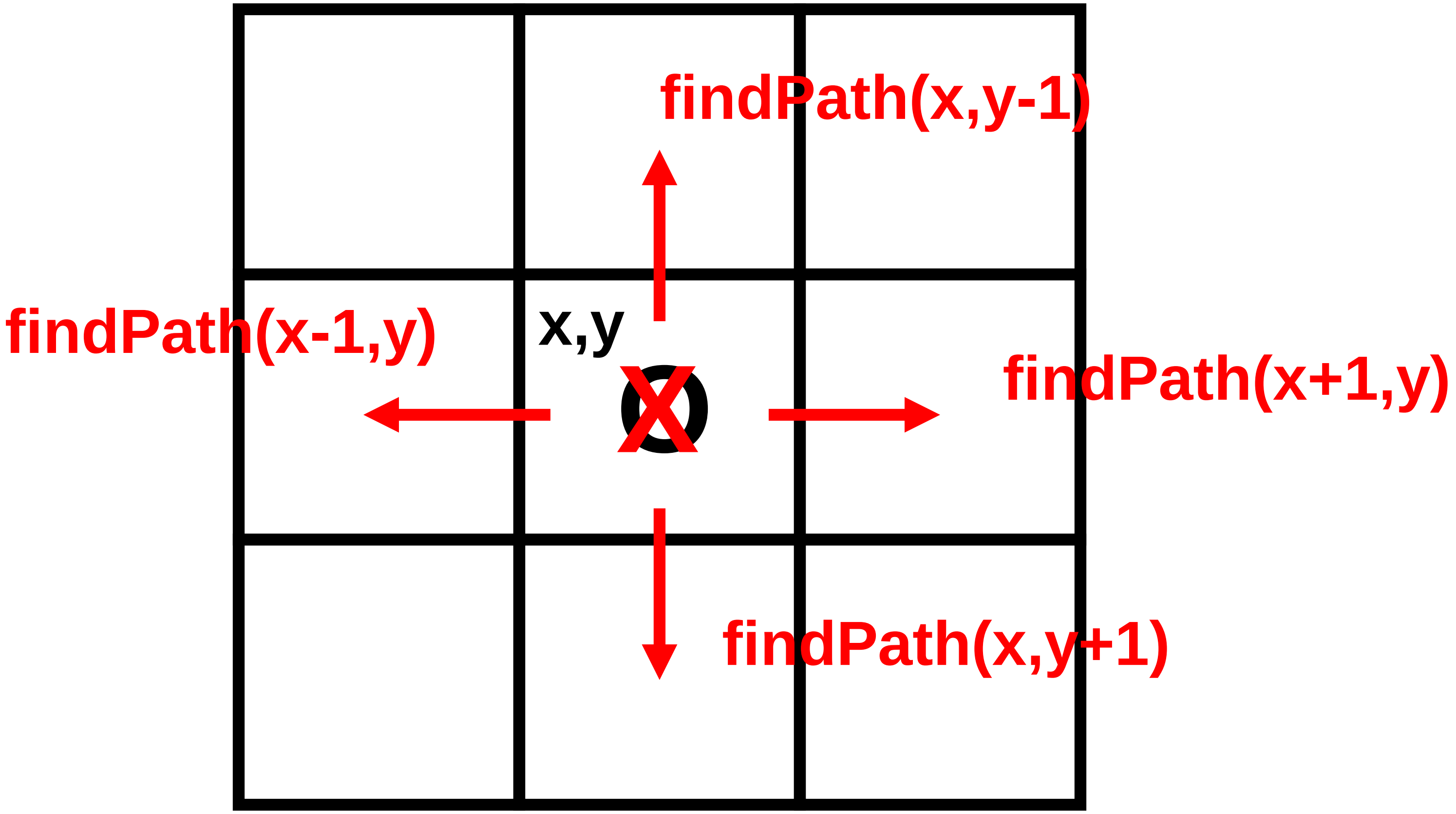
Backtracking

- An exhaustive search is a technique of generating a solution from all combinations of partial solutions.
- If any step leads to an invalid solution or infeasible solution, we backtrack to the most recent partial solution and try a different path to a full solution until we find the best solution.

Example: Searching a maze

x	x	x	x	x	x	x	x
o	x	o	x	x	o	o	x
o	x	o	o	o	o	x	o
o	o	o	x	o	x	o	o
o	x	o	x	o	x	x	o
o	x	x	o	o	o	o	o
T	x	x	x	x	x	o	x
x	o	o	o	o	o	o	o

findPath(x,y): The trick



Initial Algorithm

- **Start from position x,y such that Maze[x][y] = o**
- **if findPath(x,y)**
  output TARGET FOUND
  else
  output TARGET NOT FOUND

```
findPath(x,y)
    if Maze[x][y] = T
        output x,y
        return true
    else if Maze[x][y] = X
        return false
    else (must be an O)
        Maze[x][y] = X
        if findPath(x-1,y) OR findPath(x,y-1)
          OR findPath(x+1,y) OR findPath(x,y+1)
            output x,y
            return true
        else return false
```

This is a simplification in the algorithm. What's missing?

Dynamic Programming

- Reduce the number of recursive calls by saving the return values of recursive calls as they are determined.
- Use the saved value in place of an identical recursive call later in the execution.
- Example: let d[ ] be a global array that holds the previously determined Fibonacci values. Give these initial values of –1.

Example : Matrix Chain Multiplication

Let A_1 be a 100 x 10 matrix

Let A_2 be a 10 x 20 matrix

Let A_3 be a 20 x 30 matrix

How many operations are required to multiply

$A_1 \times A_2 \times A_3$?

Fibonacci Numbers Revisited

Using Dynamic Programming

```
public int fib(int n) {
  if (n == 0 || n == 1) {
     return n;
  }
  if (d[n-1] == -1)
     d[n-1] = fib(n-1);
  if (d[n-2] == -1)
     d[n-2] = fib(n-2);
  return (d[n-1]+d[n-2]);
}
```

Tail Recursion

- If a method is defined such that it has one recursive call as the last computational statement, then the method is called <u>tail recursive</u>.
- Every tail recursive method can be re-written as an equivalent method without recursion using a loop.
- Example: factorial is tail recursive.

Factorial Revisited

```
public int factorial(int n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
  }
```

← tail recursion

```
public int factorial(int n) {
    int product = 1;
    int i;
    for (i = n; i >= 1; i--)
        product = i * product;
    return product;
  }
```

Why should we try to eliminate tail recursion?

Reverse Print

```
public static void reversePrint(int n) {
    if (n > 0) {
        System.out.println(n);
        reversePrint(n-1);      ← tail
    }                             recursion
}
```

```
public static void reversePrint(int n) {
  L: if (n > 0) {
        System.out.println(n);
        Set up new parameters
        Jump to L
     }
}
```

```
while (n > 0) {
    System.out.println(n);
    n--;
    // no code necessary
}
```

Towers of Hanoi

```
public static void move(int n, int src, int dest, int aux) {
    if (n > 0) {
        move(n-1, src, aux, dest);
        System.out.println(src+"   "+dest);
        move(n-1, aux, dest, src);  }
}
```

← tail recursion

```
public static void move(int n, int src, int dest, int aux) {
    while (n > 0) {
        move(n-1, src, aux, dest);
        System.out.println(src+"   "+dest);
        n = n-1; exchange(aux, src);  }
}
```