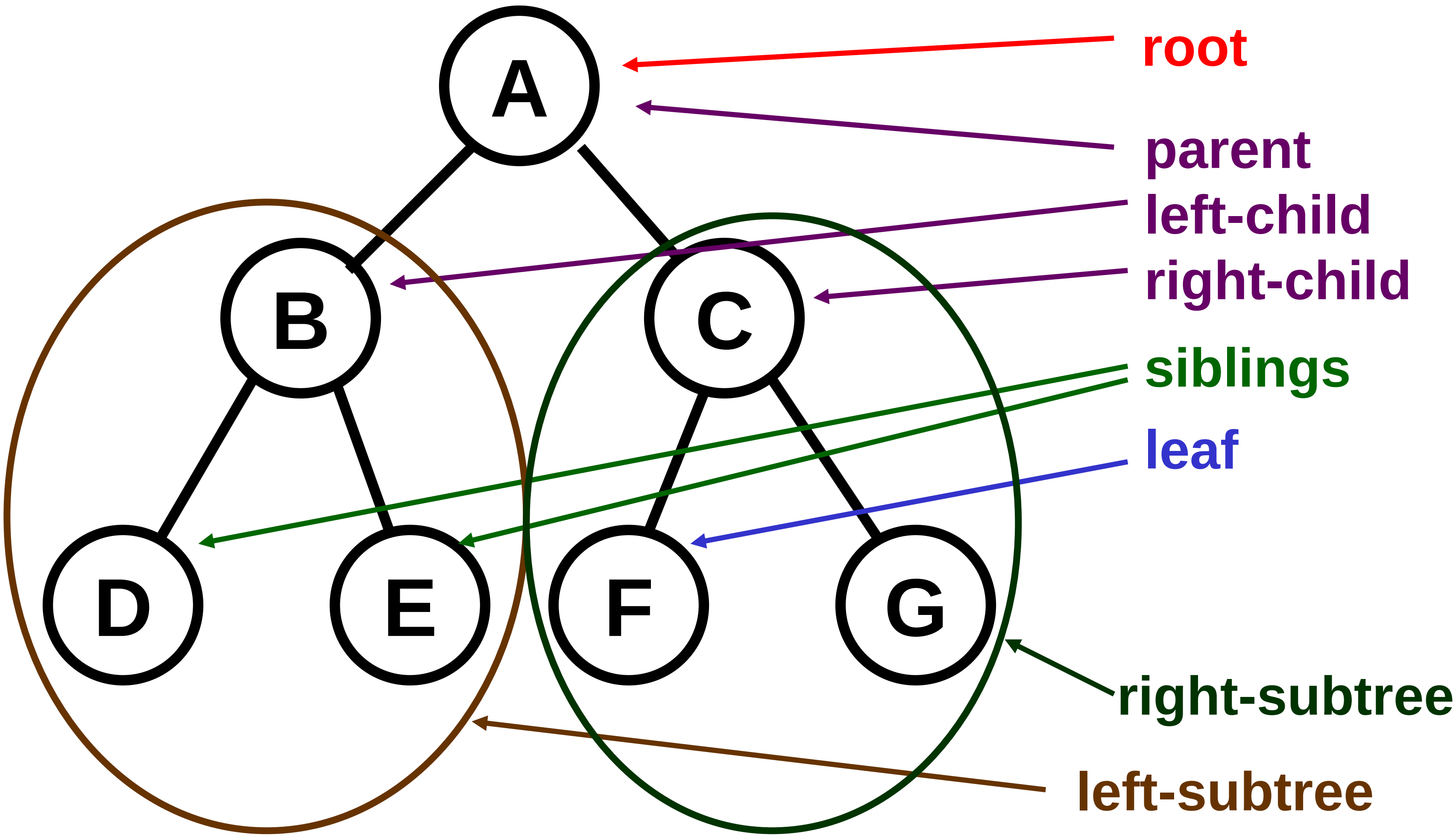
Binary Trees

Chapter 12

Fundamentals

- A binary tree is a <u>nonlinear</u> data structure.
- A <u>binary tree</u> is either empty or it contains a root node and left- and right-subtrees that are also binary trees.
- Applications: encryption, databases, expert systems

Tree Terminology



More Terminology

- Consider two nodes in a tree, X and Y.
- X is an <u>ancestor</u> of Y if

    X is the parent of Y, or

    X is the ancestor of the parent of Y.

It's RECURSIVE!

- Y is a <u>descendant</u> of X if

    Y is a child of X, or

    Y is the descendant of a child of X.

More Terminology

- Consider a node Y.
- The <u>depth of a node</u> Y is

  0, if the Y is the root, or

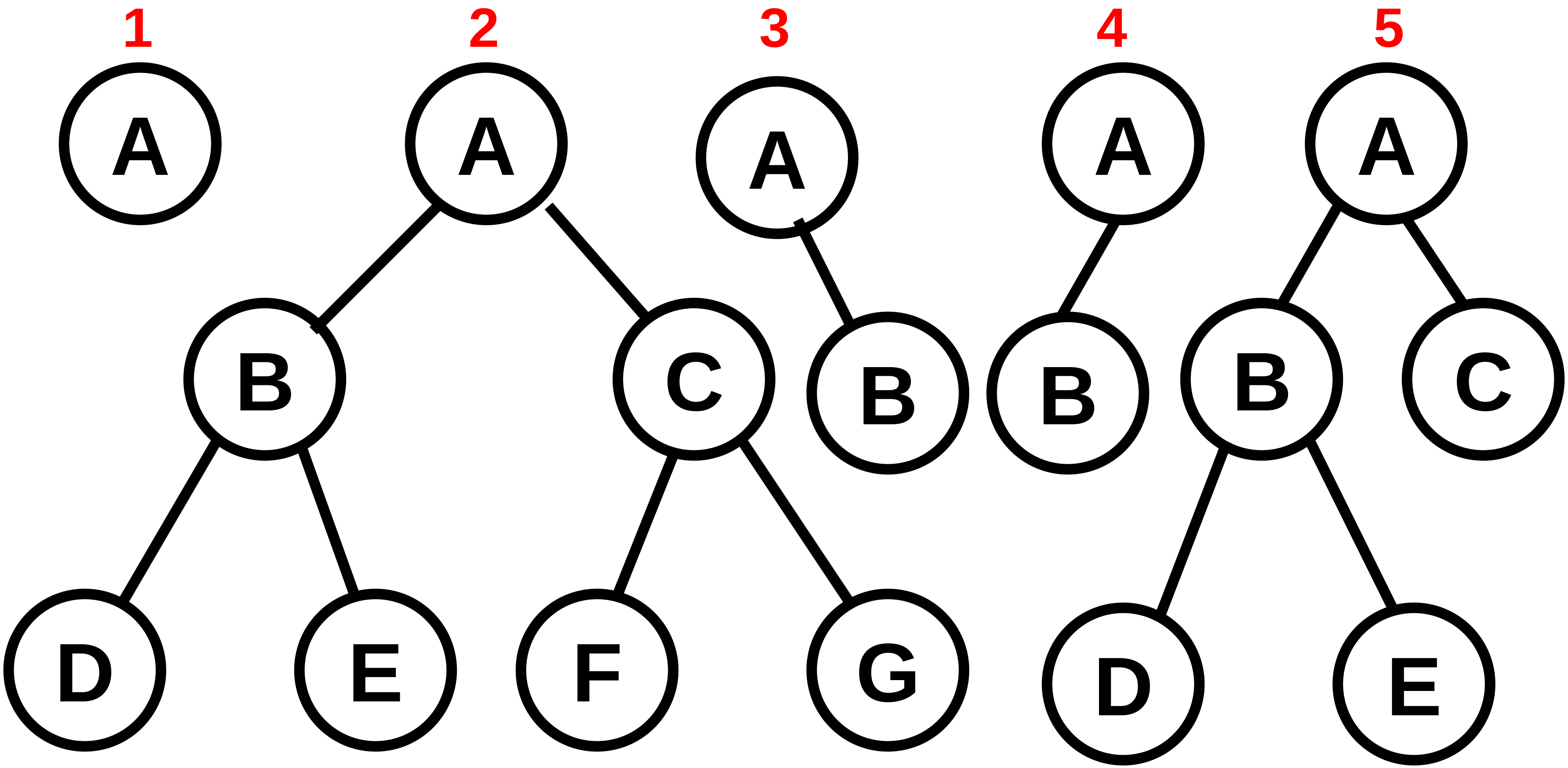
  1 + the depth of the parent of Y

- The <u>depth of a tree</u> is the maximum depth of all its leaves.

More Terminology

- A full binary tree is a binary tree such that
  - all leaves have the same depth, and
  - every non-leaf node has 2 children.
- A complete binary tree is a binary tree such that
  - every level of the tree has the maximum number of nodes possible except possibly the deepest level.
  - at the deepest level, the nodes are as far left as possible.

Tree Examples



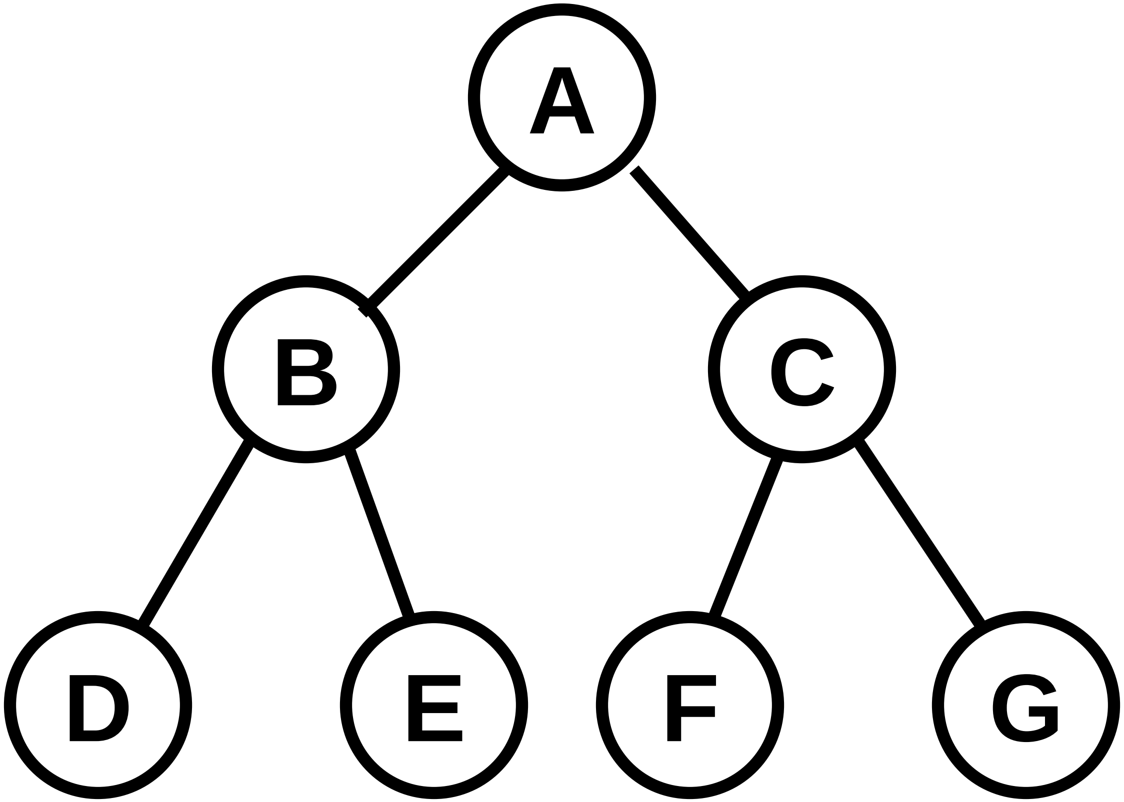
A full binary tree is always a complete binary tree.

What is the number of nodes in a full binary tree?

Tree Traversals

- preorder traversal
  1. Visit the root.
  2. Perform a preorder traversal of the left subtree.
  3. Perform a preorder traversal of the right subtree.
- inorder traversal
  1. Perform an inorder traversal of the left subtree.
  2. Visit the root.
  3. Perform an inorder traversal of the right subtree.
- postorder traversal
  1. Perform a postorder traversal of the left subtree.
  2. Perform a postorder traversal of the right subtree.
  3. Visit the root.

Traversal Example



preorder

A B D E C F G

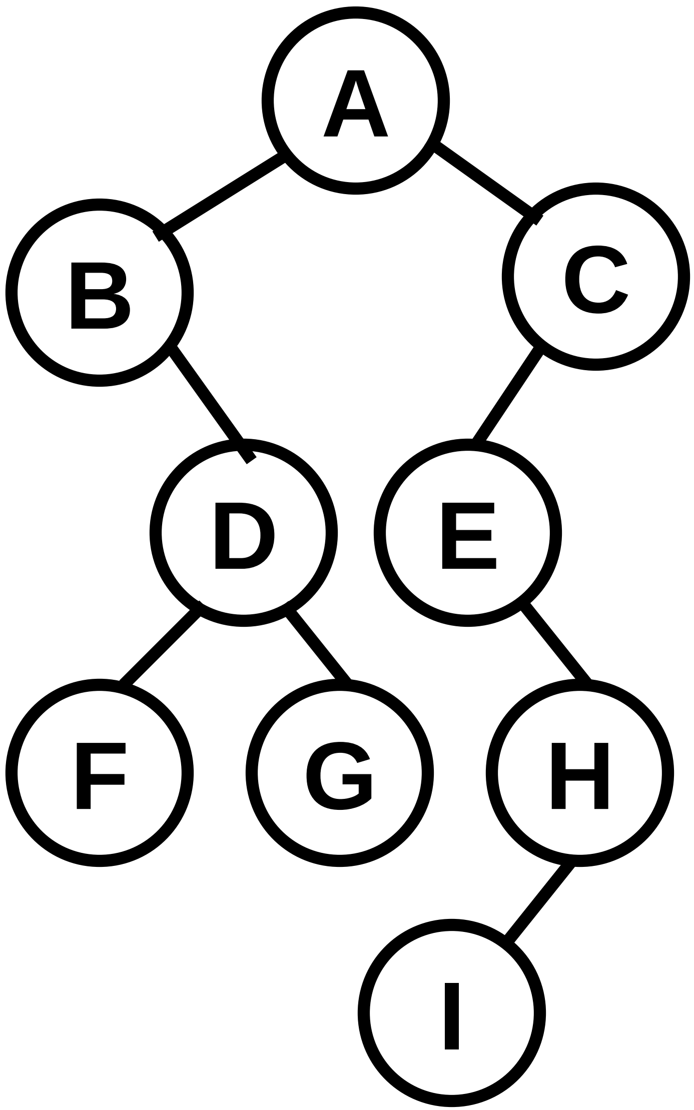
inorder

D B E A F C G

postorder

D E B F G C A

Traversal Example



preorder

ABDFGCEHI

inorder

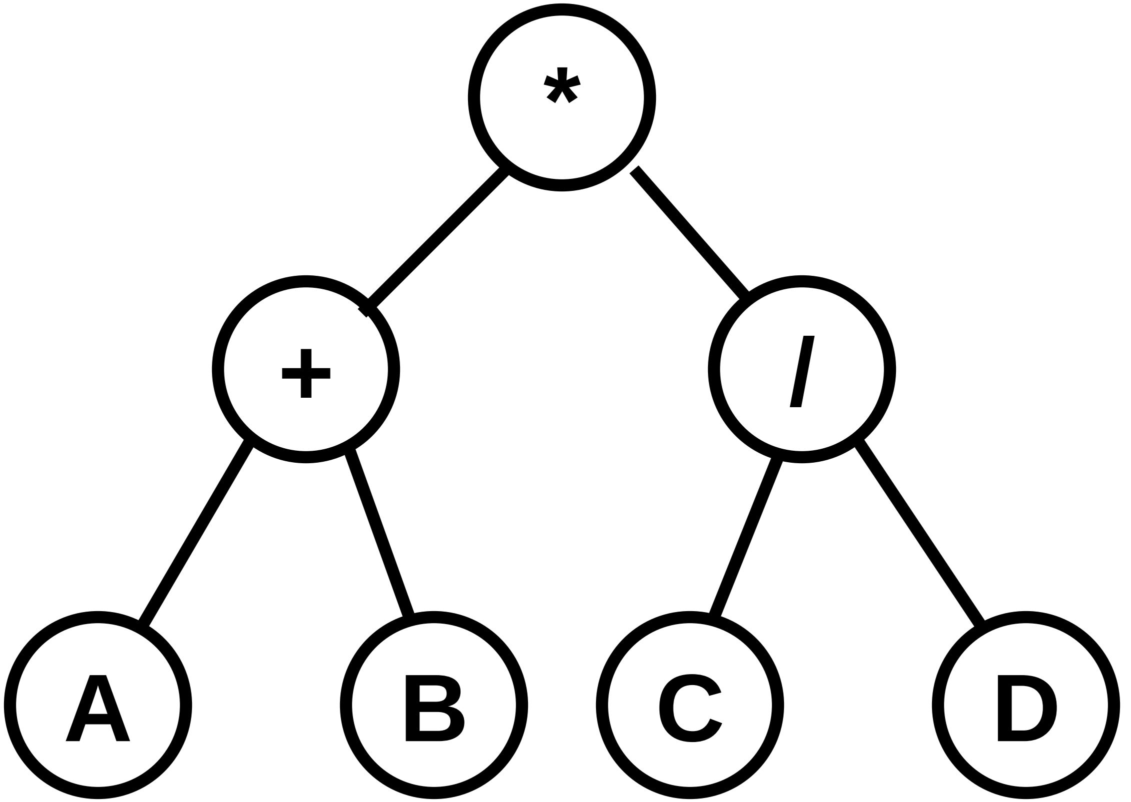
BFDGAEIHC

postorder

FGDBIHECA

Traversal Example

(expression tree)



preorder

*+AB/CD

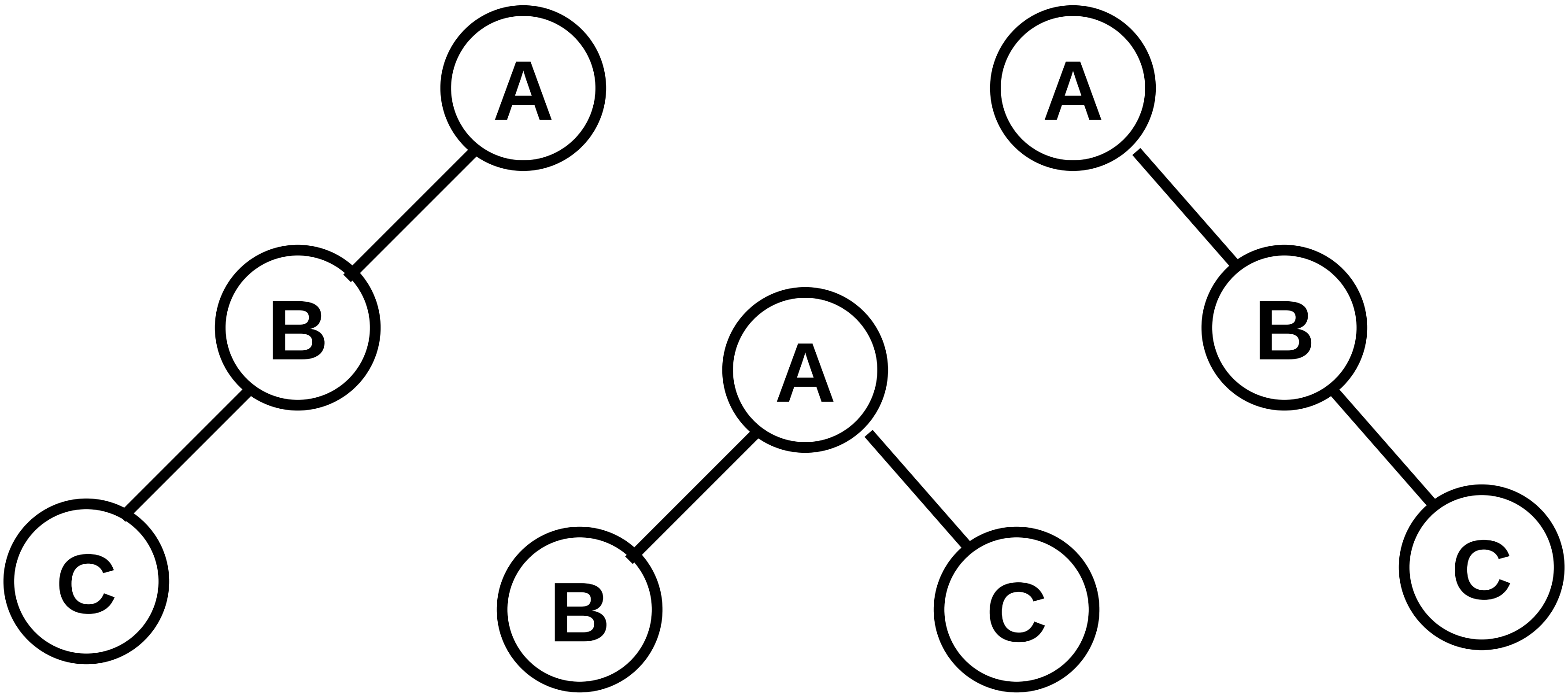
inorder

A+B*C/D

postorder

AB+CD/*

More Examples

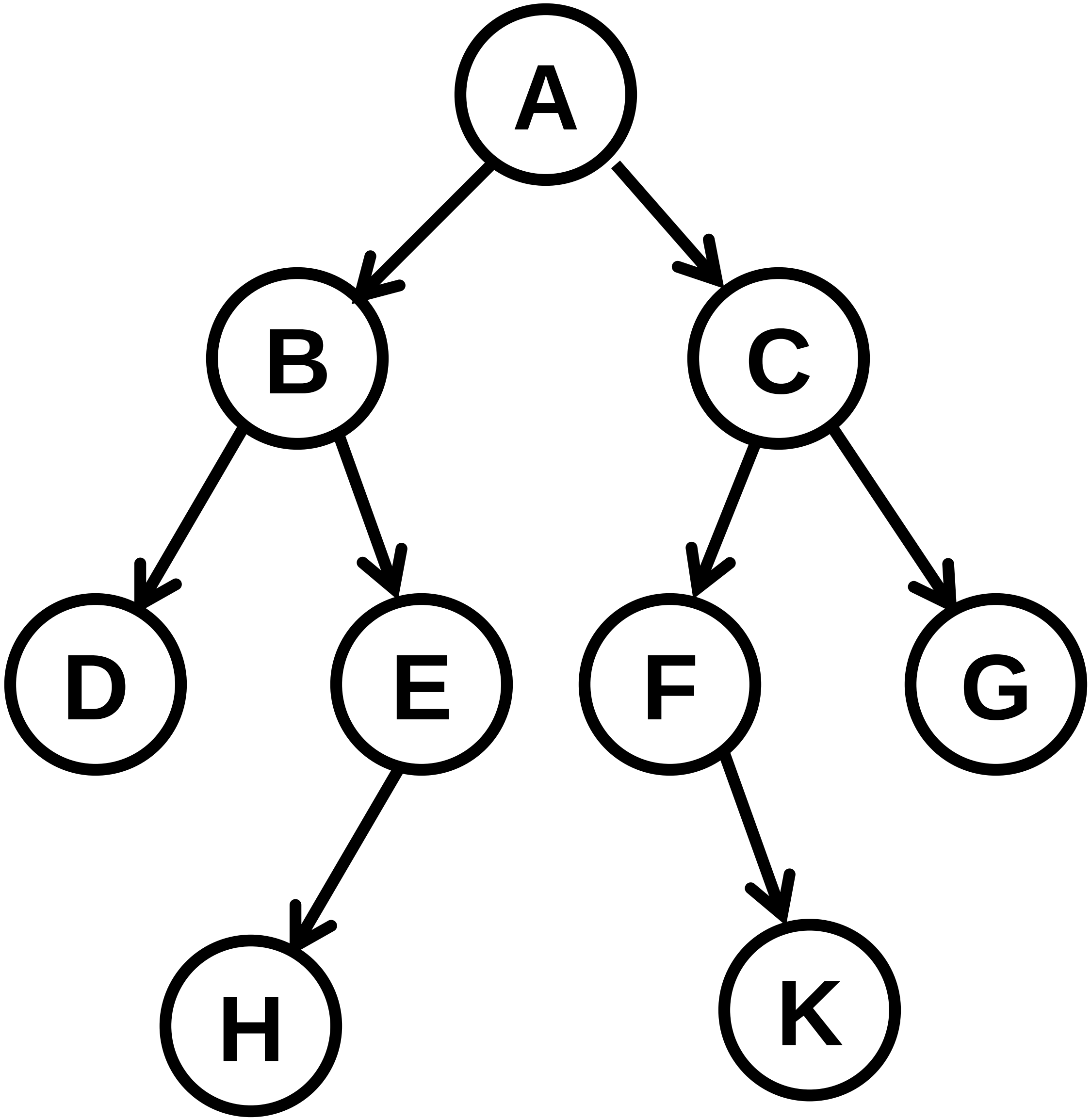


Preorder: ABC

Implementing a binary tree

- Use an array to store the nodes.
  - mainly useful for complete binary trees (next week)
- Use a variant of a singly-linked list where each data element is stored in a node with links to the left and right children of that node
- Instead of a head reference, we will use a root reference to the root node of the tree.

Conceptual Picture



Binary Tree Node (BTNode)

```
public class BTNode {
  private int data;
  private BTNode left;
  private BTNode right;

  // BTNode methods
}
```

BTNode methods

```
public BTNode(int initData) {
  data = initData;
  left = null;
  right = null;
}
public int getData()
public void setData(int newData)
public BTNode getLeft()
public void setLeft(BTNode newLeft)
public BTNode getRight()
public void setRight(BTNode newRight)
```

BTNode methods (cont'd)

```
public void inorder()
{
    if (left != null)
        left.inorder();
    System.out.println(data);
    if (right != null)
        right.inorder();
}
```

BTNode methods (cont'd)

```
public void preorder()
{
    System.out.println(data);
    if (left != null)
        left.preorder();
    if (right != null)
        right.preorder();
}
```

BTNode methods (cont'd)

```
public void traverse()
{
    //preorder
    if (left != null)
        left.traverse();
    //inorder
    if (right != null)
        right.traverse();
    //postorder
}
```

Which traversal is more appropriate?

- Evaluating an expression tree.
- Add all numbers in a tree.
- Find the maximum element of a tree.
- Print a tree rotated 90 degrees(in a counter clockwise fashion).
- Find the depth of a tree.
- Set the data field of each node to its depth.

A Binary Tree Class

- We will implement a specific type of binary tree: a binary search tree.
- A binary search tree (BST) is a binary tree such that

  - all the nodes in the left subtree are less than the root

  - all the nodes in the right subtree are greater than the root

  - the subtrees are BSTs as well

BinarySearchTree class

```
public class BinarySearchTree {
  private BTNode root;
  public BinarySearchTree() {
    root = null;
  }
  public boolean isEmpty() {
    return (root == null);
  }
```

BinarySearchTree class (cont'd)

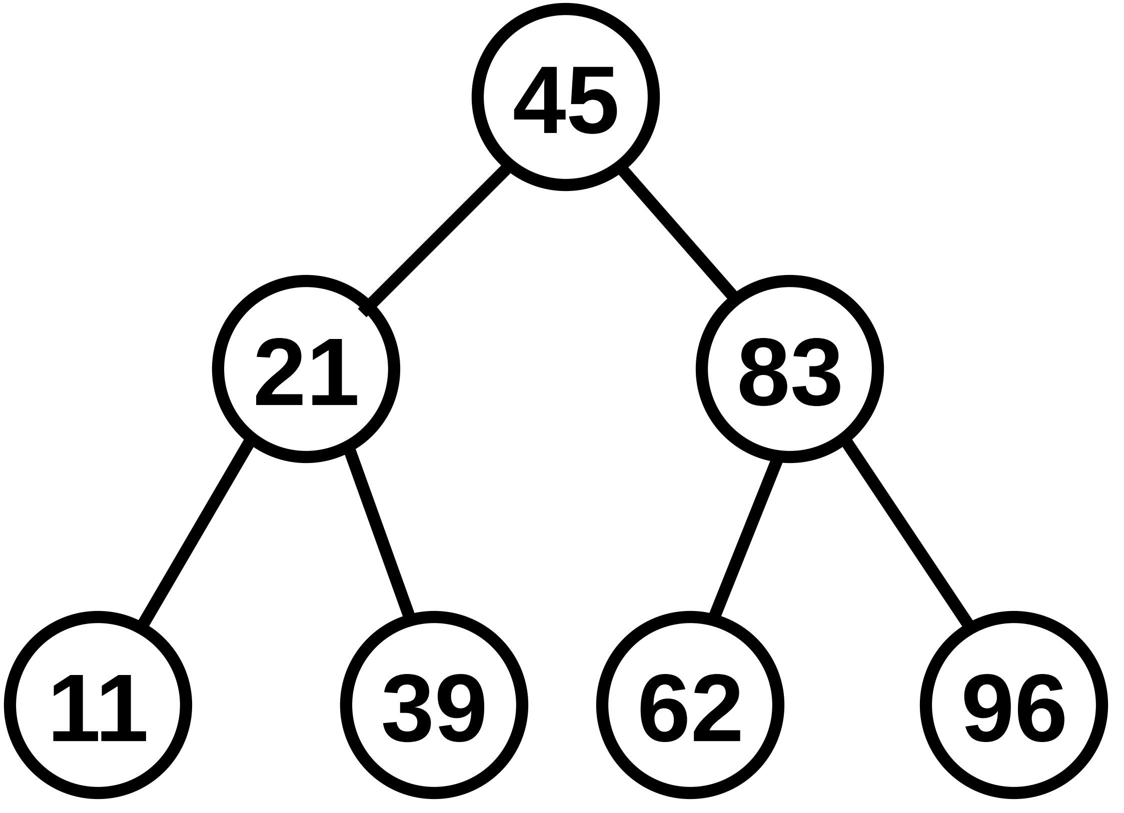
```
public void inorder() {
    if (root != null)
        root.inorder();
}


// other BinarySearchTree methods
}
```

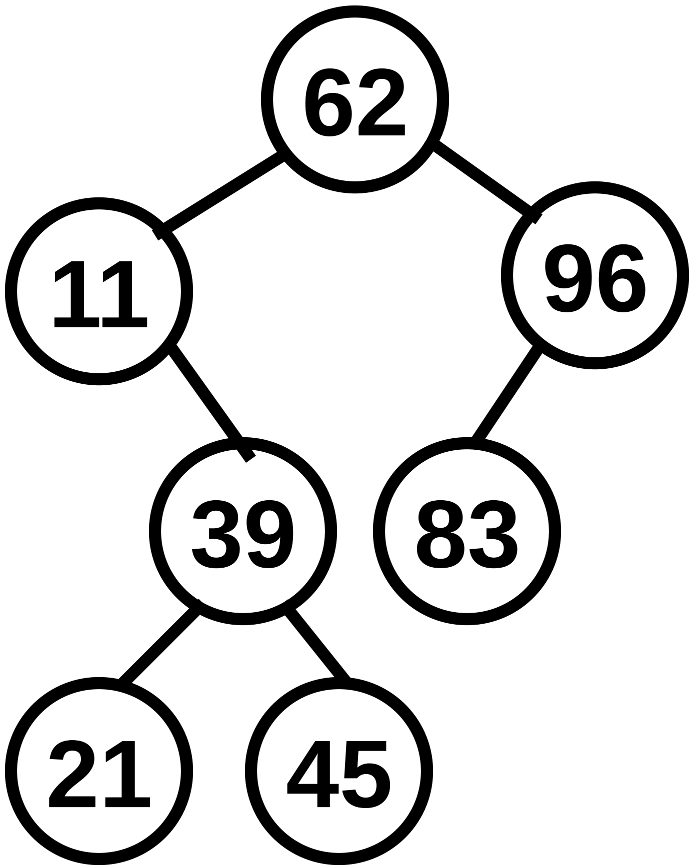
This is a call to inorder() from the BTNode class! This is not a recursive call!

Inserting into a BST

45 21 39 83 62 96 11



Inserting into a BST



62 96 11 39 21 83 45

Insert into BST

```
public void insert(int item) {
  BTNode newNode;
  BTNode cursor;
  boolean done = false;
  if (root == null) {
     newNode = new BTNode(item);
     root = newNode;
  }
```

Insert into BST (cont'd)

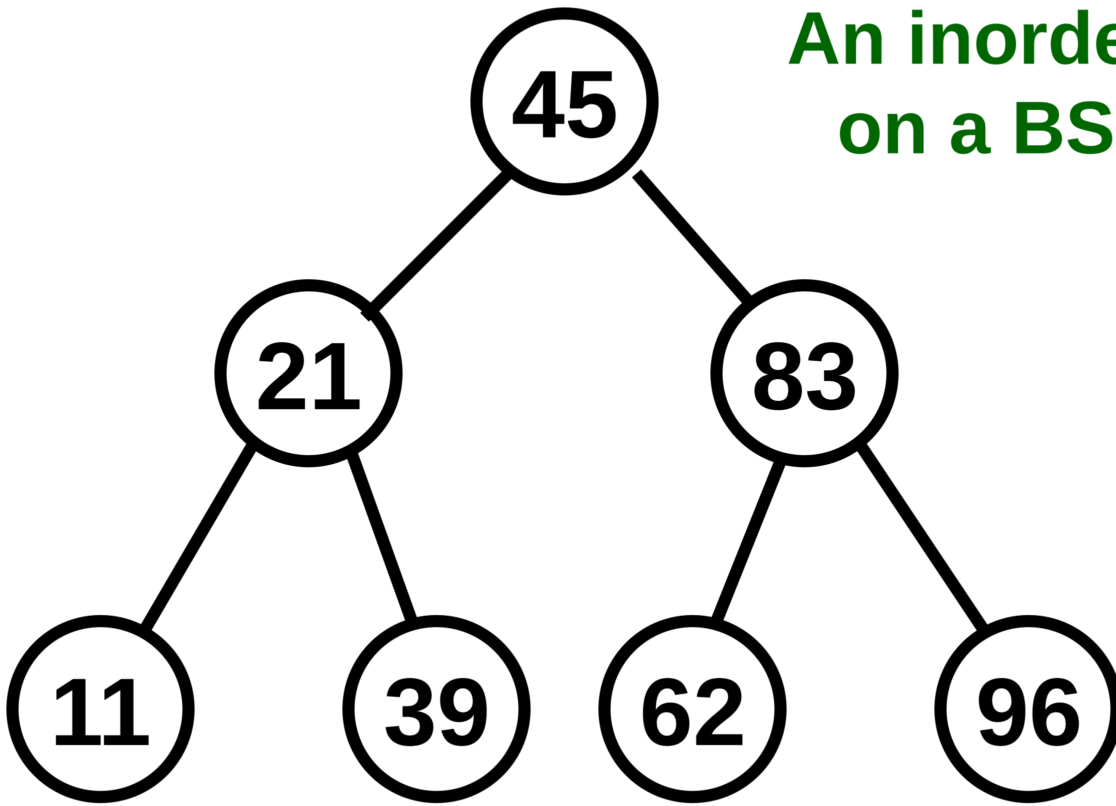
```
else {
   cursor = root;
   while (!done) {
      if (item < cursor.getData()) {
         if (cursor.getLeft() == null) {
            newNode = new BTNode(item);
            cursor.setLeft(newNode);
            done = true;
         }
         else cursor = cursor.getLeft();
      }
```

Insert into BST (cont'd)

```
    else if (item > cursor.getData()) {
        if (cursor.getRight() == null) {
            newNode = new BTNode(item);
            cursor.setRight(newNode);
            done = true;
        }
        else cursor = cursor.getRight();
    }
    else  done = true;          // Why?
  } // end while
 }
}
```

Traversing the BST

An inorder traversal on a BST sorts the numbers!



preorder	45 21 11 39 83 62 96
postorder	11 39 21 62 96 83 45
inorder	11 21 39 45 62 83 96

Efficiency of “insert”

- For a full binary tree with N nodes, what is its depth d?

  $$N = 2^0 + 2^1 + 2^2 + ... + 2^d$$

  $$= 2^{d+1} - 1$$

  Thus, $d = \log_2(N+1) - 1$.

- An insert will take O(log N) time on a full BST since we have to examine one node at each level before we find the insert point, and there are d levels.

Efficiency of “insert” (cont’d)

- Are all BSTs full?

  NO!

  Insert the following numbers in order into a BST: 11 21 39 45 62 83 96 What do you get?

- An insert will take O(N) time on an arbitrary BST since there may be up to N levels in such a tree.

Removing from a BST

General idea:

- Start at the root and search for the item to remove by progressing one level at a time until either:

  - the item is found

  - we reach a leaf and the item is not found

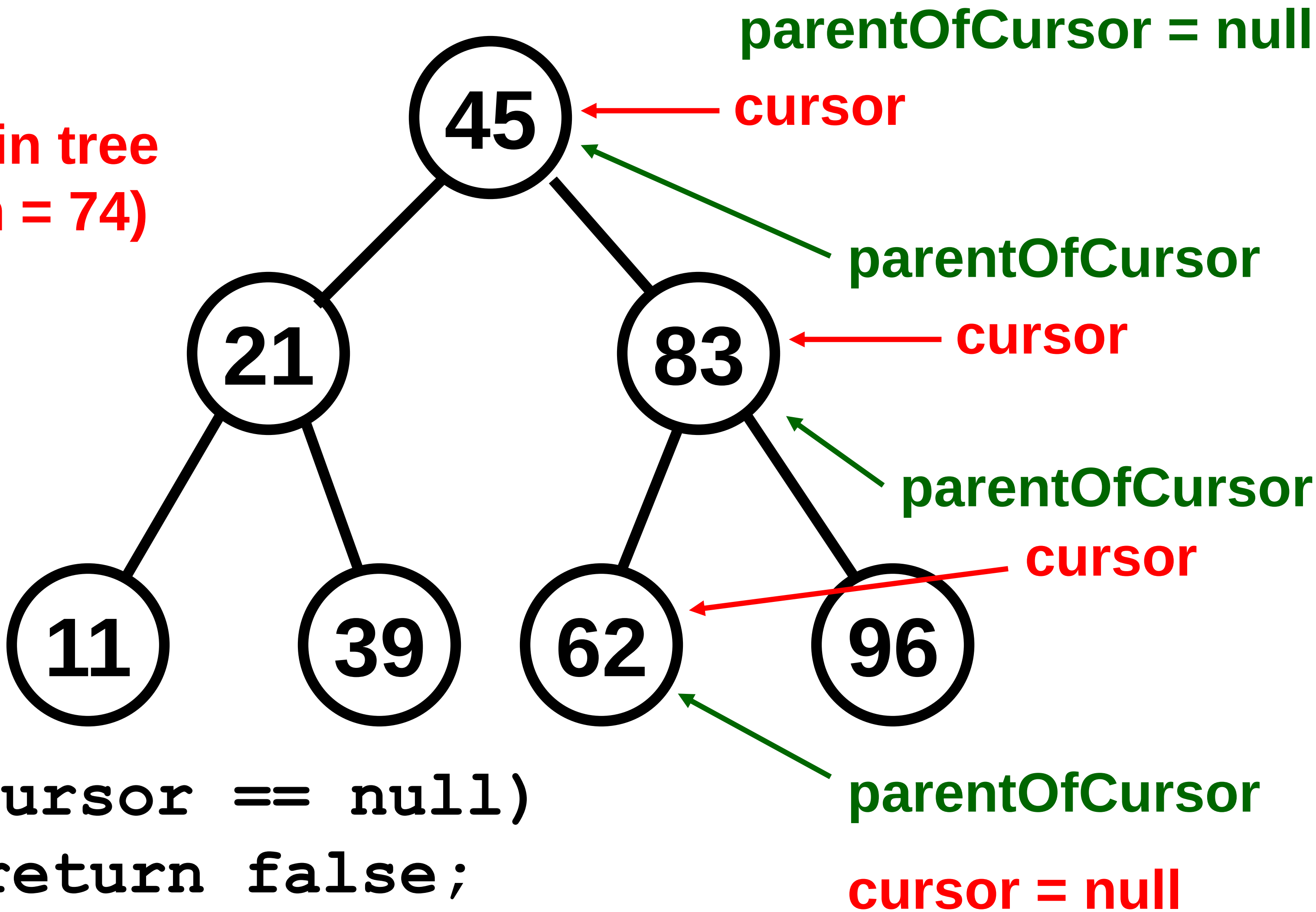
- If the item is found, remove the node and repair the tree so it is still a BST.

Removing from a BST

```
public boolean remove(int item) {
  BTNode cursor = root;
  BTNode parentOfCursor = null;
  while (cursor != null &&
         cursor.getData() != item) {
     parentOfCursor = cursor;
     if (item < cursor.getData())
         cursor = cursor.getLeft();
     else cursor = cursor.getRight();
  }
```

Removing from a BST (cont'd)

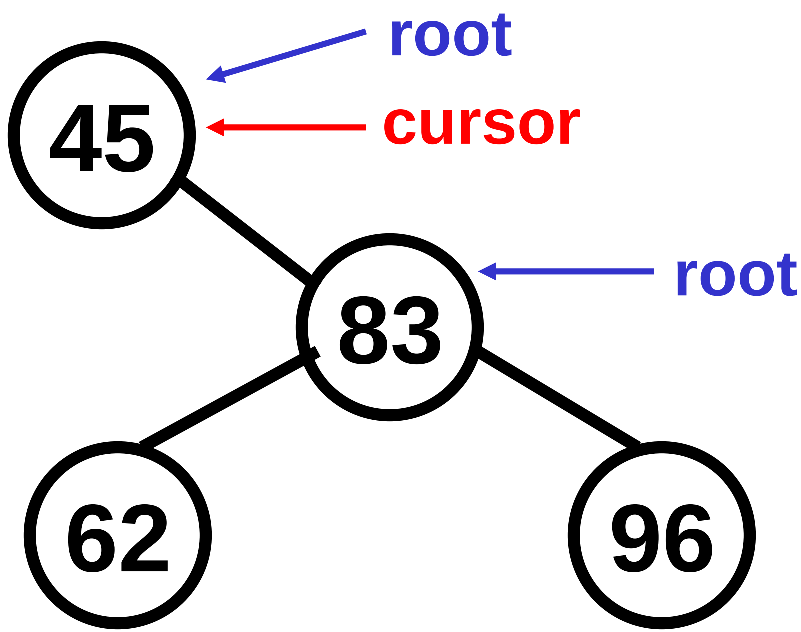
Case 1:
item not in tree
(e.g. item = 74)




```
if (cursor == null)
    return false;
```

Removing from a BST (cont'd)

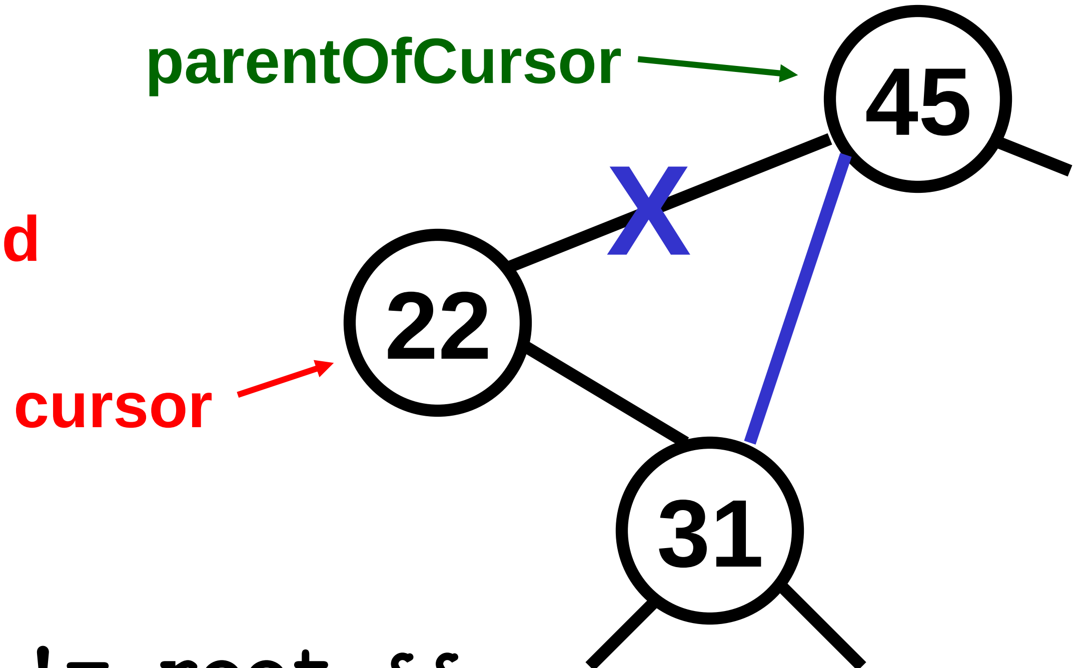
Case 2:
item is root and
root has no left child
(e.g. item=45)




```
else {
    if (cursor == root &&
               cursor.getLeft() == null) {
        root = root.getRight();
    }
```

Removing from a BST (cont'd)

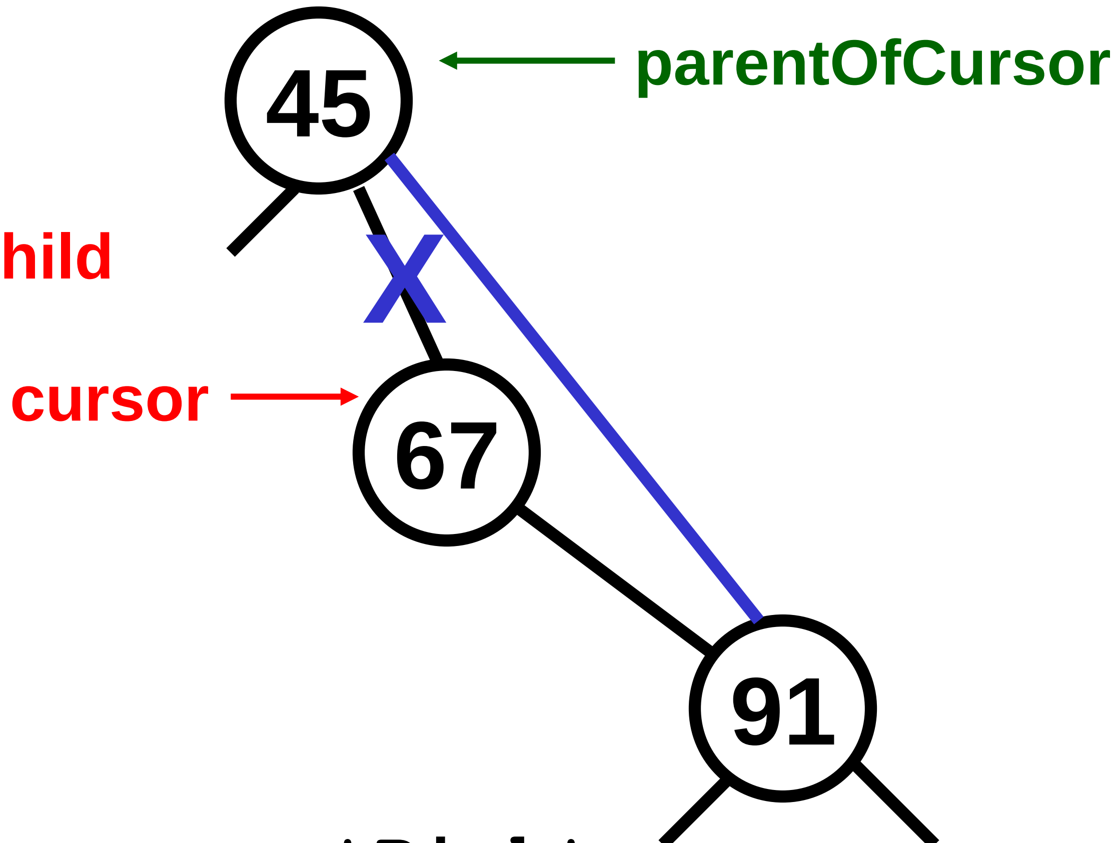
Case 3(a):
item is in a non-root node without a left child




```
else if (cursor != root &&
              cursor.getLeft() == null) {
  if (cursor == parentOfCursor.getLeft())
     parentOfCursor.setLeft
                    (cursor.getRight());
```

Removing from a BST (cont'd)

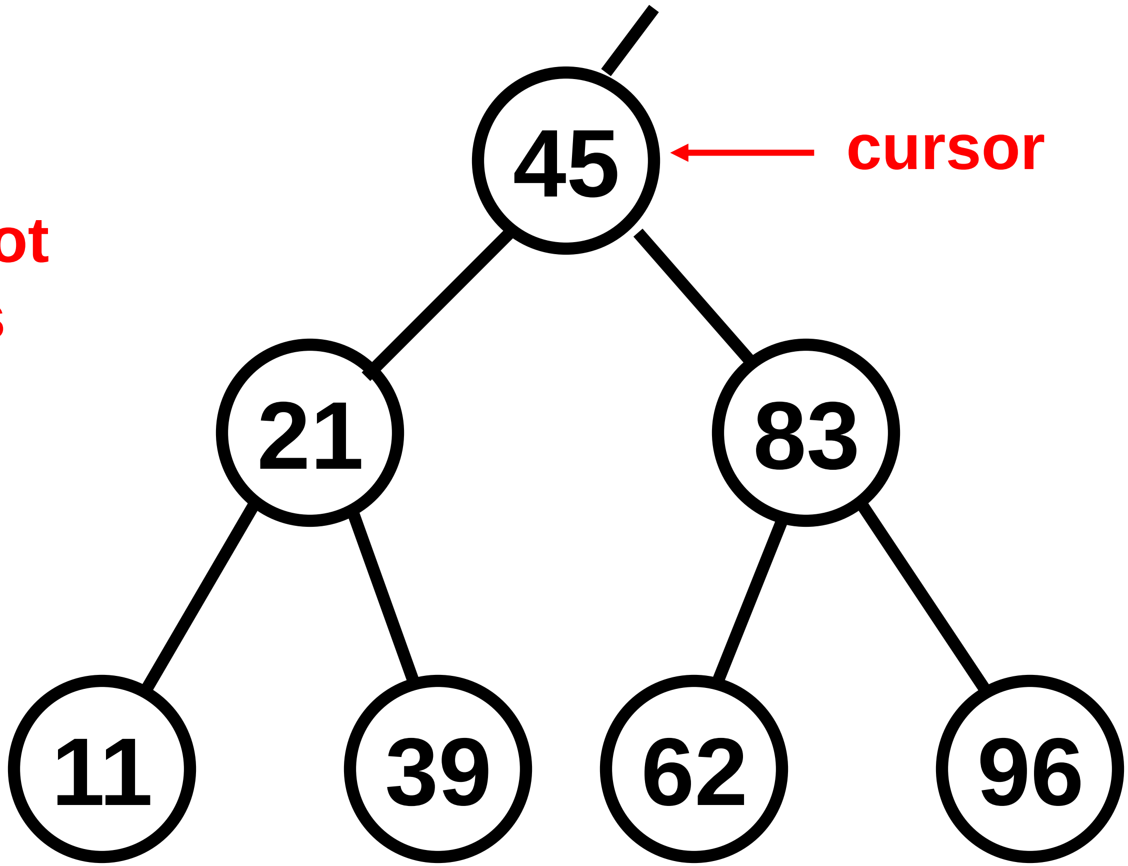
Case 3 (b):
item is in a non-root node without a left child




```
else
    parentOfCursor.setRight
                          (cursor.getRight());
}
```

Removing from a BST (cont'd)

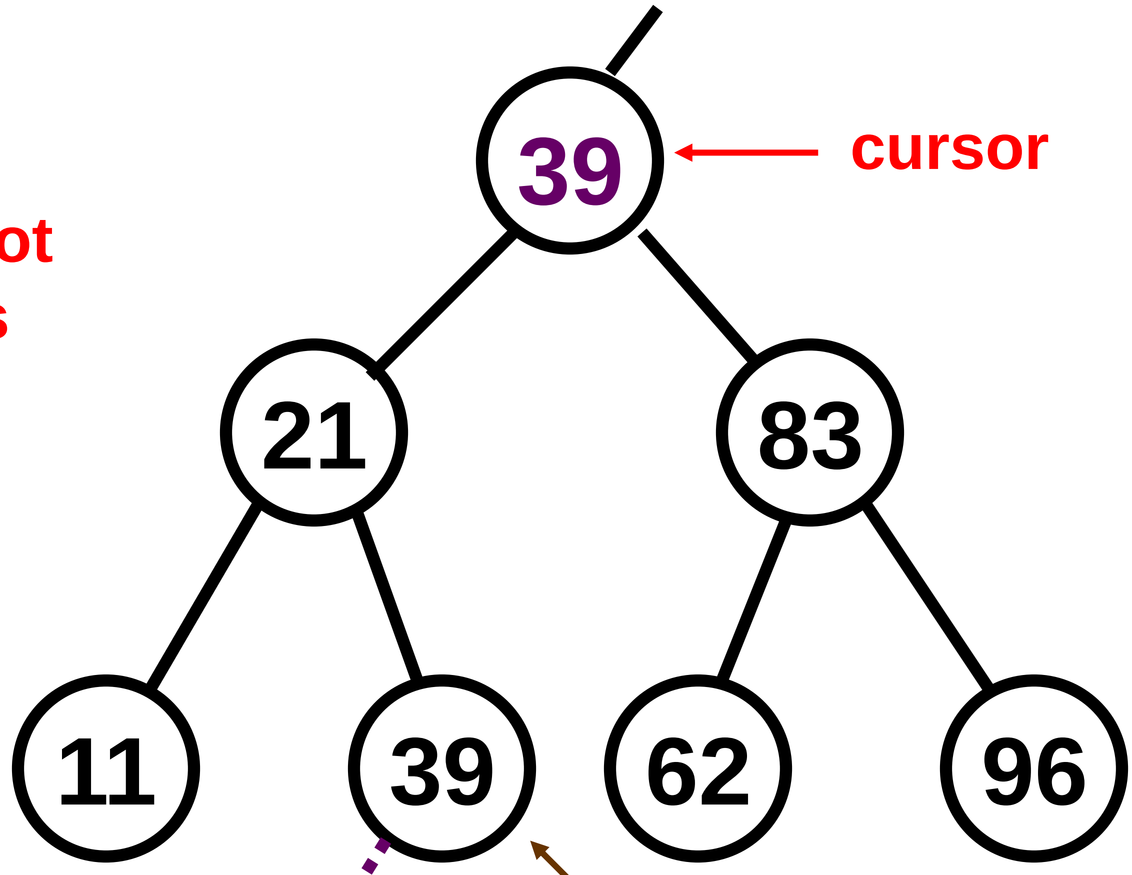
Case 4:
item is in a non-root node but node has a left child



Find the largest value in the left subtree of cursor and move it to the cursor position.
This value is the "rightmost" node in the left subtree.

Removing from a BST (cont'd)

Case 4 (cont'd):
item is in a non-root node but node has a left child



Copy the rightmost data into the cursor node.

Then remove the rightmost node (be careful: it might have a left subtree!)

rightmost

Removing from a BST (cont'd)

```
    else {
       cursor.setData(
          cursor.getLeft()
                 .getRightmostData());
       cursor.setLeft(
          cursor.getLeft()
                 .removeRightmost());
     }
   return true;
 }
}
```

Additional BTNode methods

```
public int getRightmostData() {
  if (right == null) return data;
  else return right.getRightmostData();
}
public BTNode removeRightmost() {
  if (right == null) return left;
  else {
    right = right.removeRightmost();
    return this;
  }
}
```

Order of complexity for BST removal?