This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

Auction Mechanisms Toward Efficient Resource Sharing for Cloudlets in Mobile Cloud Computing

A-Long Jin, Wei Song, Ping Wang, Dusit Niyato, and Peijian Ju

Abstract—Mobile cloud computing offers an appealing paradigm to relieve the pressure of soaring data demands and augment energy efficiency for future green networks. Cloudlets can provide available resources to nearby mobile devices with lower access overhead and energy consumption. To stimulate service provisioning by cloudlets and improve resource utilization, a feasible and efficient incentive mechanism is required to charge mobile users and reward cloudlets. Although auction has been considered as a promising form for incentive, it is challenging to design an auction mechanism that holds certain desirable properties for the cloudlet scenario. Truthfulness and system efficiency are two crucial properties in addition to computational efficiency, individual rationality and budget balance. In this paper, we first propose a feasible and truthful incentive mechanism (TIM), to coordinate the resource auction between mobile devices as service users (buyers) and cloudlets as service providers (sellers). Further, TIM is extended to a more efficient design of auction (EDA). TIM guarantees strong truthfulness for both buyers and sellers, while EDA achieves a fairly high system efficiency but only satisfies strong truthfulness for sellers. We also show the difficulties for the buyers to manipulate the resource auction in EDA and the high expected utility with truthful bidding.

Index Terms—Mobile cloud computing, cloudlet, double auction, incentive design, truthfulness, efficiency.

1 Introduction

In recent years, mobile network operators are facing unremitting demands for high date rates and ever-emerging new applications. While this growth leads to increasing revenue to the global mobile industry, there are rising concerns on environmental impact and high capital/operating expenditure. Meanwhile, cloud computing is achieving great success in empowering end users with rich experience by leveraging resource virtualization and sharing. The merging of cloud computing into the mobile domain creates the appealing paradigm of mobile cloud computing (MCC). MCC offers a promising solution not only to extend the limited capabilities of mobile devices, but also to reduce energy consumption if designed in a green manner [1].

As illustrated in Fig.1, the traditional centralized cloud [2] hosts shared resources in remote data centers and acts as an agent between the original content providers and mobile devices. To access resources/services at the data centers, mobile devices often need to go through the backbone network. The long latency and high energy consumption can hamper the capability of the cloud to support interactive applications demanded by users. In contrast, the lightweight cloudlet [3] can balance the scale of shared resources and the access overhead. A cloudlet is a trusted, resource-rich, Internet-connected computer or a cluster of computers, which can be utilized by mobile devices via a high-speed wireless local area network (WLAN). In this architecture, mobile devices function as the clients and cloudlets as the service providers. Seamless interaction between them can be more easily achieved in the cloudlet's physical proximity with the low one-hop communication latency. Due to the spatial distributions of cloudlets and their distinct capabilities or hosted resources, mobile devices have different preferences over the cloudlets. On the other hand, the cloudlets need to be incentivized to share their resources, e.g., through gaining monetary values paid by the mobile devices for using the services. As seen, there exists a trade between the mobile devices requesting the services and the cloudlets providing such services.

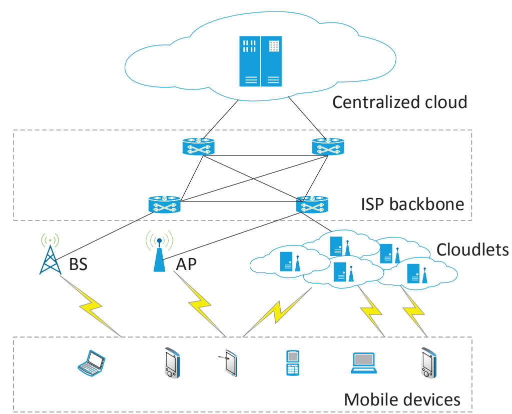


Fig. 1: Typical MCC architectures: Centralized cloud and cloudlets.

Auction is a popular trading form that can efficiently distribute resources of sellers to buyers in a market at competitive prices. Auction theory [4] is a well-researched field in economics and has been applied to other domains, e.g., radio resource management for wireless systems [5]. If the

- *A. Jin, W. Song (corresponding author), and P. Ju are with the Faculty of Computer Science, University of New Brunswick, Fredericton, Canada (emails: along.jin@unb.ca, wsong@unb.ca, i1689@unb.ca).* *This research was supported in part by Natural Sciences and Engineering Research Council (NSERC) of Canada.*
- *P. Wang and D. Niyato are with School of Computer Engineering, Nanyang Technological University, Singapore (emails: wangping@ntu.edu.sg, dniyato@ntu.edu.sg).*

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

resource trading system with buyers and sellers is viewed as an ecosystem, the auction mechanism needs to appropriately address the conflicting interests of buyers (e.g., minimizing charge for highest valuation) and sellers (e.g., maximizing reward for least cost), and internal competitions among buyers/sellers. The mechanism should fairly allocate the trading resources, determine the pricing, and optimize the payoff that each buyer/seller can achieve *with the presence of competitions*. As such, the buyers and sellers can be incentivized to participate in the auction and the ecosystem can reach an equilibrium. Such an auction mechanism is expected to hold certain desirable properties, such as *individual rationality*, *budget balance*, *truthfulness* or *incentive compatibility*, *system efficiency*, and *computational efficiency*.

Individual rationality ensures that a *buyer* is never charged more than its bid, while a *seller* is paid not less than its ask. Budget balance requires that the *auctioneer*, which acts as an intermediate agent between buyers and sellers, hosts and runs the auction without a deficit. Truthfulness is essential to resist market manipulation and ensure auction fairness. An auction mechanism is truthful (also known as incentive compatible) if revealing the private valuation truthfully is always the weakly dominant strategy for each participant to receive an optimal utility, irrespective of what strategies other participants are taking. There are various definitions of efficiency in economics from different perspectives, among which allocative efficiency is a main type that aims to maximize social welfare, i.e., the sum of valuations of the buyers who receive their desired *commodities*. In this paper, we are particularly interested in double auction, in which buyers and sellers submit to an auctioneer their bids and asks, respectively. For such bilateral trade, a mechanism is efficient if whenever a buyer's bid is greater than the seller's ask, the corresponding commodity is allocated to the buyer [4]. In addition to the above economic properties, computational efficiency ensures that the auction outcome is tractable with a polynomial time complexity, which is important to enable feasible implementation.

Many existing auction mechanisms cannot be directly applied to the cloudlet scenario without jeopardizing certain desired properties. For example, the multi-round auctions studied in [6]–[8] are not suitable due to the high communication and computation overhead. For double auction, no mechanism can be efficient, truthful, individually rational, and at the same time balance the budget [4]. Considering only homogeneous commodities, McAfee double auction [9] can achieve three desirable properties, i.e., individual rationality, budget balance, and truthfulness. The Truthful Auction Scheme for Cooperative communications (TASC) proposed in [10] extends McAfee double auction by taking into account heterogeneous trading commodities, i.e., services of relay nodes. Although TASC addresses a similar scenario, it is found that the efficiency of TASC can be further improved. In [11], the Vickrey-based double auction exploits the participant's uncertainty of bids/asks of other participants to reduce manipulation and boost allocative efficiency, while enforcing budget balance and individual rationality. This mechanism can be fairly efficient and fairly truthful, but not computationally efficient.

As seen, it is impossible to simultaneously satisfy all the aforementioned desirable properties in an auction mechanism. In many cases, certain properties (e.g., computational efficiency or truthfulness) have to be relaxed to trade for other properties (e.g., system efficiency). As a result, this leaves room for improving the relaxed properties. In this paper, we focus on designing feasible and efficient double auction mechanisms to stimulate cloudlets to serve nearby mobile devices. Here, a feasible auction mechanism needs to satisfy computational efficiency, individual rationality, budget balance, and truthfulness. Consequently, we have to relax the requirement for system efficiency, which is sub-optimal but sufficiently close to the optimum. First, we propose a truthful incentive mechanism (TIM), which is also computationally efficient, individually rational and budget-balanced. To further improve the system efficiency, we extend TIM to a more efficient design of auction (EDA), by involving randomness and bidding uncertainty. EDA maintains all desired properties of TIM except for slightly relaxed truthfulness. EDA guarantees trustfulness of sellers but it is not strongly truthful for buyers. We provide rigorous analysis regarding these properties of both mechanisms. Numerical results confirm the analysis and demonstrate their desirable properties, especially the high system efficiency achieved by EDA. It is also shown that EDA is truthful in expectation for buyers.

The remainder of this paper is structured as follows. First, we briefly review related auction mechanisms in Section 2. Then, Section 3 provides the problem formulation as a double auction and an example demonstrating the design rationale. After that, we give the design details of the proposed TIM and EDA mechanisms in Section 4 and Section 5, respectively. Numerical results are presented in Section 6, followed by conclusions in Section 7.

2 RELATED WORK

As a promising paradigm, mobile cloud computing has attracted considerable research attention and efforts. There have been a number of studies addressing various aspects of MCC, such as virtual machine migration [12], service enhancement with MCC [13], and emerging applications with MCC [14,15]. However, the research on incentive design for MCC is limited, although there have been many incentive mechanisms proposed for wireless cooperative communications [10], radio resource allocation [5], device-to-device communications [16], smartphone sensing [17], and smart grid [18].

In [10], Yang *et al.* propose a truthful double auction mechanism, TASC, to stimulate relay nodes to forward packets for other wireless nodes. TASC has two stages, namely, *Assignment* and *Winner-Determination & Pricing*. In the assignment stage, the auctioneer applies an assignment algorithm to determine the winning buyer candidates (source nodes), the winning seller candidates (relay nodes), and the mapping between these buyers and sellers. Depending on the design objective, the auctioneer can choose a different assignment algorithm. For example, the optimal relay assignment algorithm [19] can maximize the minimum valuation among all buyers; the maximum weighted matching algorithm [20] can maximize the total valuation; and the maximum matching algorithm can

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

maximize the number of successful trades (final matchings). In the winner-determination & pricing stage, TASC tightly integrates the winner determination and the pricing operation. Based on the return of the assignment stage, the auctioneer applies McAfee double auction [9] to determine the winning buyers, the winning sellers, and the corresponding clearing price and payment.

TASC overcomes the limitation of the original McAfee double auction [9] that considers only homogeneous commodities, i.e., buyers have no preference over auction items. When TASC is used in the cloudlet scenario, it can satisfy individual rationality, budget balance, and truthfulness for the sellers. However, we use an example in Section 3.4 to illustrate that the system efficiency of TASC can be further improved.

Another closely related work in [21] addresses specifically resource sharing with MCC. In [21], cloud resources are categorized into several groups (e.g., processing, storage, and communications). The resource allocation problem is formulated as a combinatorial auction with substitutable and complementary commodities. This combinatorial auction mechanism is not applicable for the cloudlet architecture since its key problem is the allocation of M resources of G groups in one MCC service provider to N users. In contrast, our system model with cloudlets focuses on distinct valuations of cloudlets to mobile users. Different from [21], we further consider computational efficiency and budget balance, which are also critical to an auction mechanism.

It is noted that many existing incentive mechanisms emphasize the property of truthfulness, which prevents market manipulation and eliminates the strategic overhead of the participants. However, there are few works on double auction design to improve system efficiency. In [11], Parkes *et al.* propose a Vickrey-based double auction, which can achieve individual rationality and budget balance. The assignment between buyers and sellers is determined to maximize social welfare (system efficiency), while the participant's utility equals the incremental contribution to the overall system, i.e., the difference between the social welfare with and without the participation. However, the Vickrey-based double auction in [11] is only fairly efficient and fairly truthful. Moreover, this mechanism cannot satisfy computational efficiency.

3 Problem Formulation

3.1 System Model

As depicted in Fig. 1, the cloudlets offer resource pools closer to the network edge. Thus, the close proximity of cloudlets can be exploited to reduce the access overhead of mobile users in terms of energy consumption and communication latency. The resources at the cloudlets may be valued differently by the mobile users depending on various factors [22], such as computation capability, communication cost, and wireless link performance (e.g., throughput, latency, and link variation). Such *valuation* of a mobile user toward a cloudlet is also associated with the application demand. For instance, when a mobile user offloads a computation-intensive task, it values high a cloudlet with rich computing resources of memory and CPU capacity. In contrast, a mobile user with a real-time task prefers a cloudlet with a short communication latency, which requires large network bandwidth and close spatial locations. As seen, the valuation of a mobile user toward a cloudlet actually takes into account the service quality that the cloudlet can provide to the user.

On the other hand, the cloudlet can be paid for sharing resources as compensation for its computation and communication *cost*. The cost covers the cloudlet's expenditures on acquiring computation resources (e.g., computing equipment, energy, and storage), and leasing communication facility from ISPs, mobile carriers, or network operators. Note that the cost implicitly incorporates the resource constraint of the cloudlet.

Clearly, the trading between the cloudlets and the mobile users should meet certain requirements to benefit both the cloudlets and mobile devices. The cloudlets need to be incentivized to provide the resources, and the demands of the mobile users should be satisfied. In particular, a cloudlet cannot be paid less than its cost, while the allocated resources of the cloudlet must fulfill a mobile user's service request. To maximize the resource utilization, the incentive mechanism should properly assign the matching between the cloudlets' resources and the mobile users' demands.

3.2 Auction Model

To assist the matching between mobile users and cloudlets, a trusted third party is necessary to administer the trading between them, e.g., in the form of auction. In particular, a double auction fits well the bilateral nature of this scenario. The trusted third party in a double auction is the *auctioneer* between mobile users (*buyers*) and cloudlets (*sellers*). The auctioneer needs to determine the matching of winning buyers and winning sellers, the price it charges the buyers and the price it rewards the sellers.

Apparently, the auctioneer should at least run the auction without a deficit and preferably benefit from the process. Taking the dominant video streaming service as an example, we can identify some network entities that would support the auctioneer functionality. In practice, content providers often pay content delivery network (CDN) operators to deliver their content, while a CDN operator in turn pays ISPs, mobile carriers, or network operators for hosting its servers in their data centers [23]. Besides, cloudlets can offer services closer to network edge, thus complementing traditional CDNs with lower delivery cost and better performance. Hence, the end users need to pay the content providers for content consumption, while the content providers in turn pay CDN operators or cloudlets for content delivery. Because cloudlets when available can deliver content with lower cost and higher quality, the content providers sitting between the end users and cloudlets can have a strong motivation and suitable position to run an auctioneer server to reduce their expenditures and sustain competitiveness.

Considering m cloudlets (sellers) that provide available resources for n mobile devices (buyers), we formulate the underlying resource allocation problem as a single-round multi-item double auction similar to [10]. Each buyer (resp. seller) can submit its bid (resp. ask) privately to the auctioneer so that everyone has no knowledge of others.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

TABLE 1: Important notations.

Symbol	Definition
b_i	Buyer (mobile device)
s_j	Seller (cloudlet)
n	Total number of buyers
m	Total number of sellers
$\mathcal{B}$	Set of buyers (mobile devices)
$\mathcal{S}$	Set of sellers (cloudlets)
$\mathcal{B}_w$	Set of winning buyers
$\mathcal{S}_w$	Set of winning sellers
$\sigma(\cdot)$	Mapping function from the indices of $\mathcal{S}_w$ to $\mathcal{B}_w$
D_i^j	Bid of buyer b_i on seller s_j
$\mathbf{D}_i$	Bid vector of buyer b_i
$\mathbf{D}^j$	Bid vector of all buyers for seller s_j
$\mathbf{D}$	Bid matrix of all buyers
A_j	Ask of seller s_j
$\mathbf{A}$	Ask vector of all sellers
$\mathbf{A}_{-j}$	Ask vector of all sellers except s_j
V_i^j	Valuation of buyer b_i on service from seller s_j
$\mathbf{V}_i$	Valuation vector of buyer b_i
C_j	Cost of seller s_j for providing service
P_i^{b}	Price charged to buyer b_i
P_j^{s}	Payment rewarded to seller s_j
P_{ij}^{b}	Price charged to buyer b_i for service of seller s_j
P_{ij}^{s}	Payment rewarded to seller s_j with assigned buyer b_i
U_i^{b}	Utility of buyer b_i
U_j^{s}	Utility of seller s_j
U_{ij}^{b}	Utility of buyer b_i with assigned seller s_j
U_{ij}^{s}	Utility of seller s_j with assigned buyer b_i

- For each buyer $b_i \in \mathcal{B}$, $\mathcal{B} = \{b_1, b_2, \ldots, b_n\}$, its bid vector is denoted by $\mathbf{D}_i = (D_i^1, D_i^2, \ldots, D_i^m)$, where D_i^j is the bid for seller $s_j \in \mathcal{S}$, $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$. The bid matrix consisting of the bid vectors of all buyers is defined as $\mathbf{D} = (\mathbf{D}_1; \mathbf{D}_2; \ldots; \mathbf{D}_n)$.
- For all sellers in $\mathcal{S}$, the ask vector is denoted by $\mathbf{A} = (A_1, A_2, \ldots, A_m)$, where A_j is the ask of seller $s_j \in \mathcal{S}$.

As seen, the asks of sellers do not differentiate among buyers since the sellers only aim at collecting payments for using their resources. On the other hand, the bids of buyers differ with respect to sellers as mobile devices have preferences over the cloudlets. Although the bids of buyers are private information to the sellers, the cloudlets do need to release certain information, such as their hosted resources, computation capabilities, network bandwidth and communication latency, to nearby mobile users, so that the users can determine their valuations toward the cloudlets. The cloudlets, however, keep their service costs confidential. The auctioneer that holds the private information thus needs to apply security mechanisms [24] to guarantee protection of privacy.

Given $\mathcal{B}, \mathcal{S}, \mathbf{D}$ and $\mathbf{A}$, the auctioneer decides the winning buyer set $\mathcal{B}_w \subseteq \mathcal{B}$, the winning seller set $\mathcal{S}_w \subseteq \mathcal{S}$, the mapping between $\mathcal{B}_w$ and $\mathcal{S}_w$, i.e., $\sigma : \{j : s_j \in \mathcal{S}_w\} \to \{i : b_i \in \mathcal{B}_w\}$, the price P_i^{b} that the winning buyer $b_i \in \mathcal{B}_w$ is charged, and the payment P_j^{s} that the winning seller $s_j \in \mathcal{S}_w$ is rewarded[1]. To highlight the utilities for the particular matching between b_i and s_j, we also use P_{ij}^{b} and P_{ij}^{s} in certain cases to denote the price and payment, respectively.

In addition to the price and payment, the utilities of the buyers and sellers further depend on the valuations of the buyers toward the acquired services and the costs for providing such services by the sellers. Let V_i^j be the valuation to buyer b_i for having the service from seller s_j, and C_j be the cost to seller s_j for providing the service. The valuation vector of buyer b_i is denoted by $\mathbf{V}_i = (V_i^1, V_i^2, \ldots, V_i^m)$. Given a buyer-seller mapping, $i = \sigma(j)$, the *utility* of buyer b_i and that of seller s_j are respectively defined as follows:

$$U_i^{\mathrm{b}} = \begin{cases} V_i^j - P_i^{\mathrm{b}}, & \text{if} \quad b_i \in \mathcal{B}_w \\ 0, & \text{otherwise} \end{cases}$$

$$U_j^{\mathrm{s}} = \begin{cases} P_j^{\mathrm{s}} - C_j, & \text{if} \quad s_j \in \mathcal{S}_w \\ 0, & \text{otherwise.} \end{cases}$$

Note that we also use U_{ij}^{b} and U_{ij}^{s} when necessary to capture that the utilities are with respect to the matching between buyer b_i and seller s_j. As seen, a utility $U_i^{\mathrm{b}} > 0$ means that the mobile user b_i as a buyer is allocated the resources of a cloudlet with a valuation greater than the charged price. Thus, U_i^{b} indicates the level of satisfaction of the mobile user on the allocated resources. On the other hand, a utility U_j^{s} of the cloudlet as a seller represents the surplus of the received payment over its cost. In other words, U_j^{s} characterizes the profit of a cloudlet for sharing its resources.

Some important notations are summarized in Table 1.

3.3 Desirable Properties and Design Objective

The auction model introduced in Section 3.2 is represented by $\Psi = (\mathcal{B}, \mathcal{S}, \mathbf{D}, \mathbf{A})$. Accordingly, the auctioneer should follow an auction mechanism to determine the set of winning buyers $\mathcal{B}_w$, the set of winning sellers $\mathcal{S}_w$, the mapping σ between $\mathcal{B}_w$ and $\mathcal{S}_w$, the set of clearing price $\mathcal{P}_w^{\mathrm{b}}$ charged to the winning buyers, and the set of clearing payment $\mathcal{P}_w^{\mathrm{s}}$ rewarded to the winning sellers. A *feasible* auction mechanism should first satisfy the following three desirable properties.

- *Computational Efficiency*: The auction outcome, which includes the winning sets of buyers and sellers, their mapping, and the clearing price and payment, is tractable with a polynomial time complexity.
- *Individual Rationality*: No winning buyer is charged more than its bid and no winning seller is rewarded less than its ask. With respect to the auction model Ψ, this means that for every winning matching between $b_i \in \mathcal{B}_w$ and $s_j \in \mathcal{S}_w$, we have $P_i^{\mathrm{b}} \leq D_i^j$ and $P_j^{\mathrm{s}} \geq A_j$.

1. To distinguish the price charged to *buyers* and the payment rewarded to *sellers*, we use b and s in the normal form as the superscript, respectively. The same naming convention is also applied to the utilities of buyers and sellers.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

- *Budget Balance*: The total price that the auctioneer charges all winning buyers is not less than the total payment that the auctioneer rewards all winning sellers, so that there is no deficit for the auctioneer. That is, $\sum_{b_i \in \mathcal{B}_w} P_i^{\mathrm{b}} \geq \sum_{s_j \in \mathcal{S}_w} P_j^{\mathrm{s}}$.

Enforcing the hard constraints on the preceding three properties, we further consider two other crucial properties which can be strictly or fairly satisfied.

- *System Efficiency*: Referring to allocative efficiency in economics, we evaluate system efficiency by the number of successful trades (the number of final matchings between winning buyers and winning sellers) and the social welfare (the total valuation of winning buyers). There are many other definitions of efficiency, such as maximizing the revenue, i.e., the total payment to winning sellers, minimizing the total charge to winning buyers, and even maximizing the profit of the auctioneer, i.e., the surplus between total charge to buyers and total reward to sellers.
  In our double auction model, the number of final matchings fits the bilateral trade nature, since a successful trade means that both the requirements of the seller (cloudlet) and the buyer (mobile user) are satisfied. Maximizing the number of successful trades can involve as many cloudlets as possible in the trading, so that the resource utilization of cloudlets can be boosted. This metric has been considered in many existing works [10] to evaluate system efficiency. Compared with maximizing sellers' revenue or minimizing buyers' charge, maximizing the number of successful trades is more realistic and desirable to maintain a stable system from which both buyers and sellers can benefit. If the system is designed toward the interest of only one side, e.g., maximum revenue to sellers or minimum charge to buyers, the bias may eventually lead to opting-out of underprivileged participants which are not offered sufficient incentives.
- *Truthfulness*: An auction mechanism is truthful if playing (bidding or asking) truthfully is a weakly dominant strategy for each player (buyer or seller) who is only concerned with its own utility. In other words, no buyer can improve its utility by submitting a bid different from its true valuation, and no seller can improve its utility by submitting an ask different from its true cost. Specifically, it implies the following for our auction model: $\forall b_i \in \mathcal{B}$, U_i^{b} is maximized when the bid $\mathbf{D}_i = \mathbf{V}_i$; and $\forall s_j \in \mathcal{S}$, U_j^{s} is maximized when the ask $A_j = C_j$. This truthfulness is very restrictive for a randomized mechanism due to the uncertainty of the actual outcome. A weaker truthful notion is *truthfulness in expectation*, which guarantees that a player's *expected utility* for truthful bidding is at least its expected utility for bidding any other value.

With the property of truthfulness, an auction mechanism can be free from market manipulation, and the strategies of the participants can be simplified accordingly. Since telling the truth produces the highest utility for each player, no rational buyer or seller would play untruthfully, even though the global utility might be improved. What is more appealing is that no player would deviate from the truth-telling strategy and the system thus reaches an equilibrium. Each player only needs to apply the simple truth-telling strategy and does not have to adapt to others' behaviors.

TABLE 2: An illustrative example.

(a) Bid matrix of 5 buyers.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
b_1	6	0	0	0	5	10	0
b_2	4	0	0	3	0	0	8
b_3	0	0	6	0	0	9	0
b_4	0	10	0	0	0	0	7
b_5	0	2	7	9	0	0	0

(b) Ask vector of 7 sellers.

Seller	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Ask	3	2	5	6	4	1	7

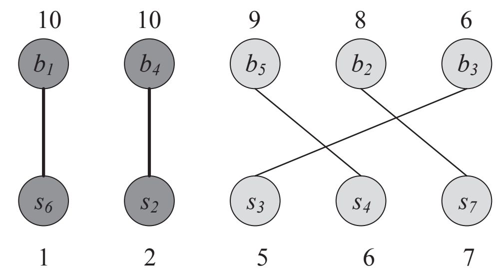


Fig. 2: Assignment result with truthful bids and asks: A bipartite graph of matched winning candidates.

Though truthfulness presents all the aforementioned merits, unfortunately, there is no double auction mechanism that is truthful, efficient[2], individually rational, and at the same time balances the budget [4]. In this paper, we design two feasible auction mechanisms that are computationally efficient, individually rational, and budget-balanced. The first mechanism TIM further satisfies truthfulness in addition to these three properties. Extending TIM by involving randomness, the second mechanism EDA can achieve a higher system efficiency than that of TIM but truthfulness is guaranteed in a weak sense, i.e., strongly truthful for sellers but truthful in expectation for buyers as demonstrated in the experiment results. Nonetheless, the difficulties in computing an effective lie, combined with the backfire risk of untruthful bid will convince the buyers not to bother to lie.

3.4 Design Rationale

As mentioned in Section 2, there is still room to improve the system efficiency of many existing truthful auction mechanisms. To illustrate this, we take the TASC double auction in [10] as an example, and consider a bid matrix of 5 buyers with true valuations in Table 2(a) and the ask vector of 7 sellers with true costs in Table 2(b).

2. This paper addresses both system efficiency and computational efficiency. To avoid confusion, we use the terms *efficient* and *efficiency* to indicate specifically the system efficiency, while we use the terms *computationally efficient* and *computational efficiency* to distinguish from system efficiency.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

Suppose that the auctioneer uses maximum matching in the assignment stage to maximize the number of matchings. According to the assignment algorithm of TASC, the winning buyer candidates, the winning seller candidates and the mapping between them are shown in Fig. 2. Then, following the TASC strategy for winner-determination & pricing, we have the set of winning buyers $\mathcal{B}_w = \{b_1, b_4\}$, the set of winning sellers $\mathcal{S}_w = \{s_6, s_2\}$, the clearing price $\mathcal{P}_w^{\mathrm{b}} = \{8\}$, and the clearing payment $\mathcal{P}_w^{\mathrm{s}} = \{6\}$. As seen, the number of successful trades is only 2.

On the other hand, if the true valuations of buyers and true costs of sellers are known *a priori*, an optimal strategy can be applied to maximize the number of successful trades. The auctioneer can assign a seller s_j to a buyer b_i as long as $V_i^j \geq C_j$. Then, the auctioneer can select a value between C_j and V_i^j as the price charged to buyer b_i and the payment rewarded to seller s_j, i.e., $V_i^j \geq P_{ij}^{\mathrm{b}} = P_{ij}^{\mathrm{s}} \geq C_j$. In the given example, it can be easily shown that this optimal strategy can maximize the number of successful trades to 5. Thus, TASC only achieves 40% of the system efficiency of the optimal strategy. Inspired by this observation, we propose two feasible auction mechanisms. TIM in Section 4 gives a truthful version, while EDA in Section 5 can achieve a higher system efficiency.

4 TRUTHFUL INCENTIVE MECHANISM (TIM)

In this section, we present a truthful auction mechanism TIM for resource sharing of cloudlets, which improves the system efficiency over TASC. We first introduce the algorithms of TIM and then give an illustrative example. The properties of TIM are also analyzed in terms of computational efficiency, individual rationality, budget balance and truthfulness.

4.1 Details of TIM

TIM in Algorithm 1 contains two sub-procedures specified in Algorithm 2 and Algorithm 3, which correspond to two stages, *Candidate-Determination & Pricing* and *Candidate-Elimination*, respectively.

Algorithm 1 TIM($\mathcal{B}$, $\mathcal{S}$, $\mathbf{D}$, $\mathbf{A}$).

Input: $\mathcal{B}$, $\mathcal{S}$, $\mathbf{D}$, $\mathbf{A}$
Output: $\mathcal{B}_w, \mathcal{S}_w, \sigma, \mathcal{P}_w^{\mathrm{b}}, \mathcal{P}_w^{\mathrm{s}}$
$(\mathcal{B}_c, \mathcal{S}_c, \hat{\sigma}, \mathcal{P}_c^{\mathrm{b}}, \mathcal{P}_c^{\mathrm{s}}) \leftarrow$ TIM-CD&P$(\mathcal{B}, \mathcal{S}, \mathbf{D}, \mathbf{A})$;
$(\mathcal{B}_w, \mathcal{S}_w, \sigma, \mathcal{P}_w^{\mathrm{b}}, \mathcal{P}_w^{\mathrm{s}}) \leftarrow$ TIM-CE$(\mathcal{B}_c, \mathcal{S}_c, \hat{\sigma}, \mathcal{P}_c^{\mathrm{b}}, \mathcal{P}_c^{\mathrm{s}}, \mathbf{D})$;
return $(\mathcal{B}_w, \mathcal{S}_w, \sigma, \mathcal{P}_w^{\mathrm{b}}, \mathcal{P}_w^{\mathrm{s}})$;

In the candidate-determination & pricing stage, the candidate determination and pricing operations are coupled together. First, the auctioneer determines the buyer candidate for each seller s_j with ask A_j. Then, the price charged to the buyer candidate and the payment paid to the seller will be determined accordingly. Let $\mathbf{A}_{-j}$ denote the ask vector excluding the ask of s_j, and A_{-j}^o be the median of $\mathbf{A}_{-j}$. The median of a vector is the value of the middle element in a non-decreasing order of the elements. If there is an even number of values, the mean of the two middle values is defined as the median. According to Algorithm 2, there are two cases in this stage to determine the winning buyer candidate for seller s_j.

Algorithm 2 TIM-CD&P$(\mathcal{B}, \mathcal{S}, \mathbf{D}, \mathbf{A})$.

Input: $\mathcal{B}$, $\mathcal{S}$, $\mathbf{D}$, $\mathbf{A}$
Output: $\mathcal{B}_c, \mathcal{S}_c, \hat{\sigma}, \mathcal{P}_c^{\mathrm{b}}, \mathcal{P}_c^{\mathrm{s}}$
$\mathcal{B}_c \leftarrow \emptyset$, $\mathcal{S}_c \leftarrow \emptyset$, $\mathcal{P}_c^{\mathrm{b}} \leftarrow \emptyset$, $\mathcal{P}_c^{\mathrm{s}} \leftarrow \emptyset$;
for $s_j \in \mathcal{S}$ **do**
  Find the median ask A_{-j}^o of the ask vector $\mathbf{A}_{-j}$;
  $\mathcal{B}^j = \{b_i : D_i^j \geq A_j, \forall b_i \in \mathcal{B}\}$;
  if $|\mathcal{B}^j| = 1$ **then**
    if $D_i^j \geq A_{-j}^o$ and $A_j \leq A_{-j}^o$ **then**
      $\hat{\sigma}(j) = i, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_i\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_j\}$;
      $P_{ij}^{\mathrm{b}} = P_j^{\mathrm{s}} = A_{-j}^o$;
      $\mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{ij}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_j^{\mathrm{s}}\}$;
    end if
  else if $|\mathcal{B}^j| > 1$ **then**
    Sort $\mathcal{B}^j$ to $\mathbb{B}^j$ such that $D_{i_{(1)}}^j \geq D_{i_{(2)}}^j \geq \cdots$;
    if $D_{i_{(1)}}^j \geq A_{-j}^o$ **then**
      if the first t $(t \geq 2)$ bids of $\mathbb{B}^j$ are the same **then**
        Randomly select b_i from first t buyers of $\mathcal{B}^j$;
      else
        Select first b_i of $\mathbb{B}^j$ with the highest bid;
      end if
      $\hat{\sigma}(j) = i, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_i\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_j\}$;
      $P_{ij}^{\mathrm{b}} = P_j^{\mathrm{s}} = \max\{A_{-j}^o, D_{i_{(2)}}^j\}$;
      $\mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{ij}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_j^{\mathrm{s}}\}$;
    end if
  end if
end for
return $(\mathcal{B}_c, \mathcal{S}_c, \hat{\sigma}, \mathcal{P}_c^{\mathrm{b}}, \mathcal{P}_c^{\mathrm{s}})$;

- Only one buyer b_i with a bid not less than A_j: If $D_i^j \geq A_{-j}^o$ and $A_j \leq A_{-j}^o$, b_i is added into the buyer candidate set $\mathcal{B}_c$ with price A_{-j}^o, and s_j is added into the seller candidate set $\mathcal{S}_c$ with payment A_{-j}^o; otherwise, b_i cannot win s_j.
- Two or more buyers with bids not less than A_j: If the highest bid is less than A_{-j}^o, no buyer wins the service of s_j; otherwise, the buyer with the highest bid (or a randomly selected buyer when there is a tie) is added into the buyer candidate set $\mathcal{B}_c$, and s_j is added into the seller candidate set $\mathcal{S}_c$. The price to the selected buyer and the payment to the corresponding seller is the same as the maximum of A_{-j}^o and the second highest bid.

In the candidate-elimination stage, since a buyer candidate in $\mathcal{B}_c$ may win two or more sellers in the seller candidate set $\mathcal{S}_c$, Algorithm 3 is run to choose only one best seller for such a buyer. Specifically, the auctioneer selects the seller so that the corresponding buyer achieves the highest utility. Likewise, when there is a tie in terms of the achievable utilities, one seller is randomly selected. At the end of the candidate elimination stage, every buyer $b_{\sigma(j)} \in \mathcal{B}_w$ has a one-to-one mapping with only one winning seller $s_j \in \mathcal{S}_w$.

4.2 A Walk-Through Example

To illustrate the procedure of TIM, we use an example with the bid matrix in Table 2(a) and the ask vector in Table 2(b).
1) Candidate-Determination & Pricing:

- $\mathcal{B}_c = \emptyset, \mathcal{S}_c = \emptyset, \mathcal{P}_c^{\mathrm{b}} = \emptyset, \mathcal{P}_c^{\mathrm{s}} = \emptyset$;
- s_1: We have the ask vector $\mathbf{A}_{-1} = \{2, 5, 6, 4, 1, 7\}$, and the median ask $A_{-1}^o = 4.5$. As $D_1^1 > A_1$ and $D_2^1 > A_1$, we have $\mathcal{B}^1 = \{b_1, b_2\}$. Since $|\mathcal{B}^1| > 1$ and $D_1^1 > A_{-1}^o > D_2^1$, b_1 is selected as the winning buyer candidate for s_1 with

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

Algorithm 3 TIM-CE($\mathcal{B}_c, \mathcal{S}_c, \hat{\sigma}, \mathcal{P}_c^{\mathrm{b}}, \mathcal{P}_c^{\mathrm{s}}, \mathbf{D}$).

Input: $\mathcal{B}_c, \mathcal{S}_c, \hat{\sigma}, \mathcal{P}_c^{\mathrm{b}}, \mathcal{P}_c^{\mathrm{s}}, \mathbf{D}$
Output: $\mathcal{B}_w, \mathcal{S}_w, \sigma, \mathcal{P}_w^{\mathrm{b}}, \mathcal{P}_w^{\mathrm{s}}$
$\mathcal{B}_w \leftarrow \mathcal{B}_c$, $\mathcal{S}_w \leftarrow \mathcal{S}_c$, $\mathcal{P}_w^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}}$, $\mathcal{P}_w^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}}$, $\sigma \leftarrow \hat{\sigma}$;
for any two sellers $s_\alpha, s_\beta \in \mathcal{S}_w, \alpha \neq \beta$ **do**
  if $\sigma(\alpha) = \sigma(\beta)$ **then**
    $U_{\sigma(j)j}^{\mathrm{b}} = D_{\sigma(j)}^{j} - P_{\sigma(j)j}^{\mathrm{b}}, j = \{\alpha, \beta\}$;
    if $U_{\sigma(\alpha)\alpha}^{\mathrm{b}} = U_{\sigma(\beta)\beta}^{\mathrm{b}}$ **then**
      $j' \leftarrow$ randomly selected from $\{\alpha, \beta\}$;
    else
      $j' \leftarrow \arg\min_{j \in \{\alpha,\beta\}} \{U_{\sigma(j)j}^{\mathrm{b}}\}$;
    end if
    $\mathcal{S}_w \leftarrow \mathcal{S}_w \setminus \{s_{j'}\}$;
    $\mathcal{P}_w^{\mathrm{b}} \leftarrow \mathcal{P}_w^{\mathrm{b}} \setminus \{P_{\sigma(j')j'}^{\mathrm{b}}\}, \mathcal{P}_w^{\mathrm{s}} \leftarrow \mathcal{P}_w^{\mathrm{s}} \setminus \{P_{j'}^{\mathrm{s}}\}, \sigma(j') = \emptyset$;
  end if
end for
return $(\mathcal{B}_w, \mathcal{S}_w, \sigma, \mathcal{P}_w^{\mathrm{b}}, \mathcal{P}_w^{\mathrm{s}})$;

$P_{11}^{\mathrm{b}} = P_1^{\mathrm{s}} = A_{-1}^{o}$. Therefore, we have $\hat{\sigma}(1) = 1, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_1\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_1\}, P_{11}^{\mathrm{b}} = P_1^{\mathrm{s}} = A_{-1}^{o}$, $\mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{11}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_1^{\mathrm{s}}\}$. As a similar procedure is applied to examine other sellers in $\mathcal{S}$, we skip the details in the following to avoid repetition;

- s_2: $\mathcal{B}^2 = \{b_4, b_5\}$, $A_{-2}^{o} = 4.5$. Since $D_5^2 < A_{-2}^{o} < D_4^2$, we have $\hat{\sigma}(2) = 4, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_4\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_2\}, P_{42}^{\mathrm{b}} = P_2^{\mathrm{s}} = A_{-2}^{o}$, $\mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{42}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_2^{\mathrm{s}}\}$;
- s_3: $\mathcal{B}^3 = \{b_3, b_5\}$, $A_{-3}^{o} = 3.5$. Since $A_{-3}^{o} < D_3^3 < D_5^3$, we have $\hat{\sigma}(3) = 5, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_5\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_3\}, P_{53}^{\mathrm{b}} = P_3^{\mathrm{s}} = D_3^3$, $\mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{53}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_3^{\mathrm{s}}\}$;
- s_4: $\mathcal{B}^4 = \{b_5\}$, $A_{-4}^{o} = 3.5$. Since $A_{-4}^{o} < A_4$, no buyer wins the service from s_4;
- s_5: $\mathcal{B}^5 = \{b_1\}$, $A_{-5}^{o} = 4$. Since $A_5 = A_{-5}^{o} < D_1^5$, we have $\hat{\sigma}(5) = 1, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_1\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_5\}, P_{15}^{\mathrm{b}} = P_5^{\mathrm{s}} = A_{-5}^{o}, \mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{15}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_5^{\mathrm{s}}\}$;
- s_6: $\mathcal{B}^6 = \{b_1, b_3\}$, $A_{-6}^{o} = 4.5$. Since $A_{-6}^{o} < D_3^6 < D_1^6$, we have $\hat{\sigma}(6) = 1, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_1\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_6\}, P_{16}^{\mathrm{b}} = P_6^{\mathrm{s}} = D_3^6$, $\mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{16}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_6^{\mathrm{s}}\}$;
- s_7: $\mathcal{B}^7 = \{b_2, b_4\}$, $A_{-7}^{o} = 3.5$. Since $A_{-7}^{o} < D_4^7 < D_2^7$, we have $\hat{\sigma}(7) = 2, \mathcal{B}_c \leftarrow \mathcal{B}_c \cup \{b_2\}, \mathcal{S}_c \leftarrow \mathcal{S}_c \cup \{s_7\}, P_{27}^{\mathrm{b}} = P_7^{\mathrm{s}} = D_4^7$, $\mathcal{P}_c^{\mathrm{b}} \leftarrow \mathcal{P}_c^{\mathrm{b}} \cup \{P_{27}^{\mathrm{b}}\}, \mathcal{P}_c^{\mathrm{s}} \leftarrow \mathcal{P}_c^{\mathrm{s}} \cup \{P_7^{\mathrm{s}}\}$;
- In summary, Algorithm 2 gives the following output:
  - $\mathcal{B}_c = \{b_1, b_2, b_4, b_5\}$;
  - $\mathcal{S}_c = \{s_1, s_2, s_3, s_5, s_6, s_7\}$;
  - $\mathcal{P}_c^{\mathrm{b}} = \{P_{11}^{\mathrm{b}} = 4.5, P_{42}^{\mathrm{b}} = 4.5, P_{53}^{\mathrm{b}} = 6, P_{15}^{\mathrm{b}} = 4, P_{16}^{\mathrm{b}} = 9, P_{27}^{\mathrm{b}} = 7\}$;
  - $\mathcal{P}_c^{\mathrm{s}} = \{P_1^{\mathrm{s}} = 4.5, P_2^{\mathrm{s}} = 4.5, P_3^{\mathrm{s}} = 6, P_5^{\mathrm{s}} = 4, P_6^{\mathrm{s}} = 9, P_7^{\mathrm{s}} = 7\}$;
  - $\hat{\sigma} = \{\hat{\sigma}(1) = 1, \hat{\sigma}(2) = 4, \hat{\sigma}(3) = 5, \hat{\sigma}(5) = 1, \hat{\sigma}(6) = 1, \hat{\sigma}(7) = 2\}$.

2) Candidate-Elimination:

The output of Algorithm 2 includes $\hat{\sigma}(1) = \hat{\sigma}(5) = \hat{\sigma}(6) = 1$, which means buyer candidate b_1 wins three seller candidates, i.e., s_1, s_5 and s_6. Algorithm 3 is run subsequently to retain one best seller for buyer b_1. The utilities of b_1 with respect to the three seller candidates are computed and given by: $U_{11}^{\mathrm{b}} = D_1^1 - P_{11}^{\mathrm{b}} = 6 - 4.5 = 1.5$, $U_{15}^{\mathrm{b}} = D_1^5 - P_{15}^{\mathrm{b}} = 5 - 4 = 1$, $U_{16}^{\mathrm{b}} = D_1^6 - P_{16}^{\mathrm{b}} = 10 - 9 = 1$. Clearly, s_1 results in the highest utility for b_1. Thus, s_5 and s_6 are eliminated from the winning seller set. In conclusion, Algorithm 3 produces the following auction outcome:

- $\mathcal{B}_w = \{b_1, b_2, b_4, b_5\}$;
- $\mathcal{S}_w = \{s_1, s_2, s_3, s_5, s_6, s_7\} \setminus \{s_5, s_6\} = \{s_1, s_2, s_3, s_7\}$;
- $\mathcal{P}_w^{\mathrm{b}} = \{P_{11}^{\mathrm{b}}, P_{42}^{\mathrm{b}}, P_{53}^{\mathrm{b}}, P_{15}^{\mathrm{b}}, P_{16}^{\mathrm{b}}, P_{27}^{\mathrm{b}}\} \setminus \{P_{15}^{\mathrm{b}}, P_{16}^{\mathrm{b}}\} = \{P_{11}^{\mathrm{b}}, P_{42}^{\mathrm{b}}, P_{53}^{\mathrm{b}}, P_{27}^{\mathrm{b}}\} = \{P_1^{\mathrm{b}} = 4.5, P_4^{\mathrm{b}} = 4.5, P_3^{\mathrm{b}} = 6, P_2^{\mathrm{b}} = 7\}$;
- $\mathcal{P}_w^{\mathrm{s}} = \{P_1^{\mathrm{s}}, P_2^{\mathrm{s}}, P_3^{\mathrm{s}}, P_5^{\mathrm{s}}, P_6^{\mathrm{s}}, P_7^{\mathrm{s}}\} \setminus \{P_5^{\mathrm{s}}, P_6^{\mathrm{s}}\} = \{P_1^{\mathrm{s}} = 4.5, P_2^{\mathrm{s}} = 4.5, P_3^{\mathrm{s}} = 6, P_7^{\mathrm{s}} = 7\}$;
- $\sigma = \{\sigma(1) = 1, \sigma(2) = 4, \sigma(3) = 5, \sigma(7) = 2\}$.

4.3 Proof of Desirable Properties

In the following, we prove that TIM holds the properties of computational efficiency, individual rationality, budget balance and truthfulness.

Theorem 1. *TIM is computationally efficient.*

Proof. To implement Algorithm 2 for the candidate-determination & pricing stage, we can first sort the sellers in $\mathcal{S}$ with a time complexity of $O(m \log m)$. Then, within the for-loop in Algorithm 2, the median of the ask vector $\mathbf{A}_{-j}$ can be obtained in $O(1)$ time. In Line 4, finding $\mathcal{B}^j$ has a time complexity of $O(n)$, while sorting $\mathcal{B}^j$ to $\mathbb{B}^j$ in Line 12 has a time complexity of $O(n \log n)$. Since there are at most n buyers in $\mathbb{B}^j$, the buyer candidate for each seller can be determined in $O(n)$ time. Thus, each round of the for-loop takes $O(n \log n)$ time. In total, Algorithm 2 has a time complexity of $O(m \log m + mn \log n)$.

In the candidate-elimination stage, the input to Algorithm 3 apparently satisfies $|\mathcal{S}_c| \leq |\mathcal{S}| \leq m$. Hence, the for-loop in Algorithm 3 spends $O(\frac{|\mathcal{S}_c|(|\mathcal{S}_c|-1)}{2}) = O(m^2)$ time. Therefore, the overall time complexity of TIM is polynomial in the order of $O(m^2 + mn \log n)$. □

Theorem 2. *TIM is individually rational.*

Proof. In Algorithm 2, there are two cases for buyer b_i to be selected as a buyer candidate ($b_i \in \mathcal{B}_c$) and for seller s_j to become a seller candidate ($s_j \in \mathcal{S}_c$).

- $|\mathcal{B}^j| = 1$: This case results in a buyer candidate b_i and a seller candidate s_j only if $D_i^j \geq A_{-j}^{o} \geq A_j$. The clearing price charged to buyer b_i and the payment to seller s_j are both A_{-j}^{o}.
- $|\mathcal{B}^j| > 1$: In this case, buyer b_i must have a bid D_i^j not less than A_{-j}^{o} and D_i^j is the highest bid for seller s_j, which implies that $A_{-j}^{o} \leq D_i^j$ and $D_{i_{(2)}}^j \leq D_i^j$. The clearing price and payment are the maximum of $D_{i_{(2)}}^j$ and A_{-j}^{o}. Besides, every qualified buyer in $\mathcal{B}^j$ must bid not less than A_j.

As seen, each buyer candidate determined in Algorithm 2 is never charged a price greater than its bid, while each seller candidate is rewarded a payment not less than its ask, which ensures individual rationality for both buyers and sellers.

It is possible for Algorithm 2 to assign multiple sellers to one buyer candidate $b_i \in \mathcal{B}_c$. Algorithm 3 eliminates redundant sellers and keeps only one best seller providing buyer b_i the highest utility. It is evident that this procedure does not change the charging price P_{ij}^{b} to the winning buyers. Thus, the buyers in $\mathcal{B}_w$ after the candidate-elimination stage

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

still satisfy individual rationality. On the other hand, a seller candidate s_j is either a winning seller with the same payment P_j^{s} determined in Algorithm 2, or eliminated with zero payment at zero cost. Therefore, individual rationality still holds for the winning sellers in $\mathcal{S}_w$ resulted from Algorithm 3.

In conclusion, TIM is individually rational. □

Theorem 3. *TIM is budget-balanced.*

Proof. After the candidate-elimination stage, every winning buyer $b_i \in \mathcal{B}_w$ has only one winning seller $s_j \in \mathcal{S}_w$. Considering this one-to-one mapping between $\mathcal{B}_w$ and $\mathcal{S}_w$, we have $|\mathcal{B}_w| = |\mathcal{S}_w|$. The clearing price and payment satisfy $P_i^{\mathrm{b}} = P_j^{\mathrm{s}}$ for each winning buyer b_i and its assigned matching seller s_j, i.e., $\sigma(j) = i$. Therefore, it can be easily shown that

$$\sum_{b_i \in \mathcal{B}_w} P_i^{\mathrm{b}} - \sum_{s_j \in \mathcal{S}_w} P_j^{\mathrm{s}} = \sum_{s_j \in \mathcal{S}_w} \left(P_{\sigma(j)}^{\mathrm{b}} - P_j^{\mathrm{s}} \right) = 0$$

which completes the proof. □

Theorem 4. *TIM is truthful.*

Theorem 4 can be proved by the following Lemma 1 and Lemma 2, which show that TIM is truthful for sellers and buyers, respectively.

Lemma 1. *TIM is truthful for sellers.*

Proof. To compare the auction outcomes with truthful or any arbitrary asks, we add tilde in the notations for the general (untruthful) cases. To prove Lemma 1, we divide the seller set $\mathcal{S}$ into three subsets: $\mathcal{S}_w$, $\mathcal{S}_c \setminus \mathcal{S}_w$ and $\mathcal{S} \setminus \mathcal{S}_c$.

1) For seller $s_j \in \mathcal{S}_w$: There are two cases when s_j asks untruthfully.
   - Seller $s_j \notin \mathcal{S}_w$: $\widetilde{U}_j^{\mathrm{s}} = 0 \le U_j^{\mathrm{s}}$.
   - Seller $s_j \in \mathcal{S}_w$: There are four subcases.
     - $|\mathcal{B}^j| = 1$ and $|\widetilde{\mathcal{B}}^j| = 1$: $\widetilde{P}_j^{\mathrm{s}} = P_j^{\mathrm{s}} = A_{-j}^o$.
     - $|\mathcal{B}^j| = 1$ and $|\widetilde{\mathcal{B}}^j| > 1$: $\widetilde{A}_j < C_j$, $\widetilde{P}_j^{\mathrm{s}} = P_j^{\mathrm{s}} = A_{-j}^o$.
     - $|\mathcal{B}^j| > 1$ and $|\widetilde{\mathcal{B}}^j| = 1$: $\widetilde{A}_j > C_j$, $\widetilde{P}_j^{\mathrm{s}} = P_j^{\mathrm{s}} = A_{-j}^o$.
     - $|\mathcal{B}^j| > 1$ and $|\widetilde{\mathcal{B}}^j| > 1$: $\widetilde{P}_j^{\mathrm{s}} = P_j^{\mathrm{s}} = A_{-j}^o$ or $D_{i_{(2)}}^j$.

     Thus, we have $\widetilde{U}_j^{\mathrm{s}} = \widetilde{P}_j^{\mathrm{s}} - C_j = P_j^{\mathrm{s}} - C_j = U_j^{\mathrm{s}}$.
2) For seller $s_j \in \mathcal{S}_c \setminus \mathcal{S}_w$: s_j cannot be in $\mathcal{S}_w$ no matter what value $\widetilde{A}_j$ is. Thus, we have $\widetilde{U}_j^{\mathrm{s}} = 0 = U_j^{\mathrm{s}}$.
3) For seller $s_j \in \mathcal{S} \setminus \mathcal{S}_c$: There are two cases when s_j asks untruthfully.
   - Seller $s_j \notin \mathcal{S}_w$: $\widetilde{U}_j^{\mathrm{s}} = 0 = U_j^{\mathrm{s}}$.
   - Seller $s_j \in \mathcal{S}_w$: Let b_i be the buyer that wins s_j at price $\widetilde{P}_{ij}^{\mathrm{b}} = \widetilde{P}_j^{\mathrm{s}}$ when s_j asks untruthfully. Here, we consider two subcases to compare the utilities.
     - $|\mathcal{B}^j| = 0$: We know that C_j is greater than any bid of the buyers. According to Theorem 2, the buyers satisfy individual rationality so that $D_i^j \ge \widetilde{P}_{ij}^{\mathrm{b}}$. Thus, we have $C_j > D_i^j \ge \widetilde{P}_{ij}^{\mathrm{b}} = \widetilde{P}_j^{\mathrm{s}}$, and $\widetilde{U}_j^{\mathrm{s}} = \widetilde{P}_j^{\mathrm{s}} - C_j < 0 = U_j^{\mathrm{s}}$.
     - $|\mathcal{B}^j| = 1$, and $A_{-j}^o < A_j = C_j \le D_i^j$: The reason that $s_j \in \mathcal{S}_w$ when s_j asks untruthfully is that $\widetilde{A}_j \le A_{-j}^o$ and/or $|\widetilde{\mathcal{B}}^j| \ge 2$. Additionally, buyer b_i needs to pay $\widetilde{P}_{ij}^{\mathrm{b}} = A_{-j}^o$ or $\widetilde{D}_{i_{(2)}}^j$, where $\widetilde{D}_{i_{(2)}}^j$ is the second highest bid of the buyers in $\widetilde{\mathcal{B}}^j$. However, we know that $A_{-j}^o < A_j = C_j$ and $\widetilde{D}_{i_{(2)}}^j < A_j = C_j$. Thus, we have $\widetilde{P}_j^{\mathrm{s}} = \widetilde{P}_{ij}^{\mathrm{b}} < C_j$, and $\widetilde{U}_j^{\mathrm{s}} = \widetilde{P}_j^{\mathrm{s}} - C_j < 0 = U_j^{\mathrm{s}}$.

As seen, telling truth provides the maximum utility for each seller in TIM. Therefore, truthful asking is a weakly dominant strategy for sellers in TIM, which proves Lemma 1. □

Lemma 2. *TIM is truthful for buyers.*

Proof. Similarly, we add tilde in the notations for the outcomes with general (untruthful) bids. Here, we divide the buyer set $\mathcal{B}$ into two subsets: $\mathcal{B}_w$ and $\mathcal{B} \setminus \mathcal{B}_w$.

1) For buyer $b_i \in \mathcal{B}_w$: Assuming buyer b_i wins seller s_j with truthful bidding, we consider three cases when b_i bids untruthfully.
   - Buyer b_i loses: $\widetilde{U}_i^{\mathrm{b}} = 0 \le U_i^{\mathrm{b}}$.
   - Buyer b_i still wins s_j: According to Algorithm 2, since the clearing price is independent of the bid of buyer b_i, we have $\widetilde{P}_{ij}^{\mathrm{b}} = P_{ij}^{\mathrm{b}}$, and thus $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij}^{\mathrm{b}} = V_i^j - \widetilde{P}_{ij}^{\mathrm{b}} = U_i^{\mathrm{b}}$.
   - Buyer b_i wins seller $s_{j'}$: There are two subcases.
     - Seller $s_{j'} \in \mathcal{S}_c$ when buyer b_i bids truthfully:
       * $\hat{\sigma}(j') = i$ with truthful bid $\mathbf{D}_i$: According to Algorithm 2, we have $\widetilde{P}_{ij'}^{\mathrm{b}} = P_{ij'}^{\mathrm{b}}$. The reason that buyer b_i wins seller s_j when it bids truthfully is $U_{ij}^{\mathrm{b}} \ge U_{ij'}^{\mathrm{b}}$, where $U_{ij'}^{\mathrm{b}} = V_i^{j'} - P_{ij'}^{\mathrm{b}}$. As a result, we have $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij'}^{\mathrm{b}} = V_i^{j'} - \widetilde{P}_{ij'}^{\mathrm{b}} = U_{ij'}^{\mathrm{b}} \le U_{ij}^{\mathrm{b}} = U_i^{\mathrm{b}}$.
       * $\hat{\sigma}(j') \ne i$ with truthful bid $\mathbf{D}_i$: According to Algorithm 2, there exists a buyer candidate i' with $D_{i'}^{j'} \ge D_i^{j'} = V_i^{j'}$. When bidding untruthfully, buyer b_i wins seller $s_{j'}$ with price $\widetilde{P}_{ij'}^{\mathrm{b}} \ge D_{i'}^{j'} \ge V_i^{j'}$. Thus, $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij'}^{\mathrm{b}} = V_i^{j'} - \widetilde{P}_{ij'}^{\mathrm{b}} \le 0 \le U_i^{\mathrm{b}}$.
     - Seller $s_{j'} \notin \mathcal{S}_c$ when buyer b_i bids truthfully:
       * Truthful bid $D_i^{j'}$ is less than ask $A_{j'}$: When buyer b_i bids untruthfully, it pays $\widetilde{P}_{ij'}^{\mathrm{b}}$ to win seller $s_{j'}$. According to Theorem 2, the sellers satisfy individual rationality so that $\widetilde{P}_{ij'}^{\mathrm{b}} \ge A_{j'} > D_i^{j'} = V_i^{j'}$. As a result, $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij'}^{\mathrm{b}} = V_i^{j'} - \widetilde{P}_{ij'}^{\mathrm{b}} < 0 \le U_i^{\mathrm{b}}$.
       * Truthful bid $D_i^{j'}$ is not less than ask $A_{j'}$, and $A_{-j'}^o > D_i^{j'} = V_i^{j'}$: When buyer b_i bids untruthfully, it pays $\widetilde{P}_{ij'}^{\mathrm{b}} = A_{-j'}^o$ to win seller $s_{j'}$. As a result, we still have $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij'}^{\mathrm{b}} = V_i^{j'} - \widetilde{P}_{ij'}^{\mathrm{b}} < 0 \le U_i^{\mathrm{b}}$.
2) For buyer $b_i \in \mathcal{B} \setminus \mathcal{B}_w$: Obviously, $U_i^{\mathrm{b}} = 0$. There are two cases when b_i bids untruthfully.
   - Buyer b_i still loses: $\widetilde{U}_i^{\mathrm{b}} = 0 = U_i^{\mathrm{b}}$.
   - Buyer b_i wins seller s_j: There are three subcases.
     - Truthful bid D_i^j is less than ask A_j: When buyer b_i with untruthful bid wins seller s_j at price $\widetilde{P}_{ij}$, the seller should satisfy individual rationality according to Theorem 2, so that $\widetilde{P}_{ij}^{\mathrm{b}} \ge A_j > D_i^j = V_i^j$. As a result, we have $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij}^{\mathrm{b}} = V_i^j - \widetilde{P}_{ij}^{\mathrm{b}} < 0 = U_i^{\mathrm{b}}$.
     - Truthful bid D_i^j is not less than ask A_j, and $A_{-j}^o > D_i^j = V_i^j$: When buyer b_i bids untruthfully, it wins seller s_j at price $\widetilde{P}_{ij}^{\mathrm{b}}$. According to Algorithm 2, we

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

know that $\widetilde{P}^{\mathrm{b}}_{ij} \geq A^o_{-j} > V^j_i$, and thus $\widetilde{U}^{\mathrm{b}}_i = \widetilde{U}^{\mathrm{b}}_{ij} = V^j_i - \widetilde{P}^{\mathrm{b}}_{ij} < 0 = U^{\mathrm{b}}_i$.

  - Truthful bid D^j_i is not less than ask A_j, $D^j_i > A^o_{-j}$, and there exists buyer $b_{i'}$ with bid $D^j_{i'} \geq D^j_i = V^j_i$: When buyer b_i bids untruthfully, it pays $\widetilde{P}^{\mathrm{b}}_{ij} \geq D^j_{i'}$ to win seller s_j. Thus, we have $\widetilde{U}^{\mathrm{b}}_i = \widetilde{U}^{\mathrm{b}}_{ij} = V^j_i - \widetilde{P}^{\mathrm{b}}_{ij} \leq 0 = U^{\mathrm{b}}_i$.

Therefore, telling truth maximizes the utility of each buyer in TIM and Lemma 2 is proved. □

5 Efficient Design of Auction (EDA)

In Section 4, we introduce the truthful incentive mechanism TIM, which is proved to hold the desirable properties of computational efficiency, individual rationality, budget balance and truthfulness. Unfortunately, a double auction mechanism cannot further achieve system efficiency (e.g., maximizing social welfare) at the same time [4]. In this section, we present another more efficient design of auction EDA, which slightly relaxes the truthfulness constraint and ensures truthfulness in a weak sense. We follow the same sequence as Section 4, giving the detailed algorithm of EDA, then an illustrative example, and the analysis of its properties at last.

5.1 Details of EDA

As illustrated in Section 4.2, the buyer candidate b_1 wins three seller candidates (s_1, s_5 and s_6), and only the seller candidate s_1 becomes a winning seller in $\mathcal{S}_w$. The other seller candidates s_5 and s_6 are not successfully matched to potential buyers, which degrades the system efficiency and thus jeopardizes the utilization of MCC resources. EDA improves the system efficiency by involving randomness and more uncertainty in the auction mechanism.

The details of EDA are given in Algorithm 4. According to EDA, the auctioneer first constructs a randomly ordered list $\mathbb{S}$ of the seller set $\mathcal{S}$, then determines the winning buyer for each seller $s_j \in \mathcal{S}$ following the order of $\mathbb{S}$. Recall that $\mathbf{A}_{-j}$ denotes the ask vector without the ask of s_j, and A^o_{-j} is the median ask of $\mathbf{A}_{-j}$. Let $\mathcal{B}^j$ be the sublist of buyers in $\mathcal{B} \setminus \mathcal{B}_w$ with bids not less than the ask of seller s_j. Then, Algorithm 4 handles two different cases as follows.

- $|\mathcal{B}^j| = 1$: Assume that b_i is the buyer in $\mathcal{B}^j$ with bid D^j_i. Among all the bids of buyers for seller s_j, denoted by $\mathbf{D}^j$, let $D^j_{i^*}$ be the highest bid in $\mathbf{D}^j$ that is less than D^j_i, and P^{bs} be the maximum of A^o_{-j} and $D^j_{i^*}$. If $D^j_i \geq P^{\mathrm{bs}} \geq A_j$, then b_i is added into $\mathcal{B}_w$ and assigned a clearing price P^{bs}, while s_j is added into $\mathcal{S}_w$ with P^{bs} as the clearing payment. If $D^j_i < P^{\mathrm{bs}}$ or $A_j > P^{\mathrm{bs}}$, b_i cannot win the service of s_j.
- $|\mathcal{B}^j| > 1$: If the highest bid of the buyers in $\mathcal{B}^j$ is less than A^o_{-j}, no buyer obtains the service of s_j; otherwise, the buyer $b_i \in \mathcal{B}^j$ with the highest bid (or a randomly selected buyer when there is a tie) wins s_j. Then, buyer b_i and seller s_j are added into $\mathcal{B}_w$ and $\mathcal{S}_w$, respectively. Let $D^j_{i_{(2)}}$ be the second highest bid of the buyers in $\mathcal{B}^j$, and $D^j_{i^*}$ be the highest bid in $\mathbf{D}^j$ that is less than D^j_i. Note that $\mathbf{D}^j$ is the bid vector of all buyers for seller s_j, while $\mathcal{B}^j$ only includes the remaining buyers that have bids not less than A_j and are still competing. The clearing price charged to b_i and the payment rewarded to s_j are both the maximum of A^o_{-j} and $D^j_{i^*}$.

Algorithm 4 EDA($\mathcal{B}, \mathcal{S}, \mathbf{D}, \mathbf{A}$).

Input: $\mathcal{B}$, $\mathcal{S}$, $\mathbf{D}$, $\mathbf{A}$
Output: $\mathcal{B}_w, \mathcal{S}_w, \sigma, \mathcal{P}^{\mathrm{b}}_w, \mathcal{P}^{\mathrm{s}}_w$

1: $\mathcal{B}_w \leftarrow \emptyset$, $\mathcal{S}_w \leftarrow \emptyset$, $\mathcal{P}^{\mathrm{b}}_w \leftarrow \emptyset$, $\mathcal{P}^{\mathrm{s}}_w \leftarrow \emptyset$;
2: Randomly order $\mathcal{S}$ to obtain $\mathbb{S} = \{s_{\mathrm{r}(1)}, s_{\mathrm{r}(2)}, \ldots, s_{\mathrm{r}(m)}\}$;
3: **for** $s_j \in \mathcal{S}$ in the order of $\mathbb{S}$ **do**
4: Find the median ask A^o_{-j} of the ask vector $\mathbf{A}_{-j}$;
5: $\mathcal{B}^j = \{b_i : D^j_i \geq A_j, \forall b_i \in \mathcal{B} \setminus \mathcal{B}_w\}$;
6: **if** $|\mathcal{B}^j| = 1$ **then**
7: Find highest bid $D^j_{i^*} \in \mathbf{D}^j$ that is not greater than D^j_i;
8: $P^{\mathrm{bs}} = \max\{A^o_{-j}, D^j_{i^*}\}$;
9: **if** $D^j_i \geq P^{\mathrm{bs}}$ and $A_j \leq P^{\mathrm{bs}}$ **then**
10: $\sigma(j) = i, \mathcal{B}_w \leftarrow \mathcal{B}_w \cup \{b_i\}, \mathcal{S}_w \leftarrow \mathcal{S}_w \cup \{s_j\}$;
11: $P^{\mathrm{b}}_{ij} = P^{\mathrm{s}}_j = P^{\mathrm{bs}}$;
12: $\mathcal{P}^{\mathrm{b}}_w \leftarrow \mathcal{P}^{\mathrm{b}}_w \cup \{P^{\mathrm{b}}_{ij}\}, \mathcal{P}^{\mathrm{s}}_w \leftarrow \mathcal{P}^{\mathrm{s}}_w \cup \{P^{\mathrm{s}}_j\}$;
13: **end if**
14: **else if** $|\mathcal{B}^j| > 1$ **then**
15: Sort $\mathcal{B}^j$ to get $\mathbb{B}^j$ such that $D^j_{i_{(1)}} \geq D^j_{i_{(2)}} \geq \cdots$;
16: Find highest bid $D^j_{i^*} \in \mathbf{D}^j$ that is not greater than $D^j_{i_{(1)}}$;
17: **if** $D^j_{i_{(1)}} \geq A^o_{-j}$ **then**
18: **if** the first t ($t \geq 2$) bids of $\mathbb{B}^j$ are the same **then**
19: Randomly select b_i from first t buyers of $\mathbb{B}^j$;
20: **else**
21: Select first buyer b_i of $\mathbb{B}^j$ with the highest bid;
22: **end if**
23: $\sigma(j) = i, \mathcal{B}_w \leftarrow \mathcal{B}_w \cup \{b_i\}, \mathcal{S}_w \leftarrow \mathcal{S}_w \cup \{s_j\}$;
24: $P^{\mathrm{b}}_{ij} = P^{\mathrm{s}}_j = \max\{A^o_{-j}, D^j_{i^*}\}$;
25: $\mathcal{P}^{\mathrm{b}}_w \leftarrow \mathcal{P}^{\mathrm{b}}_w \cup \{P^{\mathrm{b}}_{ij}\}, \mathcal{P}^{\mathrm{s}}_w \leftarrow \mathcal{P}^{\mathrm{s}}_w \cup \{P^{\mathrm{s}}_j\}$;
26: **end if**
27: **end if**
28: **end for**
29: **return** $(\mathcal{B}_w, \mathcal{S}_w, \sigma, \mathcal{P}^{\mathrm{b}}_w, \mathcal{P}^{\mathrm{s}}_w)$;

As seen in Line 5, the winning buyers in $\mathcal{B}_w$ are eliminated in the buyer sublist $\mathcal{B}^j$. Thus, a winning buyer $b_i \in \mathcal{B}_w$ will not compete with other buyers for the remaining sellers. Hence, it is not necessary to include candidate elimination as in TIM and the system efficiency is improved as a result.

5.2 A Walk-Through Example

For easy comparison, the following illustrative example for EDA is also based on the bid matrix in Table 2(a) and the ask vector in Table 2(b). Applying Algorithm 4, we have

- $\mathcal{B}_w = \emptyset, \mathcal{S}_w = \emptyset, \mathcal{P}^{\mathrm{b}}_w = \emptyset, \mathcal{P}^{\mathrm{s}}_w = \emptyset$;
- Suppose that the seller set $\mathcal{S}$ is randomly ordered to $\mathbb{S} = \{s_3, s_5, s_7, s_1, s_4, s_6, s_2\}$;
- s_3: Similar to the walk-through example for TIM, we only provide the details for this case to save space. First, we have the ask vector $\mathbf{A}_{-3} = \{3, 2, 6, 4, 1, 7\}$, and its median $A^o_{-3} = 3.5$. Since $b_3 \in \mathcal{B} \setminus \mathcal{B}_w$ with $D^3_3 > A_3$, and $b_5 \in \mathcal{B} \setminus \mathcal{B}_w$ with $D^3_5 > A_3$, we obtain $\mathcal{B}^3 = \{b_3, b_5\}$. As $|\mathcal{B}^3| > 1$, $D^3_{5^*} = 6$ and $D^3_5 > D^3_{5^*} = D^3_3 > A^o_{-3}$, b_5 is selected as the winning buyer for s_3 with $P^{\mathrm{b}}_{53} = P^{\mathrm{s}}_3 = D^3_{5^*}$. Therefore, we have $\sigma(3) = 5, \mathcal{B}_w \leftarrow \mathcal{B}_w \cup \{b_5\}, \mathcal{S}_w \leftarrow \mathcal{S}_w \cup \{s_3\}, P^{\mathrm{b}}_{53} = P^{\mathrm{s}}_3 = D^3_{5^*}$, $\mathcal{P}^{\mathrm{b}}_w \leftarrow \mathcal{P}^{\mathrm{b}}_w \cup \{P^{\mathrm{b}}_{53}\}, \mathcal{P}^{\mathrm{s}}_w \leftarrow \mathcal{P}^{\mathrm{s}}_w \cup \{P^{\mathrm{s}}_3\}$;
- s_5: $\mathcal{B}^5 = \{b_1\}$. Since $D^5_{1^*} = 0, A^o_{-5} = 4, P^{\mathrm{bs}} = A^o_{-5}$ and $A_5 = P^{\mathrm{bs}} < D^5_1$, we have $\sigma(5) = 1, \mathcal{B}_w \leftarrow \mathcal{B}_w \cup$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

$\{b_1\}, \mathcal{S}_w \leftarrow \mathcal{S}_w \cup \{s_5\}, P^{\rm b}_{15} = P^{\rm s}_5 = P^{\rm bs}, \mathcal{P}^{\rm b}_w \leftarrow \mathcal{P}^{\rm b}_w \cup \{P^{\rm b}_{15}\}, \mathcal{P}^{\rm s}_w \leftarrow \mathcal{P}^{\rm s}_w \cup \{P^{\rm s}_5\}$;

- s_7: $\mathcal{B}^7 = \{b_2, b_4\}$. Since $A^o_{-7} = 3.5 < D^7_4 = D^7_{2*} < D^7_2$, we have $\sigma(7) = 2, \mathcal{B}_w \leftarrow \mathcal{B}_w \cup \{b_2\}, \mathcal{S}_w \leftarrow \mathcal{S}_w \cup \{s_7\}, P^{\rm b}_{27} = P^{\rm s}_7 = D^7_{2*}$, $\mathcal{P}^{\rm b}_w \leftarrow \mathcal{P}^{\rm b}_w \cup \{P^{\rm b}_{27}\}, \mathcal{P}^{\rm s}_w \leftarrow \mathcal{P}^{\rm s}_w \cup \{P^{\rm s}_7\}$;
- s_1: $\mathcal{B}^1 = \emptyset$. No buyer wins the service of s_1;
- s_4: $\mathcal{B}^4 = \emptyset$. No buyer wins the service of s_4;
- s_6: $\mathcal{B}^6 = \{b_3\}$. Since $D^6_{3*} = 0, A^o_{-6} = 4.5, P^{\rm bs} = A^o_{-6}$ and $A_6 < P^{\rm bs} < D^6_3$, we have $\sigma(6) = 3, \mathcal{B}_w \leftarrow \mathcal{B}_w \cup \{b_3\}, \mathcal{S}_w \leftarrow \mathcal{S}_w \cup \{s_6\}, P^{\rm b}_{36} = P^{\rm s}_6 = P^{\rm bs}$, $\mathcal{P}^{\rm b}_w \leftarrow \mathcal{P}^{\rm b}_w \cup \{P^{\rm b}_{36}\}, \mathcal{P}^{\rm s}_w \leftarrow \mathcal{P}^{\rm s}_w \cup \{P^{\rm s}_6\}$;
- s_2: $\mathcal{B}^2 = \{b_4\}$, since $D^2_{4*} = 2, A^o_{-2} = 4.5, P^{\rm bs} = A^o_{-2}$ and $A_2 < P^{\rm bs} < D^2_4$, we have $\sigma(2) = 4, \mathcal{B}_w \leftarrow \mathcal{B}_w \cup \{b_4\}, \mathcal{S}_w \leftarrow \mathcal{S}_w \cup \{s_2\}, P^{\rm b}_{42} = P^{\rm s}_2 = P^{\rm bs}$, $\mathcal{P}^{\rm b}_w \leftarrow \mathcal{P}^{\rm b}_w \cup \{P^{\rm b}_{42}\}, \mathcal{P}^{\rm s}_w \leftarrow \mathcal{P}^{\rm s}_w \cup \{P^{\rm s}_2\}$;
- In summary, Algorithm 4 gives the auction outcome:
  - $\mathcal{B}_w = \{b_1, b_2, b_3, b_4, b_5\}$;
  - $\mathcal{S}_w = \{s_2, s_3, s_5, s_6, s_7\}$;
  - $\mathcal{P}^{\rm b}_w = \{P^{\rm b}_{42} = 4.5, P^{\rm b}_{53} = 6, P^{\rm b}_{15} = 4, P^{\rm b}_{36} = 4.5, P^{\rm b}_{27} = 7\}$;
  - $\mathcal{P}^{\rm s}_w = \{P^{\rm s}_2 = 4.5, P^{\rm s}_3 = 6, P^{\rm s}_5 = 4, P^{\rm s}_6 = 4.5, P^{\rm s}_7 = 7\}$;
  - $\sigma = \{\sigma(2) = 4, \sigma(3) = 5, \sigma(5) = 1, \sigma(6) = 3, \sigma(7) = 2\}$;

As seen, the system efficiency of EDA is improved to 5 successful matchings as opposed to 4 in TIM. It is worth noting that the performance of Algorithm 4 may depend on the ordered list $\mathbb{S}$. The auctioneer can generate multiple random lists to obtain an outcome of the highest system efficiency.

5.3 Proof of Desirable Properties

Similar to TIM, EDA also satisfies computational efficiency, individual rationality and budget balance. Nonetheless, as mentioned earlier, EDA achieves a higher system efficiency but at the cost of weaker truthfulness.

Theorem 5. *EDA is computationally efficient.*

Proof. According to the ask vector of sellers, we can first sort the sellers in $\mathcal{S}$ in a non-decreasing order of their asks, which has a time complexity of $O(m \log m)$. In each round of the for-loop in Algorithm 4, we can spend $O(1)$ time to determine the median of the ask vector $\mathbf{A}_{-j}$, and $O(n)$ time to obtain $\mathcal{B}^j$. In addition, finding $D^j_{i^*}$ runs in $O(n)$ time. When $|\mathcal{B}^j| > 1$, sorting $\mathcal{B}^j$ to $\mathbb{B}^j$ further takes $O(n \log n)$ time, while the winning buyer can be selected in $O(n)$ time since there are at most n buyers in $\mathbb{B}^j$. Thus, each round of the for-loop spends $O(n \log n)$ time. The overall time complexity of Algorithm 4 is then $O(m \log m + mn \log n)$. ☐

Theorem 6. *EDA is individually rational.*

Proof. Consider a pair of matched buyer and seller, b_i and s_j, in the winning sets $\mathcal{B}_w$ and $\mathcal{S}_w$. There are two cases for Algorithm 4 to assign this matching.

- $|\mathcal{B}^j| = 1$: In this case, b_i wins the service of s_j only when $D^j_i \geq P^{\rm bs} \geq A_j$. The clearing price and payment are both $P^{\rm bs}$, which is not greater than the bid of b_i and not less than the ask of s_j.
- $|\mathcal{B}^j| > 1$: In this case, b_i must have a bid D^j_i not less than A^o_{-j} and D^j_i is the highest bid in $\mathcal{B}^j$. Thus, we have $A^o_{-j} \leq D^j_i$ and $D^j_{i^*} \leq D^j_i$. Since the clearing price and payment are the maximum of A^o_{-j} and $D^j_{i^*}$, the preceding conditions imply that the price charged to buyer b_i is not greater than its bid D^j_i. On the other hand, we have $D^j_{i^*} \geq A_j$. This can be easily inferred from the definition that $D^j_{i^*}$ is the highest bid not greater than D^j_i in $\mathbf{D}^j$, and the fact that D^j_i is the highest bid in $\mathcal{B}^j$. Since $|\mathcal{B}^j| > 1$ and every buyer in $\mathcal{B}^j$ has a bid not less than A_j, we have $D^j_i = D^j_{i_{(1)}} \geq D^j_{i^*} \geq D^j_{i_{(2)}} \geq A_j$. Here, $D^j_{i_{(1)}}$ and $D^j_{i_{(2)}}$ are the first and second highest bids in $\mathcal{B}^j$, respectively. Thus, the clearing payment to s_j should be not less than A_j.

As seen in both cases, each winning buyer determined in Algorithm 4 never pays more than its bid, while each winning seller is paid not less than its ask, which guarantees individual rationality for buyers and sellers. ☐

Theorem 7. *EDA is budget-balanced.*

Proof. In the auction outcome, EDA keeps one-to-one mapping between $\mathcal{B}_w$ and $\mathcal{S}_w$ as TIM, while the clearing price charged to a winning buyer is equal to the clearing payment to the matched winning seller. Thus, we can easily show that EDA is budget-balanced by applying the proof of budget balance of TIM. ☐

5.3.1 Truthfulness of EDA for Sellers

Theorem 8. *EDA is truthful for sellers.*

Proof. Similar to the proof of truthfulness of TIM for sellers, we consider two subsets of the seller set $\mathcal{S}$: $\mathcal{S}_w$ and $\mathcal{S} \setminus \mathcal{S}_w$.

1) For seller $s_j \in \mathcal{S}_w$: There are two cases when seller s_j asks untruthfully.
   - Seller $s_j \notin \mathcal{S}_w$: $\widetilde{U}^{\rm s}_j = 0 \leq U^{\rm s}_j$.
   - Seller $s_j \in \mathcal{S}_w$: There are four subcases with respect to $|\mathcal{B}^j| = 1$ or $|\mathcal{B}^j| > 1$, and $|\widetilde{\mathcal{B}}^j| = 1$ or $|\widetilde{\mathcal{B}}^j| > 1$. In all these subcases, we can infer from Algorithm 4 that $\widetilde{P}^{\rm s}_j = P^{\rm s}_j = \max\{A^o_{-j}, D^j_{i^*}\}$. Therefore, we have $\widetilde{U}^{\rm s}_j = \widetilde{P}^{\rm s}_j - C_j = P^{\rm s}_j - C_j = U^{\rm s}_j$.
2) For seller $s_j \in \mathcal{S} \setminus \mathcal{S}_w$: There are two cases when seller s_j asks untruthfully.
   - Seller $s_j \notin \mathcal{S}_w$: $\widetilde{U}^{\rm s}_j = 0 = U^{\rm s}_j$.
   - Seller $s_j \in \mathcal{S}_w$: Let b_i be the buyer that wins seller s_j at price $\widetilde{P}^{\rm b}_{ij} = \widetilde{P}^{\rm s}_j$. There are two subcases to examine the utilities of different asks.
     - $|\mathcal{B}^j| = 0$: That is $D^j_i < A_j = C_j$. According to Theorem 6, the buyers satisfy individual rationality, so that $D^j_i \geq \widetilde{P}^{\rm b}_{ij}$. As a result, we have $C_j > D^j_i \geq \widetilde{P}^{\rm b}_{ij} = \widetilde{P}^{\rm s}_j$, and $\widetilde{U}^{\rm s}_j = \widetilde{P}^{\rm s}_j - C_j < 0 = U^{\rm s}_j$.
     - $|\mathcal{B}^j| = 1$, and $\max\{A^o_{-j}, D^j_{i^*}\} < A_j = C_j \leq D^j_i$: When seller s_j asks untruthfully, $s_j \in \mathcal{S}_w$ only if $\widetilde{A}_j \leq \max\{A^o_{-j}, D^j_{i^*}\}$ and/or $|\widetilde{\mathcal{B}}^j| \geq 2$. The price charged to buyer b_i is then $\widetilde{P}^{\rm b}_{ij} = \max\{A^o_{-j}, D^j_{i^*}\}$. Since $\max\{A^o_{-j}, D^j_{i^*}\} < A_j = C_j$, we have $\widetilde{P}^{\rm s}_j = \widetilde{P}^{\rm b}_{ij} < C_j$, and $\widetilde{U}^{\rm s}_j = \widetilde{P}^{\rm s}_j - C_j < 0 = U^{\rm s}_j$.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

As seen, telling truth is a weakly dominant strategy for sellers in EDA, which proves Theorem 8. □

5.3.2 *Truthfulness of EDA for Buyers*

Theorem 8 shows that EDA is strongly truthful for sellers. However, EDA cannot maintain strong truthfulness for buyers. Fortunately, the following analysis also shows that the uncertainty and randomness in EDA introduce high risks and difficulties for buyers to bid untruthfully to improve their utilities. A truthful bid can always lead to a non-negative utility, whereas bidding untruthfully may result in a non-positive utility. Therefore, EDA can ensure truthfulness for buyers in a weak sense. As further demonstrated in the experiment results, EDA achieves truthfulness in expectation.

When bidding truthfully, a buyer in $\mathcal{B}$ can fall into either subset $\mathcal{B}_w$ or subset $\mathcal{B} \setminus \mathcal{B}_w$. First, we prove that telling truth is a weakly dominate strategy for buyer $b_i \in \mathcal{B} \setminus \mathcal{B}_w$ as it maximizes the buyer's utility. There are two cases when b_i bids untruthfully.

- Buyer b_i still loses: $\widetilde{U}_i^{\mathrm{b}} = 0 = U_i^{\mathrm{b}}$.
- Buyer b_i wins seller s_j: There are three subcases.
  - Truthful bid D_i^j is less than ask A_j: According to Theorem 6, the sellers satisfy individual rationality, so we have $\widetilde{P}_{ij}^{\mathrm{b}} \geq A_j > D_i^j = V_i^j$. As a result, we have $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij}^{\mathrm{b}} = V_i^j - \widetilde{P}_{ij}^{\mathrm{b}} < 0 = U_i^{\mathrm{b}}$.
  - Truthful bid D_i^j is not less than ask A_j, and $A_{-j}^o > D_i^j = V_i^j$: When buyer b_i bids untruthfully, it wins seller s_j at price $\widetilde{P}_{ij}^{\mathrm{b}}$. According to Algorithm 4, we know that $\widetilde{P}_{ij}^{\mathrm{b}} \geq A_{-j}^o > V_i^j$, and thus $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij}^{\mathrm{b}} = V_i^j - \widetilde{P}_{ij}^{\mathrm{b}} < 0 = U_i^{\mathrm{b}}$.
  - Truthful bid D_i^j is not less than ask A_j, $D_i^j > A_{-j}^o$, and there exists a buyer $b_{i'} \in \mathcal{B} \setminus \mathcal{B}_w$ with bid $D_{i'}^j \geq D_i^j = V_i^j$: When buyer b_i bids untruthfully, it pays $\widetilde{P}_{ij}^{\mathrm{b}} \geq D_{i'}^j$ to win seller s_j. As a result, we still have $\widetilde{U}_i^{\mathrm{b}} = \widetilde{U}_{ij}^{\mathrm{b}} = V_i^j - \widetilde{P}_{ij}^{\mathrm{b}} \leq 0 = U_i^{\mathrm{b}}$.

Second, we show two situations that buyer $b_i \in \mathcal{B}_w$ can improve its utility by bidding untruthfully.

1) When buyer b_i wins seller s_j with truthful bid $D_i^j = V_i^j$ and price $P_{ij}^{\mathrm{b}} = D_{i^*}^j$, buyer b_i can submit an untruthful bid $\widetilde{D}_i^j$ which is slightly smaller than $D_{i^*}^j$, so that buyer b_i still wins seller s_j but leads to a new price $\widetilde{P}_{ij}^{\mathrm{b}} = A_{-j}^o$.
2) When bidding truthfully, buyer b_i has the highest bids for sellers s_α and s_β among all buyers in $\mathcal{B} \setminus \mathcal{B}_w$, and seller s_α appears before seller s_β in $\mathbb{S}$. In this situation, buyer b_i can bid untruthfully with $\widetilde{D}_i^\alpha = 0$ to lose seller s_α, so that it wins seller s_β for a higher utility.

Although the buyers in $\mathcal{B}_w$ can improve utility in principle by bidding untruthfully, there are some difficulties in computing an effective lie. Without the knowledge of the others' bids, a buyer has no way to determine the auction outcome, such as whether the buyer could win with truthful bid, the matched seller if the buyer wins, and the clearing price. Thus, it is very hard for a buyer to improve its utility by bidding untruthfully. As shown in the preceding discussion, when buyer $b_i \in \mathcal{B} \setminus \mathcal{B}_w$ with truthful bid, bidding untruthfully cannot result in a positive utility to exceed the zero utility with truthful bid. When buyer $b_i \in \mathcal{B}_w$ with truthful bid, we take the second situation above as an example. If buyer b_i wants to lose seller s_α to improve its utility, buyer b_i must submit a zero bid, i.e., $\widetilde{D}_i^\alpha = 0$, since buyer b_i does not know the bids of other buyers. There is also a risk that buyer b_i may achieve zero utility $\widetilde{U}_i^{\mathrm{b}} = 0$, if it cannot win a seller other than s_α.

Furthermore, supposing complete information of the bid matrix $\mathbf{D}$ and the ask vector $\mathbf{A}$ is publicly known, due to the randomness introduced in EDA, an untruthful bid may even backfire to a buyer who should win with truthful bid but lose the auction with a lie. Take the first situation above as an example. Even with the knowledge of $\mathbf{D}$ and $\mathbf{A}$, buyer b_i cannot determine $\mathcal{B}^j$, since $\mathcal{B}^j$ depends on the randomly ordered list $\mathbb{S}$. Thus, if buyer b_i wants to improve its utility with a bid $\widetilde{D}_i^j < D_{i^*}^j$, it may experience $\widetilde{U}_i^{\mathrm{b}} = 0$, since b_{i^*} is still in $\mathcal{B} \setminus \mathcal{B}_w$.

TABLE 3: Computational efficiency.

(a) Computation time of TIM.

$n = 100$	m	50	100	150	200	250	300
	Time (ms)	0.5	0.9	1.3	1.8	2.2	2.7
$m = 100$	n	50	100	150	200	250	300
	Time (ms)	0.7	0.9	1.1	1.4	1.5	1.7

(b) Computation time of EDA.

$n = 100$	m	50	100	150	200	250	300
	Time (ms)	1.0	1.7	2.6	3.3	4.3	5.1
$m = 100$	n	50	100	150	200	250	300
	Time (ms)	1.3	1.7	2.0	2.2	2.4	2.6

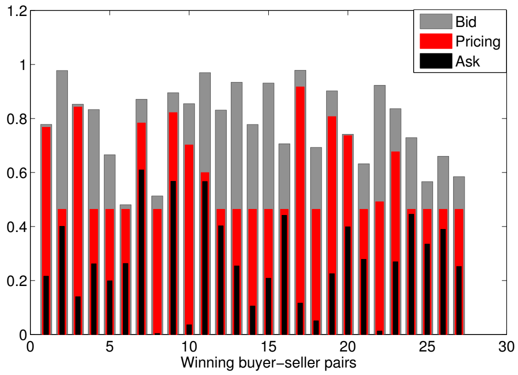
6 Numerical Results

This section presents the numerical results to evaluate the performance of TIM and EDA. As seen in the proof in Section 4.3 and Section 5.3, TIM and EDA satisfy desirable properties including computational efficiency, individual rationality, budget balance, and truthfulness. The proof does not set any presumption on the bids of buyers or the asks of sellers. Thus, the conclusions are valid for any possible data sets of the bids and asks. Thus, without loss of generality, we randomly generate the bids of buyers and the asks of sellers according to a uniform distribution within $(0, 1]$ unless otherwise specified. For the simulations regarding each desirable property, we also vary the number of buyers or sellers, the bids of buyers, or the asks of sellers, which are detailed in each subsection.

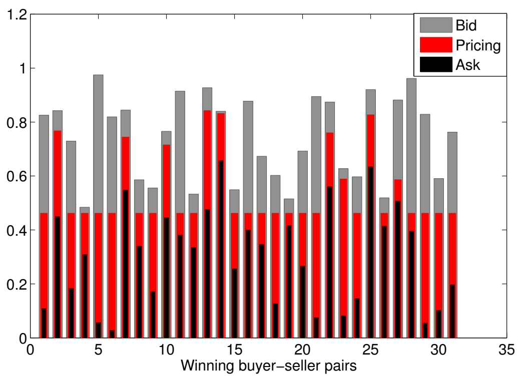
6.1 Computational Efficiency

To confirm our analysis on time complexity of TIM in Theorem 1 and that of EDA in Theorem 5, we give the computation time of TIM and EDA with different settings in Table 3. For each setting, we randomly generate 1000 instances and average the results. All the tests run on a Windows PC with 3.16 GHz Intel® Core™2 Duo processor and 4 GB memory. As seen, both TIM and EDA are subject to polynomial computation time with respect to n and m, which are the number of buyers and sellers, respectively.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

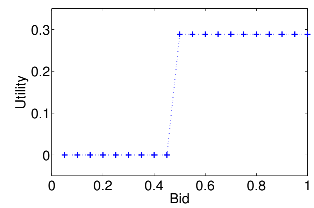


(a) TIM.

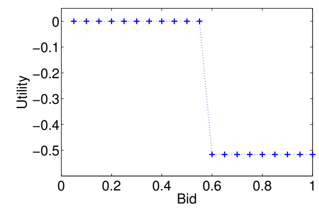


(b) EDA.

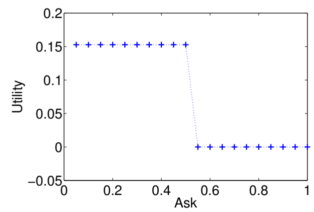
Fig. 3: Individual rationality of TIM and EDA.



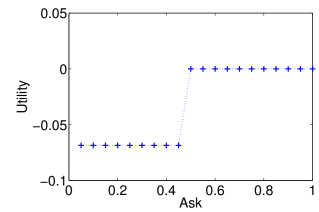
(a) Buyer $b_i \in \mathcal{B}_w$.



(b) Buyer $b_i \notin \mathcal{B}_w$.

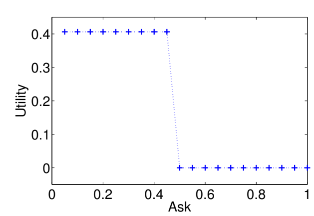


(c) Seller $s_j \in \mathcal{S}_w$.

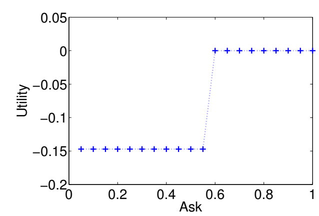


(d) Seller $s_j \notin \mathcal{S}_w$.

Fig. 4: Truthfulness of buyers and sellers with TIM.



(a) Seller $s_j \in \mathcal{S}_w$.



(b) Seller $s_j \notin \mathcal{S}_w$.

Fig. 5: Truthfulness of sellers with EDA.

6.2 Individual Rationality and Budget Balance

To validate Theorem 2 and Theorem 6 regarding individual rationality of TIM and EDA, we show the bids, pricing, and asks in Fig. 3. Since the price charged to each winning buyer P_{ij}^{b} is equal to the payment rewarded to each winning seller P_j^{s}, the pricing here presents both. As seen, for both TIM and EDA, each winning buyer is charged a price not higher than its bid, while each winning seller receives a payment not less than its ask from the auctioneer. Therefore, both TIM and EDA are individually rational. The results demonstrate that the cloudlets receive sufficient compensations to be incentivized to share their resources. On the other hand, the mobile users are allocated the demanded resources and pay no more than their valuations of these resources. Thus, the mobile users are also stimulated to request services from the cloudlets.

In addition, since $P_{ij}^{\mathrm{b}} = P_j^{\mathrm{s}}$ for all the winning pairs in TIM and EDA, budget balance is also achieved in both mechanisms, which confirms Theorem 3 and Theorem 7 for budget balance of TIM and EDA. Hence, the auctioneer can assist the resource allocation without a deficit.

6.3 Truthfulness of TIM

To verify the truthfulness of TIM, we randomly choose several buyers/sellers to examine how their utilities change when they bid or ask different values. The results are depicted in Fig. 4.

Fig. 4(a) shows the case that buyer b_i wins seller s_j and gains utility $U_i^{\mathrm{b}} = 0.2885$ when it bids truthfully with $D_i^j = V_i^j = 0.7875$. It can be seen that buyer b_i cannot improve its utility no matter what other bids it takes. Fig. 4(b) shows a different scenario that buyer b_i does not win seller s_j when it bids truthfully with $D_i^j = V_i^j = 0.0437$. Thus, b_i achieves zero utility without having the service. Fig. 4(b) shows that the utility cannot be greater than zero even when buyer b_i bids untruthfully. Fig. 4(c) shows an example that seller s_j wins when asking truthfully with $A_j = C_j = 0.3841$ and achieves utility $U_j^{\mathrm{s}} = 0.1528$. As seen, the utility with a truthful ask is the highest among all possible asks. Fig. 4(d) shows that seller s_j loses when asking truthfully with $A_j = C_j = 0.5501$ and thus obtains zero utility. For all other asks, the achievable utility is either zero or negative, but cannot be more than zero.

In summary, TIM guarantees truthfulness for both buyers and sellers since the utility cannot be improved by bidding or asking untruthfully. In addition, as seen in Fig. 4(a) and Fig. 4(c), the winning mobile user and the cloudlet that are matched successfully gain positive utilities, which means that both benefit from using or providing the demanded resources.

6.4 Truthfulness of EDA

Fig. 5 and Fig. 6 evaluate the truthfulness of EDA for sellers and buyers, respectively.

Fig. 5(a) shows an example with a randomly chosen winning seller s_j that asks truthfully with $A_j = C_j = 0.0926$ and achieves utility $U_j^{\mathrm{s}} = 0.4064$. As seen, the utility with a truthful ask is the highest among all possible asks. Fig. 5(b)

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing

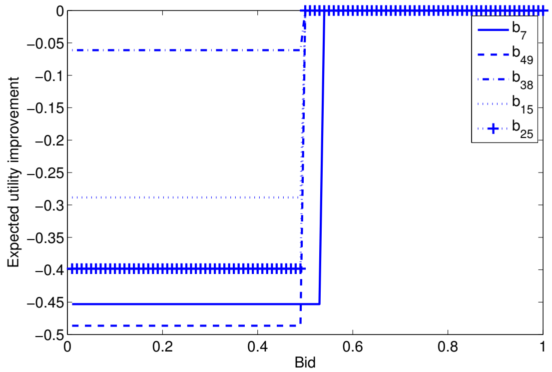


Fig. 6: Truthfulness in expectation of buyers with EDA.

shows the case that seller s_j loses when asking truthfully with $A_j = C_j = 0.6980$ and thus obtains zero utility. For all other asks, the achievable utility cannot be more than zero.

As analyzed in Section 5.3.2, EDA cannot ensure strong truthfulness for buyers. Considering several randomly chosen buyers, Fig. 6 shows their utility improvement with different bidding values over truthful bids. Since EDA involves a randomly ordered seller list in the auction, we here calculate the expected utility improvement with 1000 random instances. As seen, no buyer can achieve a positive improvement in its expected utility by bidding untruthfully. The observations in Fig. 5 and Fig. 6 confirm our conclusions on the truthfulness of EDA for sellers and its weak truthfulness for buyers.

6.5 System Efficiency

As discussed in Section 3.3, this work aims to improve system efficiency in terms of the number of successful trades or the total valuation of winning buyers. Intuitively, the system efficiency depends on the range of buyers' bids and that of sellers' asks. In practice, the bids and asks may be widely dispersed in extreme cases. For example, if the bids of buyers are far below the asks of sellers, there can be few successful trades that lead to low total valuation. To eliminate the data dependency in comparing the system efficiency of different mechanisms, we normalize the system efficiency with respect to that of the optimal strategy with complete auction information. This normalized system efficiency clearly demonstrates the gap to the optimal achievable performance.

Fig. 7 compares four incentive mechanisms, including TIM, EDA, and TASC [10] with maximum weighted matching (TASC-MWM) or maximum matching (TASC-MM). Fig. 7(a) and Fig. 7(b) show the normalized system efficiency of these mechanisms in terms of the number of successful trades and the total valuation of winning buyers, respectively. Similar trends are observed in the two figures, since the total valuation of winning buyers is proportional to the number of successful trades. As seen, TASC-MM achieves the lowest system efficiency, while EDA achieves the highest system efficiency of around 80%. The high system efficiency demonstrates that EDA improves the resource utilization of the cloudlets and the satisfaction of the mobile users' service demands.

One main reason for the low efficiency of TASC-MM is that the maximum matching algorithm used in the assignment stage may assign a seller to a buyer of a low bid. However, such a matching is eventually invalidated in the winner determination & pricing stage, which heavily depends on the ordered statistics of the bids and asks. Besides, TASC uses a universal threshold to determine the winners, which may eliminate possible matchings unnecessarily. On the other hand, TIM and EDA improve the efficiency by applying multiple thresholds (i.e., a different A^o_{-j} for each seller s_j) to determine the winners. Due to different thresholds used in TIM and EDA, more complex pricing is thus required to ensure truthfulness.

Another interesting observation of Fig. 7 is that the curves are not monotonic but fluctuating, which seems counter-intuitive since the number of successful trades and the total valuation of winning buyers should increase with the number of sellers. Indeed, more sellers can better satisfy the diverse demands of buyers and thus lead to more successful trades. Here, however, we normalize the system efficiency with respect to that of the optimal strategy in order to measure the gap to the highest achievable performance. As a result, the curves are not monotonic because the performance of the optimal strategy also varies with the number of the sellers.

7 Concluding Remarks

This paper has considered an appealing MCC application paradigm for mobile devices to acquire the resources of nearby cloudlets. The close proximity of cloudlets can reduce the energy consumption and access latency of mobile devices. We proposed two double auction mechanisms, TIM and EDA, which coordinate the resource trading between mobile devices as service users (buyers) and cloudlets as service providers (sellers). Both mechanisms are proved to be feasible, which ensures computational efficiency, individual rationality and budget balance. Furthermore, TIM guarantees truthfulness for both buyers and sellers. EDA achieves fairly high system efficiency but satisfies truthfulness in a weaker sense. EDA still maintains truthfulness for sellers, while preventing untruthful bidding of buyers with increased difficulty of computing an effective lie. The numerical results validated our theoretical analysis and demonstrated improvement in system efficiency.

References

[1] M. Rahman, J. Gao, and W.-T. Tsai, "Energy saving in mobile cloud computing," in *Proc. IEEE International Conference on Cloud Engineering (IC2E)*, Mar. 2013, pp. 285–291.

[2] Z. Liu, Y. Feng, and B. Li, "Socialize spontaneously with mobile applications," in *Proc. IEEE INFOCOM*, Mar. 2012, pp. 1942–1950.

[3] M. Satyanarayanan, P. Bahl, R. Caceres, and N. Davies, "The case for VM-based cloudlets in mobile computing," *IEEE Pervasive Computing*, vol. 8, no. 4, pp. 14–23, 2009.

[4] V. Krishna, *Auction Theory*, 2nd ed. Academic Press, Aug. 2009.

[5] Y. Zhang, C. Lee, D. Niyato, and P. Wang, "Auction approaches for resource allocation in wireless systems: A survey," *IEEE Communications Surveys & Tutorials*, vol. 15, no. 3, pp. 1020–1041, 2013.

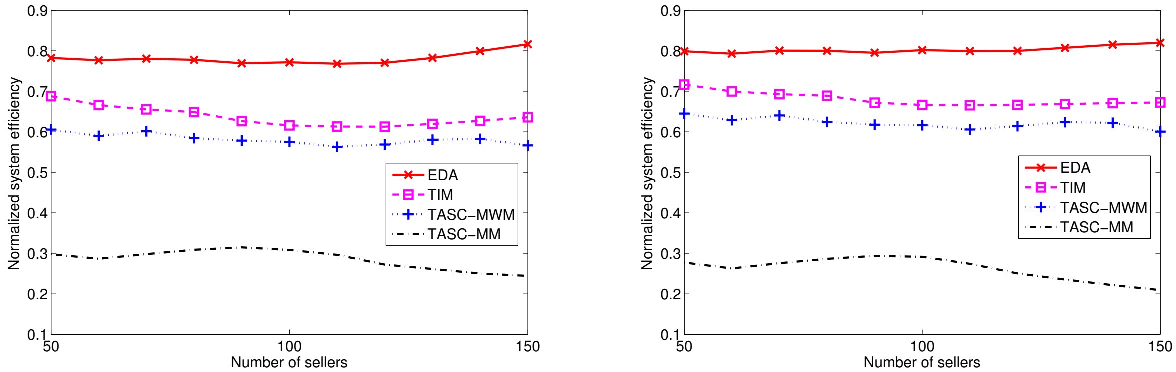
[6] L. M. Ausubel, "An efficient dynamic auction for heterogeneous commodities," *American Economic Review*, vol. 96, no. 3, pp. 602–629, 2006.

[7] G. Demange, D. Gale, and M. Sotomayor, "Multi-item auctions," *Journal of Political Economy*, vol. 94, pp. 863–872, 1986.

[8] D. Mishra and R. Garg, "Descending price multi-item auctions," *Journal of Mathematical Economics*, vol. 42, pp. 161–179, 2006.

[9] R. P. McAfee, "A dominant strategy double auction," *Journal of Economic Theory*, vol. 56, pp. 434–450, Apr. 1992.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TSC.2015.2430315, IEEE Transactions on Services Computing



(a) With respect to the number of successful trades.

(b) With respect to total valuation of winning buyers.

Fig. 7: Normalized system efficiency.

[10] D. Yang, X. Fang, and G. Xue, "Truthful auction for cooperative communications," 2011, http://inside.mines.edu/∼djyang/doc/MOBIHOC2011-TASC.pdf.

[11] D. Parkes, J. Kalagnanam, and M. Eso, "Achieving budget-balance with Vickrey-based payment schemes in exchanges," in *Proc. International Joint Conference on Artificial Intelligence*, 2001, pp. 1161–1168.

[12] L. Gkatzikis and I. Koutsopoulos, "Migrate or not? Exploiting dynamic task migration in mobile cloud computing systems," *IEEE Wireless Communications*, vol. 20, no. 3, pp. 24–32, Jun. 2013.

[13] S. Abolfazli, Z. Sanaei, E. Ahmed, A. Gani, and R. Buyya, "Cloud-based augmentation for mobile devices: Motivation, taxonomies, and open challenges," *IEEE Communications Surveys & Tutorials*, vol. 16, no. 1, pp. 337–368, 2014.

[14] Y. Xu and S. Mao, "A survey of mobile cloud computing for rich media applications," *IEEE Wireless Communications*, vol. 20, no. 3, pp. 46–53, Jun. 2013.

[15] L. Zhou, Z. Yang, J. Rodrigues, and M. Guizani, "Exploring blind online scheduling for mobile cloud multimeida services," *IEEE Wireless Communications*, vol. 20, no. 3, pp. 54–61, Jun. 2013.

[16] L. Song, D. Niyato, Z. Han, and E. Hossain, "Game-theoretic resource allocation methods for device-to-device (D2D) communication," *IEEE Wireless Communications*, vol. 21, no. 3, pp. 136–144, Jun. 2014.

[17] B. Di, T. Wang, L. Song, and Z. Han, "Incentive mechanism for collaborative smartphone sensing using overlapping coalition formation games," in *Proc. IEEE GLOBECOM*, Dec. 2013.

[18] J. Ma, J. Deng, L. Song, and Z. Han, "Incentive mechanism for demand side management in smart grid using auction," *IEEE Transactions on Smart Grid*, vol. 5, no. 3, pp. 1379–1388, May 2014.

[19] Y. Shi, S. Sharma, Y. T. Hou, and S. Kompella, "Optimal relay assignment for cooperative communications," in *Proc. ACM MobiHoc*, 2008.

[20] D. West, *Introduction to Graph Theory*, 2nd ed. Prentice Hall, 1999.

[21] Y. Zhang, D. Niyato, and P. Wang, "An auction mechanism for resource allocation in mobile cloud computing systems," in *Proc. Intl. Conf. on Wireless Algorithms, Systems, and Applications*, 2013, pp. 76–87.

[22] C. Lee, P. Wang, and D. Niyato, "A real-time group auction system for efficient allocation of cloud Internet applications," *IEEE Transactions on Services Computing*, 2014, in press.

[23] D. D. Clark, W. Lehr, and S. Bauer, "Interconnection in the Internet: The policy challenge," *Telecommunications Policy Research Conference*, 2011.

[24] J. Trevathan and W. Read, "Privacy and security concerns for online share trading," in *Proc. of IASK International Conference: E-Activity and Leading Technologies*, Dec. 2017, pp. 239–246.

A-Long Jin received his B.Eng. degree in communications engineering from Nanjing University of Posts and Telecommunications, Nanjing, China, in 2012. He received his M.Sc. degree in computer science from University of New Brunswick, Fredericton, NB, Canada, in 2015. His research interests include cooperative wireless networks, game theory for wireless networks, machine learning, and optimization.

Wei Song (M'09-SM'14) received the Ph.D. degree in electrical and computer engineering from the University of Waterloo, Waterloo, ON, Canada, in 2007. From 2008 to 2009, she was a Postdoctoral Research Fellow with the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA, USA. In 2009, she joined the Faculty of Computer Science, University of New Brunswick, Fredericton, NB, Canada, where she is now an Associate Professor. Her current research interests include cooperative wireless networking, energy-efficient wireless networks, and device-to-device communications. She was a co-recipient of a *Best Student Paper Award* from IEEE CCNC 2013, a *Top 10% Award* from IEEE MMSP 2009, and a *Best Paper Award* from IEEE WCNC 2007. She is the Communications/Computer Chapter Chair of IEEE New Brunswick Section.

Ping Wang (M'08-SM'15) received the Ph.D. degree in electrical and computing engineering from the University of Waterloo, Waterloo, ON, Canada, in 2008. She joined the School of Computer Engineering, Nanyang Technological University, Singapore in June 2008, where she is now an Associate Professor. Her current research interests include resource allocation in multimedia wireless networks, cloud computing, and smart grid. She was a co-recipient of the *Best Paper Award* from IEEE Wireless Communications and Networking Conference (WCNC) 2012 and IEEE International Conference on Communications (ICC) 2007.

Dusit Niyato (M'09-SM'15) is currently an Associate Professor in the School of Computer Engineering, at the Nanyang Technological University, Singapore. He received the Ph.D. degree in electrical and computer engineering from the University of Manitoba, Winnipeg, MB, Canada in 2008. His research interests are in the area of the optimization of wireless communication and mobile cloud computing, smart grid systems, and green radio communications.

Peijian Ju received M.Eng. and B.Eng. degrees from Huazhong University of Science and Technology, Wuhan, China, in 2011 and 2009, respectively. He is working toward the Ph.D. degree at the University of New Brunswick, Fredericton, NB, Canada. His research interests include cross-layer design and game theory for cooperative wireless networks.