Part 2 of Project in SSY125

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1 Hard vs Soft Receiver

The aim of this task is to evaluate the impact of encoder ε_2 and χ_{QPSK} with Gray mapping, and compare the performance between hard and soft receiver. The figure 1 shows the result.

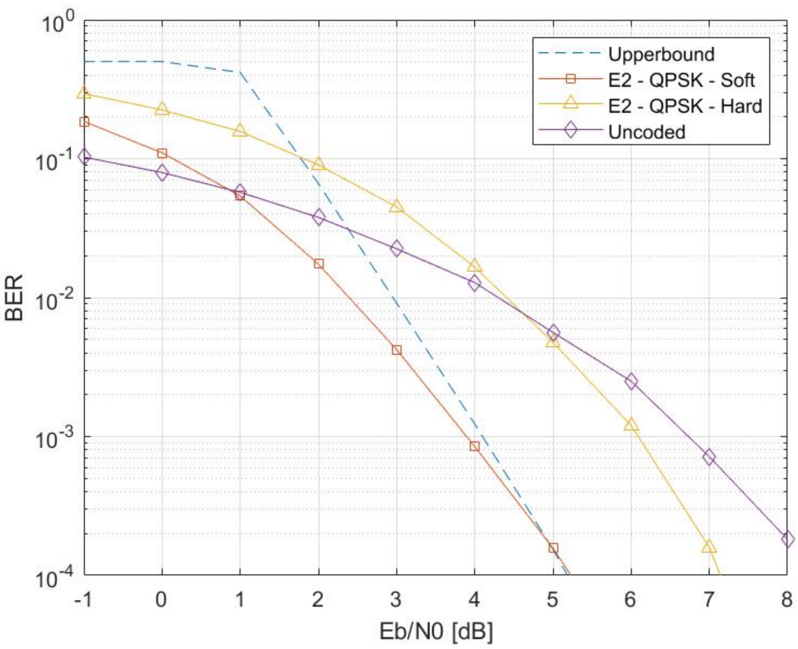

Figure 1: BER vs Eb/N0 with ε_2 and QPSK

The uncoded system shows the highest BER across all E_b/N_0 values, underscoring the impact of channel noise on uncoded transmission. In contrast, the coded system with a hard receiver demonstrates significant improvement over the uncoded system, although its performance is limited by the inability to fully utilize channel information. The soft receiver achieves the best BER performance. Additionally, the theoretical upper bound, calculated by equation (1), aligns closely with the soft receiver's performance at higher E_b/N_0 values, validating the accuracy of the simulation and the implementation of the decoding algorithms.

$$P_{\mathrm{b}}^{\mathrm{SOFT}} \leq \sum_{d=d_{\min}}^{N} \tilde{A}_d \mathrm{Q}\left(\sqrt{\frac{2dR_{\mathrm{c}}\mathsf{E}_{\mathrm{b}}}{\mathsf{N}_0}}\right) \tag{1}$$

This formula is applicable in cases where the signal-to-noise ratio (SNR) is relatively high. Theoretically, the BER of QPSK modulation should be less than $\frac{1}{2}$. The calculation of the upper bound is implemented in the attached file `calculate_bound.m`.

The coding gain at $BER = 10^{-4}$ can be calculated using equation (2).

$$G_c = \Delta(E_b/N_0) = (E_b/N_0)_{\text{uncoded}} - (E_b/N_0)_{\text{coded}} \tag{2}$$

Due to the sparsity of the BER data points and relatively small error, linear interpolation is employed to estimate the value of E_b/N_0 at $BER = 10^{-4}$. Comparing both receivers with the uncoded scenario yields a coding gain of 2.67 dB for soft decoding and 0.67 dB for hard decoding.

The asymptotic coding gain can be calculated using equation (3).

$$G_\infty = 10 \log_{10}(R_c \cdot d_{\min}) \tag{3}$$

For encoder ε_2, R_c equals 0.5, and $d_{\min}$ is obtained using the `distspec` function in MATLAB. With $d_{\min}$ equal to 5, the asymptotic gain is calculated as:

$$G_\infty = 10 \log_{10}(0.5 \cdot 5) = 10 \log_{10}(2.5) \approx 3.80\,\text{dB}$$

The spectral efficiency for the system can be calculated using equation (4).

$$R = R_c \cdot \log_2(M) \tag{4}$$

For encoder ε_2 with QPSK, where $R_c = \frac{1}{2}$ and $M = 4$, the spectral efficiency is:

$$R = \frac{1}{2} \cdot 2 = 1$$

For reliable transmission, the minimum E_b/N_0 to support a rate R is given by:

$$\frac{\mathsf{E_b}}{\mathsf{N_0}} > \frac{2^R - 1}{R} \tag{5}$$

This yields a theoretical minimum E_b/N_0 of 0 dB. The simulation results show intersection points between the coded and uncoded systems at BER = 5.92e-02, Eb/N0 = 0.88 dB for soft decoding, and BER = 8.11e-03, Eb/N0 = 4.68 dB for hard decoding.

The performance difference between hard and soft receivers stems from their information utilization. Hard receivers make discrete decisions after symbol detection, discarding confidence level information. This information loss makes them less tolerant to noise and requires a higher E_b/N_0 for reliable transmission. Soft receivers retain confidence information about the symbols, enabling more accurate bit estimation, particularly under low SNR conditions. This improved information efficiency results in better asymptotic performance and a lower minimum E_b/N_0 that approaches theoretical limits.

Receiver	**Coding gain**	**Asymptotic Gain**	**Minimum E_b/N_0**
Hard	0.67	-	4.68
Soft	2.67	3.80	0.88

Table 1: Performance Comparison between Hard and Soft Receivers

2 Encoder Comparison

Based on the simulation results and theoretical bounds shown in Figure 3, several key differences can be observed between encoders ε_1, ε_2, and ε_3 when used with QPSK modulation and soft-decision decoding. The upper bound calculation follows the same approach as used in the previous section regarding Hard vs Soft Receiver.

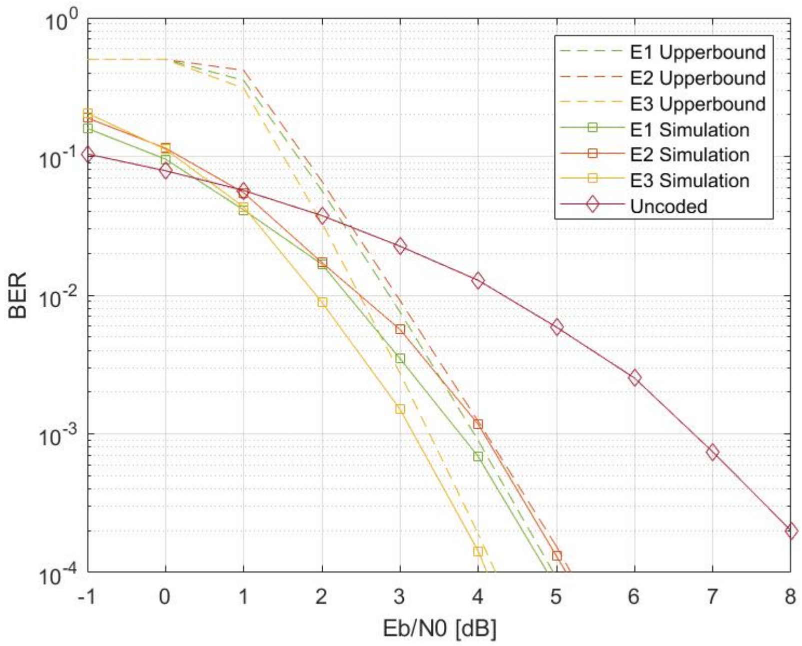

Figure 2: Performance Comparison of Different Rate-1/2 Convolutional Encoders

ε_3 shows the best performance, achieving $BER = 10^{-4}$ at approximately $E_b/N_0 = 4$ db. ε_1 and ε_2 requires about 5 dB to reach the same BER, with ε_2 performing slightly worse than ε_1. The uncoded system requires about 8 dB.

The coding gain at $BER = 10^{-4}$ can be calculated using equation (2) and linear interpolation due to the sparsity of BER data points. The analysis reveals gains of approximately 3.17 dB for both ε_1 and ε_2 over the uncoded system, while ε_3 achieves a superior gain of 4.17 dB.

For the asymptotic coding gain calculation using equation (3), these rate-1/2 encoders $R_c = 0.5$ demonstrate varying minimum distances d_{min} obtained from the distance spectrum analysis. ε_1 and ε_2 both have ($d_{min} = 5$), resulting in $G_\infty = 10\log_{10}(0.5 \cdot 5) = 3.98$ dB. ε_3 achieves ($d_{min} = 7$), yielding $G_\infty = 10\log_{10}(0.5 \cdot 7) = 5.44$ dB.

The performance differences between these encoders stem from their structural properties and distance spectrum characteristics. The constraint length K determines the encoder complexity, with ε_1 having $K = 3$ (4 states) and both ε_2 and ε_3 having $K = 5$ (16 states). While ε_1 and ε_2 share $d_{min} = 5$, their error event distributions differ with $\tilde{A}_5 = 1$ and $\tilde{A}_5 = 3$ respectively. ε_3 achieves $d_{min} = 7$ with $\tilde{A}_7 = 4$, enabling better error correction.

In terms of performance-complexity trade-off, ε_1 provides decent performance with minimal computational requirements due to its shorter constraint length. Although ε_2 and ε_3 share the same complexity, ε_3's larger d_{min} and better weight spectrum result in superior error correction capability. However, this improved performance comes at the cost of increased encoding/decoding latency in practical implementations.

3 Coding can Increase Efficiency

The following figure depicts the results from three systems. System one which was encoder three and BPSK constellation, system two which used encoder three and QPSK constellation and system three which used encoder four and AMPM. All three systems are also plotted for uncoded transmission.

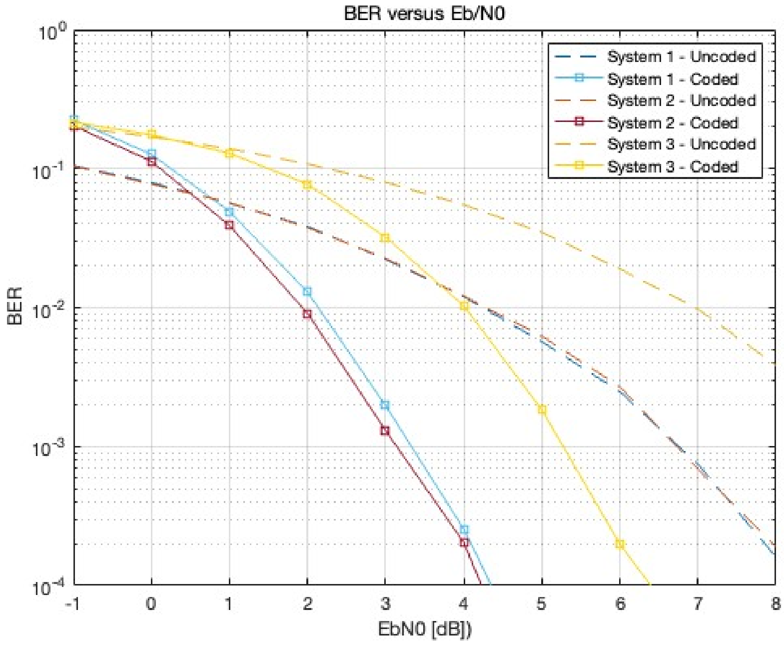

Figure 3: Performance Comparison between 3 Systems

In table 2 the spectral and power efficiency for the three systems are stated. The spectral efficiency is calculated by $R_c \cdot log_2(M)$, where R_c is the code rate and M is the number of symbols in the constellation.

System	Spectral efficiency	Power efficiency (Needed E_b/N_0 at $BER = 10^{-4}$)
Uncoded S1	1	8.2000
Uncoded S2	2	8.2000
Uncoded S3	3	8.6167
Coded S1	0.5	4.6667
Coded S2	1	4.5000
Coded S3	2	6.5000

Table 2: Table to test captions and labels.

In the table, the uncoded and coded versions of the same system can be compared. For system one, BPSK, encoding decreases the spectral efficiency by half and reduces the needed SNR to achieve the desired BER by almost as much. This result isn't surprising, since the encoding lowers the code rate and spectral efficiency. As the spectral efficiency decreases, the minimum distance between symbols increases and consequently the required E_b/N_0 to achieve a decided BER lowers, which indicates a better performance in power efficiency. The same applies when comparing systems two and three with their uncoded and coded versions.

For system one and system two, the change in spectral efficiency is similar, which is expected since the encoding half both their respective code rates. Thereby, the spectral efficiency is simply scaled by the number of symbols in the constellation. Notably, the two constellations still have the same power efficiency.

The reason is that the theoretical BER is the same for BPSK and QPSK. QPSK can be divided into two orthogonal BPSK transmissions, consequently the minimum symbol distance is the same. Additionally, the gray labelling ensures that in case of errors, at most one bit is in error, just as for BPSK! Therefore, there will be no trade-off in terms of power efficiency by choosing QPSK over BPSK. The important trade-off is the spectral efficiency, where QPSK transmits half a bit more per channel use. The higher spectral efficiency will also decrease the required bandwidth.

In system three, AMPM, the encoding adds less redundancy than encoder three, and is, therefore, more noise-sensitive. It has a higher spectral efficiency than the other systems, however, it requires a high SNR to be able to transmit at the same error rate. When comparing the three systems with capacity, QPSK is closer to capacity than AMPM.

Although there are trade-offs between the coded cases, the uncoded versions are undoubtedly worse performing. They have higher BER, and power efficiency and are further away from capacity than the coded cases.

Overall, compared to the uncoded case, the encoder reduces spectral efficiency by a factor of R_c (code rate) but improves power efficiency, requiring a lower E_b/N_0 to achieve the same BER. When comparing modulation schemes, BPSK and QPSK have similar performance. AMPM achieves higher spectral efficiency but sacrifices power efficiency, also reflecting the trade-off between the two.

4 Modern Codes

The implementation utilizes a DVB-S.2 standard LDPC code with rate 1/2 and block length of 64,800 bits, employing QPSK modulation with Gray mapping to match system 2's configuration. The MATLAB Communications Toolbox functions `comm.LDPCEncoder` and `comm.LDPCDecoder` are used for encoding and decoding, with the decoder performing a maximum of 25 iterations using soft-decision decoding. Figure 4 shows the BER performance comparison between the LDPC code and the E2 convolutional code, both using QPSK modulation.

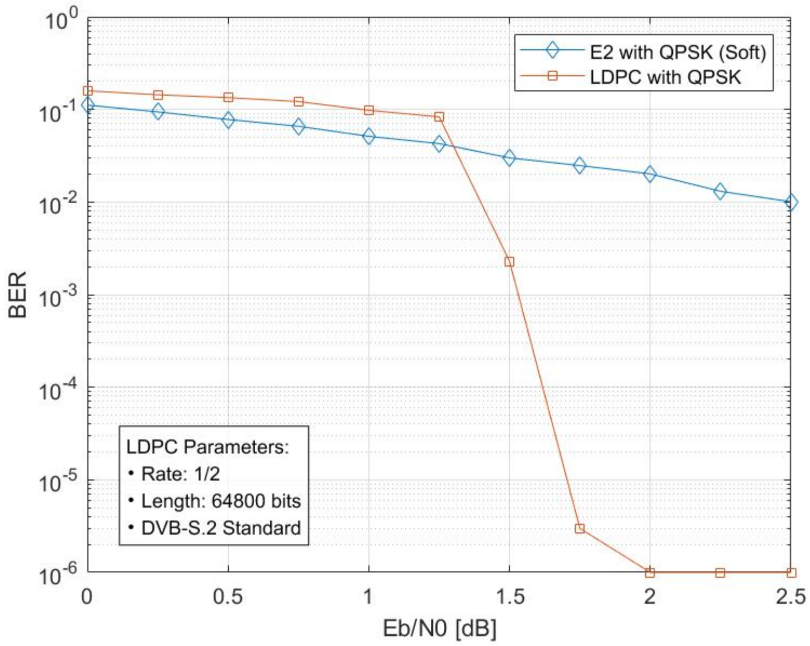

Figure 4: Performance Comparison of E2 and LDPC Systems

Performance comparison between the LDPC code and system 2 (E2 with QPSK) reveals significant differences. The LDPC code exhibits a steep waterfall region beginning at $E_b/N_0 = 1.5$ dB, where the BER drops from 10^{-2} to 10^{-6} within 0.5 dB. At E_b/N_0 of 2 dB, the LDPC implementation achieves a BER of 10^{-6}, while E2 maintains 2×10^{-2}. This four orders of magnitude improvement comes at the cost of increased complexity due to larger block sizes, though modern hardware can handle this efficiently.

Contribution Statement

Individual contributions are as stated below:

Ella Dahlström: I've worked on developing most parts of the code, although the credit for finishing the code as its current state goes elsewhere. I've made many drafts (encoder, decoder mapper, demapper) that have been edited or changed and a lot of troubleshooting on other files. I've written task 3 in collaboration with Kimia. I've also tried to lead meetings and direct tasks.
Boyi Lin: Developed, debugged, and modified code; Wrote the document for Task 2: Encoder Comparison; Simulated the LDPC system using MATLAB toolbox and wrote the document for Task 4: Modern Codes.
Kimia Safarihamid: I contributed in writing the AMPM code, Encoder 3, and editing the soft receiver, demapper and build trellis for the first part of the project, as well as commenting on most of the codes to improve readability. For the second part, I collaborated with Ella in drafting the Task 3, which consisted of making minor changes in get_uncoded_BER, AWGN, and also completing the main file. Although finalizing the code as it is, was with the help of Boyi and Terryon.
Terryon Zhao: I contributed to this project by drafting the basic structure of the project, which divided the it into different blocks. In Part I, I completed the demapper and the get_uncoded_BER block, as well as optimized the mapper and AWGN block. In Part II, I collaborated with Boyi to complete the soft decoder. I am also responsible for Task 1 and the related report.

Overall evaluation of task distribution and collaboration:

The collaboration has gone well and there has been a good atmosphere. Communication has been easy and everyone has shown a genuine desire to contribute. The work tasks were fairly assigned, but the strong motivation of some team members led them to take on tasks assigned to others, often completing them ahead of the assigned person's deadline for the task. This ensured rapid progress but also some imbalance in the workload.